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CONFINEMENT OF COLOR

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INTRODUCTION.

- QUARKS AND GLUONS ARE THE CONSTITUENTS OF HADRONS AND THE FUNDAMENTAL FIELDS OF THE QCD LAGRANGEAN

$$\mathcal{L}_{QCD} = -\frac{1}{4}\mathrm{Tr}\{G_{\mu\nu}G_{\mu\nu}\} + \sum_f \bar{\psi}_f (i\not{D} - m_f)\psi_f$$

- DETECTED BY USE OF E-W PROBES AT SMALL DISTANCE : NOT OBSERVED AS FREE PARTICLES

P.D.G.

$\frac{n_q}{n_p} \lesssim 10^{-27}$ EXPECTED IN S.C.M $\frac{n_q}{n_p} \simeq 10^{-12}$

$\sigma_q \equiv \sigma(p+p \to q(\bar{q})+X) < 10^{-40}\,\mathrm{cm}^2$ P.T. $\sigma_q \simeq 10^{-25}\,\mathrm{cm}^2$

SUPPRESSION FACTOR $\simeq 10^{-15}$!

- NATURAL EXPLANATION : $n_q = 0$, $\sigma_q = 0$

DUE TO SOME SYMMETRY $\Longrightarrow$

- CONFINEMENT OF COLOR:

ASYMPTOTIC PARTICLES ARE COLORLESS.

- HOW TO RECONCILE WITH PERTURBATION TH ?
- DOES $\mathcal{L}_{QCD}$ INCLUDE CONFINEMENT, AND IF YES BY WHAT MECHANISM (SYMMETRY)

N. CABIBBO, G. PARISI (1975): HAGEDORN LIMITING T_H COULD BE A DECONFINING TRANSITION TO A GAS OF QUARKS & GLUONS. $T_H \simeq 200$ MeV.

- EXPERIMENTS AT RHIC (AND LHC) COLLIDE HEAVY IONS TO PRODUCE THE TRANSITION. NOT EASY TO DETECT.

- VIRTUAL EXPERIMENTS: SIMULATE THE SYSTEM NUMERICALLY.

  COMPUTE AN APPROXIMANT TO FEYNMAN INTEGRAL BY DISCRETIZING SPACE TIME TO A CUBIC LATTICE.

IF THE HADRONIC SCALE λ is $\lambda \gg a$ (LATTICE SPACING) AND $\lambda \ll aL$ (LATTICE SIZE)
A GOOD APPROXIMATION TO QCD VACUUM IS OBTAINED.

FINITE TEMPERATURE FIELD THEORY

$$Z[T] = \int [\mathcal{D}\varphi]\, e^{\int_0^{1/T} dt \int d^3x\, \mathcal{L}[\varphi]}$$

P.B.C BOSONS, . A.BC FERMIONS

- ON LATTICE $T = \frac{1}{aN_T}$

$a = a(\beta, m)$ $\beta = \frac{2N}{g^2}$ LATTICE SPACING

$N_S \gg N_T$

R.G. $a \simeq \frac{1}{\Lambda_L} e^{\beta/2b_0}$ $b_0 = -\frac{1}{4\pi^2}\left\{\frac{11}{3}C_A - \frac{2}{3}N_f\right\} < 0$

$$\boxed{T = \frac{\Lambda_L}{N_T} e^{\beta/2|b_0|}}$$

HIGH g^2 LOW T
LOW g^2 HIGH T !! $\Rightarrow$

⇒ DUALITY [KRAMERS WANNIER 43, KADANOFF-CEVA 71, SEIBERG-WITTEN 94 ...]

A DEEP CONCEPT IN STATISTICAL MECHANICS AND FIELD THEORY: APPLIES TO SYSTEMS WITH TOPOLOGICALLY NON TRIVIAL EXCITATIONS (eg. 2d ISING)

TWO COMPLEMENTARY DESCRIPTIONS

DIRECT	DUAL
$\phi(x)$ $\langle\phi\rangle$ ORDER PARAM.	μ LOCAL FIELDS $\langle\mu\rangle$ (DIS)ORDER PARAMETERS
TOPOLOGICAL EXCITATIONS	$\phi(x)$ NON LOCAL EXCITATIONS
μ NON LOCAL	$g_D \simeq \frac{1}{g} \ll 1$
$g \ll 1$	CONVENIENT IN STRONG COUPLING ($g \gg 1$)

