Tehran, 15-17 May '06

Models of Neutrino Masses & Mixings

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Some recent work by our group

G.A., F. Feruglio, I. Masina, hep-ph/0402155,
G.A., F. Feruglio, hep-ph/0504165,hep-ph/0512103
G.A, R. Franceschini, hep-ph/0512202.

Reviews:

G.A., F. Feruglio, New J.Phys.6:106,2004 [hep-ph/0405048];
G.A., hep-ph/0410101; F. Feruglio, hep-ph/0410131

What if θ_{23} is really maximal?

Would be challenging!

All existing models invoke peculiar symmetries (non abelian or discrete are crucial)

Early models: Barbieri et al, Wetterich....

The most general mass matrix for $\theta_{13} = 0$ and θ_{23} maximal is given by (after ch. lepton diagonalization):

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

Grimus, Lavoura..., Ma,....

Imposing a 2-3 perm. symmetry on $L^T m_\nu L$ does not work, because $\overline{R}$ L then produces a charged lepton mixing that spoils θ_{23} max.

In some models, discrete broken symmetries are used to make charged leptons and Dirac ν masses diagonal, while the perm. symmetry is in the Majorana RR matrix

Grimus, Lavoura
Caravaglios, Morisi

An interesting particular case

Harrison, Perkins, Scott

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

A simple mixing matrix compatible with all present data

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U\mathrm{diag}(m_1, m_2, m_3) U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \to \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \to \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \to \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Same as in Fritzsch models but with 1 and 3 interchanged, so that here θ_{23} is maximal while $\sin^2 2\theta_{12} = 8/9$

Note: mixing angles independent of mass eigenvalues

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Comparison with experiment:

At 1σ: Fogli et al '05

$\sin^2\theta_{12} = 1/3$: 0.290-0.342

$\sin^2\theta_{23} = 1/2$: 0.39-0.53

$\sin^2\theta_{13} = 0$: < 0.02

The HPS mixing is clearly a very good approx. to the data!

Also called:
Tri-Bimaximal mixing

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

Two extremes:

- In anarchy all ν mixing angles and mass ratios are random. Apparent hierarchies are from fluctuations

- For the HPS mixing matrix all mixing angles are fixed to particularly symmetric values

It is interesting to construct models that can naturally produce this highly ordered structure

Models based on the A4 discrete symmetry (even permutations of 1234) are very interesting

Ma...;
GA, Feruglio hep-ph/0504165, hep-ph/0512103

Alternative models based on $SU(3)_F$ or $SO(3)_F$

Verzielas, G. Ross   King

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

An element is abcd which means 1234 --> abcd

C_1: $1 = 1234$
C_2: $T = 2314$ $ST = 4132$ $TS = 3241$ $STS = 1423$
C_3: $T^2 = 3124$ $ST^2 = 4213$ $T^2S = 2431$ $TST = 1342$
C_4: $S = 4321$ $T^2ST = 3412$ $TST^2 = 2143$

Thus A4 transf.s can be written as:

$1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2$

with: $S^2 = T^3 = (ST)^3 = 1$ [$(TS)^3 = 1$ also follows]

x, x' in same class if

C_1, C_2, C_3, C_4 are equivalence classes [$x' \sim gxg^{-1}$] g: group element

Three singlet inequivalent represent'ns:

$$\omega = \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\omega^3 = 1$$

$$1 + \omega + \omega^2 = 0$$

$$\omega^2 = \omega^*$$

Recall:

$S^2 = T^3 = (ST)^3 = 1$

$$\begin{cases} 1: \; S=1,\; T=1 \\ 1': \; S=1,\; T=\omega \\ 1'': \; S=1,\; T=\omega^2 \end{cases}$$

The only indep. 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

A4 has only 4 irreducible inequivalent represt'ns: 1,1',1",3

Table of Multiplication:
1'x1'=1"; 1"x1"=1';1'x1"=1
3x3=1+1'+1"+3+3

A4 is well fit for 3 families!

