Spectator model in D Meson Decays

Mehrdad Ghominejad[1], Physics department, Faculty of sciences, Semnan University, Semnan.
Hossein Mehrban[2], Physics department, Faculty of sciences, Semnan University, Semnan.

Abstract

In this research we describe effective Hamiltonian theory and apply this theory to the calculation of current-current ($Q_{1,2}$) and QCD penguin ($Q_{3,\ldots,6}$) decay rates. The channels of charm quark decay in the quark levels are: $c \to du\overline{d}$, $c \to du\bar{s}$, $c \to su\overline{d}$ and $c \to su\bar{s}$, that channel $c \to su\overline{d}$ is dominance.
We calculate the total decay rates of semileptonic and hadronic of charm quark in effective Hamiltonian theory. We investigate the decay rates of ***D*** **meson decays** according to **Spectator Quark Model (SQM)** for the calculation of ***D*** meson decays.
We want to make the transition from decay rates at the quark level to D meson decay rates for two body **hadronic** decays $D \to h_1 h_2$. By means that, we analyze modes of **nonleptonic** $D \to PV, D \to PP, D \to VV$ and **semileptonic** $D \to P\ell\nu_\ell$ and $D \to V\ell\nu_\ell$ decays, where V and P are light vector with $J^P = 0^-$ and pseudoscalar with $J^P = 1^-$ mesons, respectively, also ℓ, ν_ℓ refer to leptons τ, μ, e and ν_τ, ν_μ, ν_e .
We obtain the total decay rates of semileptonic and hadronic of charm quark in effective Hamiltonian according to colour Favoured (C-F) and colour Suppressed (C-S), and then to added amplitude of processes colour Favoured and colour Suppressed (F-S) and obtain the decay rates of them. Also Using Spectator Model, we obtain Branching Ratio of some D meson decays.
For ***D*** mesons, we have an isospin analysis of the spectator model for various decays and we show that this Model independent from isospin is clearly not in accord with experiment.

Introduction: Effective Hamiltonian

As a weak decay under the presence of the strong interaction, D meson decays require special techniques. The main tool to calculate such D meson decays is the effective Hamiltonian theory. It is a two step program, starting with an operator product expansion (OPE) and performing a renormalization group equation (RGE) analysis afterwards. The necessary machinery has been developed over the last years. The derivation starts as follows: If the kinematics of the decay are of the kind that the masses of the internal particle M_i are much larger than the external momenta P, $M_i^2 >> p^2$, then the heavy particle can be integrated out. This concept takes concrete form with the functional integral formalism. It means that the heavy particles are removed, as dynamical degrees

[1] mghominejad@semnan.ac.ir
[2] hmehraban@semnan.ac.ir

of freedom, from the theory. Hence their fields do not appear in the effective Lagrangian anymore. Their residual effect lies in the generated effective vertices. In this way an effective low energy theory can be constructed from a full theory like the Standard Model. A well known example is the four-Fermi interaction, where the W-boson propagator is made local for $M_W^2 >> q^2$ (q denotes the momentum transfer through the W):

$$-i(g_{\mu\nu})/(q^2 - M_W^2) \rightarrow ig_{\mu\nu}[(1/M_W^2) + (q^2/M_W^4) + ...], \tag{1}$$

where the ellipsis denote terms of higher order in $1/M_W$.

Apart from the t quark the basic framework for weak decays quarks is the effective field theory relevant for scales $M_W, M_Z, M_t >> \mu$ [1]. This framework, as we have seen above, brings in local operators, which govern "effectively" the transition in question.

It is well known that the decay amplitude is the product of two different parts, whose phases are made of a weak (Cabbibo-Kobayashi-Maskawa) and a strong (final state interaction) contribution. The weak contributions to the phases change sign when going to the CP-conjugate process, while the strong ones do not. Indeed the simplest effective Hamiltonian without QCD effects ($c \rightarrow su\bar{s}$) is

$$H_{eff}^0 = 2\sqrt{2}G_F\ V_{sc}V_{su}^*\ Q_1, \tag{2}$$

where G_F is the Fermi constant, V_{ij} are the relevant CKM factors and

$$Q_1 = (\bar{s}_\alpha c_\alpha)_{V-A}(\bar{u}_\beta s_\beta)_{V-A}, \tag{3}$$

is a $(V-A)$, $(V-A)$ is current-current local operator.

This simple tree amplitude introduces a new operator Q_2 and is modified by the QCD effect to

$$H_{eff} = 2\sqrt{2}G_F\ V_{sc}V_{su}^*(C_1Q_1 + C_2Q_2), \tag{4}$$

$$Q_2 = (\bar{s}_\beta c_\alpha)_{V-A}(\bar{u}_\alpha s_\beta)_{V-A}. \tag{5}$$

where C_1 and C_2 are Wilson coefficients. The situation in the Standard Model is, however, more complicated because of the presence of additional interactions in particular penguins which effectively generate new operators. These are in particular the gluon, photon and Z^0-boson exchanges and penguin c quark contributions as we have seen before.