ISING MODEL $g = \frac{T}{J}$ ϕ $\sigma = \pm 1$ $\langle\phi\rangle$ $\langle\sigma\rangle$

DUAL EXCITATIONS KINKS μ

$$Z[g, \{\sigma\}] = Z[g^*, \{\mu\}] \qquad \mu = \pm 1 \qquad g^* \sim \frac{1}{g}$$

$$\boxed{\text{DUALITY MAPS ORDER} \rightleftarrows \text{DISORDER.}}$$

QCD : LOOK FOR DUAL VARIABLES $\langle\mu\rangle$ AND FOR THEIR SYMMETRY, WHICH SHOULD BE RESPONSIBLE FOR CONFINEMENT.

- SUSY QCD [Seiberg Witten] μ MONOPOLES
- QCD ['t Hooft 75, Mandelstam 76]   MONOPOLES ?

$\langle\mu\rangle \neq 0$ DUAL SUPERCONDUCTIVITY OF VACUUM

$V \sim \sigma R$

DUAL MEISSNER EFFECT.

DECONFINING TRANSITION   ORDER-DISORDER

$\langle\mu\rangle \neq 0 \to \langle\mu\rangle = 0$ SUPERCONDUCTOR → NORMAL

LATTICE QCD

HOW TO DETECT CONFINEMENT (DECONFINEMENT)

- QUENCHED THEORY (NO QUARKS)

$$L(\vec{x}) = \mathrm{Tr}\, P\, e^{i\int_0^{1/T} A_0(\vec{x},t)\,dt}$$

POLYAKOV 74

$$\mathcal{D}(x) \equiv \langle L^{+}(\vec{x})\, L(0)\rangle \underset{|x|\to\infty}{\simeq} c\, e^{-\frac{\sigma x}{T}} + |\langle L(0)\rangle|^2$$

CLUSTER PROPERTY

$$V(\vec{x}) = -T\ln \mathcal{D}(x) \underset{x\to\infty}{\simeq} \begin{cases} \sigma x \\ \text{const} \end{cases}$$

IF $\langle L\rangle = 0$ CONF.

IF $\langle L\rangle \neq 0$ DECONF

$\langle L\rangle$ ORDER PARAMETER Z_3 THE SYMMETRY

F.S.S. $\rho_L = \int d^3x\, \mathcal{D}(x)$ WEAK 1st ORDER TRANSITION

AT $T_c \simeq 270$ MeV.

- ALTERNATIVE ANALYSIS: [A.DG et al 00]

μ CREATES A MONOPOLE

$\langle\mu\rangle \neq 0$ $T<T_c$; $\langle\mu\rangle = 0$ $T>T_c$ $\Rightarrow$ AGREES WITH POLYAKOV LINE.

DUAL SUPERC. NORMAL

$$\rho = \frac{\partial \ln\langle\mu\rangle}{\partial\beta} \qquad T\sim T_c \qquad \text{F.S.S.} \qquad \rho/L^{1/\nu} = f(\tau L^{1/\nu})$$

$$\tau = 1 - \frac{T}{T_c} \simeq 1 - \frac{\beta}{\beta_c} \qquad \frac{1}{\nu} = 3 \text{ 1st ORDER}$$

ν CRITICAL INDEX: DEPENDS ON ORDER AND UNIVERSALITY CLASS

$$\langle\mu\rangle \simeq L_s^{\gamma}\, f\!\left(\frac{a}{\lambda}, \frac{\lambda}{L}\right)$$

$$\lambda \sim \tau^{-\nu} \qquad \frac{a}{\lambda} \simeq 0 \qquad \frac{\lambda}{L} \sim \tau L_s^{1/\nu}$$

$$\rho_\mu \simeq \frac{\partial}{\partial\beta}\left(\ln L_s^{\gamma} f(0, \tau L_s^{1/\nu})\right) \to L_s^{1/\nu} f'(0, \tau L^{1/\nu})$$