Ch. leptons $l \sim 3$

$e^c, \mu^c, \tau^c \sim 1, 1', 1''$

In the (S-diag basis) consider 3: (a_1,a_2,a_3)

$$(a_1,a_2,a_3) \xrightarrow{S} (a_1,-a_2,-a_3)$$

$$(a_1,a_2,a_3) \xrightarrow{T} (a_3,a_1,a_2)$$

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1b_1 + a_2b_2 + a_3b_3$$

$$1' = a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3$$

$$1'' = a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3$$

$$3 \sim (a_2b_3, a_3b_1, a_1b_2)$$

$$3 \sim (a_3b_2, a_1b_3, a_2b_1)$$

e.g. $1' = a_1b_1+\omega a_2b_2+\omega^2 a_3b_3 \xrightarrow{T} a_3b_3+\omega a_1b_1+\omega^2 a_2b_2 =$
$= \omega\, [a_1b_1+\omega a_2b_2+\omega^2 a_3b_3]$

while 1' is inv. under S

Under A4

lepton doublets l ~ 3
e^c, μ^c, τ^c ~ 1, 1′, 1″ respectively
gauge singlet flavons ϕ, ϕ', ξ, (ξ') ~ 3, 3, 1,(1) respectively
driving fields (for SUSY version) ϕ_0, ϕ'_0, ξ_0 ~ 3, 3, 1

Additional symmetries: broken $U(1)_F$ symmetry (ch. lepton masses) with e^c, μ^c, τ^c charges (3-4,2,0)
and a discrete symmetry (dep. on versions) : for example
Z: (e^c, μ^c, τ^c) -> -i (e^c, μ^c, τ^c), l -> il, $\phi \to \phi$, $(\xi,\phi') \to -(\xi,\phi')$

The Yukawa interactions in the lepton sector are:

$$\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots$$

Here is without see-saw but also with see-saw is OK

Structure of the model

$$\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots$$

shorthand: Higgs and cut-off scale Λ omitted, e.g.:

$$y_e e^c(\varphi l) \sim y_e e^c(\varphi l) h_d/\Lambda, \qquad x_a \xi(ll) \sim x_a \xi(l h_u l h_u)/\Lambda^2$$

$$\begin{aligned} \langle \varphi' \rangle &= (v', 0, 0) \\ \langle \varphi \rangle &= (v, v, v) \\ \langle \xi \rangle &= u \end{aligned}$$

$$m_l = v_d \frac{v}{\Lambda} \begin{pmatrix} y_e & y_e & y_e \\ y_\mu & y_\mu \omega^2 & y_\mu \omega \\ y_\tau & y_\tau \omega & y_\tau \omega^2 \end{pmatrix}$$

Spectrum free.
Diagonalized by U_e:

the big plus of A4

$$m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}$$

$$l \rightarrow \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} l = Vl = l_{diag}$$

- From here it follows that U_{HPS} is the mixing matrix

m_ν in the basis of diagonal charged leptons is:

$$m_\nu\big|_{ldiag} \sim V^* \begin{bmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{bmatrix} V^*$$

which in turn can be written as:

$$m_\nu\big|_{ldiag} \sim U^T \begin{bmatrix} a+d & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & -a+d \end{bmatrix} U$$

with:

$$U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

The crucial issue is to guarantee the strict alignment

$$\begin{aligned} \langle \varphi' \rangle &= (v', 0, 0) \\ \langle \varphi \rangle &= (v, v, v) \\ \langle \xi \rangle &= u \end{aligned}$$

We have constructed two completely natural versions of the model:

