Effective Charges, Event Shapes and Power Corrections

M. J. Dinsdale and C. J. Maxwell (Institute for Particle Physics Phenomenology, University of Durham, U.K.)

IPPP,
University of Durham

FRIF workshop on first principles non-perturbative QCD of hadron jets, Paris, 12-14 January 2006 hep-ph/0412118 v2,

Nucl. Phys. B713 (2005) 465

Outline of talk

- Introduction to and motivation of the Method of Effective Charges

- Direct extraction of $\Lambda_{\overline{MS}}$ from e^+e^- observables

- e^+e^- Event Shape means, the DELPHI analysis

- Large Logarithms and event shape observables

- Resummation of large logarithms in the Effective Charge approach

- ECH fits for Power Corrections and $\Lambda_{\overline{MS}}$

- Extension to Event Shape means in DIS at HERA

- Conclusions

The method of Effective Charges

G. Grunberg, Phys. Rev. **D29** (1984) 2315

Consider an e^+e^- observable $\mathcal{R}(Q)$, e.g. an event shape observable- thrust or heavy-jet mass, Q being the c.m. energy.

$$\begin{aligned}\mathcal{R}(Q) &= a(\mu,\mathrm{RS}) + r_1(\mu/Q,\mathrm{RS})a^2(\mu,\mathrm{RS}) + \\ &+ r_2(\mu/Q,\mathrm{RS})a^3(\mu,\mathrm{RS}) + \cdots ,\end{aligned}$$

Here $a \equiv \alpha_s/\pi$. Normalised with the leading coefficient unity, such an observable is called an *effective charge*. The couplant $a(\mu,\mathrm{RS})$ satisfies the beta-function equation

$$\frac{da(\mu,\mathrm{RS})}{d\ln(\mu)} = \beta(a) = -ba^2(1+ca+c_2a^2+c_3a^3+\cdots)$$

$b = (33-2N_f)/6$ and $c = (153-19N_f)/12b$ are universal, the higher coefficients $c_i\,,(i \geq 2)$ are RS-dependent and may be used to label the scheme, together with dimensional transmutation parameter Λ.

The *effective charge* $\mathcal{R}$ satisfies the equation

$$\frac{d\mathcal{R}(Q)}{d\ln(Q)} = \rho(\mathcal{R}(Q)) = -b\mathcal{R}^2(1+c\mathcal{R}+\rho_2\mathcal{R}^2+\rho_3\mathcal{R}^3+\cdots$$

This corresponds to the beta-function equation in an RS where the higher-order corrections vanish and $\mathcal{R} = a$, the beta-function coefficients in this scheme are the RS-invariant combinations

$$\begin{aligned} \rho_2 &= c_2 + r_2 - r_1 c - r_1^2 \\ \rho_3 &= c_3 + 2r_3 - 4r_1 r_2 - 2r_1\rho_2 - r_1^2 c + 2r_1^3. \end{aligned}$$

The equation for $d\mathcal{R}/d\ln Q$ can be integrated to give

$$\begin{aligned} b\ln\frac{Q}{\Lambda_{\mathcal{R}}} &= \frac{1}{\mathcal{R}} + c\ln\left[\frac{c\mathcal{R}}{1+c\mathcal{R}}\right] + \\ &+ \int_0^{\mathcal{R}(Q)} dx\left[\frac{b}{\rho(x)} + \frac{1}{x^2(1+cx)}\right] \end{aligned}$$

$$\Lambda_{\mathcal{R}} = e^{r/b}\tilde{\Lambda}_{\overline{MS}}$$

$r \equiv r_1(1, \overline{MS})$ with $\mu = Q$.