Consequently the relevant effective Hamiltonian for D-meson decays involves generally several operators Q_i with various colour and Dirac structures which are different from Q_1. The operators can be grouped into three categories [2]: $i = 1,2 -$ current-current operators; $i = 3,...,6 -$ gluonic penguin operators. Moreover each operator is multiplied by a calculable Wilson coefficient $C_i(\mu)$:

$$H_{eff} = 2\sqrt{2}G_F \sum_{i=1}^{6} d_i(\mu)Q_i(\mu), \tag{6}$$

where the scale μ is discussed below, $d_i(\mu) = V_{CKM}C_i(\mu)$ and V_{CKM} denotes the relevant CKM factors that are:

$$d_{1,2} = V_{ic}V_{jk}^*C_{1,2} \qquad , \qquad d_{3,...,6} = -V_{tc}V_{tk}^*C_{3,...,6} \tag{7}$$

Effective Hamiltonian Decay Rates

The effective $\Delta C = 1$ Hamiltonian at scale $\mu = \mathrm{O}(m_c)$ for tree plus penguin term is given by,

$$H_{eff}^{\Delta C=1} = 2\sqrt{2}G_F\{[d_{1s}(\mu)Q_1^s(\mu) + d_{2s}(\mu)Q_2^s(\mu)]$$
$$+[d_{1d}(\mu)Q_1^d(\mu) + d_{2d}(\mu)Q_2^d(\mu)] - \sum_{i=3}^{6} d_i(\mu)Q_i(\mu)\}. \quad (8)$$

Here $d_1,...,_6$ are defined by Eq.(7), $d_{1,2s,d} = d_{1,2}(i = j = s,d)$ and index k refer to d or s quarks. Squaring spin average term $Q_1,...,Q_6$ is given by,

$$[(\tilde{\sigma}^\mu)(\tilde{\sigma}_\mu)_{LL} + (\tilde{\sigma}^\mu)(\sigma_\mu)_{LR}]^2_{sp-av} = \alpha_1(1/16)(1+v_i)(1+v_k)(1+v_j)[1-\cos(\theta_k - \theta_i)]$$
$$+\alpha_2(1/16)(1-v_i)(1+v_k)(1-v_j)[1+\cos(\theta_k - \theta_j)]$$
$$+\alpha_3(1/16)\sqrt{1-v_i^2}(1+v_k)\sqrt{1-v_j^2}[1+\cos(\theta_j - \theta_i)$$
$$-\cos(\theta_k - \theta_j) - \cos(\theta_k - \theta_i)]. \quad (9)$$

We must obtain the eight terms of helicity states of equation above, and then add them, so

$$[(\tilde{\sigma}^\mu)(\tilde{\sigma}_\mu)_{LL} + (\tilde{\sigma}^\mu)(\sigma_\mu)_{LR}]^2_{sp-av} = (\alpha_1/2)[1 - v_i v_k \cos(\theta_k - \theta_i)]$$
$$+(\alpha_2/2)[1 + v_k v_j \cos(\theta_j - \theta_k)]$$
$$+(\alpha_3/2)\sqrt{1-v_i^2}\sqrt{1-v_j^2}\,. \quad (10)$$

After adding all colour combinations α_1, α_2 and α_3 gives,

$$\alpha_1 = |d_1 + d_2 + d_3 + d_4|^2 + 2|d_1 + d_4|^2 + 2|d_2 + d_3|^2,$$
$$\alpha_2 = |d_5 + d_6|^2 + 2|d_5|^2 + 2|d_6|^2),$$
$$\alpha_3 = \mathrm{Re}\{(3d_1 + d_2 + d_3 + 3d_4)d_6^* + (d_1 + 3d_2 + 3d_3 + d_4)d_5^*\}. \quad (11)$$

The angle between the particle velocities must be physical, $-1 \le \cos(\theta_k - \theta_i) \le 1$ and $-1 \le \cos(\theta_j - \theta_k) \le 1$. So we should take the variable p_i and p_k, or x and y as,

$$p_i = xM_c/2, \qquad p_k = yM_c/2$$

(12)

The partial decay rate in the c rest frame is,

$$d^2\Gamma_{Q_1,...,Q_6}/dp_i dp_k = (G_F^2/\pi^3)p_i p_k E_{\bar{j}}\{\alpha_1(p_i.p_k/E_i E_k)$$
$$+\alpha_2(p_i.p_{\bar{j}}/E_i E_{\bar{j}}) + \alpha_3(m_k m_{\bar{j}}/E_k E_{\bar{j}})\}. \quad (13)$$

After the change of variable to x and y, the decay rate is given by,

$$d^2\Gamma_{Q_1,...,Q_6}/dxdy = \Gamma_{0c} I_{ps}^{EH}. \quad (14)$$
$$I_{ps}^{EH} = \alpha_1 I_{ps}^1 + \alpha_2 I_{ps}^2 + \alpha_3 I_{ps}^3. \quad (15)$$

where,

$$I_{ps}^1 = 6xy.f_{ab}.(1-h_{abc}), \qquad I_{ps}^2 = 6xy.f_{bc}.(1+h_{bca}), \qquad I_{ps}^3 = 6xy.f_{ac}.h_{xa}.h_{yc}. \quad (16)$$

f_{ab}, f_{bc}, f_{ac}, h_{abc} and h_{bca} are defined by Eq.(47).