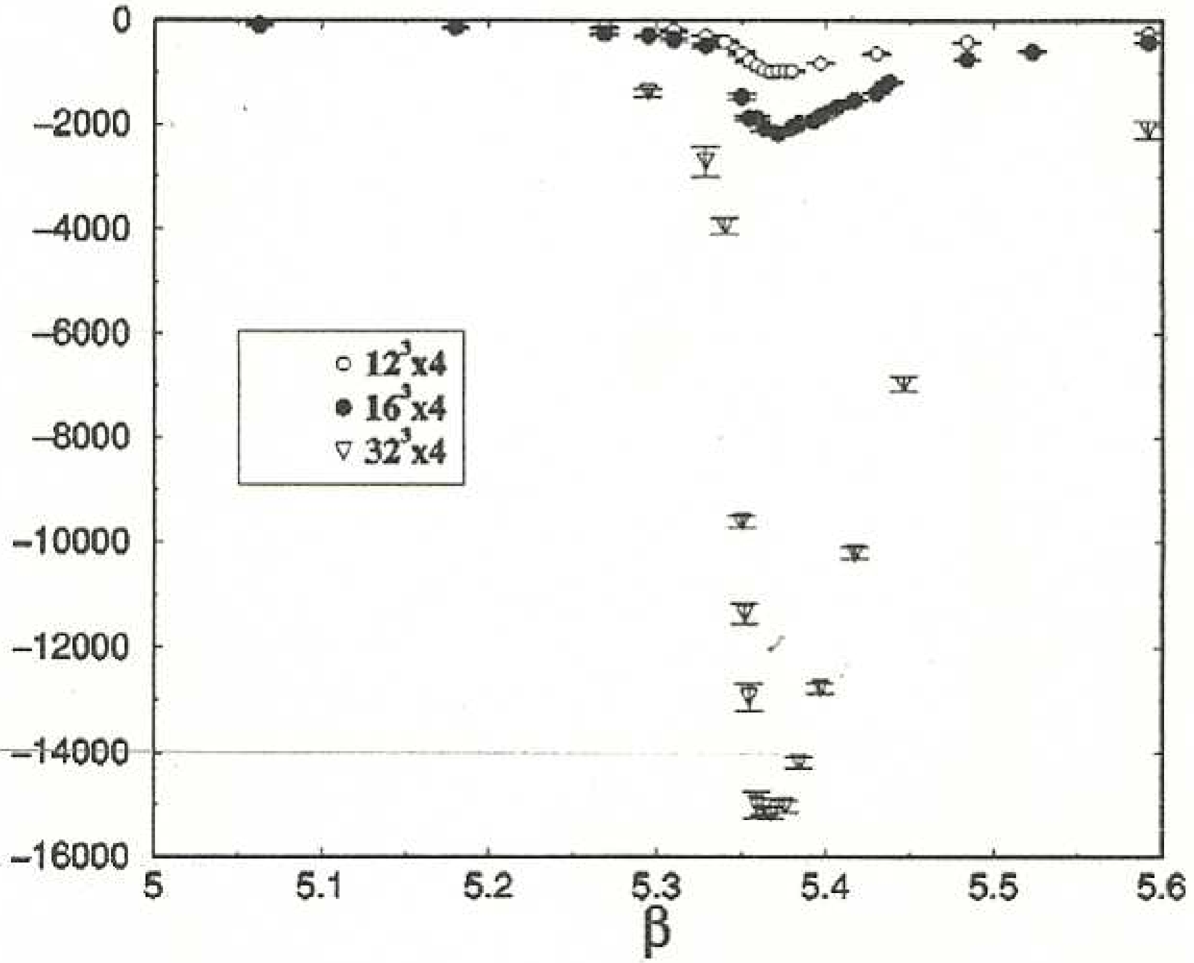


FIG. 2. Behavior of ρ around the phase transition at various lattice sizes.

- FULL QCD
  - Z_3 BROKEN
  - STRING BREAKING

$N_f = 2$ $m_u = m_d = m$, $m \neq 0$ CHIRAL SYMMETRY RESTORATION

fig. 2 : DEFINING THE TRANSITION : MAXIMA OF ρ_L, $\rho_{\langle\bar\psi\psi\rangle}$, C_V, ρ_μ

$\boxed{\begin{array}{ll} m = 0 & \text{CHIRAL} \\ m = \infty & \text{QUENCHED} \end{array}}$

$\langle\bar\psi\psi\rangle$ CHIRAL ORDER PAR.

$\rho_O = \int d^3x \langle O(\vec x) O(\vec 0)\rangle$

- FINITE SIZE SCALING ANALYSIS OF C_V CAN DETERMINE THE ORDER OF THE TRANSITION

- $\langle L\rangle$ NOT AN ORDER PARAMETER $m < \infty$
- $\langle\bar\psi\psi\rangle$ " $m \neq 0$

$\langle\mu\rangle$ CAN BE AN ORDER PARAMETER.

