Tehran, 15-17 May '06

Models of Neutrino Masses & Mixings

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Some recent work by our group

G.A., F. Feruglio, I. Masina, hep-ph/0402155,
G.A., F. Feruglio, hep-ph/0504165,hep-ph/0512103
G.A, R. Franceschini, hep-ph/0512202.

Reviews:

G.A., F. Feruglio, New J.Phys.6:106,2004 [hep-ph/0405048];
G.A., hep-ph/0410101; F. Feruglio, hep-ph/0410131

The current experimental situation is still unclear

- LSND: true or false? -> MiniBooNE soon will tell
- what is the absolute scale of ν masses?
- no detection of $0\nu\beta\beta$ (proof that ν's are Majorana)

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Different classes of models are still possible:

If LSND true

sterile ν(s)??

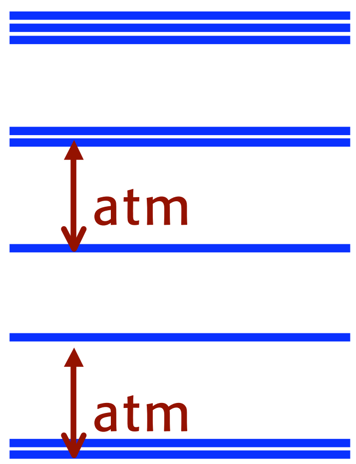
CPT violat'n??

- "3-1" or "3-n"

If LSND false → 3 light ν's are OK

We assume this case here

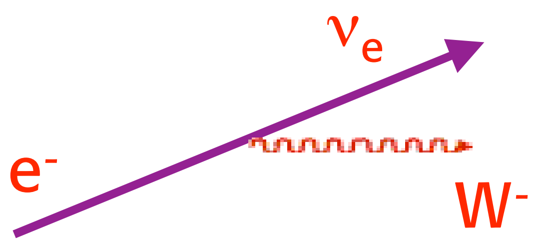
- Degenerate ($m^2 >> \Delta m^2$)
- Inverse hierarchy
- Normal hierarchy



3-ν Models

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour mass



$U = U_{P\text{-}MNS}$

Pontecorvo

Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

δ: CP violation

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

s = solar: large

CHOOZ: $|s_{13}| <\sim 0.2$

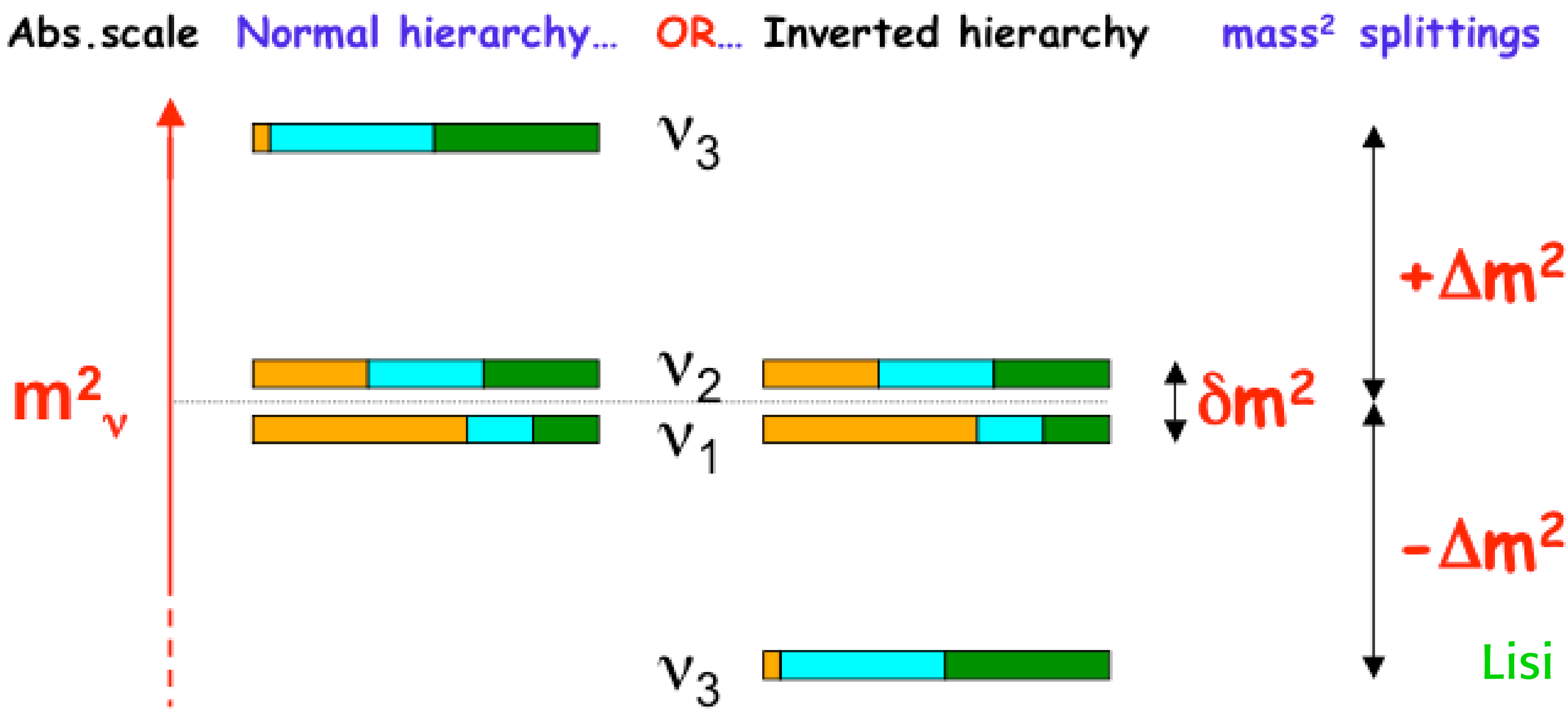
$$\sim \begin{bmatrix} c_{13}\,c_{12} & c_{13}\,s_{12} & s_{13}e^{-i\delta} \\ \dots & \dots & c_{13}\,s_{23} \\ \dots & \dots & c_{13}\,c_{23} \end{bmatrix}$$

atm.: ~ max

$$U \cong \begin{bmatrix} c & -s & 0 \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(some signs are conventional)

In general: $U = U^{+}_{e} U_{\nu}$



$\delta m^2 \sim 8 \times 10^{-5}\ \mathrm{eV}^2$ $\quad$ $\sin^2\theta_{12} \sim 0.3$

$\Delta m^2 \sim 3 \times 10^{-3}\ \mathrm{eV}^2$ $\quad$ $\sin^2\theta_{23} \sim 0.5$

