SU(5)

- Fermions not truly unified

generation : $10_F + \bar{5}_F$

minimal model: $m_\nu = 0$ (SM)

- Add ν_R (singlet) (as in SM)

  or new Higgs (15_H)

  $\Downarrow$

  no connection between neutrinos

  and charged fermions

  $\Downarrow$

  <u>no</u> information of m_ν

  <u>no</u> connection between θ_ℓ and θ_q

HIGGS AS A PSEUDO-GOLDSTONE

$$SU(6) \longrightarrow SU(3) \times SU(2) \times U(1)$$

Σ (adjoint), $H(6)$, $\bar{H}(\bar{6})$

Imagine no: $\Sigma H \bar{H}$ term

$$\Rightarrow \quad G_W = SU(6) \times SU(6)$$

$\langle \Sigma \rangle$ $\qquad\qquad\qquad$ $\langle H, \bar{H} \rangle$

$SU(4) \times SU(2) \times U(1)$ $\qquad\qquad\qquad$ $SU(5)$

$35 - 19 = 16$ $\qquad\qquad\qquad$ $35 - 24 = 11$

$$\Downarrow$$

$16 + 11 = 27$ goldstones

$35 - 12 = 23$ (eaten)

$$\Downarrow$$

$$27 - 23 = 4 = 2 + \bar{2}$$

$\uparrow$ SM doublets

get the mass only after supersymmetry breaking (no renormalization of W)

- Missing multiplet

$$24^\alpha_\beta \longrightarrow 75^{[\alpha\beta]}_{[\gamma\delta]} \qquad (10 \times \overline{10} = 75 + 24 + 1)$$

$$\Downarrow$$

$$\text{NO} \quad 75 \cdot 5_H \, \bar{5}_H$$

but: $75 \cdot 50 \cdot 5_H \Rightarrow \langle 75 \rangle \, \overline{T}_{50} \, T_5$

$$75 \cdot \overline{50} \cdot \bar{5}_H \Rightarrow \underbrace{\langle 75 \rangle}_{M_{GUT}} \, T_{50} \cdot \overline{T}_5$$

no D or $\bar{D}$ in 50 or $\overline{50}$

$$\Downarrow$$

only $T(\bar{T})$ gets a mass,

$D, \bar{D}$ massless until

supersymmetry breaking

L - R symmetry

Mohapatra, Pati, G.S.
'74, '75

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

anomaly-free
accidental global
symmetry in SM

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

$$W_L \qquad W_R$$

$$Q_{em} = T_{3L} + T_{3R} + \frac{B-L}{2}$$

Higgs: $\langle \Delta_L \rangle = 0 \qquad \langle \Delta_R \rangle = M_R \gg M_L$

$$\mathcal{L}_Y = Y_\nu \left(\nu_L \nu_L \Delta_L + \nu_R \nu_R \Delta_R \right) + Y_D \bar{\nu}_L \nu_R H \quad + h.c.$$

$$\Downarrow$$

$$M_{\nu_R} = Y_\nu M_R$$

$$\Downarrow$$

Mohapatra, G.S
'80

$$\langle H \rangle \simeq M_W, \quad \langle \Delta_L \rangle \simeq M_W^2 / M_R$$

see - saw

$$m_{\nu_L} \simeq \underbrace{\frac{Y_D^2 M_W^2}{Y_\nu M_R}}_{\mathrm{I}} + \underbrace{Y_\nu \, M_W^2 / M_R}_{\mathrm{II}}$$

• NEUTRINO MASS

Minkowski ;
Mohapatra, G. S.
'79

ν_R : singlet of $SU(2)_L \times U_Y(1)$

$\Downarrow$

$M_R \, \nu_R^T C \nu_R$ (Majorana mass) ALLOWED

ONLY SINGLETS CAN
HAVE MAJORANA MASS

$\Downarrow$

$$\begin{array}{c} \\ \nu_L \\ \nu_R \end{array} \begin{pmatrix} 0 & \underset{}{m_D} \overset{= y_D M_W}{} \\ m_D & M_R \end{pmatrix}$$

