# GRAND UNIFICATION

OR

# UNIFICATION OF STRONG AND ELECTRO-WEAK INTERACTIONS

TEHRAN, 2006

# IN FAVOUR :

- QUARK and LEPTON FAMILIES FIT REMARKABLY INTO MULTIPLETS OF SU(5), SO(10)

- UNIFICATION OF GAUGE COUPLINGS @

  $M_{GUT} \simeq 10^{16}$ GeV

  ( just enough for p stability ; low enough ($\ll M_{Pl}$) for pert. — tiny window )

  - SUPERSYMMETRY ( hierarchy )
  - LEFT-RIGHT SYMMETRY ( $m_\nu \neq 0$ )

- NEUTRINO MASSES

  - $\nu_R$ predicted by L-R, Pati-Salam, SO(10)
  - $M_{\nu_R}$ (see-saw) $\leq M_{GUT}$

- CORRELATION BETWEEN q and l masses and mixings

# RESEARCH

- PROTON DECAY

  ~~[illegible]~~

- LEPTO GENESIS

- FERMION MASSES and MIXINGS

  (NEUTRINO)

- RARE (FLAVOR VIOLATION) PROCESSES

  ($\mu \to e \gamma$)

- EXTRA DIMENSIONS

- DOUBLET-TRIPLET SPLITTING

These lectures:

- BASIC GROUP THEORY
- MINIMAL MODELS:

  SU(5), Pati - Salam, SO(10)

- BASIC ISSUES

  - PROTON DECAY (d=5)
  - UNIFICATION OF GAUGE COUPLINGS
  - DOUBLET- TRIPLET SPLITTING
  - NEUTRINO MASSES

  - q and ℓ masses and mixings

    ↑ seminar

# STANDARD MODEL

$$G_{SM}: \quad \overset{g_c}{SU(3)_c} \times \overset{g}{SU(2)_L} \times \overset{g'}{U(1)_Y}$$

$$Q = T_{3L} + Y/2$$

- MATTER (quarks and leptons)

15 fields (16?)

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R, u_R, u_R$$

$$d_R, d_R, d_R$$

maximal L-R breaking

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R, (\nu_R !?)$$

. construction: charges ($q_d = -1/3$, $q_e = -1$)

$\Rightarrow q_u = 2/3 \quad q_\nu = 0$

fixed by arbitrary Y

- needs symmetry breaking

(all particle massless)

- Higgs:

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \rightsquigarrow SU(2)_L \text{ doublet}$$

( $Y = 1$ hypercharge)

Vacuum:

$$\langle \varphi^0 \rangle = v \neq 0$$

↑ spont. sym. breaking

- $|D_\mu \Phi|^2 \longrightarrow$

$$M_W = g v \quad (W^+, W^-)$$

$$M_Z = \sqrt{g^2 + g'^2}\, v \quad (Z^0)$$

($g$: $SU(2)_L$; $g'$: $U_Y(1)$)

$$m_A = 0 \quad (\text{photon})$$

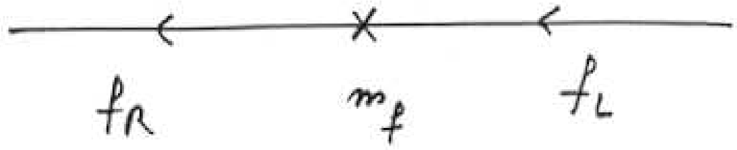
- $$\mathcal{L}_Y = y_d \, \bar{q}_L \Phi d_R + y_u \, \bar{q}_L i\sigma_2 \Phi^* u_R + y_e \, \bar{l}_L \Phi e_R$$

$$m_f = y_f \langle \varphi^0 \rangle = y_f v$$

$$y_f = g \frac{m_f}{M_W}$$

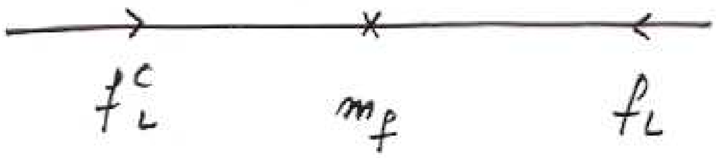
TWO WAYS OF WRITING f MASSES
(YUKAWAS)