$m_\nu < O(1)\ \mathrm{eV}$ $\quad$ $\sin^2\theta_{13} < \mathrm{few}\%$

$\mathrm{sign}(\pm\Delta m^2)$ unknown $\quad$ δ (CP) unknown

$0\nu\beta\beta$ would prove that L is not conserved and ν's are Majorana
Also can tell degenerate, inverted or normal hierarchy

$|m_{ee}|=c_{13}^2\,[m_1 c_{12}^2+e^{i\alpha}m_2 s_{12}^2]+m_3 e^{i\beta}s_{13}^2$

Degenerate: $\sim|m||c_{12}^2+e^{i\alpha}s_{12}^2|\sim|m|(0.3\text{-}1)$

$|m_{ee}|\sim|m|\,(0.3\,\text{-}1)\ <0.23\text{-}1$ eV

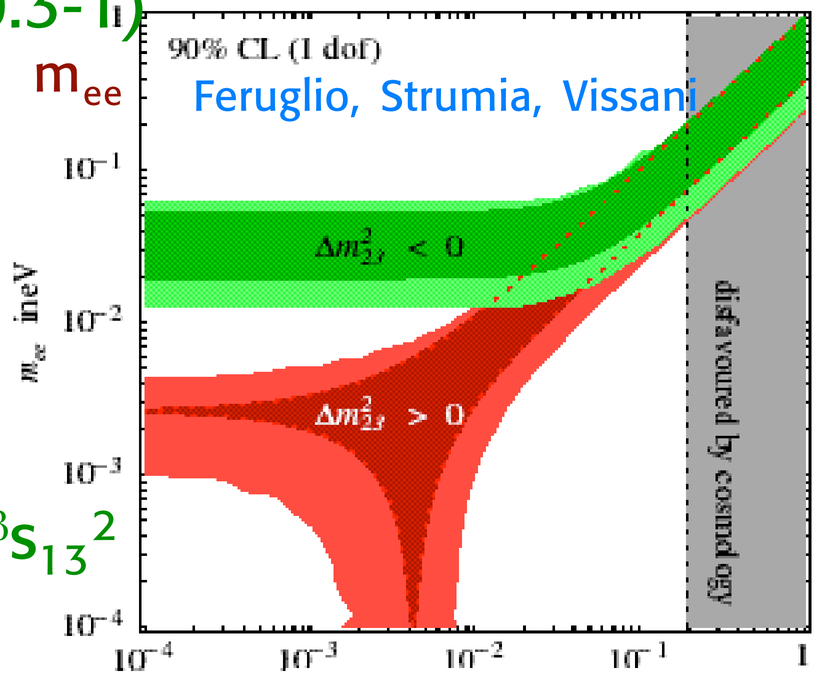
IH: $\sim(\Delta m^2_{atm})^{1/2}|c_{12}^2+e^{i\alpha}s_{12}^2|$

$|m_{ee}|\sim\ (1.6\text{-}5)\ 10^{-2}$ eV

NH: $\sim(\Delta m^2_{sol})^{1/2}s_{12}^2+(\Delta m^2_{atm})^{1/2}e^{i\beta}s_{13}^2$

$|m_{ee}|\sim$ (few) 10^{-3} eV

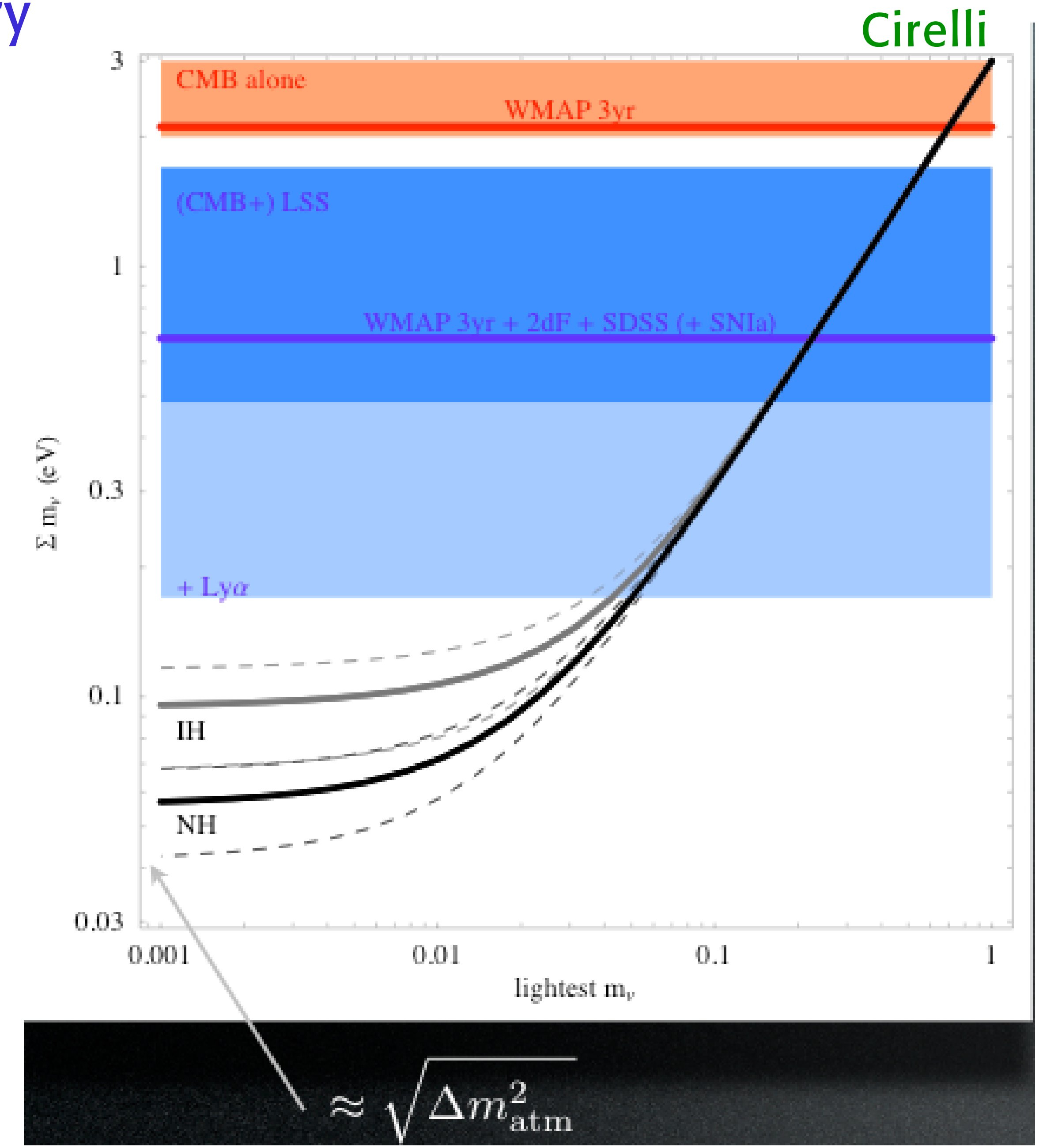
Full dependence on min m_ν



lightest m_ν (eV)

