

Soft-gluon and hadron-mass effects on fragmentation functions

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We review recent progress in the development of an approach valid to any order which unifies the fixed-order DGLAP evolution of fragmentation functions at large x with soft-gluon logarithmic resummation at small x . At leading order, this approach, implemented with the Double Logarithmic Approximation, reproduces exactly the Modified Leading Logarithm Approximation, but it is more complete due to the degrees of freedom given to the quark sector and the inclusion of the fixed-order terms. We find that data from the largest x values to the peak region can be better fitted than with other approaches. In addition, we develop a treatment of hadron mass effects that leads to additional improvements at small x .

1. Introduction

The perturbative approach to Quantum Chromodynamics (QCD) is believed to solve all problems within its own limitations provided the correct choices of the expansion variable and the variable(s) to be fixed are used. However, perturbative QCD (pQCD) currently has the status of being a large collection of seemingly independent approaches, since a single unified approach valid for all processes is not known. This is problematic when one wants to use a range, qualitatively speaking, of different processes to constrain the same parameters, *e.g.* in global fits. What is needed is a single formalism valid over the union of all ranges that the various pQCD approaches allow. This unification must be consistent, *i.e.* it must agree with each approach in the set, when the expansion of that approach is used up to the order being considered.

In this presentation, we review recent progress in establishing a generalized formalism for the evolution of fragmentation functions (FFs) from the smallest x values probed so far way up to unity [1]. After explaining how to treat soft-gluon and hadron-mass effects in Secs. 2 and 3, respectively, we study their phenomenological implications in Sec. 4. Our conclusions are contained in Sec. 5.

2. Resummation of soft-gluon logarithms

The current optimum description of single-hadron inclusive production is provided by the QCD parton model, which requires knowledge of the FFs $D_a^h(x, Q^2)$, each corresponding at leading order (LO) to the probability for the parton a produced at short distance $1/Q$ to form a jet that includes the hadron h carrying a fraction x of the longitudinal momentum of a . From now on we omit the label h and put the FFs for all a into the vector $D(x, Q^2)$. The evolution of the FFs in the factorization scale Q^2 is also required, which at large and intermediate x is well described

[2] by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [3], given at LO by

$$\frac{d}{d \ln Q^2} D(x, Q^2) = \int_x^1 \frac{dz}{z} a_s(Q^2) P^{(0)}(z) D\left(\frac{x}{z}, Q^2\right), \quad (1)$$

where $P^{(0)}(z)$ are the LO splitting functions calculated from fixed-order (FO) pQCD. We define $a_s = \alpha_s/(2\pi)$, which at LO obeys $a_s(Q^2) = 1/[\beta_0 \ln(Q^2/\Lambda_{\text{QCD}}^2)]$, where $\beta_0 = (11/6)C_A - (2/3)T_R n_f$ is the first coefficient of the beta function and Λ_{QCD} is the asymptotic scale parameter of QCD. For the color gauge group SU(3), the color factors appearing in this presentation are $C_F = 4/3$, $C_A = 3$, and $T_R = 1/2$; n_f is the number of active quark flavors.

On the other hand, at small x , the FO approximation fails due to unresummed *soft-gluon logarithms* (SGLs). The most singular SGLs are the *double logarithms* (DLs), which give a $z \rightarrow 0$ singularity in the LO splitting function $a_s P^{(0)}(z)$ of the form a_s/z . These DLs occur at all orders in the FO splitting function, being generally of the form $(1/z)(a_s \ln z)^2 (a_s \ln^2 z)^r$ for $r = -1, \dots, \infty$, implying that, as x decreases, Eq. (1) becomes a poor approximation once $\ln(1/x) = O(a_s^{-1/2})$. An improvement of the small- x description should be obtained by accounting for all DLs, which is provided by the Double Logarithmic Approximation (DLA) [4, 5],

$$\frac{d}{d \ln Q^2} D(x, Q^2) = \int_x^1 \frac{dz}{z} \frac{2C_A}{z} A z^{2\frac{d}{d \ln Q^2}} \left[a_s(Q^2) \times D\left(\frac{x}{z}, Q^2\right) \right], \quad (2)$$

where $A = 0$ for valence-quark or non-singlet FFs, while

$$A = \begin{pmatrix} 0 & \frac{2C_F}{C_A} \\ 0 & 1 \end{pmatrix}, \quad (3)$$

when $D = (D_\Sigma, D_g)$, where $D_\Sigma = (1/n_f) \sum_{q=1}^{n_f} (D_q + D_{\bar{q}})$ is the singlet FF. The Modified Leading Logarithm Approximation (MLLA) [5, 6, 7] improves the

description over the DLA only by including a part of the FO contribution that is known to be important at small x . With certain qualifications [8], the MLLA leads to a good description of all data down to the smallest x values.