- 'DIRAC' : $\overline{f_R}\, f_L\, m_f \quad (+ h.c.)$




- 'MAJORANA' : $\overline{(f^c)_L}^{\,T}\, C\, f_L\, m_f$

$\left( (f^c)_L \equiv C\, \overline{f_R}^{\,T} \right)$ (charge conjugation flips chirality)




useful in GUT and supersymmetry

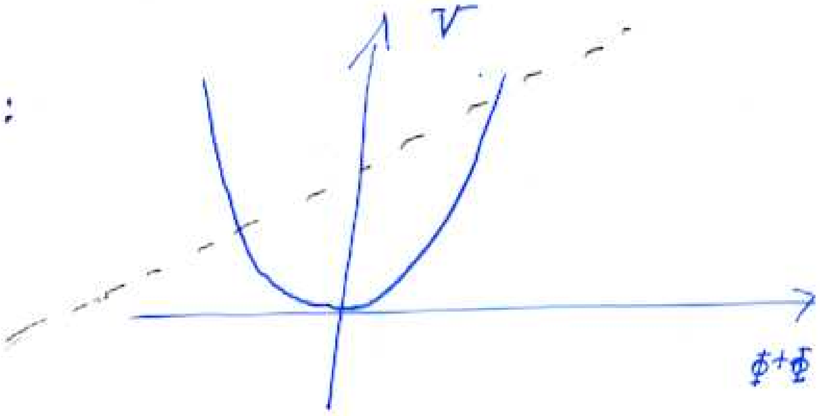
- $u^c_L$, $d^c_L$ $\qquad \begin{pmatrix} u \\ d \end{pmatrix}_L$

$e^c_L$, $\nu^c_L$ (?) $\qquad \begin{pmatrix} \nu \\ e \end{pmatrix}_L$

- $V(\Phi) = \frac{\lambda^2}{4}(\Phi^+\Phi \pm v^2)^2$ (+ const ?)

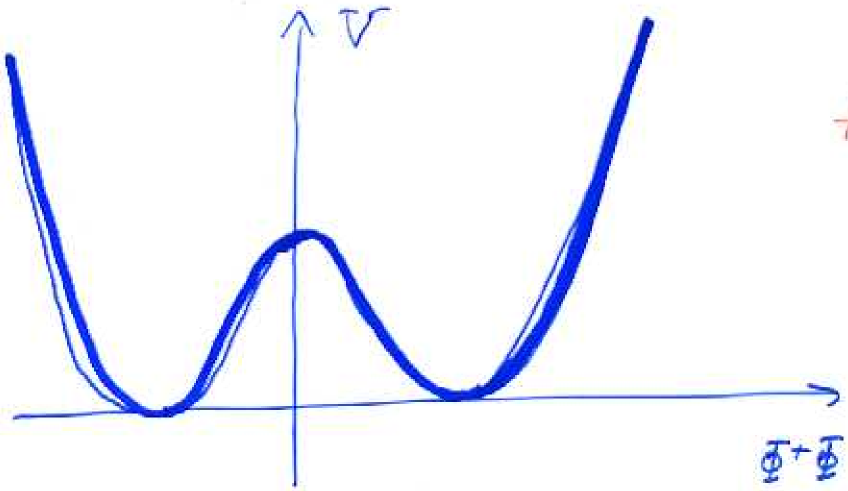
↑ most general renormalizable potential

+ :




bad : no symmetry breaking

− :




the chosen one

(half of the parameter space)

remarkably simple and efficient

$m_h = \lambda v$ ← not predicted

$m_h > 110$ GeV LEP

# STANDARD MODEL: POSITIVE

- $M_W$, $\sin\theta_W$ ($e \equiv g \sin\theta_W$ ; $\tan\theta_W = g'/g$):

  all weak interaction data

- Higgs remarkably simple (minimal)

$$V = \lambda/4 \left( \Phi^+ \Phi \pm v^2 \right)^2$$

most general renormalizable potential

50% parameter space ⇒ all masses

- B, L automatic in perturbation theory

  B−L anomaly free

  ⇓ gauged?