- CHIRAL TRANSITION [Pisarski Wilczek 84]

RENORMALIZATION GROUP ANALYSIS + $4-\epsilon$ exp

$\Phi_{ij} = \langle\bar\psi_i (1+\gamma_5)\psi_j\rangle$ Rdn

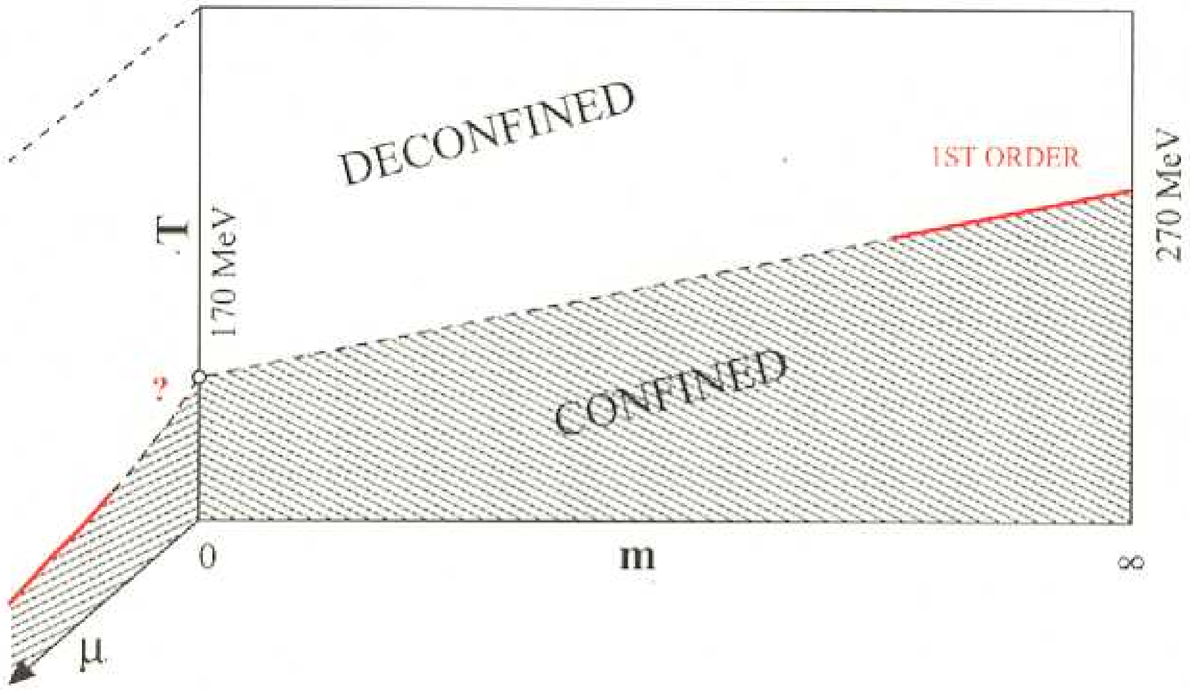
$N_f \geq 3$ 1st ORDER

$N_f = 2$
- (i) 2nd ORDER $O(4)$ $m_{\eta'} \neq 0$ $T \to T_c$ $m > 0$ CROSSOVER
- (ii) 1st ORDER $m_{\eta'} \to 0$ $T \to T_c$ 1ST ORDER

(i) THE DECONFINING TRANSITION IS NOT ORDER-DISORDER

(ii) THE TRANSITION CAN BE ORDER-DISORDER

WHAT IS THE CHOICE OF QCD? A FUNDAMENTAL QUESTION WHICH CAN BE ASKED TO LATTICE.


DECONFINED
1ST ORDER
270 MeV
T
170 MeV
?
CONFINED
0
m
∞
μ

A VERY DEMANDING PROGRAM NUMERICALLY.