Present exp. limit: $m_{ee}<0.3\text{-}0.5$ eV
(and a hint of signal????? Klapdor Kleingrothaus)

Summary



Cirelli
CMB alone
WMAP 3yr
(CMB+) LSS
WMAP 3yr + 2dF + SDSS (+ SNIa)
+ Lyα
IH
NH
3
1
0.3
0.1
0.03
0.001
0.01
0.1
1
lightest m_ν
$\Sigma\, m_\nu$ (eV)
$\approx \sqrt{\Delta m^2_{\rm atm}}$

Model building

Quality factors for models:

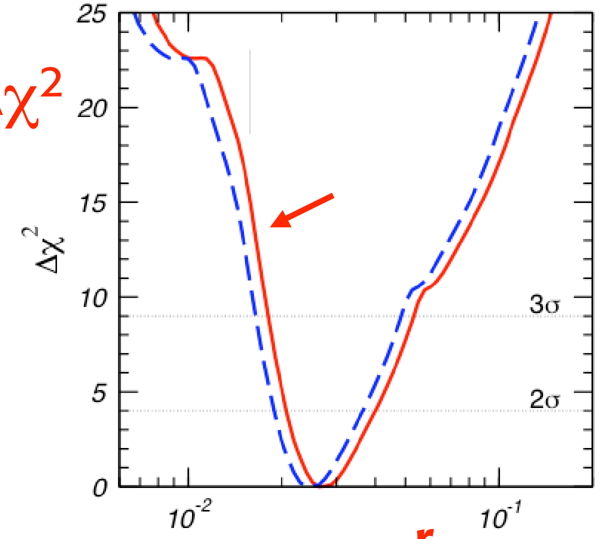
- Based on the most general lagrangian compatible with some simple symmetry or dynamical principle
- Should be complete: address at least charged leptons and neutrinos ($U_{P\text{-}NMS} = U^+_e U_\nu$, and the gauge symmetry connects ch. leptons and LH neutrinos)
- As many as possible small parameters (masses and mixings) should be naturally explained as a consequence.
- The necessary vev configuration should be a minimum of the most general potential for a region of parameter space
- Checked stability under radiative corrections and higher dim operators
- Elegance, simplicity, economy of fields and parameters, predictivity

General remarks

• After KamLAND, SNO and WMAP.... not too much hierarchy is needed for ν masses:

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/35$

$\Delta\chi^2$



r

Precisely at 3σ: $0.018 < r < 0.053$

or

$m_{heaviest} < 0.2$ - 0.7 eV
$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν

$\longrightarrow$ e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

$$3\text{-}\sigma \text{ interval } 0.31 < \sin^2\theta_{23} < 0.72$$

Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small

  probably accessible to exp.

Very small θ_{13} theoretically hard

Normal models: θ_{23} large but not maximal,
θ_{13} not too small (θ_{13} of order λ_C or λ_C^2)

Exceptional models: θ_{23} maximal and/or θ_{13} very small
or: a special value for θ_{12}....

Natural models of the "normal" type are not too difficult to build up

It is reasonable to attribute hierarchies in masses and mixings to differences in some flavour quantum number(s).

A simplest flavour (or horizontal) symmetry is $U(1)_F$

For example, some models based on see-saw and $U(1)_F$ work for all quark and lepton masses and mixings, are natural and compatible with (SUSY) GUT's, e.g $SU(5)xU(1)_F$.

Hierarchy for masses and mixings via horizontal $U(1)_F$ charges.

Froggatt, Nielsen '79

Principle:

A generic mass term

$\bar{R}_1 m_{12} L_2 H$

is forbidden by U(1)

if $q_1+q_2+q_H$ not 0

q_1, q_2, q_H:
U(1) charges of
$\bar{R}_1, L_2, H$

U(1) broken by vev of "flavon" field θ with U(1) charge $q_\theta = -1$.
If vev $\theta = w$, and $w/M=\lambda$ we get for a generic interaction:

Δ_{charge}

$\bar{R}_1 m_{12} L_2 H \,(\theta/M)^{\,q1+q2+qH}$ $\qquad m_{12} \rightarrow m_{12}\,\lambda^{q1+q2+qH}$

Hierarchy: More Δ_{charge} -> more suppression (λ small)

One can have more flavons (λ, λ', ...)
with different charges (>0 or <0) etc -> many versions

Degenerate ν's

$m^2 \gg \Delta m^2$

- Limits on m_{ee} from $0\nu\beta\beta$

$m_{ee} < 0.3\text{-}0.7$ eV (Exp)

$m_{ee} = c^2_{13}\,(m_1 c^2_{12} + m_2 s^2_{12}) + s^2_{13} m_3 \sim m_1 c^2_{12} + m_2 s^2_{12}$

are not very demanding: for $\sin^2\theta \sim 0.3$ $\cos^2\theta - \sin^2\theta \sim 0.4$

and $|m_1| \sim |m_2| \sim |m_3| \sim 1\text{-}2$ eV (with $m_1 = -m_2$) would be perfectly fine

However, WMAP&LSS: $|m| < 0.23$ eV, is very constraining

Only a moderate degeneracy is still allowed:

$m/(\Delta m^2_{atm})^{1/2} < 5$, $m/(\Delta m^2_{sol})^{1/2} < 30$.