$m_D \Longleftrightarrow m_f$

small Dirac mass

$M_R \gg M_W , m_D$

$\Downarrow$

$m_N \simeq M_R$ $\qquad$ $N = \nu_R + \varepsilon \, \nu_L$

$$\boxed{m_\nu \simeq m_D^2 / M_R}$$

$\nu = \nu_L - \varepsilon \, \nu_R$

$\varepsilon \simeq m_D / M_R$

FAMILY OF FERMIONS

$$SU(2)_{L,R} \downarrow \quad \underbrace{\overbrace{\begin{pmatrix} u & u & u \\ d & d & d \end{pmatrix}}^{}}_{} \!\!\begin{array}{c} \nu \\ e \end{array}\Bigg)_{L,R}$$

$$\begin{pmatrix} u & u & u & \vdots & \nu \\ d & d & d & \vdots & e \end{pmatrix}_{L,R}$$

$$\longleftrightarrow$$

$SU(3)_c$

$$\underbrace{\hphantom{u\quad u\quad u\quad \nu}}_{SU(4)_c}$$

(Pati - Salam)

$$B-L = \begin{pmatrix} 1/3 & 1/3 & 1/3 & -1 \end{pmatrix}$$

BROKEN @

HIGH ENERGY SCALE

$$F = \underbrace{(2_L, 1_R, 4_c) + (1_L, 2_R, 4_c)}_{}$$

16 elem. fermions in a family

$$16_F = \text{spinor of } SO(10)$$

analogy : Lorentz group

$SU(2)_L \times SU(2)_R \times SU(4)_c$

$$\Big\downarrow \text{unify}$$

$SO(10)$ minimal unified theory of forces and <u>matter</u>

$\uparrow SU(5)$

$$SO(10) \supseteq \underset{\underset{SU(2)_L \times SU(2)_R}{\parallel}}{SO(4)} \times \underset{\underset{SU(4)_c}{\parallel}}{SO(6)}$$

SO(10) :

the MSGUT ?

$$SO(10) \supseteq SO(4) \times SO(6)$$

$a = 1, \dots, 10$ $\qquad i = 1, 2, 3, 4$ $\qquad \alpha = 5, 6, \dots, 10$

$$\begin{cases} SO(4) = SU(2)_L \times SU(2)_R \\ \frac{4 \cdot 3}{2} = 3 + 3 \end{cases}$$

$$\begin{cases} SO(6) = SU(4) \\ \frac{6 \cdot 5}{2} = 4^2 - 1 \end{cases}$$

$$SO(10) \supseteq SU(2)_L \times SU(2)_R \times SO(4)_c$$

$L \leftrightarrow R$ symmetry

$f_L \leftrightarrow f_R$ every fermions

$\Rightarrow \quad \exists \, \nu_R \leftrightarrow \nu_L$

SPINORS IN $SO(2n)$

- $\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}$ $\quad\quad$ $i,j = 1,\dots,2n$

Clifford algebra

$$\Sigma_{ij} = \frac{1}{4i}[\Gamma_i, \Gamma_j]$$

generate $SO(2n)$ $\quad$ $(Spin(2n))$

$$\psi \to e^{i\theta_{ij}\Sigma_{ij}}\psi$$

dimension 2^n

- $\Gamma_{FIVE} = (-i)^n \Gamma_1 \cdots \Gamma_{2n} \propto \Sigma_{12}\Sigma_{34}\cdots\Sigma_{2n-1,2n}$

$$\psi_{+(-)} \equiv \frac{1 \pm \Gamma_{FIVE}}{2}\psi$$

$$\Gamma_{FIVE}^2 = 1, \quad [\Gamma_5, \Sigma_{ij}] = 0$$

$$\{\Gamma_5, \Gamma_i\} = 0$$

irreducible spinor : ψ_+ $\quad$ (2^{n-1} comp.)