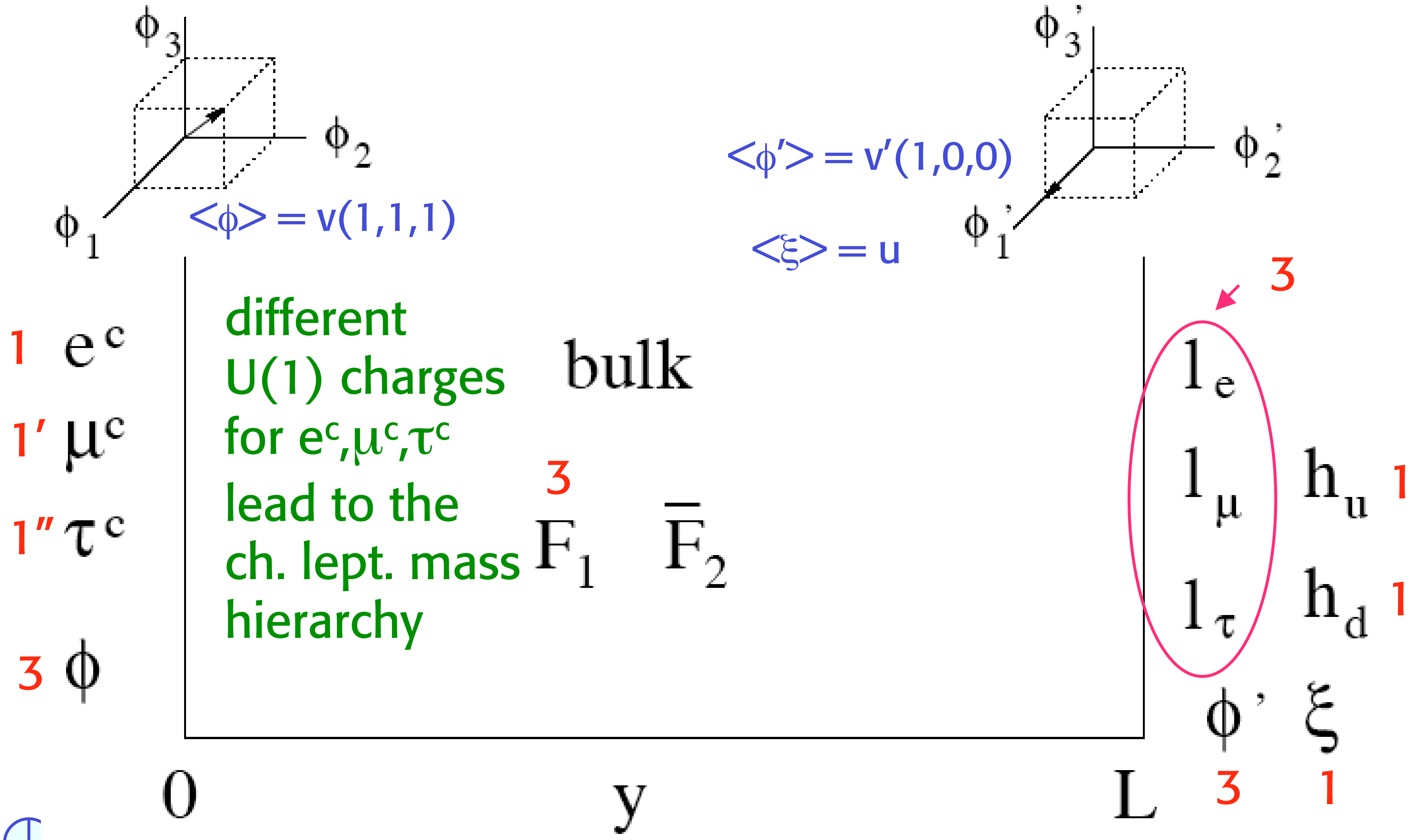
- a version in 5 dimensions
- a SUSY version in 4-dim (with more fields)

We first briefly discuss the 5-dim version

GA, Feruglio hep-ph/0504165

The model has 1 compactified extra dim. and 2 branes

(crucial issue: guarantee and protect the vev alignment)



In lowest approximation the action is:

$$
\begin{aligned}
S &= \int d^4x dy \Big\{ \Big[iF_1\sigma^\mu \partial_\mu \overline{F}_1 + iF_2\sigma^\mu \partial_\mu \overline{F}_2 + \frac{1}{2}(F_2\partial_y F_1 - \partial_y F_2 F_1 + h.c.) \Big] \\
&- M(F_1F_2 + \overline{F}_1\overline{F}_2) \\
&+ V_0(\varphi)\delta(y) + V_L(\varphi', \xi)\delta(y-L) \\
&+ \left[Y_e e^c(\varphi F_1) + Y_\mu \mu^c(\varphi F_1)'' + Y_\tau \tau^c(\varphi F_1)' + h.c. \right] \delta(y) \\
&+ \left[\frac{x_a}{\Lambda^2}\xi(ll)h_u h_u + \frac{x_d}{\Lambda^2}(\varphi' ll)h_u h_u + Y_L(F_2 l)h_d + h.c. \right] \delta(y-L) \Big\} + \ldots
\end{aligned}
$$

a Z-parity has also been imposed

$$
(f^c, l, F, \varphi, \varphi', \xi) \xrightarrow{Z} (-if^c, il, iF, \varphi, -\varphi', -\xi)
$$