If so, constraints from $0\nu\beta\beta$ are satisfied (both $m_1 = \pm m_2$ allowed)

It is difficult to marry degenerate models with see-saw

$$m_\nu \sim m_D^T M^{-1} m_D$$

(needs a sort of conspiracy between M and m_D)

So most degenerate models deny all relation to m_D and directly work in the LL Majorana sector

For $|m| < 0\nu\beta\beta$ bound we can have $m_1 = m_2 = - m_3$ and

$$m_\nu \sim m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

equality of $m_{11} = m_{23} = m_{32}$ requires a symmetry

eg O(3) Barbieri, Hall, Kane, Ross

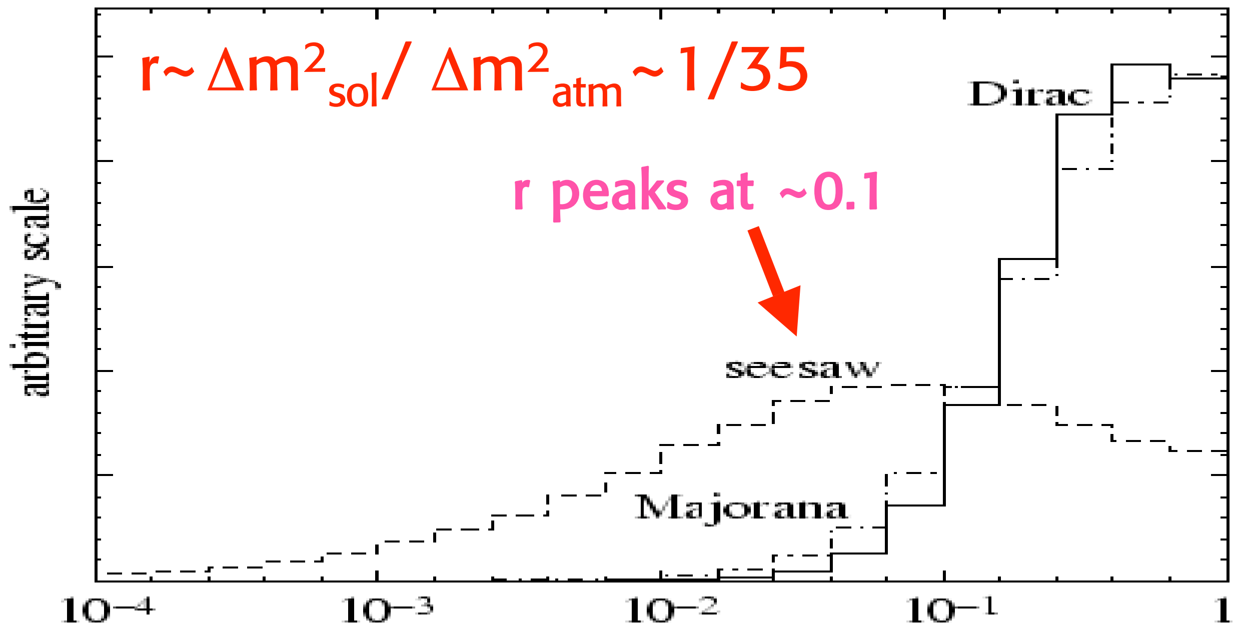
θ_{23} maximal, θ_{12} large, $\theta_{13} = 0$ in symmetric limit

Anarchy (or accidental hierarchy):
No structure in the leptonic sector

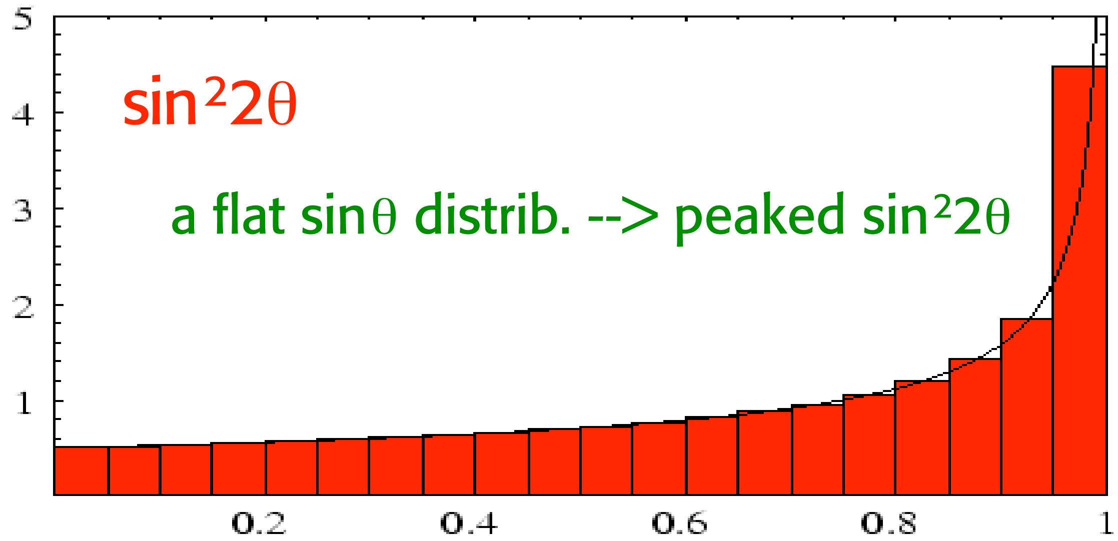
Hall, Murayama, Weiner

See-Saw:
$m_\nu \sim m^2/M$
produces hierarchy
from random m,M

could fit the data



But: all mixing angles
should be not too large,
not too small



Marginal: predicts
θ_{13} near bound

Semianarchy: no structure in 23

Consider a matrix like $m_\nu \sim \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix}$

Note: $\theta_{13} \sim \varepsilon$
$\theta_{23} \sim 1$

with coeff.s of o(1) and det23~o(1)
[ε~1 corresponds to anarchy]

After 23 and 13 rotations $m_\nu \sim \begin{pmatrix} \varepsilon^2 & \varepsilon & 0 \\ \varepsilon & \eta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Normally two masses are of o(1) or r ~1 and $\theta_{12} \sim \varepsilon$
But if, accidentally, $\eta \sim \varepsilon$, then r is small and θ_{12} is large.

The advantage over anarchy is that θ_{13} is small, but θ_{12} large and the hierarchy $m^2_3 \gg m^2_2$ are accidental

Ramond et al, Buchmuller et al

$q(\bar{5}) \sim (2, 0, 0)$ with no see-saw --> no structure in 23

Consider a matrix like $m_\nu \sim L^T L \sim \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix}$

Note: $\theta_{13} \sim \lambda^2$
$\theta_{23} \sim 1$

with coeff.s of o(1) and det23~o(1)
[semianarchy, while $\lambda \sim 1$ corresponds to anarchy]

After 23 and 13 rotations $m_\nu \sim \begin{bmatrix} \lambda^4 & \lambda^2 & 0 \\ \lambda^2 & \eta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Normally two masses are of o(1) or r ~1 and $\theta_{12} \sim \varepsilon$
But if, accidentally, $\eta \sim \varepsilon$, then r is small and θ_{12} is large.