However, what has been lacking is a single approach which can describe data from the largest to the smallest x values. Such an approach can be simply and consistently constructed from the knowledge provided by the DLA and the DGLAP equation. The result reproduces the FO result when expanded in a_s , while all small- ω singularities are resummed into a non-singular expression in Mellin space (see later). As shown in detail in Ref. [1], this approach is unique up to higher-order terms in the FO series and up to less singular SGLs, *e.g.* the *single logarithms* (SLs). Such a formalism may be constructed simply by evolving according to Eq. (1), but with the splitting function modified as

$$a_s P^{(0)}(z) \rightarrow P^{\text{DL}}(z, a_s) + a_s \bar{P}^{(0)}(z), \quad (4)$$

where $P^{\text{DL}}(z, a_s)$ contains the complete resummed contribution to the splitting function to all orders from the DLs, while $a_s \bar{P}^{(0)}(z)$ is the remaining FO contribution at LO. It is obtained by subtracting the LO DLs, already accounted for in P^{DL} , from $a_s P^{(0)}(z)$ to prevent double counting. We work to LO, since the less singular SGLs that occur at next-to-leading order (NLO) are not known to all orders. However, we leave the SL at LO unresummed, since it is not singular as $z \rightarrow 0$. We now use Eq. (2) to gain some understanding of P^{DL} . For this, we transform to Mellin space, defined by

$$f(\omega) = \int_0^1 dx x^\omega f(x). \quad (5)$$

Upon Mellin transformation, Eq. (2) becomes

$$\begin{aligned} \left(\omega + 2 \frac{d}{d \ln Q^2}\right) \frac{d}{d \ln Q^2} D(\omega, Q^2) \\ = 2C_A a_s(Q^2) A D(\omega, Q^2). \end{aligned} \quad (6)$$

Substituting Eq. (4) with the FO term $a_s \bar{P}^{(0)}(z)$ neglected in Eq. (1), taking the Mellin transform,

$$\frac{d}{d \ln Q^2} D(\omega, Q^2) = P^{\text{DL}}(\omega, a_s(Q^2)) D(\omega, Q^2), \quad (7)$$

and inserting this in Eq. (6), we obtain

$$2(P^{\text{DL}})^2 + \omega P^{\text{DL}} - 2C_A a_s A = 0. \quad (8)$$

We choose the solution

$$P^{\text{DL}}(\omega, a_s) = \frac{A}{4} \left(-\omega + \sqrt{\omega^2 + 16C_A a_s} \right), \quad (9)$$

since its expansion in a_s yields at LO the result

$$a_s P^{\text{DL}(0)}(\omega, a_s) = \begin{pmatrix} 0 & a_s \frac{4C_F}{\omega} \\ 0 & a_s \frac{2C_A}{\omega} \end{pmatrix}, \quad (10)$$

which agrees with the LO DLs from the literature [3]. Equation (9) contains all terms in the splitting function of the form $(a_s/\omega)(a_s/\omega^2)^{r+1}$, being the DLs in Mellin space, and agrees with the results of Refs. [5, 6]. We now return to x space, where Eq. (9) reads

$$P^{\text{DL}}(z, a_s) = \frac{A\sqrt{C_A a_s}}{z \ln(1/z)} J_1 \left(4\sqrt{C_A a_s} \ln \frac{1}{z} \right), \quad (11)$$

with J_1 being the Bessel function of the first kind.

To summarize our approach, we evolve the FFs according to Eq. (1), but with the replacement of Eq. (4), where $P^{\text{DL}}(z, a_s)$ is given by Eq. (11), and $a_s \bar{P}^{(0)}(z)$ is given by $a_s P^{(0)}(z)$ after the terms proportional to a_s/z have been subtracted.