After integrating out of the F fields one obtains the required effective 4-dim action

$$
\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots
$$

In the flavour basis:

$$m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} a+2d/3 & -d/3 & -d/3 \\ -d/3 & 2d/3 & a-d/3 \\ -d/3 & a-d/3 & 2d/3 \end{pmatrix}$$

$m_\nu = U\,\mathrm{diag}(a+d, a, -a+d)\,U^T$ (in units of v_u^2/Λ) and $U = U_{HPS}$

In terms of physical param.s (moderate normal hierarchy):

$$|m_1|^2 = \left[-r + \frac{1}{8\cos^2\Delta(1-2r)}\right]\Delta m^2_{atm} \sim (0.017\ \mathrm{eV})^2$$

$$|m_2|^2 = \frac{1}{8\cos^2\Delta(1-2r)}\Delta m^2_{atm} \sim (0.017\ \mathrm{eV})^2$$

$$|m_3|^2 = \left[1 - r + \frac{1}{8\cos^2\Delta(1-2r)}\right]\Delta m^2_{atm} \sim (0.053\ \mathrm{eV})^2$$

- A moderate fine tuning is needed for r

The model crucially depends on the precise vev alignment

$$\begin{array}{rcl} \langle\varphi'\rangle & = & (v',0,0) \\ \langle\varphi\rangle & = & (v,v,v) \\ \langle\xi\rangle & = & u \end{array}$$

The extra dimension with 2 branes allows the decoupling of the ϕ and ξ,ϕ' potentials.

A discrete symmetry is also essential: a separate continuous rotation symmetry on the 2 branes would make any disalignment illusory.

An alternative in 4 dimensions is a SUSY model with driving fields and a superpotential where all terms allowed by symmetry are present (with added fields ξ', ϕ_0, ϕ'_0, ξ_0).

In our models

- all terms allowed by symmetry are present
- all correct'ns are under control and can be made negligible

The 4-dim SUSY version (written in the T-diag basis)

$$w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + (x_a\xi + \tilde{x}_a\tilde{\xi})(ll) + x_b(\varphi_S ll) + h.c. + ...$$

One more singlet is needed for vacuum alignment

The superpotential (at leading order):

$$\begin{aligned} w_d &= M(\varphi_0^T\varphi_T) + g(\varphi_0^T\varphi_T\varphi_T) \\ &+ g_1(\varphi_0^S\varphi_S\varphi_S) + g_2\tilde{\xi}(\varphi_0^S\varphi_S) + g_3\xi_0(\varphi_S\varphi_S) + g_4\xi_0\xi^2 + g_5\xi_0\xi\tilde{\xi} + g_6\xi_0\tilde{\xi}^2 \end{aligned}$$

and the potential $V = \sum_i \left|\frac{\partial w}{\partial \phi_i}\right|^2 + m_i^2|\phi_i|^2 + ...$

The assumed simmetries are summarised here

Field	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	1	$1'$	$1''$	1	3	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2
$U(1)_F$		2q	q	1								

The driving field have zero vev. So the minimization is:

$$\frac{\partial w}{\partial \varphi_{01}^T} = M\varphi_{T1} + \frac{2g}{3}(\varphi_{T1}^2 - \varphi_{T2}\varphi_{T3}) = 0 \qquad \frac{\partial w}{\partial \varphi_{01}^S} = g_2\tilde{\xi}\varphi_{S1} + \frac{2g_1}{3}(\varphi_{S1}^2 - \varphi_{S2}\varphi_{S3}) = 0$$

$$\frac{\partial w}{\partial \varphi_{02}^T} = M\varphi_{T3} + \frac{2g}{3}(\varphi_{T2}^2 - \varphi_{T1}\varphi_{T3}) = 0 \qquad \frac{\partial w}{\partial \varphi_{02}^S} = g_2\tilde{\xi}\varphi_{S3} + \frac{2g_1}{3}(\varphi_{S2}^2 - \varphi_{S1}\varphi_{S3}) = 0$$

$$\frac{\partial w}{\partial \varphi_{03}^T} = M\varphi_{T2} + \frac{2g}{3}(\varphi_{T3}^2 - \varphi_{T1}\varphi_{T2}) = 0 \qquad \frac{\partial w}{\partial \varphi_{03}^S} = g_2\tilde{\xi}\varphi_{S2} + \frac{2g_1}{3}(\varphi_{S3}^2 - \varphi_{S1}\varphi_{S2}) = 0$$

$$\frac{\partial w}{\partial \xi_0} = g_4\xi^2 + g_5\xi\tilde{\xi} + g_6\tilde{\xi}^2 + g_3(\varphi_{S1}^2 + 2\varphi_{S2}\varphi_{S3}) = 0$$