APE-NEXT COMPUTERS OF INFN: (≃ TERAFLOP × YEAR (PO-TI KS))

A TWO SCALE PROBLEM [Bielefeld, MILC, Tsukuba]

$$\left.\begin{aligned} C_V - C_0 &= L_s^{\alpha/\nu}\,\Phi_c(\tau L_s^{1/\nu}, m L_s^{y_h}) \\ \chi - \chi_0 &= L^{\gamma/\nu}\,\Phi_\chi(\tau L_s^{1/\nu}, m L_s^{y_h}) \end{aligned}\right\} (A)$$

$$\left.\begin{aligned} C_V - C_0 &\simeq m^{-\alpha/\nu y_h}\, f_c(\tau L_s^{1/\nu}) \\ \chi - \chi_0 &\simeq m^{-\gamma/\nu y_h}\, f_\chi(\tau L_s^{1/\nu}) \end{aligned}\right\} (B)\ m L_s >> 1,\ \tau L_s^{1/\nu} \text{ fixed}$$

	ν	α	γ	y_h
O(4)	.75	−.24	1.48	2.49
1st ord	1/3	1	1	3
O(2)	.668	−.005	1.31	2.49

(A) O(4) ASSUMED $m L_s^{2.49}$ fixed $\chi^2/dof \simeq 25$
O(2)
1st ORDER ASSUMED $m L_s^3$ fixed NOT FINISHED

$(C_V - C_0)/L_s^{\alpha/\nu} = f_c(\tau L_s^{1/\nu})$; $(\chi - \chi_0)/L_s^{\gamma/\nu} = f_\chi(\tau L_s^{1/\nu})$

(B) O(4) O(2) EXCLUDED $\chi^2/dof \simeq 20$
1st ORDER $\chi^2/dof \simeq 1.8$

PHYS REV 72D 114510 (2005)

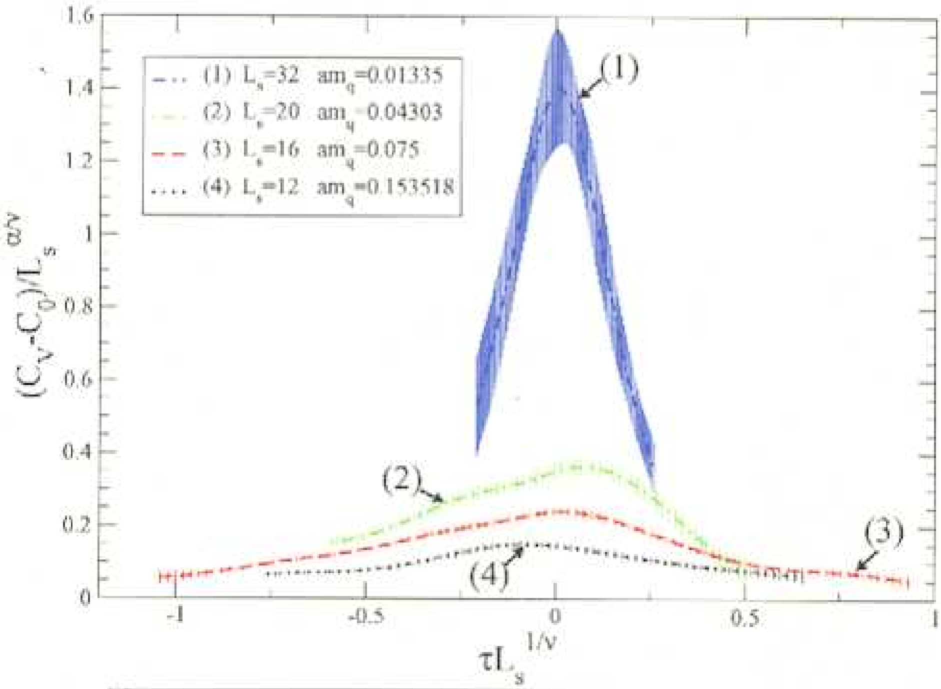
- IMPROVED ALGORITHM. IMPROVED ACTION ⟹.

- $m_{\eta'}$ $T \simeq T_c$: ANISOTROPIC LATTICE. $a_t << a_s$
$m_{\eta'} \simeq 4\,T_c$ NOT FINISHED.