- "charge" conjugation B :.

$$\psi^T B \psi = \text{inv.} \quad \Longleftrightarrow \quad \psi^c = B\psi^*$$

$$\Rightarrow \Sigma^T B + B\Sigma = 0$$

$$B_{(1)} = \Gamma_1\Gamma_3 \cdots \Gamma_{2n-1} \qquad B_{(2)} = \Gamma_2 \cdots \Gamma_{2n}$$

- Cartan Σ_{12}; Σ_{34}; ..., $\Sigma_{2n-1,2n}$

eigenvalues $\pm(1/2)$

$$L_{12} = \begin{pmatrix} 0 & -i & & \\ i & 0 & & \\ & & \ddots & \end{pmatrix}, \quad L_{34} = \begin{pmatrix} 0 & 0 & & - & - \\ 0 & 0 & 0 & -i \\ & & i & 0 \\ - & - & - & \end{pmatrix}$$

denote :

$\varepsilon_1 \cdots \varepsilon_n \ (\pm)$

$$(L_{ij})_{ke} = -i(\delta_{ik}\delta_{je} - \delta_{ie}\delta_{jk})$$

$\Downarrow$

$$\Gamma_{FIVE} = \prod_{i=1}^{n} \varepsilon_1 \cdots \varepsilon_n$$

ψ_+ : $\Gamma_{FIVE} = +1 \Rightarrow \prod \varepsilon_1 \cdots \varepsilon_n = +1$

one can denote the states

$|\varepsilon_1 \cdots \varepsilon_n\rangle$

e.g. $SO(10) \Rightarrow |+++++\rangle$

$\Gamma_{FIVE} = +1$

$$\underbrace{|+++--\rangle,\ |++-+-\rangle \cdots}_{10}$$

$$\underbrace{|+----\rangle,\ |-+---\rangle,\ |--+--\rangle,\ |---+-\rangle,\ |----+\rangle}_{5}$$

$$16 = \underbrace{10 + \bar{5}}_{SU(5)} + 1$$

ν_R

SO(2): a prototype for SO(4n+2) (SO(10))

$$\{ \Gamma_i, \Gamma_j \} = 2\delta_{ij}$$

$$\Gamma_1 = \sigma_1, \quad \Gamma_2 = \sigma_2 \quad \Rightarrow \quad \Gamma_{FIVE} \equiv -i\sigma_1\sigma_2 = \sigma_3$$

$$\Sigma_{12} \equiv \frac{1}{4i}[\Gamma_1, \Gamma_2] = \sigma_3/2$$

$$\psi \equiv \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \rightarrow e^{i\sigma_3/2\,\theta} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$$\psi_+ \rightarrow e^{i\theta/2}\psi_+ \quad , \quad \psi_- \rightarrow e^{-i\theta/2}\psi_-$$

- $\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ vector :
$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\phi_1 \pm i\phi_2 \rightarrow e^{\mp i\theta}(\phi_1 \pm i\phi_2)$$

complex U(1) version

- $\psi^T B \psi =$ inv. $\Rightarrow$ $\underline{B_{(1)} = \sigma_1}$, $B_{(2)} = i\sigma_2$

$$\Downarrow$$

$$\psi^T B \psi = \psi_+ \psi_-$$

but we chose only ψ_+ say (irred.)

$\Rightarrow$ no mass term for the spinor in SO(2) dual representations

use $\varepsilon_{ij} \det O = O_{ik} O_{jl} \varepsilon_{kl}$

$$\Downarrow$$

$\phi_i \sim \varepsilon_{ij} \phi_j$ (same transf.)

$\Rightarrow \quad \phi(\pm) = \phi_i \pm i\, \varepsilon_{ij} \phi_j \simeq \phi_1 \pm i \phi_2$

complex

$$+\text{sign} \begin{cases} \phi_1 + i\phi_2 \\ \phi_2 - i\phi_1 = -i(\phi_1 + i\phi_2) \end{cases}$$

$$\Downarrow$$

in $SO(2n)$ def.

$$\underbrace{\Phi(\pm)_{[ij\dots]}}_{n} = \Phi_{[ij\dots]}{}_{n} \pm \frac{(i)^n}{n!} \underbrace{\varepsilon_{ij\dots mn}}_{2n} \underbrace{\Phi_{[mn]}}_{n}$$

e.g. $SO(10)$ – 5 index anti-symmetric

$$\underbrace{\Sigma^{(\pm)}_{ijklm} = \Sigma_{ijklm} \pm \frac{i}{5!} \varepsilon_{ijklmpqrst} \Sigma_{pqrst}}_{\text{complex irreducible}}$$