Also, $$h_{xa} = [1-(x^2/(x^2+a^2))]^{1/2},$$
$$h_{yc} = [1-(y^2/(y^2+c^2))]^{1/2}. \tag{17}$$

Spectator Model

In the spectator model [3] the spectator quark is given a non-zero momentum having in this work a Gaussian distribution, represented by a free (but adjustable) parameter, Λ:
$$P(|p_s|^2) = (1/\pi^{3/2}\Lambda^3)\ e^{-(p_s^2/\Lambda^2)}. \tag{18}$$
Probability distribution of three momentum for spectator quarks are given by
$$dP(p_s) = P(|p_s|^2)d^3p_s = P(|p_s|^2)d\Omega p_s^2 dp_s. \tag{19}$$
and $P(|p_s|^2)$ is normalized according to,
$$4\pi\int_0^\infty P(|p_s|^2)\ p_s^2 dp_s = 1. \tag{20}$$
We, however, consider here a tentative spectator model based upon the idea of duality between quark and hadron physics at the high energies of c quark and D meson decays. The decays at the quark level, even including the penguins, are basically short distance processes. In our proposed spectator quark model the long distance hadronization is largely a matter of incoherently assigning regions of the final quark phase space to the different mesonic systems. For example, we consider a D meson $c\bar{u}$ or $c\bar{d}$ to be a heavy stationary c quark accompanied by a light spectator constituent antiquark, which has a spherically symmetric normalised momentum distribution $P(|p_s|^2)d^3p_s$. The total meson decay rate through a particular mode is then assumed to be
$$\Gamma_{total} = \int \frac{d^2\Gamma}{dp_i dp_k} P(|p_s|^2)d^3p_s dp_i dp_k, \tag{21}$$
equal to the initiating decay rate. Neglecting, for the moment, any constraints due to quark colour, we suppose the spectator antiquark q_s to combine with the quark q ($q = q_i$ *or* q_k) to form the meson system. For example, if $q = q_i$, we assign a mass $M_{q_i q_s}$ to the system such that
$$M_{is}^2 = (p_i + p_s).(p_i + p_s) = m_i^2 + m_s^2 + 2(E_iE_s - p_ip_s\cos\theta_{is}). \tag{22}$$
Constraining p_i and p_s to have mass M_{is}, we can infer from Eq.(21) that
$$\frac{d\Gamma}{dM_{is}} = 2M_{is}\int \frac{d^2\Gamma}{dp_i dp_k} P(|p_s|^2)\ \delta(M_{is}^2 - m_i^2 - m_s^2 - 2(E_iE_s - p_ip_s\cos\theta_{is}))$$
$$\times 2\pi\ p_s^2 dp_s d(\cos\theta_{is})dp_i dp_k. \tag{23}$$
Hence
$$\frac{d\Gamma}{dM_{is}} = 2\pi\ M_{is}\int \frac{p_s dp_s}{p_i}\ \frac{d^2\Gamma}{dp_i dp_k} P(|p_s|^2)\ dp_i dp_k. \tag{24}$$
where, the integration region is restricted by the condition $|\cos\theta_{is}| \le 1$. We also assign a mass $M_{k\bar{j}}$ to the second quark-antiquark system such that,

$$M_{k\bar{j}}^2 = (p_k + p_{\bar{j}}).(p_k + p_{\bar{j}}) = m_k^2 + m_{\bar{j}}^2 + 2(E_k E_{\bar{j}} - p_k p_{\bar{j}} \cos\theta_{k\bar{j}}) . \tag{25}$$

or,

$$M_{k\bar{j}}^2 = (p_k + p_{\bar{j}}).(p_k + p_{\bar{j}}) = (p_b - p_i).(p_c - p_i) = m_c^2 + m_i^2 - 2m_c E_i . \tag{26}$$

The variable E_i , or p_i, determines the mass $M_{k\bar{j}}$. Taking this mass to be the independent variable, we have

$$\frac{d^2\Gamma}{dM_{is}dM_{k\bar{j}}} = \frac{2\pi\ M_{is}\ M_{k\bar{j}}}{m_c} \int \frac{E_i p_s}{p_i^2}\ \frac{d^2\Gamma}{dp_i dp_k} P(|p_s|^2)\ dp_s dp_k\ . \tag{27}$$

The integration range is restricted by $|\cos\theta_{kj}| \le 1$,

$$\cos\theta_{k\bar{j}} = (m_k^2 + m_{\bar{j}}^2 - M_{k\bar{j}}^2 + 2E_k E_{\bar{j}}) / 2 p_k p_{\bar{j}} . \tag{28}$$

We call this mode of quark and antiquark combination ***process(C - F)*** (colour favoured). Finally, by integration, we compute the partial decay rates $\Gamma(M_{is}, M_{k\bar{j}})$ into quark systems with masses less than M_{is} and $M_{k\bar{j}}$. With suitable bindning, we equate these partial decay rates with corresponding rates into mesons. Of particular interest are the quark antiquark systems forming the lowest mass 0^- and 1^- meson states as data exists for charmed quark systems which can be used to test the spectator quark model.

It is also possible that the spectator antiquark combines with the quark q_k , for which we get

$$\frac{d^2\Gamma}{dM_{ks}dM_{i\bar{j}}} = \frac{2\pi M_{ks}\ M_{i\bar{j}}}{M_c} \int \frac{E_k p_s}{p_k^2}\ \frac{d^2\Gamma}{dp_i dp_k} P(|p_s|^2)\ dp_s dp_i\ . \tag{29}$$