  would need $\nu_R$ (per generation)

  ⇒ $m_\nu \neq 0$

  for $m_{\nu_R} \gg m_W$ (gauge singlet)

  ⇒ see-saw mechanism:

  $\nu_L$ naturally light

Issues

- Yukawa arbitrary : no connection between $q$ and $l$ masses and mixings

GETS IMPROVED IN GRAND UNIFICATION

• SO(10) •

- no connection between $g_c$, $g$, $g'$

⇒ GRAND UNIFICATION

- Higgs mass not protected by symmetries (unlike $m_f$) from large scales

gets worse in GUT (why $M_H \ll M_{GUT}$)

⇒ supersymmetry

- origin of Parity breaking ?

⇒ L − R symmetry, Pati − Salam

• SO(10) •

related to $\nu$ mass

- why is charge quantized? $Y$ arbitrary

SM: anomaly cancellation

$\Rightarrow \quad q_e = +3\, q_d$

generation mixing: $q_e = q_\mu = q_\tau$

no prediction for new particles:

- vector-like $(f_L + f_R)$ - arbitrary
- new family? (decoupled): $q_{\tau'} = ?$

NO magnetic monopoles

$\Rightarrow$ GRAND UNIFICATION

- why Higgs tachyonic?

not really an issue to me

→ natural in low-energy supersymmetry

- New (high energy) physics: **operator analysis** — Weinberg, Wilczek-Zee

$$\mathcal{L}_{effective} = \mathcal{L}_{ren}\,(d \le 4) + \mathcal{L}_{nren}\,(d > 4)\,(1/\Lambda)$$

- $d = 5$

$$\mathcal{L}_{neutrino} = c_\nu \frac{(\ell^T i\sigma_2 \Phi)(\Phi^T i\sigma_2 C \ell)}{\Lambda_\nu}$$

$$\hookrightarrow c_\nu \frac{\nu^T C \nu\, M_W^2}{\Lambda_\nu}$$

$$m_\nu \simeq c_\nu \frac{M_W^2}{\Lambda_\nu} \le 1\,eV$$

$$c_\nu = O(1) \Rightarrow \Lambda_\nu \gtrsim 10^{13}\,GeV$$

- $d = 6$

$$\mathcal{L}_{proton} = c_p \frac{qqq\ell}{\Lambda_p^2} \quad \begin{cases} \epsilon_{\alpha\beta\gamma} \dfrac{(q_L^\alpha C \sigma_2 q_L^\beta)(q_L^\gamma C \sigma_2 \ell_L)}{\Lambda_p^2} \\ \epsilon_{\alpha\beta\gamma} \dfrac{u_R^{\alpha T} C\, d_R^\beta\; u_R^{\gamma T} C\, e_R}{\Lambda_p^2} \end{cases}$$

$$\tau_p \propto \frac{\Lambda_p^4}{c_p\, m_p^5} > 10^{33}\,yr$$

$$\Rightarrow \Lambda_p > 10^{15}\,GeV \quad (c_p \sim O(1))$$

UNIFY WHAT?

'LOW ENERGY' ($\sim$ TeV) EFFECTIVE THEORY?

- STANDARD MODEL

NO indication of new physics $\gtrsim$ TeV

($m_\nu \neq 0$ : INDICATION FOR NEW PHYSICS at $E \gg$ TeV ($\gg 10^{10}$ GeV))

THEORETICAL PREJUDICE:

