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Towards classification of simple dark energy cosmological models

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We characterize a class of simple FRW models filled by both dark energy and dark matter in notion of a single potential function of the scale factor $a(t)$; t is the cosmological time. It is representing potential of fictitious particle - Universe moving in 1-dimensional well $V(a)$ which the positional variable mimics the evolution of the Universe. Then the class of all dark energy models (called a multiverse) can be regarded as a Banach space naturally equipment in the structure of the Sobolev metric. In this paper we explore notion of C^1 metric introduced in the multiverse which measure distance between any two dark energy models. If we choose cold dark matter as a reference one then we can find how so far apart are different models offering explanation of present accelerating expansion phase of the Universe. We consider both models with dark energy (models with the generalized Chaplygin gas, models with variable coefficient equation of state $w_X = \frac{p_X}{\rho_X}$ parameterized by redshift z, models with phantom matter) as well as models basing on some modification of the Friedmann equation (Cardassian models, Dvali-Gabadadze-Porati brane models). We argue that because observational data still favor the ΛCDM model all reasonable dark energy models should belong to the nearby neighborhood of this model.

Keywords: geometry of dynamical behaviour, dark energy

1. Introduction – dark energy models as a dynamical systems of Newtonian type

Let us assume a class of cosmological models with the Robertson-Walker symmetry (i.e. homogeneous and isotropic) of space slices filled by perfect fluid described by the general form of the equation of state $w_{\text{eff}} = \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = w_{\text{eff}}(a)$, where p_{eff} and ρ_{eff} are effective pressure and energy density. Then dynamics of the models under consideration, i.e. the FRW models are governed by two basic equations

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p), \tag{1}$$

$$\dot{\rho} = -3H(\rho + p), \tag{2}$$

where $\rho(a)$ and $p(a)$ plays a role of the effective energy density and pressure respectively, a is the scale factor, $H = (\ln a)^{\cdot}$ is the Hubble function, a dot means differentiation with respect to the cosmological time t; $8\pi G = c = 1$.

The first equation is called the acceleration equation or Raychaudhuri equation and the second one is the conservation condition. The system of two basic equation will be in closed form after postulating the form of equation of state (E.Q.S). We assume that Universe is filled by mixture of i noninteracting fluids (in general) such that

$$p_{\text{eff}} = \sum_i p_i \equiv \sum_i w_i(a)\rho_i. \tag{3}$$

For example if we assume dust mater (baryonic and dark mater) and dark energy X then effective pressure and energy density are function of the scale factor only

$$\begin{aligned} p_{\text{eff}} &= 0 + w_X(a)\rho_X(a), \\ \rho_{\text{eff}} &= \rho_{\text{m},0}a^{-3} + \rho_{X,0}a^{-3}\exp\left[-3\int_1^a \frac{w_X(a)}{a}da\right], \end{aligned} \tag{4}$$

where $w_X(a) \equiv \frac{p_X}{\rho_X}$ is a coefficient of E.Q.S for dark energy, $1 + z = a^{-1}$ and $a = 1$ is corresponding to a present value of the scale factor.

Hence for assumed form of E.Q.S equation (1) can be written down in the form analogous to the Newtonian equation of motion

$$\ddot{a} = -\frac{\partial V}{\partial a}, \quad V \equiv -\frac{\rho_{eff}a^2}{6}. \tag{5}$$

In equation 5 $V(a)$ plays the role of the potential function of the particle of unit mass moving in the 1-dimensional potential well. Equation (5) reduces problem of dynamics cosmological models with dark energy to the problem of classical mechanics if only energy density satisfies conservation condition (2).

Of course system (5) admits the first integral in the form

$$\frac{\dot{a}^2}{2} + V(a) = E = \text{const}, \tag{6}$$

which is usually called the Friedmann first integral.