Direct Extraction of $\Lambda_{\overline{MS}}$ from data $\mathcal{R}$

S.J. Burby and C.J.M. NPB **609** (2001)

One can recast the last equation as

$$\Lambda_{\overline{MS}} = Q\mathcal{F}(\mathcal{R}(Q))\mathcal{G}(\mathcal{R}(Q))e^{-r/b}(2c/b)^{c/b}\,.$$

The final factor converts to the standard convention for Λ. Here $\mathcal{F}(\mathcal{R})$ is the *universal* function

$$\mathcal{F}(\mathcal{R}) = e^{-1/b\mathcal{R}}(1 + 1/c\mathcal{R})^{c/b}\,,$$

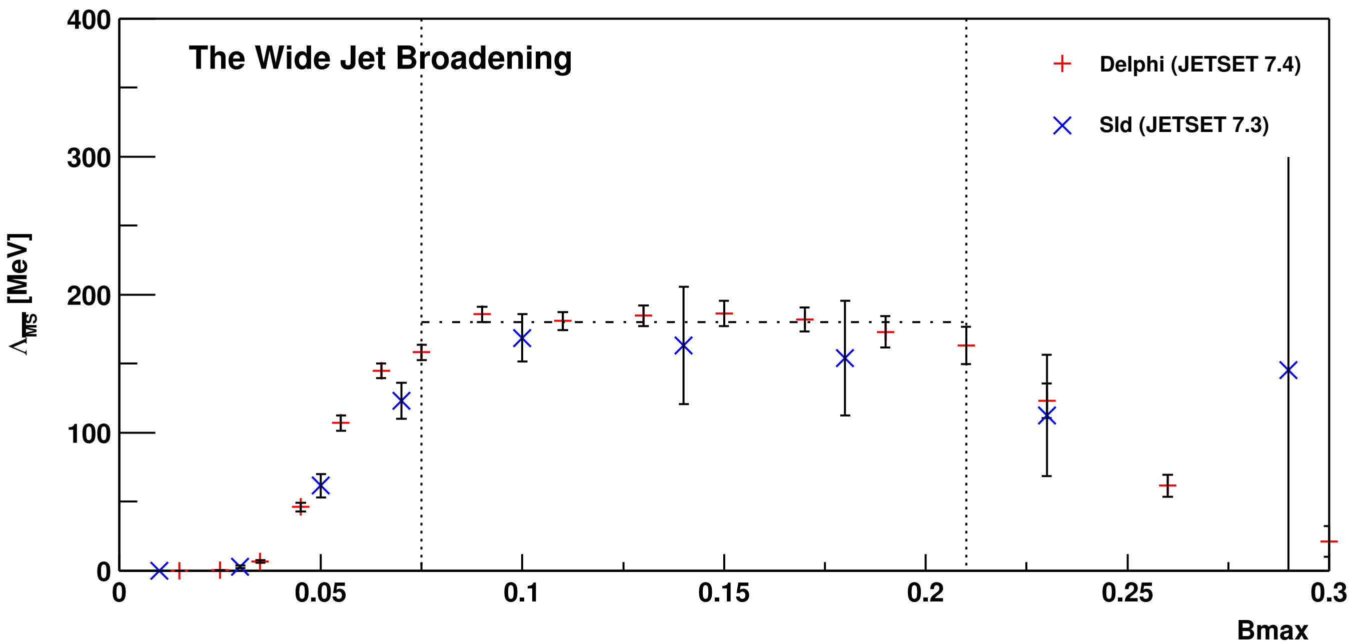
and $\mathcal{G}(\mathcal{R})$ is

$$\mathcal{G}(\mathcal{R}) = 1 - \frac{\rho_2}{b}\mathcal{R} + O(\mathcal{R}^2) + \ldots\,,$$