SO(4)

$$\Gamma_1 = \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix} \quad \Gamma_2 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix} \quad \Gamma_3 = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix}$$

$$\Gamma_4 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

- $\Gamma_5 = -\Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$B = \Gamma_1 \Gamma_3 \quad \{(\Gamma_2 \Gamma_4)\} = \begin{pmatrix} i\sigma_2 & \\ & i\sigma_2 \end{pmatrix}$

$\psi^T B \psi = \psi_+^T i\sigma_2 \psi_+ + \quad + \to -$

mass term possible $\Rightarrow$

no chiral fermions possible

true of SO(4n) theories

- $\Gamma_5 = 1 \Rightarrow \epsilon_1 \epsilon_2 = + \Rightarrow \underbrace{|++\rangle, \ |--\rangle}$

$T_{3L} = \frac{1}{2}(\Sigma_{12} + \Sigma_{34}) \qquad T_{3R} = \frac{1}{2}(\Sigma_{12} - \Sigma_{34})$

$SU(2)_L$ doublet; $SU(2)_R$ singlet

$\Gamma_5 = -1 \Rightarrow |+-\rangle, \ |-+\rangle$

opposite

$SO(6) = SU(4)$

$$\downarrow$$

$$\frac{6.5}{2} = 4^2 - 1 \qquad (\#\text{ of generators})$$

$$4: \quad 2^3 = 8 = 4_+ + 4_-$$

- 4_+ : $\underbrace{|+++\rangle}_{1_c} \qquad \underbrace{|+--\rangle,\ |-+-\rangle,\ |--+\rangle}_{3_c}$

$$B-L = -2/3\,(\Sigma_{12} + \Sigma_{34} + \Sigma_{56})$$

$$4 = 1_{(-1)} + 3_{c\,(1/3)}$$

$$T_{3c} = (\Sigma_{12} - \Sigma_{34})\,1/2\ , \ldots$$

- 6 of $SO(6)$ (fundamental) : ϕ_i

$$\left.\begin{array}{l} \phi_1 \mp i\phi_2 \\ \phi_3 \mp i\phi_4 \\ \phi_5 \mp i\phi_6 \end{array}\right\} \quad \begin{array}{l} 6 = \underset{(\mp 2/3)}{3_c} + 3_c^*{}_{(+2/3)} \\ B-L = -2/3\,(L_{12} + L_{34} + L_{56}) \end{array}$$

- 6 of $SU(4)$ = antisymmetric

$$4\times 4 = \underset{AS}{6} + \underset{S}{10}$$

$$4 \times 4 = (1_{-1} + 3_{1/3}) \times (1_{-1} + 3_{1/3})$$

$$= \underbrace{1_{-2} + 6_{2/3} + 3_{-2/3}}_{10 \text{ of } SU(4)} + \underbrace{3_{-2/3} + 3^*_{2/3}}_{6 \text{ of } SU(4)}$$

- 10 of $SO(6)$?

$\phi_{[ijk]}$ $\qquad \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$

$$\phi^{(\pm)}_{ijk} = \phi_{ijk} \pm \frac{i}{3!} \varepsilon_{ijk\ell mn} \phi_{\ell mn}$$

$$20 = 10 + \overline{10}$$

self-dual $\qquad$ anti self-dual

Higgs : Yukawa sector

- SM (standard Model)

$$\mathcal{L}_Y = y_d \, \bar{Q}_L \, \Phi \, d_R + \cdots$$

doublet

singlet

doublet

- SO(10)

$$16_F = \Psi_+$$

$$16 \times 16 = 10 + 120 + 126$$

10_i $\quad$ $120_{[ijk]}$ $\quad$ $[126]_{ijklm}$

$$\underbrace{120_{[ijk]} \quad [126]_{ijklm}}_{\text{anti-symmetric}}$$

$$\mathcal{L}_Y = 16_F \, \underbrace{\left(y_{10} \, 10_H + y_{120} \, 120_H + y_{126} \, 126_H \right)}_{} \, 16_F$$

how many needed ?