It is convenient to rewrite equation (6) to the new (equivalent) form in which appears dimensionless density parameters $\Omega_i = \frac{\rho_i}{3H_0^2}$. For this aims it is sufficient to reparameterize the cosmological time: $t \mapsto \tau$: $|H_0|dt = d\tau$. Then we obtain

$$\frac{y^2}{2} + V(x) = \frac{1}{2}\Omega_{k,0}, \tag{7}$$

where $x \equiv \frac{a}{a_0}$, $y = x'$, $' \equiv \frac{d}{d\tau}$ (a_0 is present value of the scale factor), $\Omega_k = \Omega_{k,0}a^{-2} \equiv \frac{-3\frac{k}{a^2}}{3H_0^2}$, $1 + z = \frac{a_0}{a} = \frac{1}{a}$. Without lost degree of generality, k (curvature index) and density parameter for curvature can be formally incorporated into potential function.

Therefore the motion of the system is restricted to the distinguished energy level $E = \frac{1}{2}\Omega_{k,0}$. In the phase (x, y) space trajectories of the system are determined from relation (7). Therefore the classical motion of the system in configuration space is restricted to the domain admissible for motion

$$\mathcal{D}_E = \{x: E - V(x) \geq 0\}.$$

In the special case of the ΛCDM model ($\Omega_{k,0} = 0$) we have

$$V(x) = -\frac{1}{2}\{\Omega_{\mathrm{m},0}x^{-1} + \Omega_{\Lambda,0}x^2\}, \tag{8}$$
$$V(x(z)) = -\frac{1}{2}\{\Omega_{\mathrm{m},0}(1+z) + (1-\Omega_{\mathrm{m},0})(1+z)^{-2}\},$$

where we use constraint relation $\Omega_{m,0} + \Omega_{\Lambda,0} = 1$.

Formally curvature term can be included into potential but then system should be considered on zero energy $E = 0$ level instead of $E = \frac{1}{2}\Omega_{k,0}$. If we include the curvature term into the potential V then an additional additive term appears.

The phase space $(x, y = x')$ is divided by trajectory of the flat model ($\Omega_{k,0} = 0$) $y^2 = -2V(x)$ on domains occupied by close ($\Omega_{k,0} < 0$) and open models ($\Omega_{k,0} > 0$).

In the general case because first integral is valid for any z we obtain constraint condition $H(z = 0) = H_0$ in the form

$$\Omega_{\mathrm{m},0} + \Omega_{X,0} + \Omega_{k,0} = 1. \tag{9}$$

We complete the different FRW models with dark energy (see Table 1) and different generalized FRW models based on modification of the Friedmann equation (see Table 2), appeared in the context of discussion in the origin of the present accelerating phase of the Universe [1,2]. For all these models the dynamics is determined from the potential function $V(x(z))$ and corresponding dynamical system is of the form

$$x' = y; \quad y' = -\frac{\partial V(x)}{\partial x}; \quad \text{where } \frac{y^2}{2} + V(x) \equiv \frac{1}{2}\Omega_{k,0} \tag{10}$$

i.e. a Newtonian type. Therefore in the phase space (at finite domain) only critical points of the saddle or centre types are admissible. It is simple consequence of the fact that characteristic equation of the linearization matrix $\lambda^2 + V_{xx}(x_0) = 0$ calculated at the critical points ($y_0 = 0,\ V(x_0) = \frac{1}{2}\Omega_{k,0}$) are admissible. In any case they are corresponding to extremum of the potential. If $V_x(x_0) = 0$ and $V_{xx}(x_0) < 0$ then we obtain saddle points (eigenvalues are real of opposite signs). If at the critical point $V_{xx}(x_0) > 0$ then we obtain centre.

From the physical point of view max is corresponding moment of switching decelerate phase dominate by matter on accelerate dominating by dark energy effects.