HIGGS MASS HIERARCHY

$\Rightarrow$ SUPERSYMMETRY at $\sim$ TeV

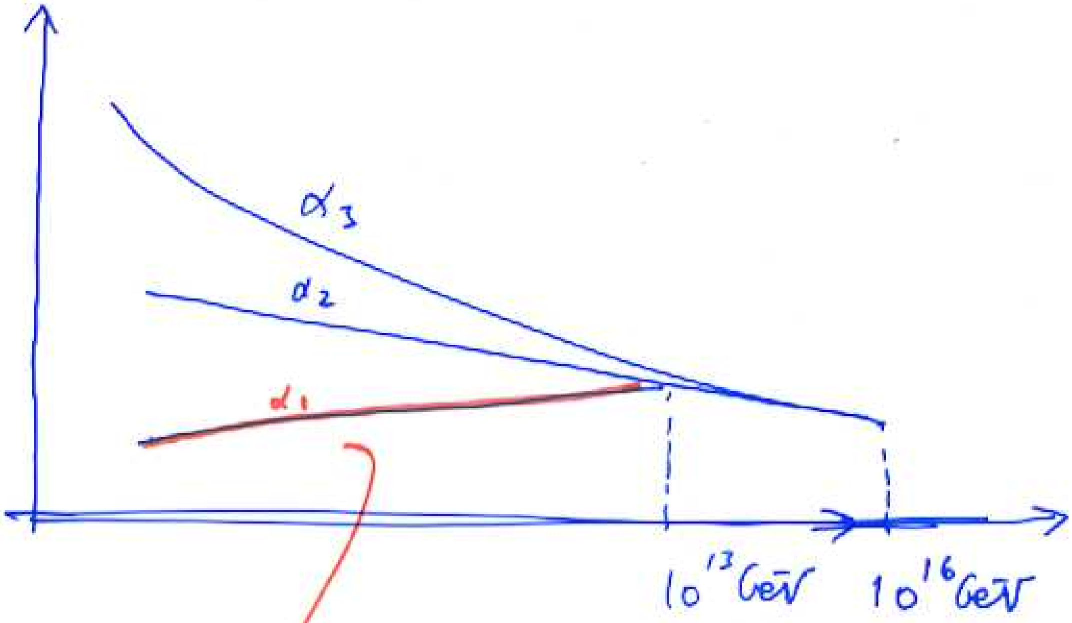
NATURALNESS

$$m_H^2 = m_{H0}^2 + \frac{Y_t^2}{16\pi^2}\left(m_t^2 - m_{\tilde{t}}^2\right)$$

$Y_t \simeq O(1) \Rightarrow$ Higgs naturally 'tachyonic'

# GAUGE COUPLING UNIFICATION

SM




↑ nice

(proton stability)

($M_X \gtrsim 10^{15.5}$ GeV)

problem if ym assume a desert

⇕

(minimal G) SU(5) : ( not good for fermion (neutrino) masses )

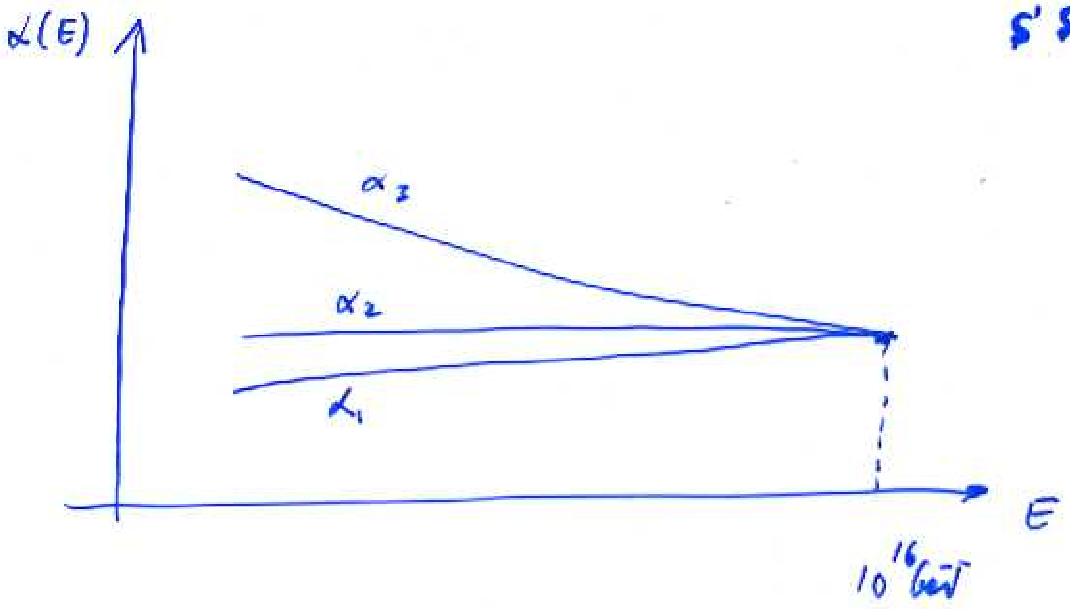
⇓

$\sin^2\theta_W = .2$

exp. $\sin^2\theta_W = .23$

# GAUGE COUPLING UNIFICATION

S'SM




$\Longleftrightarrow \quad \sin^2\theta_W = .23$

ANTICIPATED (predicted)

Einhorn, Jones

* Marciano, G.S.