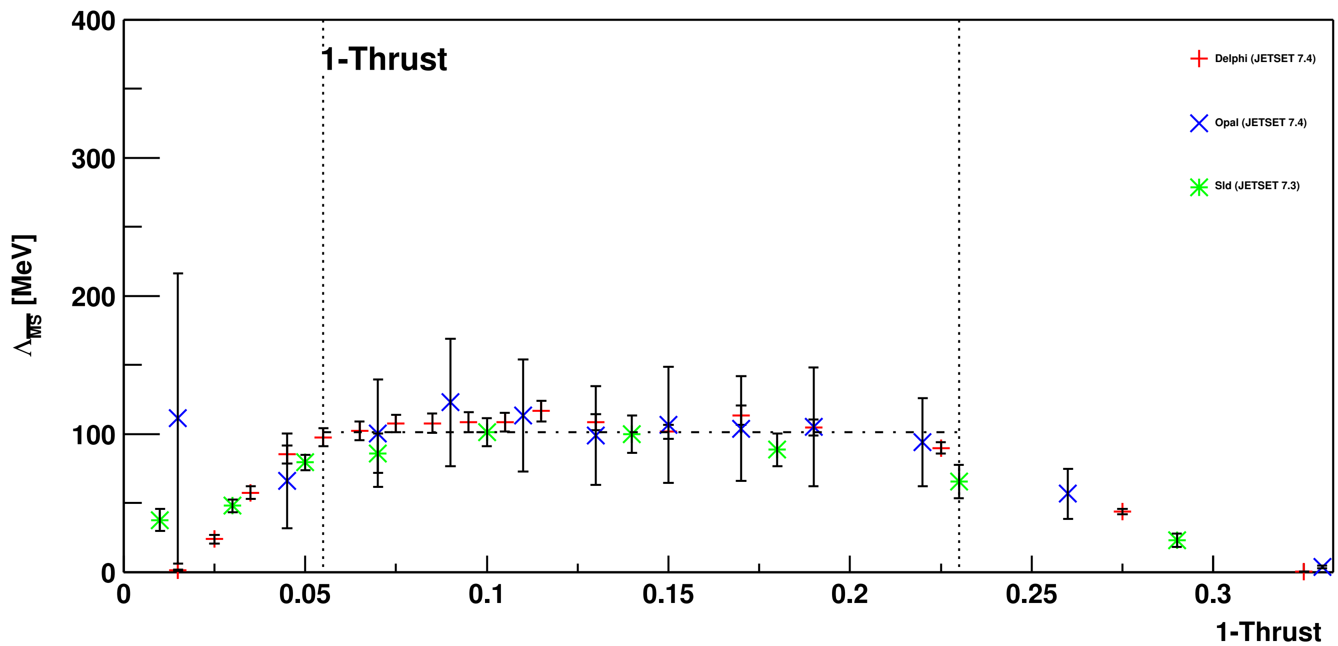
Here ρ_2 is the NNLO ECH RS-invariant. If only a NLO calculation is available, as is the case for e^+e^- jet observables, then $\mathcal{G}(\mathcal{R}) = 1$, and

$$\Lambda_{\overline{MS}} = Q\mathcal{F}(\mathcal{R}(Q))e^{-r/b}(2c/b)^{c/b}\,.$$