While potential function $V(a)$ contain all what is needed to characterize the system evolution it is possible to reconstruct this modulo $\Omega_{k,0}$ immediately from

Table 1. The potential function for 10 prototypes of cosmological models explaining the present acceleration of the Universe in terms of dark energy

case	model	$V(x(z))$ relation
1	ΛCDM model	$V(x(z)) = -\frac{1}{2}\{\Omega_{\mathrm{m},0}(1+z) + (1-\Omega_{\mathrm{m},0})(1+z)^{-2}\}$
2	non-flat FRW model with Λ	$V(x(z)) = -\frac{1}{2}\{\Omega_{\mathrm{m},0}(1+z) + \Omega_{k,0} + \Omega_{\Lambda,0}(1+z)^{-2}\}$
3	FRW model with 2D topological defects $p_X = -\frac{2}{3}\rho_X$	$V(x(z)) = -\frac{1}{2}\{\Omega_{\mathrm{m},0}(1+z) + \Omega_{k,0} + \Omega_{\mathrm{top},0}(1+z)^{-1}\}$
4	FRW model with phantom dark energy $p_X = -\frac{4}{3}\rho_X$	$V(x(z)) = -\frac{1}{2}\{\Omega_{\mathrm{m},0}(1+z) + \Omega_{k,0} + \Omega_{ph,0}(1+z)^{-3}\}$
5	FRW model with phantom dark energy $p_X = w_X\rho_X$, $w_X < -1$ fitted	$V(x(z)) = -\frac{1}{2}\{\Omega_{\mathrm{m},0}(1+z) + \Omega_{k,0} + \Omega_{\mathrm{ph},0}(1+z)^{3(1+w_X)-2}\}$
6	FRW model with Chaplygin gas $p_X = -\frac{A}{\rho_X}$, $A > 0$	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{\mathrm{m},0}(1+z) + \Omega_{k,0} + \Omega_{\mathrm{Ch},0}(1+z)^{-2}[A_S + (1-A_S)(1+z)^6]^{\frac{1}{2}}\right\}$
7	FRW model with generalized Chaplygin gas $p_X = -\frac{A}{\rho_X^\alpha}$, $A > 0$, $\alpha = \mathrm{const}$	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{\mathrm{m},0}(1+z) + \Omega_{k,0} + \Omega_{\mathrm{Ch},0}(1+z)^{-2}[A_S + (1-A_S)(1+z)^{3(1+\alpha)}]^{\frac{1}{1+\alpha}}\right\}$
8a	FRW models with dynamical E.Q.S. parameterized by z $p_X = (w_0 + w_1 z)\rho_X$	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{\mathrm{m},0}(1+z) + \Omega_{k,0} + \Omega_{X,0}(1+z)^{3(w_0-w_1+1)-2}\exp[3w_1 z]\right\}$
8b	FRW models with dynamical E.Q.S parameterized by scale factor a $p_X = (w_0 + w_1(1-a))\rho_X$	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{\mathrm{m},0}(1+z) + \Omega_{k,0} + \Omega_{X,0}(1+z)^{3(w_0+w_1+1)-2}\exp[-\frac{3w_1 z}{1+z}]\right\}$
9	FRW model with quantum effects origin from massless scalar field at low temperature (Casimir effect) $\rho_X = -\frac{\rho_{X,0}}{a^4}$, $\rho_{X,0} > 0$	$V(x(z)) = -\frac{1}{2}\{\Omega_{\mathrm{m},0}(1+z) + \Omega_{k,0} + \Omega_{\Lambda,0}(1+z)^{-2} - \Omega_{\mathrm{Cass},0}(1+z)^2\}$
10	flat FRW model with dissipative dark energy $p_{\mathrm{eff}} = 0 - 3\bar{\alpha}\rho^m H$; $m = -1.5$ is fixed	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{m,0}(1+z) + \Omega_{\mathrm{diss},0}(1+z)^{-2}[A_S + (1-A_S)(1+z)^{3(\frac{1}{2}-m)}]^{\frac{2}{1-2m}}\right\}$