• $\sin^2\theta_W = .2$ in '81

• Dimopoulos et al
Ibañez, Ross

* suggest: $m_t \simeq 200$ GeV

$\Rightarrow \rho > 1 \Rightarrow \sin^2\theta_W \uparrow$

$SU(2)_L \times U(1) \times SU(3)_c$

$\cap$

$G$

$(d=4)$

- UNIFICATION OF ELECTRO-WEAK and STRONG FORCE at $M_{GUT} \gg M_W$

$$G \xrightarrow[M_{GUT}]{} SM$$

$$\alpha_i = \alpha_{GUT} \quad (M_{GUT})$$

$i = 1, 2, 3$

- QUANTIZATION OF CHARGE

$\mathrm{Tr}\, Q_{em} = 0$ CONNECTS q AND l CHARGES

example : $SU(2)$ Schwinger '57

$Q = T_3$

$1/2, -1/2$

$1, 0, -1$

$3/2, 1/2, 0, -1/2, -3/2$

$\vdots$

$SU(2) \downarrow U(1)$ } PROTOTYPE

• CHARGE QUANTIZATION

$\Downarrow$

'tHooft Polyakov

MAGNETIC MONOPOLES

$m_M \simeq M_{GUT}$

NEVER FOUND    MACRO    Gran Sasso

$\Phi_m \leq 10^{-15} / \mathrm{cm}^2\,\mathrm{sec}$

( DIRAC: quantum mechanics

monopole (s) $\Rightarrow$ charge quantization

$\psi_f \propto \exp(i q_f g_m) \Rightarrow q_f g_m \propto n$ )

14 Feb. '82 (Cabrera)?

• COSMOLOGY: too many (PROBLEM)

Kibble
Preskill

SOLUTIONS:

TOO FEW (inflation: $\sim 1$)

• Callan-Rubakov: monopoles catalyze $p$ decay $\propto 1/\Lambda^2_{QCD}$

$\Rightarrow \Phi_m \leq 10^{-27} / \mathrm{cm}^2\,\mathrm{sec}$    pulsars

# BASIC GROUP THEORY : SU(N)

- fundamental representation

$N$: $\Phi \longrightarrow U\Phi$ $\qquad U^\dagger U = U U^\dagger = 1$

$\det U = 1$

$U = e^{-i\theta_a T_a}$ $\qquad T_a = T_a^\dagger, \quad \mathrm{Tr}\, T_a = 0$

$(N \times N)$ $\qquad (N^2-1)$

$N^*$: $\Phi^* \longrightarrow U^* \Phi^*$

- adjoint representation $(N^2-1)$

↖ # of generators

$A(x) \longrightarrow U A(x) U^\dagger$

$A \equiv A_a T_a \qquad (\mathrm{Tr}\, A = 0)$

↑ irreducible

$[T_a, T_b] = i f_{abc} T_c$

$A_a \longrightarrow A_a + f_{abc}\, \theta_b A_c$

- $(\hat{T}_a)_{bc} = -i f_{abc}$

  ↑ provides an adjoint representation of $[\hat{T}_a, \hat{T}_b] = i f_{abc} \hat{T}_c$

- Exercise : show that the above is true.