- YUKAWA SECTOR

$$16 \times 16 = 10 + 120 + 126$$

center

$16\,\Gamma_i\,16 = 10_i$ $\quad 10 \to -10,\; 16 \to i16$

$16\,\Gamma_i\Gamma_j\Gamma_k\,16 = 120_{(ijk)}$ $\quad 120 \to -120,$

$16\,\Gamma_i\Gamma_j\Gamma_k\Gamma_\ell\Gamma_m\,4 = 126_{(ijk\ell m)}$ $\quad 126 \to -126$

$$10 = 5 + \bar{5}$$

SU(5)

$$126 = (5+\bar{5})^5_{AS} = 5^5_{AS} + \cdots$$

singlet

$M(R) = C^2$: $16 \to -16$, $10 \to +10$, $126 \to +126$

ν_R (ν^c) singlet

Minkowski
Gell-Mann et al; Glashow
Mohapatra, G. S.
Yanagida

SU(5)

$\nu^c \nu^c \langle 126 \text{ singlet} \rangle$

see-saw $\quad \nu^c$ mass

has B-L=-2

BREAKS B-L, but not M

- B-L BROKEN

alternative $16_H \sim 16_F$ $(\sim \tilde{\nu}_c)$

$(B-L=-1)$

- $\underline{126_H}$ rather interesting (5-index AS)

$$126_H = \underline{(3,1,10)} + \underline{(1,3,\overline{10})} + (1,1,6)$$

$$+ \underline{(2,2,15)}$$

Higgs doublet:

$3\, m_l = -\, m_e$

- $(3,1,10)$: Δ_L triplet (II) Mohapatra, G.S. Lazarides et al

$(\ell_L^T \Delta_L \ell_L + \ell_R^T \Delta_R \ell_R)$

- $(1\; 3\; \overline{10})$: Δ_R – gives mass to ν_R (I)

both present

126: complete?

Both ordinary Higgs + see-saw

$\Big\}$ A MUST!!

HIGHER DIMENSIONAL OPERATORS ?

$$Y_{tree} + c \frac{\langle \Phi_{GUT} \rangle}{M_{Pl}} \simeq 10^{-3}$$

?

If $c = O(1)$ $\Rightarrow$

changes all the predictions

(except $b-\tau$)

*

- Hint:

$d=5$ proton decay

$$W_{\rm eff} = c\,\frac{QQQL}{M_{\rm pe}}$$

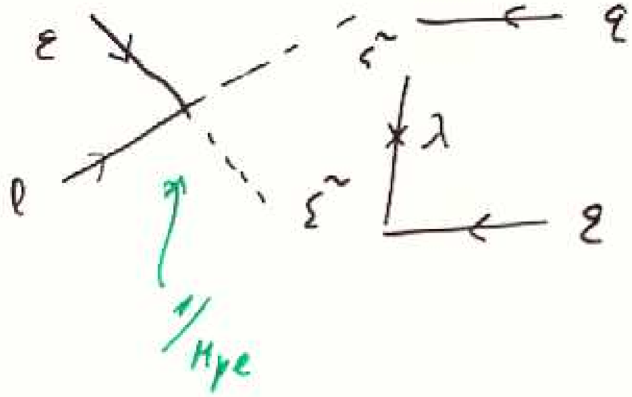
$\Rightarrow \quad c < 10^{-6} \qquad (\tau_p > 10^{33}\,\mathrm{yr})$

- use discrete symmetries and appeal to strings

NO!!!

- accident: c small, but others large

- maybe others small
(self-consistency)



$$\frac{1}{16\pi^2}\,\frac{1}{M_{pe}}\,\frac{m_\lambda}{m_{\tilde{\xi}}^2}\,(qq\xi\ell)$$

$$\Downarrow$$

$$\tau_p \simeq 10^{23}\,\mathrm{yr}$$

Needs a suppression : $c_{pe} \leq 10^{-6}$

Could it be that $1/M_{pe}$ suppressed?

Maybe a renormalizable theory only?