Table 2. The potential function for 10 cosmological models beyond the standard general relativity

case	model	$V(x(z))$ relation
1	Cardassian type of Friedmann equation, $\Omega_{r,0} = 10^{-4}$ is fixed	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{k,0} + \Omega_{\rm m,0}(1+z)^2\left[\frac{1}{1+z} + (1+z)^{-4+4n}\left(\frac{1-\Omega_{r,0}-\Omega_{\rm m,0}}{\Omega_{\rm m,0}}\right)\left(\frac{\frac{1}{1+z}+\frac{\Omega_{r,0}}{\Omega_{\rm m,0}}}{1+\frac{\Omega_{r,0}}{\Omega m,0}}\right)^n\right]\right\}$
2	Dvali-Gabadadze-Porrati brane models (DGP)	$V(x(z)) = -\frac{1}{2}\left\{(1+z)^{-2}\left[\sqrt{\Omega_{\rm m,0}(1+z)^3+\Omega_{rc,0}}+\sqrt{\Omega_{rc,0}}\right]^2 + \Omega_{k,0}\right\}$
3	Deffayet-Dvali-Gabadadze brane models with λ (DDG)	$V(x(z)) = -\frac{1}{2}\left\{(1+z)^{-2}\left[-\frac{1}{2r_0H_0}+\sqrt{\Omega_{\rm m,0}(1+z)^3+\Omega_{\lambda,0}+\frac{1}{4r_0^2H_0^2}}\right]^2\right\}$
4	Randall-Sundrum brane models with dark radiation and $\Lambda = 0$	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{\rm m,0}(1+z) + \Omega_{k,0} + \Omega_{dr,0}(1+z)^2 + \Omega_{\lambda,0}(1+z)^4\right\}$
5	Randall-Sundrum brane models with dark radiation and Λ (RSB)	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{\rm m,0}(1+z) + \Omega_{k,0} + \Omega_{dr,0}(1+z)^2 + \Omega_{\lambda,0}(1+z)^4 + \Omega_{\Lambda,0}(1+z)^{-2}\right\}$
6a	Shtanov brane models (Brane1)	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{\rm m,0}(1+z) + \Omega_{\sigma,0}(1+z)^{-2} + 2\Omega_{l,0}(1+z)^{-2} - 2(1+z)^{-2}\sqrt{\Omega_{l,0}}\sqrt{\Omega_{\rm m,0}(1+z)^3+\Omega_{\sigma,0}+\Omega_{l,0}+\Omega_{\Lambda b,0}}\right\}$
6b.	Shtanov brane models (Brane2)	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{\rm m,0}(1+z) + \Omega_{\sigma,0}(1+z)^{-2} + 2\Omega_{l,0}(1+z)^{-2} + 2(1+z)^{-2}\sqrt{\Omega_{l,0}}\sqrt{\Omega_{\rm m,0}(1+z)^3+\Omega_{\sigma,0}+\Omega_{l,0}+\Omega_{\Lambda b,0}}\right\}$
7	modified affine gravity (MAG) model	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{\rm m,0}(1+z) + \Omega_{k,0} + \Omega_{\psi,0}(1+z)^4 + \Omega_{\Lambda,0}(1+z)^{-2}\right\}$
8	FRW models of nonlinear gravity with Lagrangian density proportional to Ricci scalar R (NG)	$V(x(z)) = -\frac{1}{2}\left\{\left\{\Omega_{\rm m,0}(1+z)\frac{2n}{3-n} + \Omega_{r,0}(1+z)^2\frac{4n(2-n)}{(n-3)^2}\right\}\Omega_{\rm nonl,0}(1+z)^{\frac{3(1-n)}{n}-2}\right\}$
9	bouncing models with Λ (BΛCDM)	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{\rm m,0}(1+z) + \Omega_{k,0} - \Omega_{n,0}(1+z)^{n-2} + \Omega_{\Lambda,0}(1+z)^{-2}\right\}$
10a	models with energy transfer (dark matter $\leftrightarrow$ vacuum energy ρ_1 sector)	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{\rm m,0}(1+z) + \Omega_{\rm int}(1+z)^{n-2} + \Omega_{\Lambda,0}(1+z)^{-2}\right\}$
10b	models with energy transfer (dark matter $\leftrightarrow$ phantom dark energy sector)	$V(x(z)) = -\frac{1}{2}\left\{\Omega_{\rm m,0}(1+z) + \Omega_{\rm int}(1+z)^{n-2} + \Omega_{\rm ph}(1+z)^{3(1+w_X)-2}\right\}$