  Hint: use Jacoby identity for $\hat{T}_a$

- $A \to U A U^{+} = A - i\, \theta^{a} [T_a, A] + \cdots$

  $\hat{T}_a A = [T_a, A]$

  - gauge bosons : adjoint $\left( D_\mu = \partial_\mu - i g \hat{T}^a \hat{A}^a_\mu \right)$
  - Higgs $(\Sigma)$ $\Rightarrow \langle \Sigma \rangle \to U \langle \Sigma \rangle U^{+}$

    $\Rightarrow \langle \Sigma \rangle = \text{diagonal}$

  $\Rightarrow [\langle \Sigma \rangle, T \in \text{Cartan}] = 0$

  Adjoint Higgs preserves the rank

MINIMAL $G \supseteq SU(3) \times SU(2) \times U(1)$

## SU(5) ( rank = 4 = rank $G_{SM}$ )

- FERMIONS

$$\bar{5}_F = \begin{pmatrix} d^c \\ d^c \\ d^c \\ \hline \nu \\ e \end{pmatrix}_L \quad \begin{matrix} \} \text{ color} \\ \} \; SU(2)_L \end{matrix}$$

$Tr\, Q_{em} = 0$

anomaly cancellation

$$10_F = \begin{pmatrix} 0 & u^c & u^c & u & d \\ & 0 & u^c & u & d \\ & & 0 & u & d \\ & & & 0 & e^c \\ & & & -e^c & 0 \end{pmatrix}_L$$

$[ij]_{AS}$

NO $\nu^c$ ! $m_\nu = 0$

- All other repr. built out of $\Phi$ (fundamental)
(and/or $\Phi^*$)

example : $\phi_i \phi_j = \phi_{[ij]} + \phi_{\{ij\}}$

$\phi_{[ij]} \rightarrow \frac{N(N-1)}{2}$, $\phi_{\{ij\}} \rightarrow \frac{N(N+1)}{2}$

SU(5): $\Phi = \underline{5} \Rightarrow$

$\underline{10}$ = anti-symmetric

$\underline{15}$ = symmetric

- charges sum up :

$$Q(\phi_i \phi_j) = Q(\phi_i) + Q(\phi_j)$$

- Exercise : Show that $\underline{15}$ cannot accomodate the generation of fermions.

• **YUKAWA** ( Higgs = $5_H$ ($\bar{5}_H$))

$$W_Y = Y_u\, 10_F\, 10_F\, 5_H + Y_d\, 10_F\, \bar{5}_F\, \bar{5}_H$$

$$Y_u^T = Y_u \ , \quad Y_d = Y_e$$

accidental

⇓

$m_b = m_\tau$ ← very good ($M_{GUT}$)

$m_s = m_\mu$ ← bad (~1/3)

$m_d = m_e$ ← bad (~3)

**ALSO:**

$m_\nu = 0$ (later)

• higher dimensional operators

• more Higgs (lose predictions)

$45_H$

$$3\, Y_d = -Y_e$$

# SU(5): GAUGE BOSONS

ADJOINT OF SU(5): $24_V$

$$24_V = \left(\begin{array}{c|c} \text{gluons} & X^\alpha, Y^\alpha \\ \hline X^+_\alpha,\ Y^+_\alpha & \vec{W}, B \end{array}\right) \quad \updownarrow SU(3)_c, \quad \overset{SU(2)}{\longleftrightarrow}$$

$(5 \times \bar{5})$

$$24 = (\text{glue} + \vec{W}, B) + \overset{(4/3,\ 1/3)}{(X, Y)} + (\bar{X}, \bar{Y})$$

$$8 + 3 + 1 = 12 \qquad (3+3) \times 2 = 12$$

$\begin{pmatrix} X \\ Y \end{pmatrix}^\alpha$ : lepto-quarks (diquarks)