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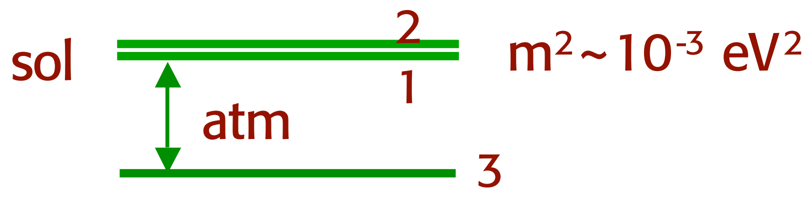
Ramond et al, Buchmuller et al

⊕ With see-saw, one can do much better (see later)

Inverted Hierarchy

Zee, Joshipura et al;
Mohapatra et al; Jarlskog et al;
Frampton,Glashow; Barbieri et al
Xing; Giunti, Tanimoto.......



An interesting model:

An exact U(1) L_e-L_μ-L_τ symmetry for m_ν predicts:

(a good 1st approximation)

$$m_\nu = U m_{\nu diag} U^T = m \begin{bmatrix} 0 & a & -b \\ a & 0 & 0 \\ -b & 0 & 0 \end{bmatrix} \quad \text{with} \quad m_{\nu diag} = \begin{bmatrix} m & 0 & 0 \\ 0 & -m & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- $\theta_{13} = 0$
- $\theta_{12} = \pi/4$
- $\sin^2\theta_{23} = b^2$

θ_{sun} maximal!

θ_{atm} generic

Can arise from see-saw or dim-5 $L^T H H^T L$

- 1-2 degeneracy stable under rad. corr.'s

1st approximation

$$m_{\nu diag} = \begin{bmatrix} m & 0 & 0 \\ 0 & -m & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad m_\nu = U m_{\nu diag} U^T = m \begin{bmatrix} 0 & a & -b \\ a & 0 & 0 \\ -b & 0 & 0 \end{bmatrix}$$

- Data? This texture prefers θ_{sol} closer to maximal than θ_{atm}

In fact: 12-> $\begin{bmatrix} 0 & a \\ a & 0 \end{bmatrix} \rightarrow$ Pseudodirac θ_{12} maximal 23-> $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow \theta_{23} \sim o(1)$

With perturbations: $\begin{bmatrix} 0 & a & -b \\ a & 0 & 0 \\ -b & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} \delta & 1 & 1 \\ 1 & \eta & \eta \\ 1 & \eta & \eta \end{bmatrix}$ (modulo o(1) coeff.s)

one gets $1- tg^2\,\theta_{12} \sim o(\delta + \eta) \sim (\Delta m^2_{sol}/\Delta m^2_{atm})$

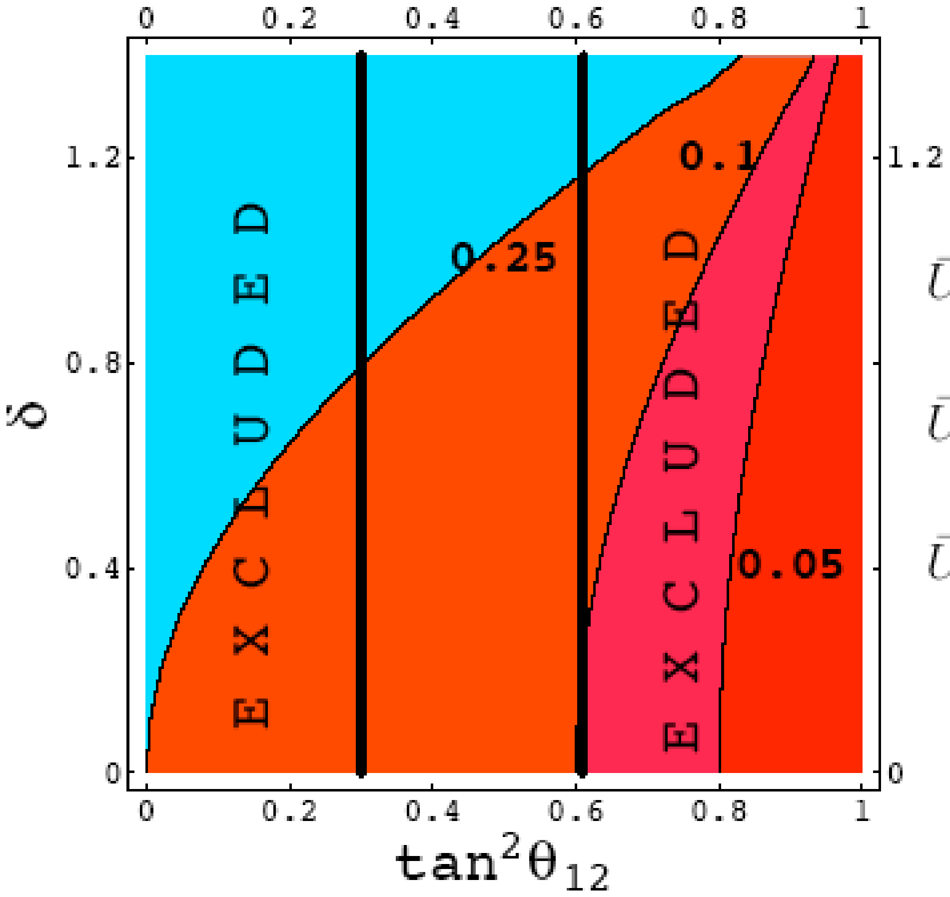
Exp. (3σ): 0.39-0.70 0.018-0.053

- In principle one can use the charged lepton mixing to go away from θ_{12} maximal.
  In practice constraints from θ_{13} small ($\delta\theta_{12} \sim \theta_{13}$)

Frampton et al; GA, Feruglio, Masina '04

For the corrections from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1-\tan^2\theta_{12})/4\cos\delta \sim 0.15$

GA, Feruglio, Masina '04



$$\bar{U}_{12} = -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s^e_{12}e^{-i\alpha_2} + s^e_{13}e^{i\delta_e}}{2}$$

$$\bar{U}_{13} = \frac{s^e_{12}e^{-i\alpha_2} - s^e_{13}e^{i\delta_e}}{\sqrt{2}}$$

$$\bar{U}_{23} = -e^{-i\alpha_2}\frac{1 + s^e_{23}e^{i\alpha_2}}{\sqrt{2}}$$

Corr.'s from s^e_{12}, s^e_{13} to U_{12} and U_{13} are of first order (2nd order to U_{23})

But: a realistic model (eg $\tan^2 2\theta_{12}$ large) with IH, θ_{13} small can be obtained from see-saw, if L_e-L_μ-L_τ is badly broken in M_{RR}

Grimus, Lavoura; G.A., Franceschini

As ν_R are gauge singlets the large soft breaking in M_{RR} does not invade all other sectors when we do rad. corr's

By adding a small flavon breaking of $U(1)_F$ symmetry with parameter $\lambda \sim m_\mu/m_\tau$ the lepton spectrum is made natural and leads to $\theta_{13} \sim m_\mu/m_\tau \sim 0.05$ or even smaller.