the SNIa observations due to the simple relation between the luminosity distance and the Hubble function (for flat models)

$$H(z) = \left[\frac{d}{dz}\left(\frac{d_L(z)}{1+z}\right)\right]^{-1}, \tag{11}$$

where

$$H^2(x(z)) = -2V(x)x^{-2}, \tag{12}$$
$$1 + z = x^{-1}.$$

Now we can define notion of space all dark energy cosmological models in terms of space of all dynamical systems of Newtonian type with potential describing dark energy model.

Definition 1. *By multiverse of cosmological models with dark energy we understood functional space of all dynamical systems of Newtonian type with potential functions $V(x)$ of C^1 class.*

Introduced before notion of the multiverse of dark energy models satisfying equation of state (3) can be generalized on the case of cosmological models with scalar fields. Let us consider for example the FRW model with single minimally coupled to gravity scalar field. They can be defined in the form of perfect fluid with energy density $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$. Therefore potential function of the system is $U(\phi) = -\frac{1}{12}\dot{\phi}^2 a^2 - \frac{1}{6}V(\phi)a^2$ and then term $-\frac{1}{12}\dot{\phi}^2 a^2$ can be shifted into kinetic part because of conventional character of division on kinetic and potential parts for the systems with natural Lagrangian. Finally we obtain the dynamical system in the form of energy function

$$\epsilon(a, \dot{a}, \phi, \dot{\phi}) = \frac{\dot{a}^2}{2} - \frac{1}{12}\dot{\phi}^2 a^2 + U(a, \phi), \tag{13}$$

where $U(a, \phi) = -\frac{1}{6}V(\phi)a^2 - \frac{1}{6}\rho a^2$.

The analogous construction can be performed for conformally coupled scalar field. Therefore if we fix the form of kinetic energy then the potential function $U(a, \phi)$ identify quintessence model.

2. Metric structure multiverse of dark energy cosmological systems

The space of all dynamical systems on the plane can be naturally equipment in the structure of Banach space after introducing the Sobolev norm within a space of vector field which are tangent to phase curve

$$||f||_r = \max_{x \in C} \{|f(x)|, |\partial_1 f|, \ldots, |\partial_r f|\}, \tag{1}$$

where C is closed subspace of the phase space, f is the vector field defined on the phase space; $f = [x, -\frac{\partial V}{\partial x}]^T$ in our case.

Let us consider C^1 metric defined as $d(f,g) = ||f-g||$. If we consider two models of the multiverse then C^0 distance between any two dark energy models are

$$d_{C^0}(1,2) = \max_{x\in C}\left\{|(V_1)_x - (V_2)_x|\right\}, \tag{2}$$

where the curvature term $\frac{\Omega_{k,0}}{2}$ we shift into potential then system is considered on $E=0$ level. Therefore two dark energy models are close if their potential function and first derivatives are close. The choice of subset C of configuration space can be performed in different way. If we are interesting in comparison any two dark energy models from view point of present day observational data. At the present epoch $x=1$ ($z=0$) and $V(1) = -\frac{1}{2} - \frac{1}{2}\Omega_{k,0}$ and (2) reduces to