To extend NLO calculations to small x_p , the complete resummed DL contribution given by Eq. (11) must be added to the NLO splitting functions. These contain SGLs belonging to the classes $m = 1, \dots, 4$, which must be subtracted. Note that the NLO $m = 1$ term is accounted for by the resummed DL contribution. The $m = 4$ term is a type $p = 0$ term, and hence does not need to be subtracted.

Before we outline the phenomenological investigation of our approach, we note that it is more complete than the MLLA, which can be shown as follows. With $a_s \bar{P}^{(0)}(z)$ accounted for, Eq. (6) is modified to

$$\begin{aligned} \left(\omega + 2 \frac{d}{d \ln Q^2}\right) \frac{d}{d \ln Q^2} D(\omega, Q^2) \\ = 2C_A a_s(Q^2) A D(\omega, Q^2) + \left(\omega + 2 \frac{d}{d \ln Q^2}\right) \left[a_s(Q^2) \right. \\ \left. \times \bar{P}^{(0)}(\omega) D(\omega, Q^2) \right], \end{aligned} \quad (12)$$

up to terms which are being neglected in this presentation and which are neglected in the MLLA. If we approximate $a_s \bar{P}^{(0)}(\omega)$ by its SLs, defined at LO to be the coefficients of ω^0 , equal to those in $a_s P^{(0)}(\omega)$,

$$P^{\text{SL}(0)}(\omega) = \begin{pmatrix} 0 & -3C_F \\ \frac{2}{3} T_R n_f & -\frac{11}{6} C_A - \frac{2}{3} T_R n_f \end{pmatrix}, \quad (13)$$

and apply the approximate result that follows from the DLA at large Q ,

$$D_{q,\bar{q}} = \frac{C_F}{C_A} D_g, \quad (14)$$

which can be derived, *e.g.*, from Eq. (11), the gluon component of Eq. (12) becomes precisely the MLLA differential equation. Therefore, we conclude that, since we do not use these two approximations, our approach is more complete and accurate than the MLLA.

3. Incorporation of hadron mass effects

We now incorporate hadron mass effects into our calculations, using a specific choice of scaling variable.

For this purpose, it is helpful to work with light cone coordinates, in which any four-vector V is written in the form $V = (V^+, V^-, \mathbf{V}_T)$ with $V^\pm = (V^0 \pm V^3)/\sqrt{2}$ and $\mathbf{V}_T = (V^1, V^2)$. In the center-of-mass (CM) frame, the momentum of the electroweak boson takes the form

$$q = \left(\frac{\sqrt{s}}{\sqrt{2}}, \frac{\sqrt{s}}{\sqrt{2}}, \mathbf{0} \right). \quad (15)$$

In the absence of hadron mass, x_p (whose definition $x_p = 2p/\sqrt{s}$ applies only in the CM frame) is identical to the light cone scaling variable $\eta = p_h^+/q^+$. However, the definition $x_p = 2p/\sqrt{s}$ applies only in the CM frame. So, η is a more convenient scaling variable for studying hadron mass effects, since it is invariant with respect to boosts along the direction of the hadron's spatial momentum. Taking this direction to be the three-axis, and introducing a mass m_h for the hadron, the momentum of the hadron in the CM frame reads

$$p_h = \left(\frac{\eta\sqrt{s}}{\sqrt{2}}, \frac{m_h^2}{\sqrt{2}\eta\sqrt{s}}, \mathbf{0} \right). \quad (16)$$

Therefore, the relation between the two scaling variables in the presence of hadron mass is

$$x_p = \eta \left(1 - \frac{m_h^2}{s\eta^2} \right). \quad (17)$$

Note that these two variables are approximately equal when $m_h \ll x_p\sqrt{s}$, *i.e.* hadron mass effects cannot be neglected when x_p (or η) are too small.