GA, Franceschini; hep-ph/0512202

$U(1)_F$ charges

$$l_i \sim L_e - L_\mu - L_\tau \sim (1,-1,-1)$$
$$l_{Ri} \sim (Q_e, Q_\mu, -1)$$
$$\nu_{Ri} \sim (-Q_R, Q_R, 0)$$

λ	Q_e	Q_μ	Q_τ
$0.25 \sim \xi^{\frac{1}{2}}$	7	-3	-1
$0.15 \sim \xi^{\frac{2}{3}}$	$\frac{11}{2}$	$-\frac{5}{2}$	-1
$0.06 \sim \xi$	4	-2	-1
$4 \times 10^{-3} \sim \xi^2$	$\frac{5}{2}$	$-\frac{3}{2}$	-1
$2 \times 10^{-4} \sim \xi^3$	2	$-\frac{4}{3}$	-1

Charged lepton sector

$$m^l \sim \bar{l} l_R \sim m_\tau \begin{pmatrix} \lambda^{|-1+Q_e|} & \lambda^{|-1+Q_\mu|} & \lambda^2 \\ \lambda^{|1+Q_e|} & \lambda^{|1+Q_\mu|} & 1 \\ \lambda^{|1+Q_e|} & \lambda^{|1+Q_\mu|} & 1 \end{pmatrix} \sim m_\tau \begin{pmatrix} \xi' & \xi\epsilon & \epsilon \\ \xi'\epsilon & \xi & 1 \\ \xi'\epsilon & \xi & 1 \end{pmatrix}$$

$$\epsilon \sim \lambda^2, \quad \xi \sim \lambda^{|1+Q_\mu|} \sim \frac{m_\mu}{m_\tau} \sim 6\ 10^{-2}, \quad \xi' \sim \lambda^{|-1+Q_e|} \sim \frac{m_e}{m_\tau} \sim 3\ 10^{-4}$$

Diagonalisation

$$\begin{pmatrix} \xi' & \xi\epsilon & \epsilon \\ \xi'\epsilon & \xi & 1 \\ \xi'\epsilon & \xi & 1 \end{pmatrix} = U_l \begin{pmatrix} \xi' & 0 & 0 \\ 0 & \xi & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad U_l = R_{23}(\theta_l) R_{13}(\epsilon) R_{12}(\epsilon)$$

θ_l --> large shift to θ_{23}, $O(\lambda^2)$ contrib'ns to θ_{13} , θ_{12}

Neutrino sector

$$\nu_{Ri} \sim (-Q_R, Q_R, 0)$$

Q_R=1

Dirac: $$m_\nu^D \sim \bar{\nu}_R\, l \sim m \begin{pmatrix} y_{11}\lambda^2 & a & b \\ d & y_{22}\lambda^2 & y_{23}\lambda^2 \\ y_{31}\lambda & y_{32}\lambda & y_{33}\lambda \end{pmatrix}$$

a,b,d
W,Z do not
break U(1)

Majorana:

$$m_{RR}^{-1} \sim \frac{1}{M} \begin{pmatrix} x_{11}\lambda^2 & W & x_{13}\lambda \\ W & x_{22}\lambda^2 & x_{23}\lambda \\ x_{13}\lambda & x_{23}\lambda & Z \end{pmatrix}$$

no soft breaking

$$m_{RR}^{-1} \sim \frac{1}{M} \begin{pmatrix} A & W & B \\ W & C & D \\ B & D & Z \end{pmatrix}$$

with soft breaking

$$m_\nu = m_\nu^{DT} m_{RR}^{-1} m_\nu^D \sim m_{\nu 0} + \lambda m_{\nu 1} + \ldots \sim$$

$$\sim \frac{m^2}{M} \begin{pmatrix} d^2C & adW & bdW \\ adW & a^2A & abA \\ bdW & abA & b^2A \end{pmatrix} +$$

after see-saw

$$+ \lambda \frac{m^2}{M} \begin{pmatrix} 2y_{31}dD & y_{31}aB + y_{32}dD & y_{31}bB + y_{33}dD \\ y_{31}aB + y_{32}dD & 2y_{32}aB & y_{32}bB + y_{33}aB \\ y_{31}bB + y_{33}dD & y_{32}bB + y_{33}aB & 2y_{33}bB \end{pmatrix} + o(\lambda^2)$$

Various stages:

exact U(1)   r=0

$$m_1 = - m_2; \quad m_3 = 0; \quad \theta_{13} = 0; \quad \theta_{12} = \frac{\pi}{4}; \quad \tan\theta_{23} = \frac{b}{a}$$

pure L_e-L_μ-L_τ

only soft breaking (λ=0)

$$m_3 = 0; \quad m_1 + m_2 = \bar{C} + \bar{A}; \quad m_1 - m_2 = \sqrt{(\bar{A} - \bar{C})^2 + \bar{W}^2};$$

$$\theta_{13} = 0; \quad \tan\theta_{23} = \left|\frac{b}{a}\right|; \quad \tan^2 2\theta_{12} \sim \frac{\bar{W}^2}{(\bar{A} - \bar{C})^2}$$

with

$$\bar{A} = \frac{m^2}{M} A\,(a^2 + b^2); \quad \bar{C} = \frac{m^2}{M} C\, d^2; \quad \bar{W}^2 = \frac{m^2}{M} 4W^2 (a^2 + b^2) d^2$$

θ_{12} $\rightarrow$ $$\frac{|\bar{A} - \bar{C}|}{|\bar{W}|} \sim 0.40.$$

requires large soft breaking

r~1/30 $\rightarrow$ $$\left|\frac{\bar{A} + \bar{C}}{\bar{A} - \bar{C}}\right| \cos\delta \sim 0.02$$

requires some fine tuning

Summarising this model with IH

In the limit of exact $U(1)_F$ $\theta_{12}=\pi/4$ and r , $\theta_{13,}$ as well as m_e/m_τ and m_μ/m_τ (for our choice of charges) are all zero.

In general a small symmetry breaking will make them different from zero but small. And θ_{12} will only be sligthly displaced from $\pi/4$ (bad)

A large soft explicit mixing in the M_{RR} sector can decouple θ_{12}, which gets a large shift, from θ_{13}, m_e/m_τ and m_μ/m_τ which remain small.

The only remaining imperfection is that a moderate fine tuning is needed for r.