$$d(1,2) = \frac{1}{2}\max_{C=\{1\}x\{y:\ 0\leq\sqrt{\Omega_{k,0}^{max}-2V(1)}\}}\left\{\left|(\Omega_{m,0}^{(1)} - \Omega_{m,0}^{(2)}) + \right.\right. \tag{3}$$

$$\left.\left. + \left(\Omega_X^{(1)}(1+3w_X^{(1)}(1)) - \Omega_X^{(2)}(1+3w_X^{(2)}(1))\right)\right|\right\},$$

where we arbitrary assume roughly constraint to $\Omega_{k,0}$ for example that $-0.1 \leq \Omega_{k,0} \leq 0.1$, and a set C chosen around $\Omega_{k,0}=0$ on the line $x=1$. Of course if we introduce the C^1 metric as

$$d_{C^1}(1,2) = \max_{(x,y)\in C}\left\{|(V_1)_x - (V_2)_x|,\ |(V_1)_{xx} - (V_2)_{xx}|\right\} \tag{4}$$

it will be precisely differentiate between the models. From the observational point of view this metric controls both deceleration and jerk parameters. We say that any two models are close in sense of the C^1 metric if both deceleration and jerk parameters are close. The C^1- metric can be naturally generalized on the case of the C^r metric which can be expressed in dimensionless parameters controlling all $(r+1)$ time derivatives of the scale factor.

It is convent to rewrite C^0 metric to the new form

$$d_{C^0}(1,2) = \max_{C}\left\{\left|\left[\frac{d}{dz}\left(\frac{H_{(1)}^2(z)}{H_0^2}\right)\right]_{z=0} - \left[\frac{d}{dz}\left(\frac{H_{(2)}^2(z)}{H_0^2}\right)\right]_{z=0}\right|\right\}, \tag{5}$$

where we apply relation

$$V_x(z=0) = \left[\frac{\frac{d}{dz}H^2(z)}{H_0^2} - 1\right]_{z=0}$$

and index in the parent denotes kind of model taken from Table 1 or Table 2.

The form of the metric (5) can be simply generalized on the case of C^1 metric, namely

$$d_{C^1}(1,2) = H_0^{-2}\max_{(1,y)\in C}\left\{\left|\left[\frac{d}{dx}\left(H_{(1)}^2(x) - H_{(2)}^2(x)\right)\right]_{x=1}\right|, \tag{6}$$

$$\left|\left[\frac{d^2}{dx^2}\left(H_{(1)}^2(x) - H_{(2)}^2(x)\right)\right]_{x=1} + 4\left[\frac{d}{dx}\left(H_{(1)}^2(x) - H_{(2)}^2(x)\right)\right]_{x=1}\right|\right\}.$$

For our aims it is more suitable the notion of C^k norm defined on the class of $C^k(E)$ functions, i.e. for $f \in C^k(E)$, where E is an open subset of $\mathbb{R}^n$ we define

$$||f||_k = \sup_E |f(x)| + \sup_E ||Df(x)|| + \ldots + \sup_E ||D^k f(x)||. \tag{7}$$

The norm $||\ ||$ on the right-hand side of this equation is defined in the following way

$$||D^k f(x)|| = \max \left| \frac{\partial f^k(x)}{\partial x_{j1} \ldots \partial x_{jk}} \right|$$

, the maximum being taken over $j1, \ldots, jk = 1, \ldots, n$. Each of the spaces $C^k(E)$ is then the Banach space [3].

The above metric of relatively character can be useful in comparing to different models with the concordance ΛCDM one.