We call this ***process(C - S)*** (colour suppressed). In some meson decays, for example $D^+(c\bar{d}) \to (d\bar{d}) + (\bar{d}u)$ which is initiated by the quark decay $c \to du\bar{d}$, the spectator $\bar{d}$ could have combined with the $_s$ or the u . In this case, we show results if the processes add incoherently and also assuming coherence. Summing, the decay rates of B mesons for ***process(C - F)*** and ***process(C - S)*** are:

$$\Gamma_{(C-F)} = \int_{m_{\min is}}^{m_{cutis}} \int_{m_{\min k\bar{j}}}^{m_{cutk\bar{j}}} \frac{d^2\Gamma}{dM_{is}dM_{k\bar{j}}} dM_{is}dM_{k\bar{j}} , \qquad \Gamma_{(C-S)} = \int_{m_{\min ks}}^{m_{cutks}} \int_{m_{\min i\bar{j}}}^{m_{cuti\bar{j}}} \frac{d^2\Gamma}{dM_{ks}dM_{i\bar{j}}} dM_{ks}dM_{i\bar{j}} . \tag{30}$$

where $m_{\min is} = (m_{q_i} + m_{q_s})$, $m_{cutis} = M_{q_i q_s}$ and so on. Turning now to the colour factors, we examine what may be regarded as two extreme possibilities. In the first, called here ***model(A)*** , we take the Eq.(13),(27) and (29) at face value, that is we make no attempt to follow the flow of colour and assume that all colour flow is looked after by the gluon fields in the meson system. In the second, called here ***model(B)*** , we consider the possibility that the lowest mass meson states are only formed if the quark-antiquark pairs are in a colour singlet state. That is, we assume that the colour distribution caused by a quark-antiquark pair in an octet state will result in more complex meson systems than the lowest mass 0^- and 1^- states.

Projecting out the colour singlet states results only in a modification of the coefficients α_1, α_2 and α_3 of Eq.(11). Physical hadrons are singlets in colour space, and are termed colourless or white. Colour symmetry is absolute, and weak quark currents are, as hadrons, white. This means, for example, that $\bar{q}_s q_i$ is in fact a sum of three terms:

$$q_s \bar{q}_i = q_{s\alpha}(h)\bar{q}_{i\alpha}(h) = q_s(1)\bar{q}_i(\bar{1}) + q_s(2)\bar{q}_i(\bar{2}) + q_s(3)\bar{q}_i(\bar{3})\ . \qquad (31)$$

Where suffices 1,2,3 stand for yellow, blue and red colour, respectively. The spectator is a colour singlet with the c quark. Hence, the colour factors for $Q_1, Q_2 Q_3, Q_4, Q_5$ and Q_6 are given by,

$$Q_1, Q_4, Q_6 \rightarrow process(C-F) \equiv \sqrt{3}\,, \qquad process(C-S) \equiv 1/\sqrt{3}$$

$$Q_2, Q_3, Q_5 \rightarrow process(C-F) \equiv 1/\sqrt{3}\,, \qquad process(C-S) \equiv \sqrt{3}$$

Consequently, the factors α_1, α_2 and α_3 of Eq.(11) are different for the $process(C-F)$ and $process(C-S)$ of *model(B)*. For $process(C-F)$ represented by Eq.(27) they become,

$$\alpha_1 = 3\left|d_1 + (d_2/3) + (d_3/3) + d_4\right|^2,$$

$$\alpha_2 = 3\left|(d_5/3) + d_6\right|^2. \qquad (32)$$

While, for $process(C-S)$ represented by Eq.(58) they are

$$\alpha_1 = 3\left|(d_1/3) + d_2 + d_3 + (d_4/3)\right|^2,$$

$$\alpha_2 = 3\left|d_5 + (d_6/3)\right|^2. \qquad (33)$$

Effective Hamiltonian Spectator Model

Now we want to try calculation of spectator model for general case that we called effective Hamiltonian spectator model. According to Eq.(14), the total decay rates for current-current plus penguin operators in the Effective Hamiltonian is given by,

$$\Gamma_{Q_1,\ldots,Q_6} = \Gamma_{0c} I_{ps}^{EH}\ . \qquad (34)$$

The differential decay rates for two boson system in the spectator quark model for current-current plus penguin operators in the Effective Hamiltonian is given by,

$$\frac{d^2\Gamma_{Q_1,\ldots,Q_6}}{d(q_{si}/M_c)d(q_{k\bar{j}}/M_c)} = \Gamma_{0c}\frac{8q_{si}q_{k\bar{j}}}{\sqrt{\pi}M_c}\frac{\beta^2}{\Lambda}\frac{\sqrt{(2m_i/M_c)^2 + x^2}}{x^2} \times \int_0^1 dy \int_0^1 dz\ \zeta^{eff}_{ps(q,z)}\ z\ e^{-\beta^2 z^2}\ . \qquad (35)$$

Where

$$\zeta^{eff}_{ps(q,z)} = \alpha_1\zeta_1^{eff} + \alpha_2\zeta_2^{eff} - \alpha_3\zeta_3^{eff}\ . \qquad (36)$$

The integration region is restricted by the condition $\cos\theta_{is} \leq 1$, thus

$$\zeta_1^{eff}, \zeta_2^{eff}, \zeta_3^{eff} = \begin{cases} \zeta_{1ps}^{eff}, \zeta_{2ps}^{eff}, \zeta_{3ps}^{eff} & if \quad (f_{si(z)})^2 \leq 1 \\ 0 & otherwise \end{cases}. \qquad (37)$$

Where $f_{si(z)} = \Big([(m_i + m_s)/M_c]^2 - (q_{si}/M_c)^2 + (1/M_c)\sqrt{m_s^2 + (\beta\Lambda z)^2}$
$\times\sqrt{(2m_i/M_c)^2 + x^2} \Big)/(\beta\Lambda xz/M_c)$.