Solution:

$$\varphi_T = (v_T, 0, 0) \quad , \qquad v_T = -\frac{3M}{2g}$$

$$\begin{aligned} \tilde{\xi} &= 0 \\ \xi &= u \\ \varphi_S &= (v_S, v_S, v_S) \quad , \qquad v_S^2 = -\frac{g_4}{3g_3}u^2 \end{aligned}$$

In the paper
w at NLO is also
studied

NLO corrections studied in detail

to m_l

LO is $1/\Lambda$

$$\frac{1}{\Lambda^2}(f^c l \varphi_T \varphi_T) h_d \quad , \qquad (f^c = e^c, \mu^c, \tau^c)$$

to m_ν

LO is $1/\Lambda^2$

$$\frac{x_c}{\Lambda^3}(\varphi_T \varphi_S)'(ll)'' h_u h_u \qquad \frac{x_d}{\Lambda^3}(\varphi_T \varphi_S)''(ll)' h_u h_u \qquad \frac{x_e}{\Lambda^3}\xi(\varphi_T ll) h_u h_u$$

to vevs

LO is 1

$$\begin{aligned}
\langle \varphi_T \rangle &\to (v'_T + \delta v_T, \delta v_T, \delta v_T) \\
\langle \varphi_S \rangle &\to (v_S + \delta v_1, v_S + \delta v_2, v_S + \delta v_3) \\
\langle \xi \rangle &\to u \\
\langle \tilde{\xi} \rangle &\to \delta u'
\end{aligned}$$

δv_T, δv_S, δv_i, $\delta u' \sim o(1/\Lambda)$

All observables get a correction of order $1/\Lambda$

From exp (eg r, θ_{12}) must be less than 5%

$$0.0022 < \frac{v_S}{\Lambda} \approx \frac{v_T}{\Lambda} \approx \frac{u}{\Lambda} < 0.05$$

In particular $\theta_{13} < \sim 0.05$

Extension to quarks problematic

If we take all fermion doublets as 3 and all singlets as 1, 1′, 1″
(as for leptons): $Q_i \sim 3$, $u^c, d^c \sim 1$, $c^c, s^c \sim 1'$, $t^c, b^c \sim 1''$

Then u and d quark mass matrices are BOTH diagonalised by

$$U_u, U_d \sim \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$$

As a result VCKM is unity: $V_{CKM} = U_u^+ U_d \sim 1$

So, in first approx. (broken by loops and higher dim operators),
ν mixings are HPS and quark mixings ~identity

Corrections are too small to reproduce quark mixings eg λ_C

Note: it not possible to embed this in a GUT:
with these assignments A4 does not commute with SU(5)

Conclusion

We favour:

Normal models: θ_{23} large but not maximal, θ_{13}
not too small (θ_{13} of order λ_C or $\lambda_C{}^2$ vs θ_{12}, θ_{23} ~o(1))
- Semi anarchy
- Inverse hierarchy

In particular
-Normal hierarchy with suppressed 23 determinant

Well compatible with GUT's (simplest: $SU(5)XU(1)_F$)

Exceptional models: θ_{23} maximal or θ_{13} very small
or also: all mixing angles fixed as in HPS (intriguing!!)
are peculiar but very interesting (eg A4).

From experiment: a good first approximation

For quarks:

$$V_{CKM} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and for neutrinos

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

All this is highly non trivial but no real illumination has followed!!

Main lessons from ν masses and mixings

- ν's are not all massless but their masses are very small
- probably masses are small because ν's are Majorana particles
- then masses are inv. prop. to the large scale M of L n. viol.
- $M \sim m_{\nu R}$ is empirically close to 10^{14}-10^{15} GeV $\sim M_{GUT}$
  -> ν masses fit well in the SUSY GUT picture
- decays of ν_R with CP & L violation can produce a B-L asymm.
  -> baryogenesis via leptogenesis
- ν's are not a significant component of dark matter in Universe
- detecting $0\nu\beta\beta$ would prove ν's are Majorana and L is viol.
- ν mixing angles are large except for θ_{13} that is small
- there is no contradiction between large ν mixings and small
  q mixings, even in GUT's