Normal hierarchy

- A crucial point: in the 2-3 sector we need both large m_3-m_2 splitting and large mixing.

$$m_3 \sim (\Delta m^2_{atm})^{1/2} \sim 5\ 10^{-2}\ \mathrm{eV}$$
$$m_2 \sim (\Delta m^2_{sol})^{1/2} \sim 8\ 10^{-3}\ \mathrm{eV}$$

- The "theorem" that large Δm_{32} implies small mixing (pert. th.: $\theta_{ij} \sim 1/|E_i-E_j|$) is not true in general: all we need is (sub)det[23]~0

- Example: $m_{23} \sim \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$

Det = 0; Eigenvl's: 0, $1+x^2$
Mixing: $\sin^2 2\theta = 4x^2/(1+x^2)^2$

For x~1
large splitting
and large mixing!

So all we need are natural mechanisms for det[23]=0

Examples of mechanisms for Det[23]~0

based on see-saw: $m_\nu \sim m^T_D M^{-1} m_D$

1) A ν_R is lightest and coupled to μ and τ

King; Allanach; Barbieri et al......

$$M \sim \begin{bmatrix} \varepsilon & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow M^{-1} \sim \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_\nu \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \approx 1/\varepsilon \begin{bmatrix} a^2 & ac \\ ac & c^2 \end{bmatrix}$$

2) M generic but m_D "lopsided" $\quad m_D \sim \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix}$

Albright, Barr; GA, Feruglio, ...

$$m_\nu \sim \begin{bmatrix} 0 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix} = c \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$$

An important property of SU(5)

Left-handed quarks have small mixings (V_{CKM}),
but right-handed quarks can have large mixings (unknown).

In SU(5): LH for d quarks	$\Longleftrightarrow$	RH for l^- leptons

$$\overset{\bar{5}}{\bar{d}_R}\quad \overset{10}{d_L}: \quad m_d \sim \bar{d}_R d_L$$

$$\overset{10}{\bar{e}_R}\quad \overset{\bar{5}}{e_L}: \quad m_e \sim \bar{e}_R e_L$$

$$\bar{5} : (\underbrace{\bar{d},\bar{d},\bar{d},}_{R}\ \underbrace{\nu,e^-}_{L})$$

$$\boxed{m_d = m_e^T}$$

cannot be exact, but approx.

Most "lopsided" models are based on this fact. In these models large atmospheric mixing arises (at least in part) from the charged lepton sector.

- The correct pattern of masses and mixings, also including ν's, is obtained in simple models based on

  SU(5)xU(1)$_{\text{flavour}}$

Ramond et al; GA, Feruglio+Masina; Buchmuller et al; King et al; Yanagida et al, Berezhiani et al; Lola et al.......

Offers a simple description of hierarchies, but it is not very predictive (large number of undetermined o(1) parameters

- SO(10) models could be more predictive, as are non abelian flavour symmetries, eg O(3)

Albright, Barr; Babu et al; Buccella et al; Barbieri et al; Raby et al; King, Ross

With suitable charge assignments all relevant patterns can be obtained

Recall: u ~ 10 10
$d = e^T \sim \bar{5}\, 10$
$\nu_D \sim \bar{5}\ 1$; $M_{RR} \sim 1\ 1$

1st fam. 2nd 3rd

$$\begin{cases} \Psi_{10}: (5, 3, 0) \\ \Psi_{5}: (2, 0, 0) \\ \Psi_{1}: (1,-1, 0) \end{cases}$$

Equal 2,3 ch. for lopsided

No structure for leptons

No automatic det23 = 0

Automatic det23 = 0

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchical (A)	(3,2,0)	(0,0,0)	(0,0,0)	(0,0)
Semi-Anarchical (SA)	(2,1,0)	(1,0,0)	(2,1,0)	(0,0)
Hierarchical (H_I)	(6,4,0)	(2,0,0)	(1,-1,0)	(0,0)
Hierarchical (H_{II})	(5,3,0)	(2,0,0)	(1,-1,0)	(0,0)
Inversely Hierarchical (IH_I)	(3,2,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)
Inversely Hierarchical (IH_{II})	(6,4,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)

all charges positive

not all charges positive

All entries are a given power of λ times a free o(1) coefficient

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix}$$

In a statistical approach we generate these coeff.s as random complex numbers $\rho e^{i\phi}$ with ϕ =[0,2π] and ρ= [0.5,2] (default) or [0.8,1.2], or [0.95,1.05] or [0,1] (real numbers also considered for comparison)

For each model we evaluate the success rate (over many trials) for falling in the exp. allowed window:

(boundaries ~3σ limits)

Maltoni et al, hep-ph/0309130

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm}$

$0.018 < r < 0.053$
$|U_{e3}| < 0.23$
$0.30 < \tan^2\theta_{12} < 0.64$
$0.45 < \tan^2\theta_{23} < 2.57$

for each model the λ,λ' values are optimised

The optimised values of λ are of the order of λ_C or a bit larger (moderate hierarchy)

model	$\lambda(=\lambda')$
A_{SS}	0.2
SA_{SS}	0.25
$H_{(SS,II)}$	0.35
$H_{(SS,I)}$	0.45
$IH_{(SS,II)}$	0.45
$IH_{(SS,I)}$	0.25

Example: Normal Hierarchy

G.A., Feruglio, Masina

1st fam. 2nd 3rd

$q(10)$: $(5, 3, 0)$
$q(\bar{5})$: $(2, 0, 0)$
$q(1)$: $(1,-1, 0)$

Note: not all charges positive --> det23 suppression

$q(H) = 0$, $q(\bar{H}) = 0$
$q(\theta) = -1$, $q(\theta') = +1$

In first approx., with $<\theta>/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$$\underbrace{m_u}_{10_i 10_j} \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix}, \qquad \underbrace{m_d}_{10_i \bar{5}_j} = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$$\underbrace{m_{\nu D}}_{\bar{5}_i 1_j} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix}, \qquad \underbrace{M_{RR}}_{1_i 1_j} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

Note: coeffs. O(1) omitted, only orders of magnitude predicted

$$\bar{5}_i 1_j \qquad\qquad 1_i 1_j$$

$$m_{\nu D} \sim v_u \begin{pmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{pmatrix}, \qquad M_{RR} \sim M \begin{pmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{pmatrix}$$

see-saw $\quad m_\nu \sim m_{\nu D}{}^T M_{RR}{}^{-1} m_{\nu D}$

$$m_\nu \sim v_u{}^2/M \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix},$$