Therefore using Eq.(16) the phase space parameters will be defined by,

$$\zeta^{eff}_{1ps} = 6xy.f_{ab}.(1-h_{abc}) ,$$
$$\zeta^{eff}_{2ps} = 6xy.f_{bc}.(1+h_{bca}) ,$$
$$\zeta^{eff}_{3ps} = 6xy.f_{ac}.h_{xa}.h_{yc} . \tag{38}$$

Now, we can integrate over the two mass cuts (two boson systems), and obtain the hadronic decay rates as follows,

$$\Gamma'_{Q_1,\ldots,Q_6} = \int_{\min}^{m_{cut}}\int_{\min'}^{m'_{cut}} \frac{d^2\Gamma_{Q_1,\ldots,Q_6}}{d(q_{si}/M_c)d(q_{k\bar{j}}/M_c)} dm_{cut}dm'_{cut} ,$$

$$= \Gamma_{0c}\int_{\min}^{m_{cut}}\int_{\min'}^{m'_{cut}} \frac{8q_{si}q_{k\bar{j}}}{\sqrt{\pi}M_c}\frac{\beta^2}{\Lambda}\frac{\sqrt{(2m_i/M_c)^2+x^2}}{x^2}\int_0^1 dy\int_0^1 dz\ \zeta^{eff}_{ps(q,z)}\ z\ e^{-\beta^2 z^2} dm_{cut}dm'_{cut} . \tag{39}$$

Decay Rates of Processes C-F plus C-S (F+S) of Effective Hamiltonian

Now we want to calculate the decay rates of Effective Hamiltonian ($Q_1,\ldots,Q_6$) for F+S at quark-level and spectator model. The Effective Hamiltonian for F+S, is given by

$$H^{A+B}_{eff} = H^{b\to ik\bar{j}}_{eff} + H^{b\to i\bar{j}k}_{eff} . \tag{40}$$

where $H^{c\to ik\bar{j}}_{eff}$ defined by Eq.(8) and we can obtain $H^{c\to i\bar{j}k}_{eff}$. So adding the amplitudes of these two terms for b quark spin 1/2 and -1/2 gives,

$$\begin{aligned}[(\tilde{\sigma}^{\mu})(\tilde{\sigma}_{\mu})]^{F+S}_{1/2} = {} & A_1[\sin((\theta_k-\theta_j-\theta_i)/2)+\sin((\theta_k+\theta_j-\theta_i)/2)] \\ & A_2[\sin((\theta_j-\theta_k-\theta_i)/2)+\sin((\theta_k-\theta_j-\theta_i)/2)] \\ & A_1[\sin((\theta_j-\theta_k-\theta_i)/2)+\sin((\theta_j+\theta_k-\theta_i)/2)] \\ & A_3[\sin((\theta_k-\theta_j-\theta_i)/2)+\sin((\theta_j-\theta_k-\theta_i)/2)] .\end{aligned}$$

$$\begin{aligned}[(\tilde{\sigma}^{\mu})(\tilde{\sigma}_{\mu})]^{F+S}_{-1/2} = {} & A_1[\cos((\theta_k-\theta_j-\theta_i)/2)-\cos((\theta_k+\theta_j-\theta_i)/2)] \\ & A_2[\cos((\theta_j-\theta_k-\theta_i)/2)+\cos((\theta_k-\theta_j-\theta_i)/2)] \\ & A_1[\cos((\theta_j-\theta_k-\theta_i)/2)-\cos((\theta_j+\theta_k-\theta_i)/2)] \\ & A_3[\cos((\theta_k-\theta_j-\theta_i)/2)+\cos((\theta_j-\theta_k-\theta_i)/2)] .\end{aligned} \tag{41}$$

Here A_1, A_2 and A_3 refer to colour and helicity factor,

$$A_1 = (\sqrt{\alpha_1}/4)\sqrt{1+v_i}\sqrt{1+v_k}\sqrt{1+v_j} ,$$
$$A_2 = (\sqrt{\alpha_2}/4)\sqrt{1-v_i}\sqrt{1+v_k}\sqrt{1-v_j} ,$$

$$A_3 = (\sqrt{\alpha_2}/4)\sqrt{1-v_i}\sqrt{1-v_k}\sqrt{1+v_j}\,. \tag{42}$$

We must square these terms and averaging over the b quark spin 1/2 and -1/2,

$$\begin{aligned}([(\tilde{\sigma}^{\mu})(\tilde{\sigma}_{\mu})]^{F+S}_{sp-av})^2 = {} & [(3/2)\alpha_1 + \alpha_2 - \alpha_3\sqrt{1-v_i^2}\sqrt{1-v_k^2} - \alpha_1 v_i v_k \cos(\theta_k - \theta_i)] \\ & -[\alpha_3\sqrt{1-v_i^2}\sqrt{1-v_j^2} + \alpha_1 v_i v_j \cos(\theta_j - \theta_i)] \\ & +[\alpha_1\sqrt{1-v_k^2}\sqrt{1-v_j^2} + (1/2)(\alpha_1 - 2\alpha_2) v_k v_j \cos(\theta_k - \theta_j)]\,. \end{aligned} \tag{43}$$

where α_1, α_2 and α_3 defined by Eq.(11). The decay rates of current-current plus penguin for F+S is given by,