Let us illustrate how the Banach metric works for the different dark energy models completed in the Table. Let us consider for simplicity the C^0 metric. Then we obtain

$$d_{C^0}(1,2) = |V_{1x} - V_{2x}| = \left| \Omega^{(1)}_{\mathrm{m},0} + (1 - \Omega^{(1)}_{\mathrm{m},0}) - \Omega^{(2)}_{\mathrm{m},0} - (1 - \Omega^{(2)}_{\mathrm{m},0}) - \Omega_{k,0} \right|.$$

Therefore if we take into account independent prior on density parameter for matter $\Omega^{(1)}_{\mathrm{m},0} = \Omega^{(2)}_{\mathrm{m},0} = \Omega^{*}_{\mathrm{m},0}$ from independent galactic observation then we obtain $d_{C^0}(1,2) = |\Omega_{k,0}|$. The results of calculation of the C^0 metric for other models (flat for simplicity) are

$$\begin{aligned}
&d_{C^0}(3,1) = \Omega_{\mathrm{top},0} = (1 - \Omega^{*}_{\mathrm{m},0}) \\
&d_{C^0}(4,1) = \Omega_{\mathrm{ph},0} = (1 - \Omega^{*}_{\mathrm{m},0}) \\
&d_{C^0}(5,1) = 3|1 + w_X|(1 - \Omega^{*}_{\mathrm{m},0}) \\
&d_{C^0}(6,1) = 3(1 - A_s)(1 - \Omega^{*}_{\mathrm{m},0}) \\
&d_{C^0}(7,1) = 3(1 - A_s)(1 - \Omega^{*}_{\mathrm{m},0} \\
&d_{C^0}(8a,1) = 3(1 - \Omega^{*}_{\mathrm{m},0})|w_0 + 1| \\
&d_{C^0}(8b,1) = d(8a,1) \\
&d_{C^0}(9,1) = 4(1 - \Omega^{*}_{\mathrm{m},0} - \Omega_{\Lambda,0}) \\
&d_{C^0}(10,1) = 3(1 - A_s)(1 - \Omega^{*}_{\mathrm{m},0})
\end{aligned}$$

where $w_X, A_s, w_0, \Omega_{\Lambda,0}$ should be taken from estimation with fixed value of $\Omega^{*}_{\mathrm{m},0}$.

The metric d describe residuals with respect to the reference ΛCDM model controlled by second derivatives of the scale factor. The higher derivatives of the scale factor will be controlled by a more subtle C^1 metric:

$$d_{C^1}(1,2) = |(V_1)_x(x=1) - (V_2)_x(x=1)| + |(V_1)_{xx}(x=1) - (V_2)_{xx}(x=1)| =$$

$$= H_0^{-2} \left\{ \left| \left[\frac{d}{dx} \left(H^2_{(1)}(x) - H^2_{(2)}(x) \right) \right]_{x=1} \right| + \right.$$

$$\left. \left| \left[\frac{d^2}{dx^2} \left(H^2_{(1)}(x) - H^2_{(2)}(x) \right) \right]_{x=1} + 4 \left[\frac{d}{dx} \left(H^2_{(1)}(x) - H^2_{(2)}(x) \right) \right]_{x=1} \right| \right\}$$

Table 3. Values of d_{C^0} and d_{C^1} for flat models from Table 1

Case	d_{C^0}	d_{C^1}
2	0.00	0.00
3	0.70	2.10
4	0.70	3.50
5	0.60	2.84
6	0.02	0.08
7	0.02	0.02
8a	1.34	12.16
8b	1.22	13.29
9	0.00	0.00
10	0.02	0.08