In the leading-twist component of the cross section after factorization, the hadron h is produced by fragmentation from a real, massless parton of momentum

$$k = \left(\frac{p_h^+}{y}, 0, \mathbf{0} \right). \quad (18)$$

The + component of everything other than this parton and of everything produced by the parton other than the observed hadron h must be positive, implying that $y \geq \eta$ and $y \leq 1$, respectively. As a generalization of the massless case, we assume the cross section we have been calculating is $(d\sigma/d\eta)(\eta, s)$, *i.e.*

$$\frac{d\sigma}{d\eta}(\eta, s) = \int_{\eta}^1 \frac{dy}{y} \frac{d\sigma}{dy}(y, s, Q^2) D\left(\frac{\eta}{y}, Q^2\right), \quad (19)$$

which is related to the measured observable $(d\sigma/dx_p)(x_p, s)$ via

$$\frac{d\sigma}{dx_p}(x_p, s) = \frac{1}{1 + m_h^2/[s\eta^2(x_p)]} \frac{d\sigma}{d\eta}(\eta(x_p), s). \quad (20)$$

Note that the effect of hadron mass is to reduce the size of the cross section at small x_p (or η).

The data we are studying are described by the sum of the production cross sections for each light charged

hadron species, being the charged pion, the charged kaon, and the (anti)proton, whose masses (140, 494, and 938 MeV, respectively) are substantially different. Therefore, separate FFs and hadron masses for each of the three species are needed. Clearly, so many free parameters would not be constrained by these data. However, since most of the produced particles are pions, it is reasonable to take the final-state hadrons to have the same mass, so that in our approach it is reasonable to use a single hadron mass parameter m_h , whose fitted result should be closer to the pion mass than the proton mass, *i.e.* around 300 MeV. Any significant deviation from this value would imply that some physics has not been accounted for.

4. Numerical analysis

In a first test of our approach, we compare its effects on fits of quark and gluon FFs to e^+e^- data with the standard FO DGLAP evolution. We use normalized differential-cross-section data for the process $e^+e^- \rightarrow (\gamma, Z) \rightarrow h + X$, where h is a light charged hadron and X is anything else, from TASSO at $\sqrt{s} = 14, 35, 44$ [9], and 22 GeV [10], MARK II [11] and TPC [12] at 29 GeV, TOPAZ at 58 GeV [13], ALEPH [14], DELPHI [15], L3 [16], OPAL [17], and SLC [18] at 91 GeV, ALEPH [19] and OPAL [20] at 133 GeV, DELPHI at 161 GeV [21], and OPAL at 172, 183, 189 [22], and 202 GeV [23]. We place a small- x cut [8] on our data of

$$\xi = \ln \frac{1}{x} < \ln \frac{\sqrt{s}}{2M}, \quad (21)$$

where M is a mass scale of $O(1 \text{ GeV})$. We fit the gluon FF, $D_g(x, Q_0^2)$, as well as the quark FFs,

$$\begin{aligned} D_{dsb}(x, Q_0^2) &= \frac{1}{3} [D_d(x, Q_0^2) + D_s(x, Q_0^2) \\ &\quad + D_b(x, Q_0^2)], \\ D_{uc}(x, Q_0^2) &= \frac{1}{2} [D_u(x, Q_0^2) + D_c(x, Q_0^2)], \end{aligned} \quad (22)$$

where $Q_0 = 14 \text{ GeV}$. Since the hadron charge is summed over, we set $D_{\bar{q}} = D_q$. For each of these three FFs, we choose the parameterization

$$D(x, Q_0^2) = N \exp(-c \ln^2 x) x^\alpha (1-x)^\beta, \quad (23)$$

which, at small x , is a Gaussian in ξ for $c > 0$ with center positive in ξ for $\alpha < 0$ as is found to be the case, while it reproduces the standard parameterization (*i.e.* that without the $\exp(-c \ln^2 x)$ factor) used in global fits at intermediate and large x . We use Eq. (14) to motivate the simplification

$$c_{uc} = c_{dsb} = c_g, \quad \alpha_{uc} = \alpha_{dsb} = \alpha_g \quad (24)$$

to our parameterization. We also fit Λ_{QCD} , so that there is a total of 9 free parameters. Since we only use data for which $\sqrt{s} > m_b$, where $m_b \approx 5$ GeV is the mass of the bottom quark, and since $Q_0 > m_b$, we take $n_f = 5$. While the precise choice for n_f does not matter in the DLA, calculations in the FO approach strongly depend on it.