$$d^2\Gamma^{F+S}_{EH}/dxdy = \Gamma_{0c} I^{F+S}_{pc}\,, \tag{44}$$

$$I^{F+S}_{pc} = I_{1ps} + I_{2ps} + I_{3ps}\,. \tag{45}$$

where

$$\begin{aligned} I_{1ps} &= 6xy.f_{ab}.[\alpha_1((3/2) - h_{abc}) + \alpha_2 - \alpha_3 h_{xa} h_{yb}]\,, \\ I_{2ps} &= -6xy.f_{ac}.[\alpha_1 h_{acb} + \alpha_3 h_{xa} h_{yc}]\,, \\ I_{3ps} &= 6xy.f_{bc}.[(\alpha_1/2) h_{bca} + \alpha_2 (h_{xb} h_{yc} - h_{bca})]\,. \end{aligned} \tag{46}$$

and,

$$\Gamma_{0c} = G_F^2 M_c^5 / 192\pi^3\,,$$

$$f_{ab} = 2 - \sqrt{x^2+a^2} - \sqrt{y^2+b^2}\,, \qquad h_{abc} = \frac{(f_{ab})^2 - (c^2+x^2+y^2)}{2\sqrt{x^2+a^2}\sqrt{y^2+b^2}}\,,$$

$$f_{bc} = 2 - \sqrt{x^2+b^2} - \sqrt{y^2+c^2}\,, \qquad h_{bca} = \frac{(f_{bc})^2 - (a^2+x^2+y^2)}{2\sqrt{x^2+b^2}\sqrt{y^2+c^2}}\,,$$

$$f_{ac} = 2 - \sqrt{x^2+a^2} - \sqrt{y^2+c^2}\,, \qquad h_{acb} = \frac{(f_{ac})^2 - (b^2+x^2+y^2)}{2\sqrt{x^2+a^2}\sqrt{y^2+c^2}}\,.$$

$$h_{xa} = [1 - (x^2/(x^2+a^2))]^{1/2}\,, \qquad h_{yc} = [1 - (y^2/(y^2+c^2))]^{1/2}\,.$$

$$h_{xb} = [1 - (x^2/(x^2+b^2))]^{1/2}\,, \qquad h_{yb} = [1 - (y^2/(y^2+b^2))]^{1/2}\,. \tag{47}$$

Numerical Results

As an example of the use of the formalism above, we use the standard Particle Data Group [4] parameterization of the CKM matrix with the central values

$$\theta_{12} = 0.221\,, \qquad \theta_{13} = 0.0035\,, \qquad \theta_{23} = 0.041\,,$$

and choose the CKM phase δ_{13} to be $\pi/2$. Following Ali and Greub [5] we treat internal quark masses in tree-level loops with the values (GeV) $m_b = 4.88$, $m_s = 0.2$, $m_d = 0.01$, $m_u = 0.005$, $m_c = 1.5$, $m_e = 0.0005$, $m_\mu = 0.1$, $m_\tau = 1.777$ and $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$.

Following G.Buccella [6] we choose the effective Wilson coefficients C_i^{eff} for the various $c \to q$ transitions.

a) The total decay rate and branching ratios of semileptonic and hadronic modes according to Effective Hamiltonian theory (see Eq.(14)) shown in Table 1. We see that mode $c \to su\bar{d}$ is dominant. The total c-quark decay rate of the Effective Hamiltonian is given by

$$\begin{aligned}\Gamma_{total}^{EH}(c \to anything) &= \Gamma(c \to s\ anything) + \Gamma(c \to d\ anything) \\ &= 9.261 \times 10^{-13} + 0.606 \times 10^{-13} GeV, \\ &= 9.867 \times 10^{-13} GeV,\end{aligned}$$

b) Now we can obtain the mean lives of the charm quark (D meson) theoretically and compare with the experimental mean life of $D^\pm, D^0$ and D_s^+, so

$$Mean\ life_{theory}^{EH}(D) = \hbar / \Gamma_{total}^{EH} = 1.067 \times 10^{-12} Sec. \tag{48}$$

and

$$\begin{aligned}Mean\ life_{\exp}(D^+) &= (1.040 \pm 0.007) \times 10^{-12} Sec, \\ Mean\ life_{\exp}(D^0) &= (0.410 \pm 0.001) \times 10^{-12} Sec, \\ Mean\ life_{\exp}(D_s^+) &= (0.461 \pm 0.015) \times 10^{-12} Sec.\end{aligned} \tag{49}$$

Also, we can compare the branching ratio of the semileptonic theoretically and experimentally, so

$$\begin{aligned}BR(c \to e^+\ anything)_{theory} &= BR(c \to se^+\nu_e) + BR(c \to de^+\nu_e) \\ &= 147.96 \times 10^{-3} + 8.702 \times 10^{-3} = 15.67E-2,\end{aligned} \tag{50}$$

and

$$\begin{aligned}BR_{\exp}(D^+ \to e^+\ anything) &= (17.2 \pm 1.9) \times 10^{-2}, \\ BR_{\exp}(D^0 \to e^+\ anything) &= (6.87 \pm 0.8)E-2, \\ BR_{\exp}(D_s^+ \to e^+\ anything) &< 20 \times 10^{-2}.\end{aligned} \tag{51}$$

We see that the theoretical and experimental results are close.

c) We have used in Spectator Quark Model the value $\Lambda = 0.6$ GeV [7]. For the maximum mass of the quark-antiquark systems (m_{cut}) we take a value midway between the lowest mass 1^- state and the next most massive meson. Thus we take, for $(s\bar{u})$ or $(s\bar{d})$, $m_{cut(u\bar{d})} = 0.877 GeV$ between the $\rho(0.770)$ and the $a_0(0.984)$; for $(u\bar{u})$ and $(d\bar{d})$, $m_{cut(u\bar{u})} = m_{cut(d\bar{d})} = 0.870 GeV$ between $\omega(0.782)$ and $\eta'(0.958)$.