The results of calculation C^1 metric for flat models from Table 1 are

$$
\begin{aligned}
&d_{C^1}(2,1) = 0\\
&d_{C^1}(3,1) = 3(1-\Omega^*_{\mathrm{m},0})\\
&d_{C^1}(4,1) = 5(1-\Omega^*_{\mathrm{m},0})\\
&d_{C^1}(5,1) = 3(1-\Omega^*_{\mathrm{m},0})|1+w_X|(1+3|w_X|)\\
&d_{C^1}(6,1) = 3(1-\Omega^*_{\mathrm{m},0})(1-A_S)(1+3A_S)\\
&d_{C^1}(7,1) = 3(1-\Omega^*_{\mathrm{m},0})(1-A_S)(1+3A_S\alpha)\\
&d_{C^1}(8a,1) = 3(1-\Omega^*_{\mathrm{m},0})|w_0+1|\left\{1+\left|\tfrac{3w_0(w_0+1)+w_1}{w_0+1}\right|\right\}\\
&d_{C^1}(8b,1) = d_{C^1}(8a,1)\\
&d_{C^1}(9,1) = 8(1-\Omega^*_{\mathrm{m},0}-\Omega_{\Lambda,0})\\
&d_{C^1}(10,1) = 3(1-\Omega^*_{\mathrm{m},0})(1-A_S)(1+3A_S|m+\tfrac{1}{2}|)
\end{aligned}
$$

In Table 3 we presented values of d_{C^0} and d_{C^0} for models from Table 1. The values of unknown parameters were estimating from SNIa data (Gold sample [**?**]) with fixed value of $\Omega^*_{\mathrm{m},0} = 0.3$. Distances from the ΛCDM model are also illustrated on Figures 1A and 1B, respectively.

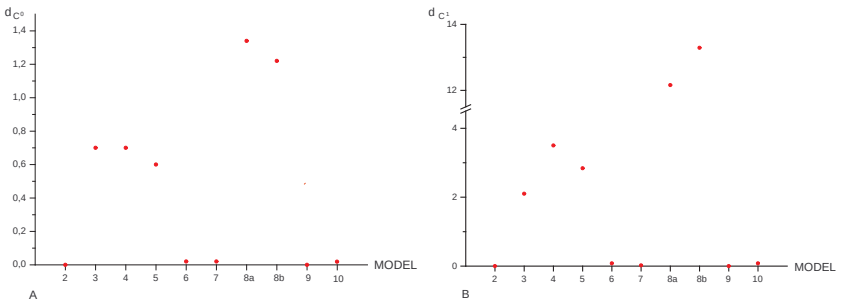

Fig. 1. A. d_{C^0} and B. d_{C^1} for flat models from Table 1

3. Conclusion

In the modern observational cosmology appeared the so-called degeneration problem manifested by the fact that many cosmological models which predict different evolution scenario in the past and in the future becomes still in good agreements with observational data. In the classical approach to model selection problem conclusion are made due to the best fit likelihood function for models under consideration, i.e. so called likelihood ratio test. Unfortunately this method allows to compare only nested models therefore it is not appropriate to compare cosmological models. In this context the Bayesian framework of model selection may be useful in decision which cosmological models is supported by observational data.

Recent observational data suggest that the coefficient of E.Q.S is very close to constant value $w \simeq -1$ but only $w = -1$ imply that energy density of dark matter (vacuum) is constant.

In this paper we characterize class of candidates models with dark energy in tools of Sobolev metric in the space of all dynamical system on the plane.

We believe that adequate model with dark energy should lies within the open ball at the centre in the ΛCDM model and with the small radius ϵ.

Our convince based on the observation that the ΛCDM model is structurally stable, i.e. small changes r.h.s of the system (potential) does not disturb qualitative evolutional scenario of the model. Following the Peixoto theorem structurally stable dynamical systems on the plane form an open and dense subset in the space of all dynamical systems. Therefore structurally stable systems are typical (or generic) (structurally unstable becomes exceptional one) and can be approximated by adding small perturbations.

The metric introduced in the Banach space measure how so far we are from the ΛCDM - concordance model favored by the present observational data. With the help of this metric we can compare different proposition in a similar way like with the help of the Kullback-Leibler metric [5].

Our main conclusion is that all simple dark energy models can be successfully unified within a scheme of dynamical systems of Newtonian type and potential function uniquely characterize different dark energy models. Some of them are generic like ΛCDM model and another exceptional. The models within close neighborhood of the ΛCDM model are typical because its structurally stable.

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