Since the theoretical error in our LO approach is larger than that at NLO, some of our results should be interpreted somewhat qualitatively. In particular, the result for Λ_{QCD} has a theoretical error of a factor of $O(1)$.

Without the $(1-x)^\beta$ factors, Eq. (14) implies that

$$N_{uc} = N_{dsb} = \frac{C_F}{C_A} N_g. \quad (25)$$

Since the $(1-x)^\beta$ factors are important at large x , where Eq. (14) is no longer valid, we do not impose Eq. (24), but rather test its validity over the whole x range.

Table I Parameter values for the FFs at $Q_0 = 14$ GeV parameterized as in Eq. (23) from a fit, with $\chi_{\text{DF}}^2 = 3.0$, to all data listed in the text using DGLAP evolution in the FO approach to LO. The fit also yields $\Lambda_{\text{QCD}} = 388$ MeV.

FF	N	β	α	c
g	0.22	-0.43	-2.38	0.25
$u + c$	0.49	2.30	[-2.38]	[0.25]
$d + s + b$	0.37	1.49	[-2.38]	[0.25]

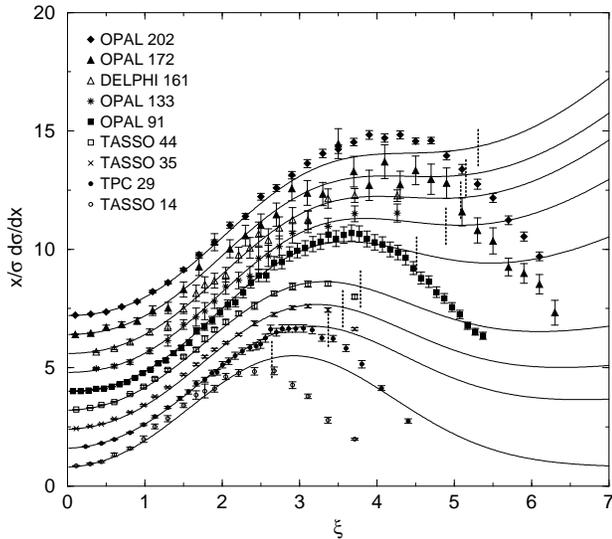


Figure 1: Fit to data as described in Table I. Some of the data sets used for the fit are shown, together with their theoretical predictions from the results of the fit. Data to the right of the vertical dotted lines are not used. Each curve is shifted up by 0.8 for clarity.

We first perform a fit to all data sets listed above using standard LO DGLAP evolution, *i.e.* Eq. (1) without the replacement in Eq. (4). We fit to those data for which Eq. (21) is obeyed with $M = 0.5$ GeV. This gives a total of 425 data points out of the available 492. We obtain $\chi_{\text{DF}}^2 = 3.0$ (or 2.1 after subtraction of the contribution to χ^2 from the TOPAZ data, which is the only data set whose individual χ_{DF}^2 is greater than 6), and the results are shown in Table I and Fig. 1. It is clear that FO DGLAP evolution fails in the description of the peak region and shows a different trend outside the fit range. The $\exp(-c \ln^2 x)$ factor does at least allow for the fit range to be extended to x values below that of $x = 0.1$, the lower limit of most global fits, to around $x = 0.05$ ($\xi = 3$) for data at the larger \sqrt{s} values. Note that β_g is negative, while kinematics require it to be positive. However, this clearly does not make any noticeable difference to the cross section.

Table II Parameter values for the FFs at $Q_0 = 14$ GeV parameterized as in Eq. (23) from a fit, with $\chi_{\text{DF}}^2 = 2.1$, to all data listed in the text using DGLAP evolution in the DLA-improved approach to LO. The fit also yields $\Lambda_{\text{QCD}} = 801$ MeV.