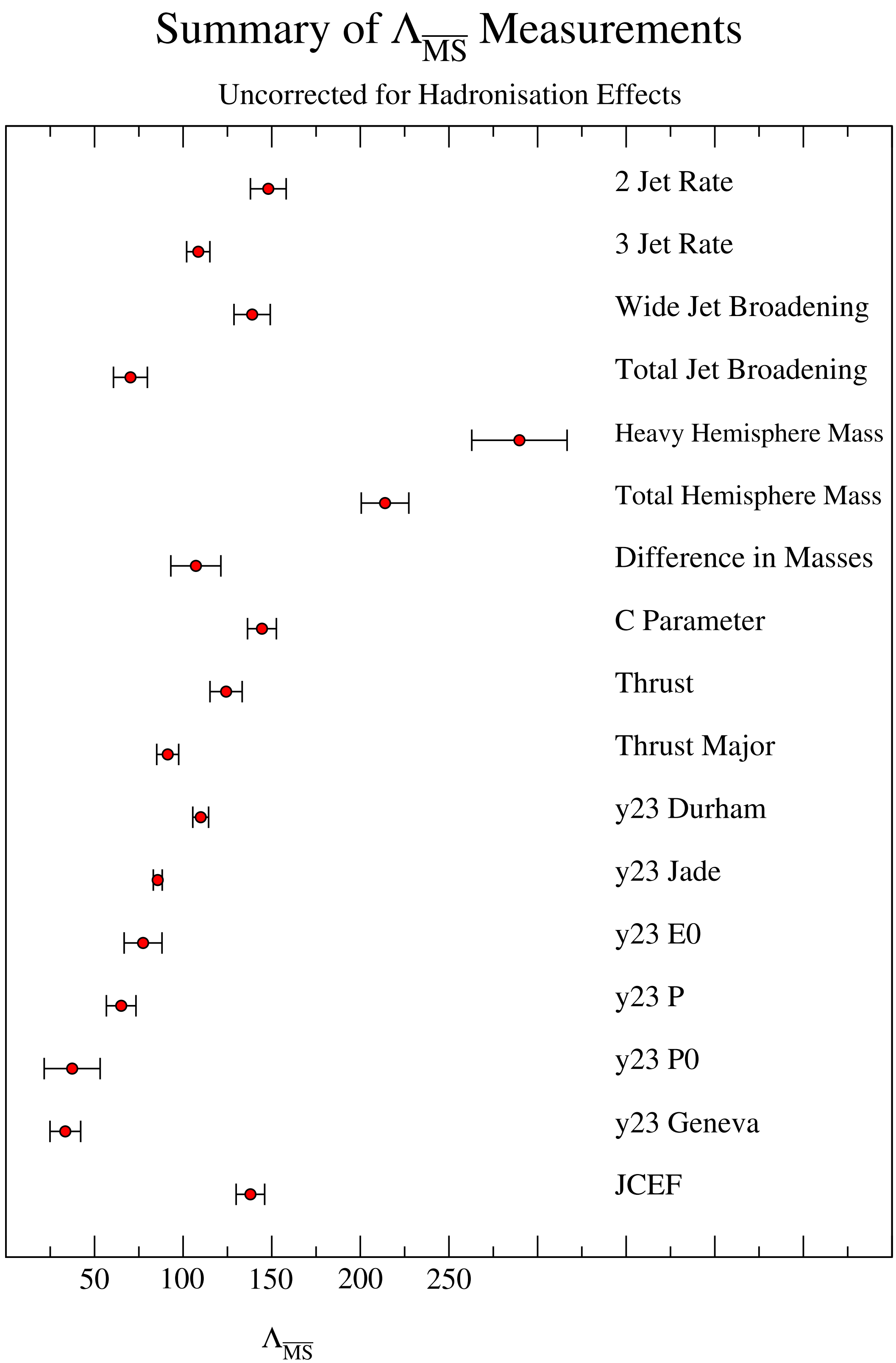
This formula has been used to directly extract $\Lambda_{\overline{MS}}$ from (hadronization corrected) data for a variety of e^+e^- event shape observables



The Wide Jet Broadening
Delphi (JETSET 7.4)
Sld (JETSET 7.3)
$\Lambda_{\overline{MS}}$ [MeV]
Bmax
0
100
200
300
400
0
0.05
0.1
0.15
0.2
0.25
0.3



1-Thrust
Delphi (JETSET 7.4)
Opal (JETSET 7.4)
Sld (JETSET 7.3)
$\Lambda_{\overline{MS}}$ [MeV]
400
300
200
100
0
0
0.05
0.1
0.15
0.2
0.25
0.3
1-Thrust



Summary of $\Lambda_{\overline{MS}}$ Measurements
Uncorrected for Hadronisation Effects
2 Jet Rate
3 Jet Rate
Wide Jet Broadening
Total Jet Broadening
Heavy Hemisphere Mass
Total Hemisphere Mass
Difference in Masses
C Parameter
Thrust
Thrust Major
y23 Durham
y23 Jade
y23 E0
y23 P
y23 P0
y23 Geneva
JCEF
50
100
150
200
250
$\Lambda_{\overline{MS}}$

Summary of $\Lambda_{\overline{MS}}$ Measurements
Corrected for Hadronisation Effects
3 Jet Rate
Wide Jet Broadening
Total Jet Broadening
Heavy Hemisphere Mass
Total Hemisphere Mass
C Parameter
Thrust
y23 Durham
y23 Jade
y23 P
y23 P0
y23 Geneva
JCEF
50
100
150
200
250
$\Lambda_{\overline{MS}}$