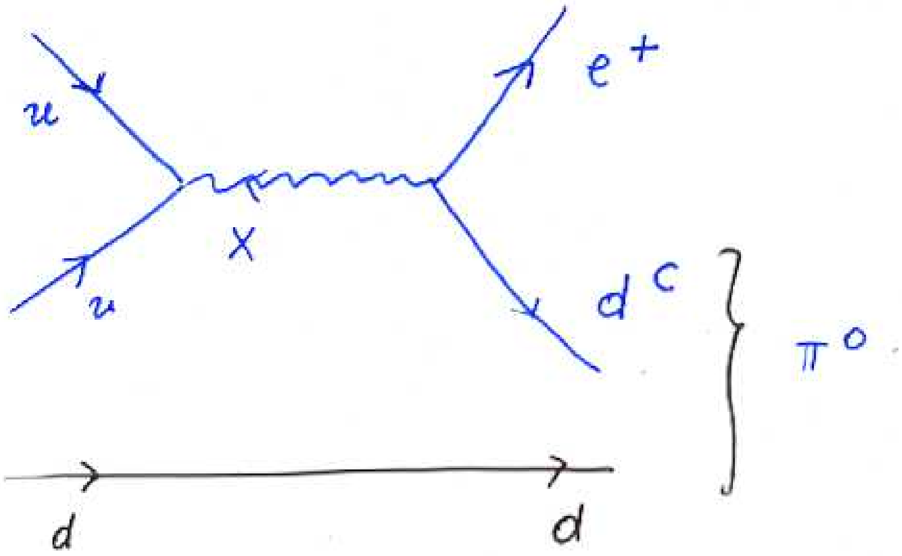
$$X^+_\mu \left[ \bar{u}^c_L \gamma^\mu u_L + \bar{e}_L \gamma^\mu d^c_L \right] \quad \text{charge } 4/3 \quad (Y: 1/3)$$

NO B, L conservation

( but good : $B - L = +2/3$ )

$\Downarrow$

p decay?




$p \to \pi^0 + e^+$ (B-L!)

$\mathcal{M} \propto \left( 1/M_X^2 \right)$

$\Downarrow$

$\tau_p \propto \dfrac{M_X^4}{m_p^5}$ (~ μ decay)

$\tau_p > 10^{33}\ \text{yr} \Longrightarrow M_X > 10^{15}\ \text{GeV}$

• $M_{X,Y} \simeq M_{GUT} \sim 10^{16}\ \text{GeV}$

$\tau_p \simeq 10^{36}\ \text{yr}$ expected, but

# HIGGS : Minimal

- $24_H$ $\quad (SU(5) \xrightarrow[M_{GUT}]{} SM)$

- $5_H$ $(\bar{5}_H)$ $\quad SM \xrightarrow[M_W]{} U(1)$

$$W = \frac{m}{2} \, Tr \, 24_H^2 + \lambda/_3 \, Tr \, 24_H^3$$

$$+ (\mu + \alpha \, 24_H) \, 5_H \, \bar{5}_H$$

$M_{GUT}$ : $\langle 5_H \rangle = \langle \bar{5}_H \rangle = 0$

$F_{24} = 0$ $\qquad D_{24} \propto [24_H, 24_H^+] = 0$

$24_H$ diagonal

degenerate

- $SU(3) \times SU(2) \times U(1)$ $(321)$ :

$$\langle 24_H \rangle = \text{diag} \, \frac{m}{\lambda} (2, 2, 2, -3, -3) \quad \left(\frac{m}{\lambda} \simeq M_{GUT}\right)$$

- $SU(4) \times U(1)$ $(41)$ :

$$\langle 24_H \rangle = \text{diag} \, \frac{m}{3\lambda} (1, 1, 1, 1, -4)$$

- $SU(5)$ $(5)$ $\qquad \langle 24_H \rangle = 0$

- PROTON DECAY ?

- $d = 4$

$$\Delta W = \lambda\, 10_F\, \bar{5}_F\, \bar{5}_F + \mu'\, 5_H\, \bar{5}_F$$

$$= \lambda\, (Q L D^c + U^c D^c D^c) + \cdots$$

exchange of $\tilde{d}^c$ ($\sim T$) :

p decay at TeV

$$\Rightarrow \quad \lambda < 10^{-12}$$

Matter parity: $F \to -F,\ H \to H$

ad-hoc

## • d = 5 p decay

$$W_Y = y_u\, 10_F\, 10_F\, 5_H + y_d\, 10_F\, \bar{5}_F\, \bar{5}_H +$$

$$\downarrow \qquad\qquad \downarrow$$

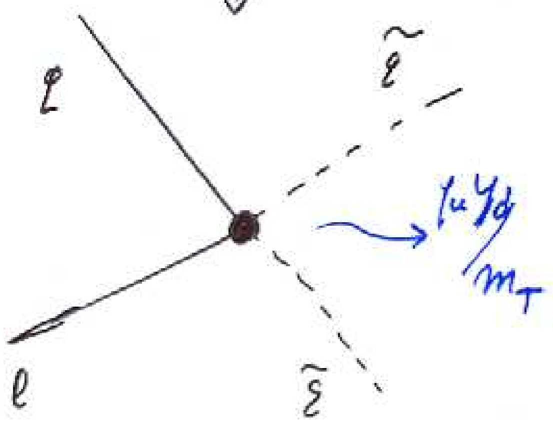
$$y_u\, Q Q T \qquad\qquad y_d\, Q L \bar{T}$$