FF	N	β	α	c
g	1.60	5.01	-2.63	0.35
$u + c$	0.39	1.46	[-2.63]	[0.35]
$d + s + b$	0.34	1.49	[-2.63]	[0.35]

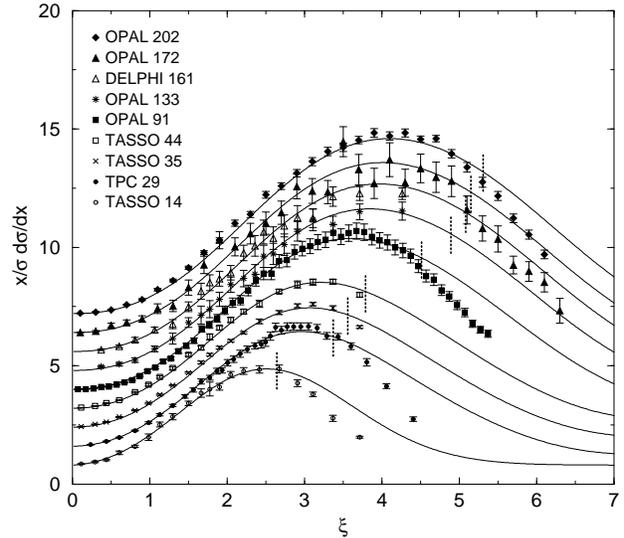


Figure 2: Fit to data as described in Table II.

Now we perform the same fit again, but using our approach, *i.e.* Eq. (1) with the replacement in Eq. (4), for the evolution. The results are shown in Table II and Fig. 2. We obtain $\chi_{\text{DF}}^2 = 2.1$ (or 1.4 without

the TOPAZ data, the individual χ_{DF}^2 for each remaining data set being less than 3), a significant improvement relative to the fit above with FO DGLAP evolution. This should also be compared with the fit to the same data in Ref. [24], where DL resummation is used within the MLLA but with neither FO terms nor quark freedom (*i.e.* Eq. (14) is imposed over the whole x range) and $\chi_{\text{DF}}^2 = 4.0$ is obtained. The data around the peak is now much better described. The energy dependence is well reproduced up to the largest \sqrt{s} value, 202 GeV. At $\sqrt{s} = 14$ GeV, the low- x description of the data is extended from $x = 0.1$ in the unresummed case down to 0.06 in the resummed case, and from $x = 0.05$ to 0.005 at $\sqrt{s} = 202$ GeV.

We conclude that, relative to the MLLA, the FO contributions in the evolution, together with freedom from the constraint of Eq. (14), make a significant improvement to the description of the data for ξ from zero to just beyond the peak. The value $\Lambda_{\text{QCD}} \approx 800$ MeV is somewhat larger than the value 480 MeV, which we obtain from a DGLAP fit in the large- x range ($x > 0.1$). Had we made the usual DLA (MLLA) choice $Q = \sqrt{s}/2$ instead of our choice $Q = \sqrt{s}$ as is done in analyses using the DGLAP equation, we would have obtained half this value for Λ_{QCD} . In an analysis at NLO, the choice of Q is less relevant, since the theoretical error on Λ_{QCD} is smaller. In addition, while the DL resummation greatly improves the description around the peak, it is still not perfect. N_g exceeds the value expected from Eq. (14) by a factor of about 2. However, note that N_g is weakly constrained, since the gluon FF couples to the data only through the evolution, requiring gluon data to be properly constrained. These problems are related to the worsening of the description of the data on moving beyond the peak, since fits in which the cuts are moved to larger ξ values give an increase in Λ_{QCD} and N_g , as well as in χ_{DF}^2 .

Figure 2 is repeated in Fig. 3, to show more clearly the good quality of the fit at intermediate and large x . A couple of points at $x = 0.9$ are not well described, although the data here are scarce and have larger errors. This shows that the DL resummation does not deteriorate the quality of the FO description at large x . This is further confirmed by noting that, for the above fit in our approach, the subsample of the data at $x > 0.1$ yields $\chi_{\text{DF}}^2 = 3.0$, which is close to the result $\chi_{\text{DF}}^2 = 2.9$ obtained from fitting to the same large- x data using only the FO approach.