Further comments on Effective Charges

It is commonly stated that the method of effective charges is inapplicable to exclusive quantities which depend on multiple scales. However given an observable $\mathcal{R}(Q_1, Q_2, Q_3, \ldots, Q_n)$ depending on n scales it can always be written as

$$\mathcal{R} = \mathcal{R}(Q_1, Q_2/Q_1, \ldots, Q_n/Q_1) \equiv \mathcal{R}_{x_2 x_3 \ldots x_n}(Q_1)$$

Here the $x_i \equiv Q_i/Q_1$ are *dimensionless* quantities that can be held fixed, allowing the Q_1 evolution of $\mathcal{R}$ to be obtained as before. In the 2-jet region for e^+e^- observables large logarithms $L = \ln(1/x_i)$ arise and need to be resummed to all-orders.

Effective Charges from resumming UV logarithms

CORGI approach CJM

Can write the NLO coefficient $r_1(\mu)$ as

$$r_1(\mu) = b\ln\frac{\mu}{\tilde{\Lambda}_{\overline{MS}}} - b\ln\frac{Q}{\Lambda_{\mathcal{R}}} \; .$$

So can identify scale-dependent μ-logs and RS-invariant "physical" UV Q-logs. Higher coefficients are polynomials in r_1.

$$\begin{aligned} r_2 &= r_1^2 + r_1 c + (\rho_2 - c_2) \\ r_3 &= r_1^3 + \frac{5}{2} c r_1^2 + (3\rho_2 - 2c_2) r_1 + (\frac{\rho_3}{2} - \frac{c_3}{2}) \; . \end{aligned}$$

Given a NLO calculation of r_1, parts of $r_2, r_3, \ldots$ are "RG-predictable". Usually choose $\mu = xQ$ then r_1 is Q-independent, and so are all the r_n. The Q-dependence of $\mathcal{R}(Q)$ comes entirely from the RS-dependent coupling $a(Q)$!

However, if we insist that μ is held constant *independent of* Q the only Q-dependence resides in the "physical" UV Q-logs in r_1. Asymptotic freedom then arises only if we resum these Q-logs to *all-orders*. Given only a NLO calculation, and assuming for simplicity that that we have a trivial one loop beta-function $\beta(a) = -ba^2$ so that $a(\mu) = 1/b\ln(\mu/\tilde{\Lambda}_{\overline{MS}})$ the RG-predictable terms will be

$$\mathcal{R} = a(\mu)\left(1 + \sum_{n>0}(a(\mu)r_1(\mu))^n\right) .$$

Summing the geometric progression one obtains

$$\begin{aligned}\mathcal{R}(Q) &= a(\mu)/\left[1 - \left(b\ln\frac{\mu}{\tilde{\Lambda}_{\overline{MS}}} - b\ln\frac{Q}{\Lambda_{\mathcal{R}}}\right)a(\mu)\right] \\ &= 1/b\ln(Q/\Lambda_{\mathcal{R}}) ,\end{aligned}$$