$$Q = \begin{pmatrix} U \\ D \end{pmatrix} \qquad L = \begin{pmatrix} \nu \\ E \end{pmatrix}$$

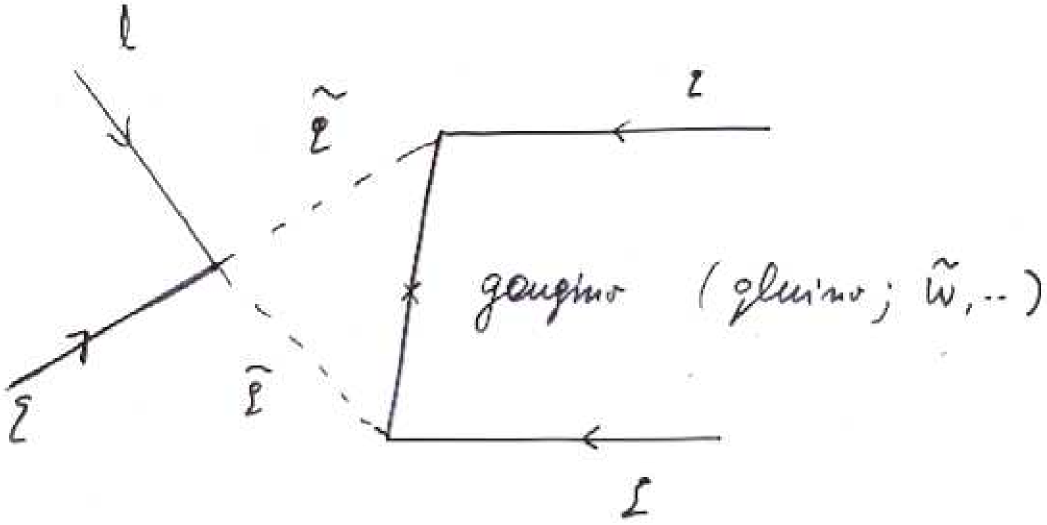
$$m_T\, T \bar{T}$$

$$\Rightarrow \quad W_{eff} = y_u\, y_d\, \frac{QQQL}{m_T}$$

$$\Downarrow$$




d = 5 operator (not yet proton decay)




$$\mathcal{M} \propto \frac{Y_u Y_d g^2}{16\pi^2} \frac{m_\lambda}{m_T m_{\tilde{q}}^2}$$

↑

border - line : $m_T \simeq M_{GUT}$

$m_{\tilde{q}} \gtrsim \text{TeV}$ , $m_\lambda \sim 100\ \text{GeV}$

under scrutiny

- needs a theory (realistic) of $Y_u$, $Y_d$

$\tau_p \simeq 10^{31} - 10^{34} (10^{35})\ \text{yr}$

only possible if you believe in naturalness ($m_{\tilde{q}} \leq \text{TeV}$) ?

- AFTER SU(5) BREAKING

$$5_H = \begin{pmatrix} D \\ T \end{pmatrix}, \quad \bar{5}_H = \begin{pmatrix} \bar{D} \\ \bar{T} \end{pmatrix}$$

fine-tuning →

$$M_D = \mu + 2 M_{GUT} = 0 \quad (\sim M_W)$$

$$M_T = \mu - 3 M_{GUT} \simeq M_{GUT}$$

↑

D - T splitting problem

A number of ideas (SUSY not sufficient)

D - pseudo Goldstone bosons

A second order question. First:

ESTABLISH THE THEORY

↳ analog to the SM: a

SG : STANDARD GUT