For example, we can calculate the Branching Ratios of mode $c \to du\bar{d}$ in the Tree-level and Effective Hamiltonian spectator model. The modes $c \to du\bar{d}$ is for decays

$D^+ \to \pi^0 \pi^+$ ، $D^+ \to \eta\ \pi^+$ ، $D^+ \to \rho^0 \pi^+$ ، $D^+ \to \omega\ \pi^+$ ،
$D^+ \to \pi^0 \rho^+$ ، $D^+ \to \eta\ \rho^+$ ، $D^+ \to \rho^0 \rho^+$ and $D^+ \to \omega\ \rho^+$ **.**We told that, in this cases got a two-boson system and, therefore two masses of cut for bosons system. We choose masses of cut for two boson system $m_{cut1} = 0.870 / M_c$ and $m_{cut2} = 0.877 / M_c$.
Theoretically, the Branching Ratio of Effective Hamiltonian spectator model is given by,

$$BR_{EH}(c \to du\overline{d}) = \Gamma_{EH}(c \to du\overline{d})_{cut1,cut2} / \Gamma_{total\ EH}(c \to anything)$$
$$= 1.2502 \times 10^{-14} / 9.867 \times 10^{-13}$$
$$= 1.2671 \times 10^{-2}$$

The masses of some mesons and the masses of cut are presented in Table 2. The results are presented in Table 3 and compared, where data is available, with the sum of the branching ratios into mesons with masses less than the above cutoff masses. Also all the experimental and theoretical D meson decays in spectator model classified and given by Table 3.

d) The decay rates of c quark for F+S shown in the Table.4 and the total decay rates of F+S is given by

$(c \to du\overline{d})$ $D^+ \to (\pi^0, \eta, \rho^0, \omega), (\pi^+, \rho^+)$ $BR_{EH}^{F+S} = 2.1023 \times 10^{-2}$,
$(c \to su\overline{d})$ $D^+ \to (\pi^+, \rho^+), (\overline{K}^0, \overline{K}^{*+})$ $BR_{EH}^{F+S} = 51.2871 \times 10^{-2}$,
$(c \to su\overline{s})$ $D^+ \to (\eta', \phi), (K^+, K^{*+})$ $BR_{EH}^{F+S} = 3.8671 \times 10^{-2}$.

Conclusions

We used in this research, effective Hamiltonian theory & spectator quark model for C quark and calculated hadronic decays of *D* mesons. in this model we added decays of channel hadronic decays of *D* mesons. For colour favoured and suppressed we consider the channel $c \to du\overline{d}$ (e.g. $D^+ \to (\pi^0, \pi^+)$) and achieved theoretical values very close to experimental ones. Finally it has been shown the case, in which the theoretical values are better than the amplitude of all the dacay rates have been calculated.

TABLE 1. Decay rates (Γ) and Branching Ratio (BR) of Effective Hamiltonian of c-quark.

Process	$\Gamma_{EH} \times 10^{-15}$	$BR_{EH} \times 10^{-3}$
$c \to du\overline{d}$	31.689	32.12
$c \to du\overline{s}$	1.0785	1.093
$c \to su\overline{d}$	409.44	414.95
$c \to su\overline{s}$	23.836	24.157

TABLE 2. The masses of some mesons and the masses of cut for D meson decay processes.

System of Quark	Particle	Mass (GeV)	Cutoff Mass (GeV)
$s\bar{u}$, $s\bar{d}$	K	0.494	1.081
	K^{*}	0.892	
	K^{**}	1.270	
$u\bar{d}$	π	0.140	0.877
	ρ	0.770	
	a_0	0.984	
$u\bar{u}$, $d\bar{d}$	π^0	0.140	0.870
	η	0.547	
	ρ^0	0.770	
	ω	0.782	
	η'	0.958	
$s\bar{s}$	η'	0.958	1.150
	ϕ	1.020	
	ϕ	1.680	

TABLE 3. Experimental and theoretical of spectator model of Branching Ratios of semileptonic and hadronic for D meson decays.

1- Decay of c Quark
2- Decay of D Meson, $process(C-F)$
3- Decay of D Meson, $process(C-S)$
4- Experimental Branching Ratio, $process(C-F)$
5- Total Experimental Branching Ratio, $process(C-F)$
6- Experimental Branching Ratio, $process(C-S)$
7- Total Experimental Branching Ratio, $process(B)$
8- $m_{cut1} \times M_c$ GeV, $m_{cut2} \times M_c$ GeV, $process(C-F)$
9- $m_{cut1} \times M_c$ GeV, $m_{cut2} \times M_c$ GeV, $process(C-S)$
10- Effective Hamiltonian Branching Ratio, $Model(A)$, $process(C-F)$
11- Effective Hamiltonian Branching Ratio, $Model(A)$, $process(C-S)$

1- $c \rightarrow du\bar{d}$

2- $D^+ \to (\pi^0, \eta, \rho^0, \omega), (\pi^+, \rho^+)$, $D^0 \to (\pi^-, \rho^-), (\pi^+, \rho^+)$, $D_s^+ \to (K^0, K^{*0}), (\pi^+, \rho^+)$