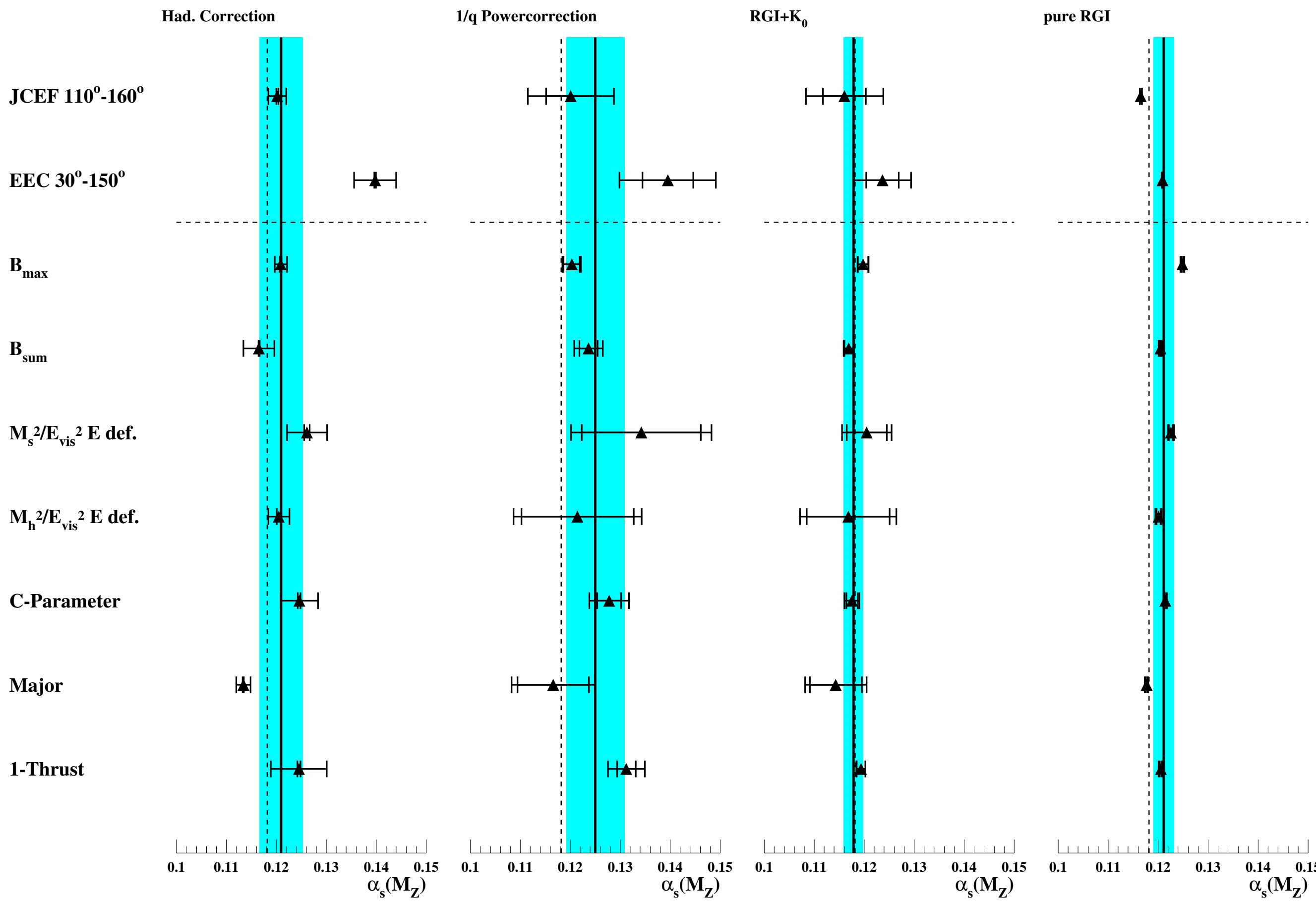
The μ-logs "eat themselves" and one arrives at the NLO ECH result $\mathcal{R}(Q) = 1/b\ln(Q/\Lambda_{\mathcal{R}})$.

Means of e^+e^- Event Shape Observables-

DELPHI analysis

DELPHI collaboration hep-ex/0307048

DELPHI analysed the means of event shapes measured at LEP and found that excellent fits could be obtained using the NLO ECH predictions (RGI Perturbation Theory). These were consistent with there being *no* non-perturbative power corrections $1/Q$, and were of higher quality than the fits obtained using the $\overline{MS}$ scheme with a “physical scale choice $\mu = Q$ together with the model for power corrections of Webber and Dokshitzer.