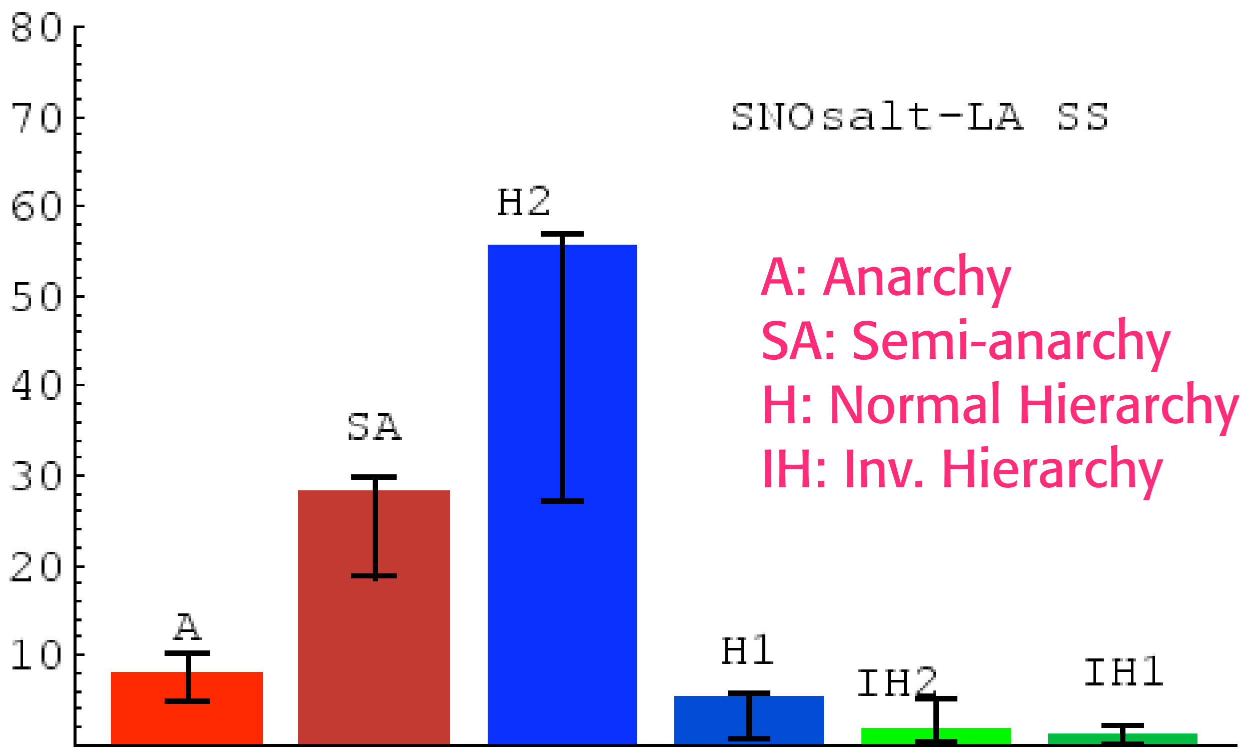
$$\det_{23} \sim \lambda^2$$

The 23 subdeterminant is automatically suppressed,
$\theta_{13} \sim \lambda^2$, θ_{12}, $\theta_{23} \sim 1$

This model works, in the sense that all small parameters are due to various degrees of suppression.
But too many free parameters!!

Results with see-saw dominance (updated in Nov. '03):

Scale: Σrates=100



1 or 2 refer to models with 1 or 2 flavons of opposite ch.

With charges of both signs and 1 flavon some entries are zero

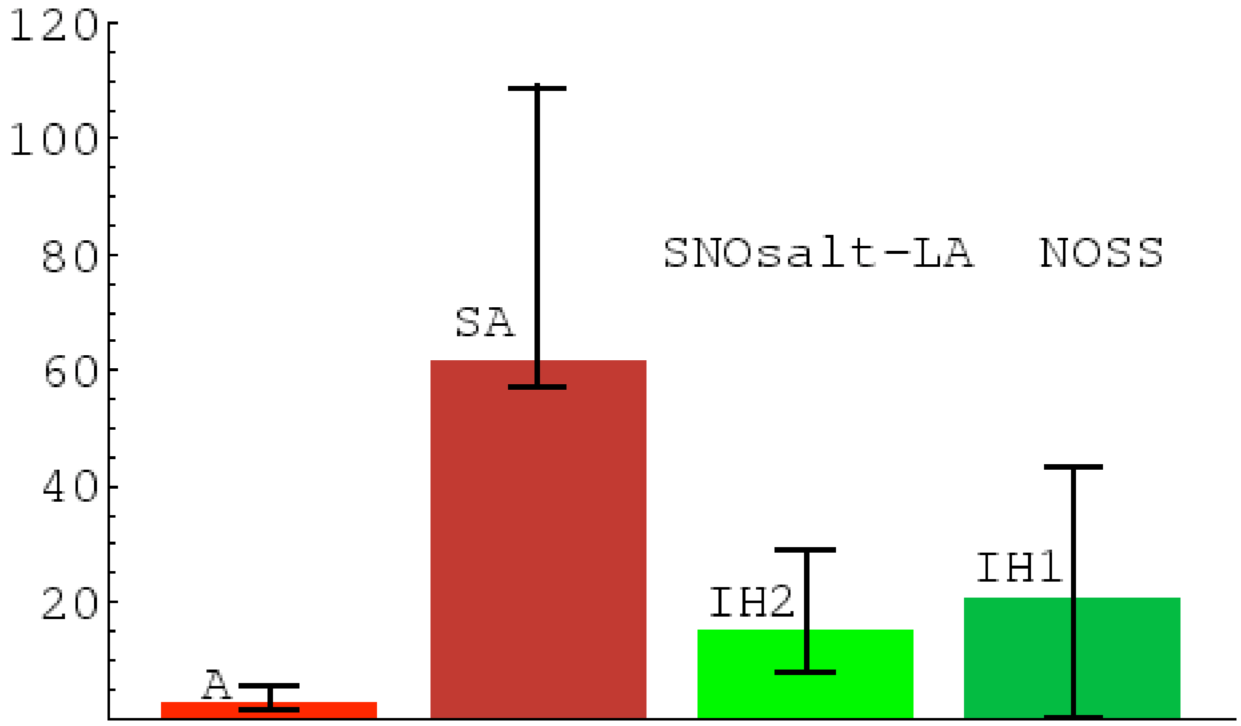
A: Anarchy
SA: Semi-anarchy
H: Normal Hierarchy
IH: Inv. Hierarchy

Errors are linear comb. of stat. and syst. errors (varying the extraction procedure: interval of ρ, real or complex)

H2 is better than SA, better than A, better than IH

With no see-saw (m_ν generated directly from $L^T m_\nu L \sim \overline{5}\,\overline{5}$) IH is better than A

[With no-see-saw H coincide with SA]



Note: we always include the effect of diagonalising charged leptons