We now perform the last fit again, but with m_h included in the list of free parameters. We obtain the results in Table III and Fig. 4, and the values $\Lambda_{\text{QCD}} = 399$ MeV and $m_h = 252$ MeV. Treatment of hadron mass effects renders the value of Λ_{QCD} obtained in the fit with DL resummation more reasonable and slightly improves the latter, yielding $\chi_{\text{DF}}^2 = 2.0$. We conclude, therefore, that to improve the large- ξ description and to achieve a reasonable value for Λ_{QCD} , both DL re-

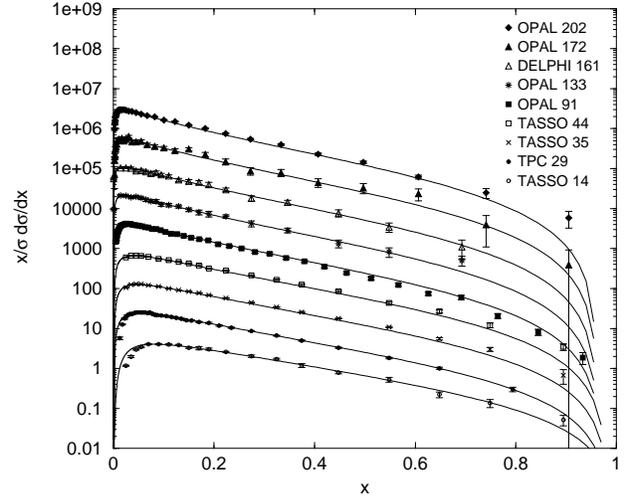


Figure 3: As in Fig. 2, but with the cross section on a logarithmic scale versus x . Each curve, apart from the lowest one, has been rescaled relative to the one immediately below it by a factor of 5 for clarity.

Table III Parameter values for the FFs at $Q_0 = 14$ GeV parameterized as in Eq. (23) from a fit, with $\chi_{\text{DF}}^2 = 2.0$, to all data listed in the text using DGLAP evolution in the DLA-improved approach to LO incorporating hadron mass effects. The fit also yields $\Lambda_{\text{QCD}} = 399$ MeV and $m_h = 252$ MeV.

FF	N	β	α	c
g	1.59	7.80	-2.65	0.33
$u + c$	0.62	1.43	[-2.65]	[0.33]
$d + s + b$	0.74	1.60	[-2.65]	[0.33]

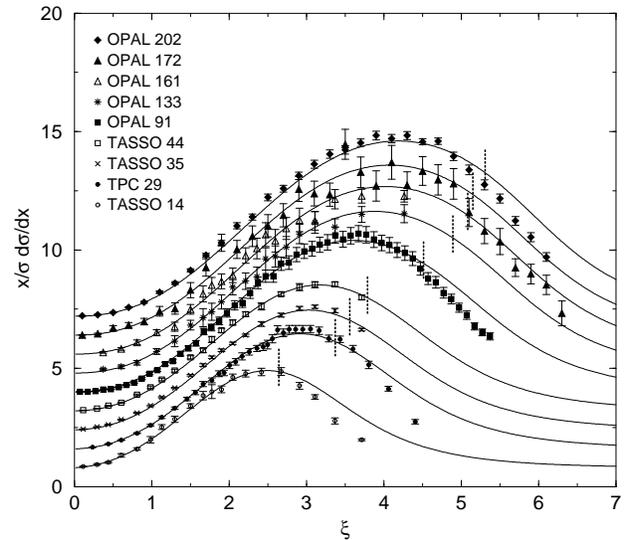


Figure 4: Fit to data as described in Table III.

summation and treatment of hadron mass effects are

required.

5. Conclusions

In conclusion, we have proposed a single unified scheme which can describe a larger range in x than either FO DGLAP evolution or the DLA, by implementing SGL resummation in the standard DGLAP formalism in a minimal and unambiguous way through any order of perturbation theory. Our present limitation to LO is caused by the lack of knowledge on SGL resummation necessary to match NLO DGLAP evolution. The actual inclusion of higher orders, although conceptually straightforward in our scheme, requires a separate analysis, which lies beyond the scope of Ref. [1] and is left for future work. Further improvement in the small- x region can be expected from the inclusion of resummed SLs. Our scheme allows for a determination of quark and gluon FFs over a wider range of data than previously achieved, and should be incorporated into global fits of FFs such as that in Ref. [25], since the current range of $0.1 < x < 1$ is very limited.

Since FFs and their DGLAP evolution are universal, our approach should be expected to also improve the description of inclusive hadron production from reactions other than e^+e^- annihilation, such as those involving at least one proton in the initial state.

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