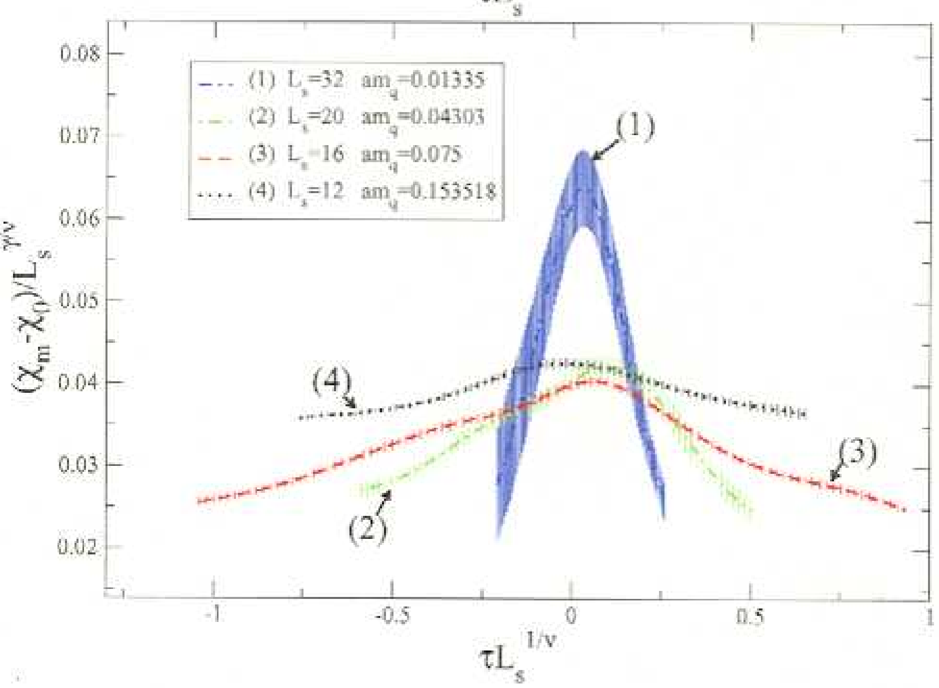
- $\rho_\mu / L_s^{1/\nu} = f_\mu(\tau L_s^{1/\nu})$ ⟹ 1ST ORDER