3- $D^+ \to (\pi^0, \eta, \rho^0, \omega), (\pi^+, \rho^+)$, $D^0 \to (\pi^0, \eta, \rho^0, \omega), (\pi^0, \eta, \rho^0, \omega)$,

$D_s^+ \to (\pi^0, \eta, \rho^0, \omega), (K^+, K^{*+})$

4- $\pi^0\pi^+, (2.5 \pm 0.7)E-3$ $\pi^-\pi^+, (1.25 \pm 0.11)E-3$ $K^0\pi^+, < 8.0E-3$

$\rho^0\pi^+, < 1.4E-3$

$K^*(892)^0\pi^+, (6.5 \pm 2.8)E-3$

$\eta\pi^+, (7.5 \pm 2.5)E-3$

$\omega\pi^+, < 7.0E-3$

$\eta\rho^+, 1.2E-2$

5- $< (3.04 \pm 0.25)E-2$ $(1.25 \pm 0.11)E-3$ $(14.5 \pm 2.8)E-3$

6- $\pi^0\pi^+, (2.5 \pm 0.7)E-3$ $\pi^0\pi^0, (8.4 \pm 2.2)E-4$ $K^+\rho^0, < 2.9E-3$

$\rho^0\pi^+, < 1.4E-3$

$\eta\pi^+, (7.5 \pm 2.5)E-3$

$\omega\pi^+, < 7.0E-3$

$\eta\rho^+, 1.2E-2$

7- $< (3.04 \pm 0.25)E-2$ $(8.4 \pm 2.2)E-4$ $< 2.9E-3$

8-0.870,0.877 0.877,0.877 1.081,0.877

9-0.870,0.877 0.870,0.870 0.870,1.081

10-1.2671E-2 ***(F+S 2.1023E-2)*** 1.2834E-2 1.2723E-2

11-1.2931E-2 1.2543E-2 1.2398E-2

1- $c \to su\bar{s}$

2- $D^+ \to (\bar{K}^0, \bar{K}^{*0}), (K^+, K^{*+})$, $D^0 \to (K^-, K^{*-}), (K^+, K^{*+})$, $D_s^+ \to (\eta', \phi), (K^+, K^{*+})$

3- $D^+ \to (\eta', \phi), (\pi^+, \rho^+)$, $D^0 \to (\eta', \phi), (\pi^0, \eta, \rho^0, \omega)$, $D_s^+ \to (\eta', \phi), (K^+, K^{*+})$

4- $\bar{K}^0K^+, (7.2 \pm 1.2)E-3$ $K^+K^-, (4.33 \pm 0.27)E-3$ $\phi K^+, < 5.0E-4$

$\bar{K}^*(892)^0K^+, (4.2 \pm 0.5)E-3$ $K^*(892)^+K^-, (3.5 \pm 0.8)E-3$

$\bar{K}^0K^*(892)^+, (3.0 \pm 1.4)E-2$ $K^+K^*(892)^-, (1.8 \pm 1.0)E-3$

$\bar{K}^*(892)^0K^*(892)^+, (2.6 \pm 1.1)E-2$

5- $(6.74 \pm 2.62)E-2$ $(9.63 \pm 2.07)E-3$ $< 5.0E-4$

6- $\eta'(958)\pi^+, < 9.0E-3$ $\phi\pi^0, < 1.4E-3$ $\phi K^+, < 5.0E-4$

$\eta'(958)\rho^+, < 1.5E-2$ $\phi\eta, < 2.8E-3$

$\phi\pi^+, (6.1 \pm 0.6)E-3$ $\phi\omega, < 2.1E-3$

$\phi\rho^+, < 1.5E-2$ $\phi\rho^0, (1.07 \pm 0.29)E-3$

7- $(4.51 \pm 0.1)E-2$ $(7.37 \pm 0.29)E-3$ $< 5.0E-4$

8-1.081,1.081 1.081,1.081 1.510,1.081

9-1.150,0.877 1.150,0.870 1.150,1.081

10-1.9543E-2 1.9378E-2 1.9859E-2 ***(F+S 3.8671 E-2)***

11-1.9213E-2 1.8975E-2 1.9453E-2

TABLE.4. Decay rates and Branching Ratio of F+S of Effective Hamiltonian

Process	$\Gamma_{HE} \times 10^{-15}$	$BR_{EH} \times 10^{-2}$
$c \to du\bar{d}$	35.611	31.262
$c \to du\bar{s}$	1.4608	1.2824
$c \to su\bar{d}$	554.45	486.74
$c \to su\bar{s}$	26.927	23.638

Bibliography

[1] A. J. Buras, Nucl Instrum Meth. A368 (1995) 1-20
[2] A. Ali and C. Greub, Phys. Rev. D 57. N 5 , 2996 (1998)
[3] [26] G. Altarelli, N. Cabibbo, G. Corbo, L. Maiani and G. Martinelli, Nucl. Phys. B208. 365 (1982), A. Ali and C. Greub, Phys. Lett. B 259, 182 (1991), D. Atwood and A.Soni, ibid. B405, 150 (1997) and C. S. Kim, Y. G. Kim and K. Y. Lee, Phys. Rev. D 57, 4002 (1998)
[4] Particle Data Group, European physical J. C 3, 1 (2005)
[5] A. Ali and C. Greub, Phys. Rev. D 57. N 5 , 2996 (1998)
[6] G. Buchalla, Nucl Phys, B391 (1993) 501-514
[7] W. N. Cottingham, H. Mehrban and I. B. Whittingham, Phys. Rev. D60 (1999) 114029