PHYS REV D73, 114502, 2005

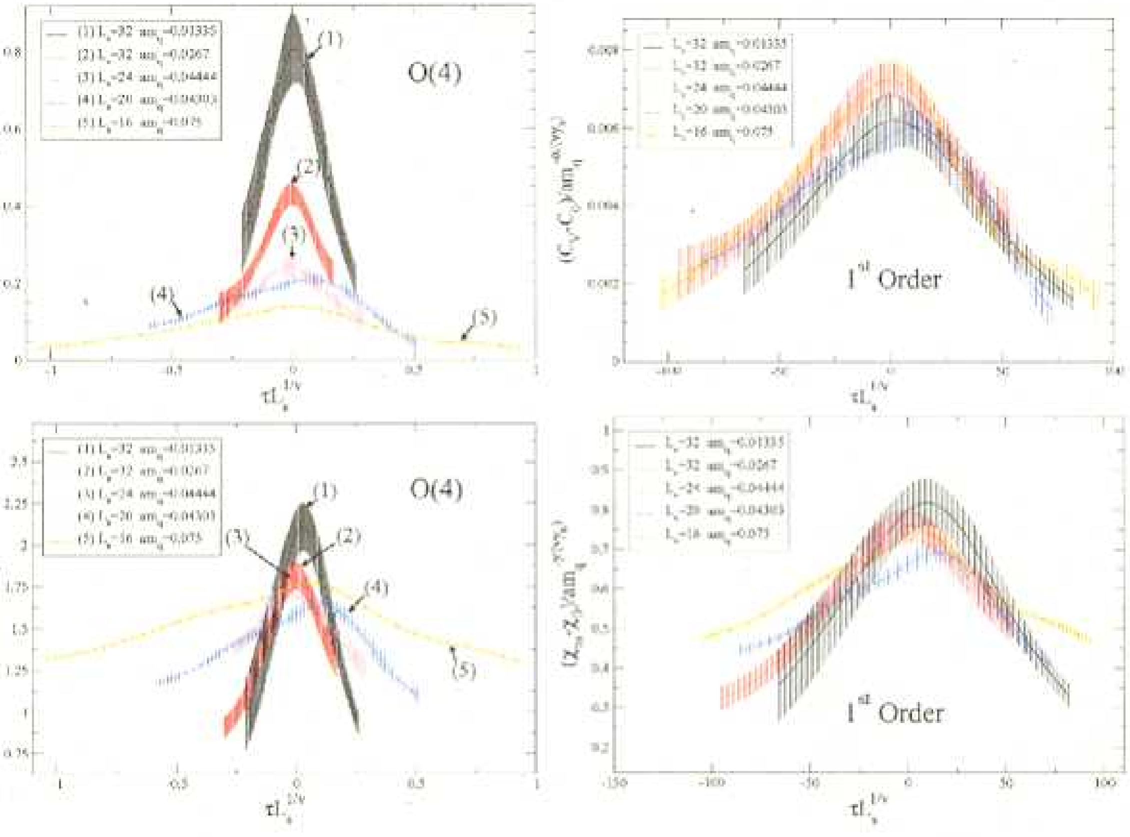
Run1



(1) L_s=32 am_q=0.01335
(2) L_s=20 am_q=0.04303
(3) L_s=16 am_q=0.075
(4) L_s=12 am_q=0.153518
$(C_V-C_0)/L_s^{\alpha/\nu}$
$\tau L_s^{1/\nu}$



(1) L_s=32 am_q=0.01335
(2) L_s=20 am_q=0.04303
(3) L_s=16 am_q=0.075
(4) L_s=12 am_q=0.153518
$(\chi_m-\chi_0)/L_s^{\gamma/\nu}$
$\tau L_s^{1/\nu}$



O(4)
(1) L_s=32 am_q=0.01335
(2) L_s=32 am_q=0.0267
(3) L_s=24 am_q=0.04444
(4) L_s=20 am_q=0.04303
(5) L_s=16 am_q=0.075
1st Order
τL_s^{1/ν}

DISCUSSION

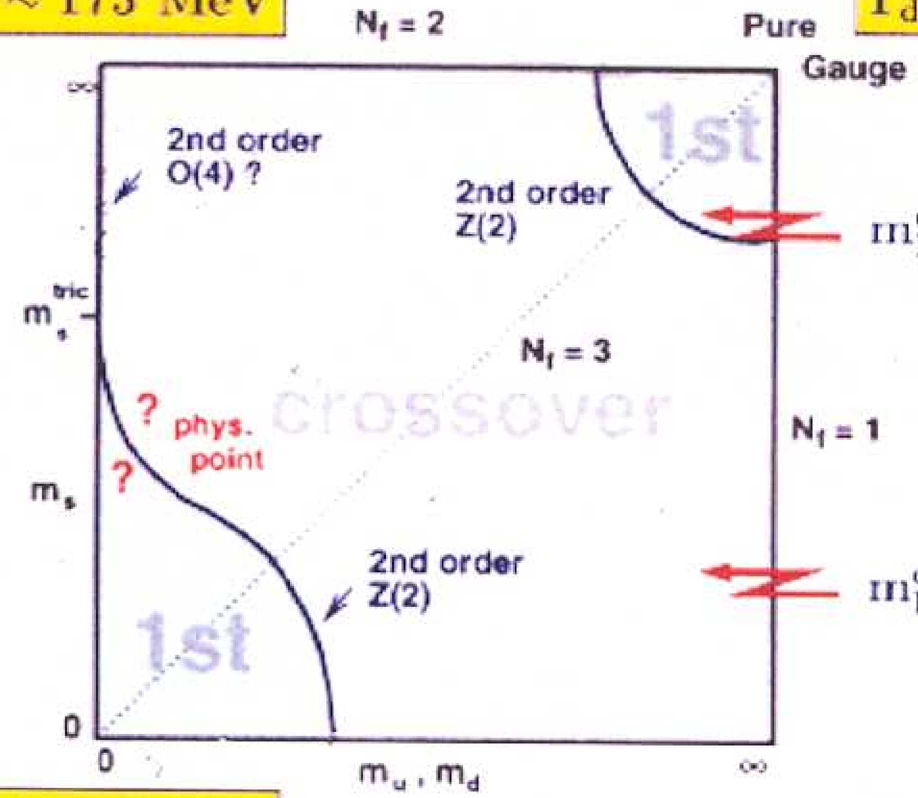
- CONFINEMENT IS A TYPICAL LARGE DISTANCE PROBLEM IN QCD WHICH CAN BE STUDIED ON LATTICE

- ORDER OF THE DECONFINING PHASE TRANSITION : IMPORTANT TO UNDERSTAND THE MECHANISM OF CONFINEMENT.
  - THE ANSWER IS AT HAND.

3-flavor phase diagram

$T_\chi^{n_f=2} \sim 175$ MeV

$T_d \sim 270$ MeV



$m_{PS}^{crit} \simeq 2.5$ GeV

$m_{PS}^{crit} \simeq 300$ MeV

$T_\chi^{n_f=3} \sim 155$ MeV

BIELEFELD GROUP