Had. Correction
1/q Powercorrection
RGI+K_0
pure RGI
JCEF 110^o-160^o
EEC 30^o-150^o
B_{max}
B_{sum}
M_s^2/E_{vis}^2 E def.
M_h^2/E_{vis}^2 E def.
C-Parameter
Major
1-Thrust
0.1 0.11 0.12 0.13 0.14 0.15
$\alpha_s(M_Z)$

Large Logarithms and event shape distributions

Event shape distributions for thrust (T) or heavy-jet mass (ρ_h) contain large kinematical logarithms $L = \ln(1/y)$, where $y = (1-T), \rho_h, \cdots$. These must be resummed to all-orders in the two-jet region

$$\frac{1}{\sigma}\frac{d\sigma}{dy} = A_{LL}(aL^2) + L^{-1}A_{NLL}(aL^2) + L^{-2}A_{NNLL}(aL^2) +$$

Here LL, NLL, $NNLL$ denote leading log, next-to-leading log, etc. For thrust and heavy-jet mass the distributions *exponentiate*

$$\begin{aligned} R_y(y') &\equiv \int_0^{y'} dy \frac{1}{\sigma}\frac{d\sigma}{dy} = C(a\pi)\exp(Lg_1(a\pi L) \\ &+ g_2(a\pi L) + ag_3(a\pi L) + \cdots) + D(a\pi, y) \end{aligned}$$

Here g_1 contains the LL and g_2 the NLL. $C = 1 + O(a)$ is independent of y and D contains terms that vanish as $y \to 0$.

It is natural to define an effective charge $\mathcal{R}(y')$ so that

$$R_y(y') = \exp(r_0(y')\mathcal{R}(y'))$$

This effective charge will have the expansion

$$r_0(L)\mathcal{R}(L) = r_0(L)(a+r_1(L)a^2+r_2(L)a^3+\cdots)$$

Here $r_0(L) \sim L^2$, and the higher coefficients $r_n(L)$ have the structure

$$r_n = r_n^{\mathrm{LL}} L^n + r_n^{\mathrm{NLL}} L^{n-1} + \cdots$$

Usually one resums these logarithms to all-orders using the known closed-form expressions for $g_1(aL)$ and $g_2(aL)$, where a is taken to be the $\overline{MS}$ coupling with a "physical" scale choice $\mu = Q$ ($\overline{MS}$PS). Instead we want to re-sum logarithms to all-orders in the $\rho(\mathcal{R})$ function (ECH).The form of the ρ_n RS-invariants means that the ρ_n have the structure

$$\rho_n = \rho_n^{\mathrm{LL}} L^n + \rho_n^{\mathrm{NLL}} L^{n-1} + \cdots$$

Resummation of the $\rho(\mathcal{R})$ function

From the structure of the ρ_n invariants it follows that we can define RS-invariant approximations to $\rho(\mathcal{R})$ such as

$$\begin{aligned}
\rho_{\mathrm{LL}}(\mathcal{R}) &= -b\mathcal{R}^2(1 + c\mathcal{R} + \sum_{n=2}^{\infty} \rho_n^{\mathrm{LL}} L^n \mathcal{R}^n) \\
\rho_{\mathrm{NLL}}(\mathcal{R}) &= -b\mathcal{R}^2(1 + c\mathcal{R} + \sum_{n=2}^{\infty} (\rho_n^{\mathrm{LL}} L^n \\
&+ \rho_n^{\mathrm{NLL}} L^{n-1})\mathcal{R}^n)
\end{aligned}$$

The resummed $\rho_{\mathrm{NLL}}(\mathcal{R})$ can then be used to solve for $\mathcal{R}_{NLL}$

$$\begin{aligned}
b \ln \frac{Q}{\Lambda_{\mathcal{R}}} &= \frac{1}{\mathcal{R}} + c \ln\left[\frac{c\mathcal{R}}{1 + c\mathcal{R}}\right] \\
&+ \int_0^{\mathcal{R}(Q)} dx \left[\frac{b}{\rho_{NLL}(x)} + \frac{1}{x^2(1 + cx)}\right]
\end{aligned}$$

Since $\Lambda_{\mathcal{R}}$ involves the *exact* value of $r_1(1, \overline{MS})$ there is no matching problem as in the standard $\overline{MS}$PS approach.

Numerical Calculation of the resummed $\rho(\mathcal{R})$

$\rho_{LL}(\mathcal{R})$ can be straightforwardly numerically computed using

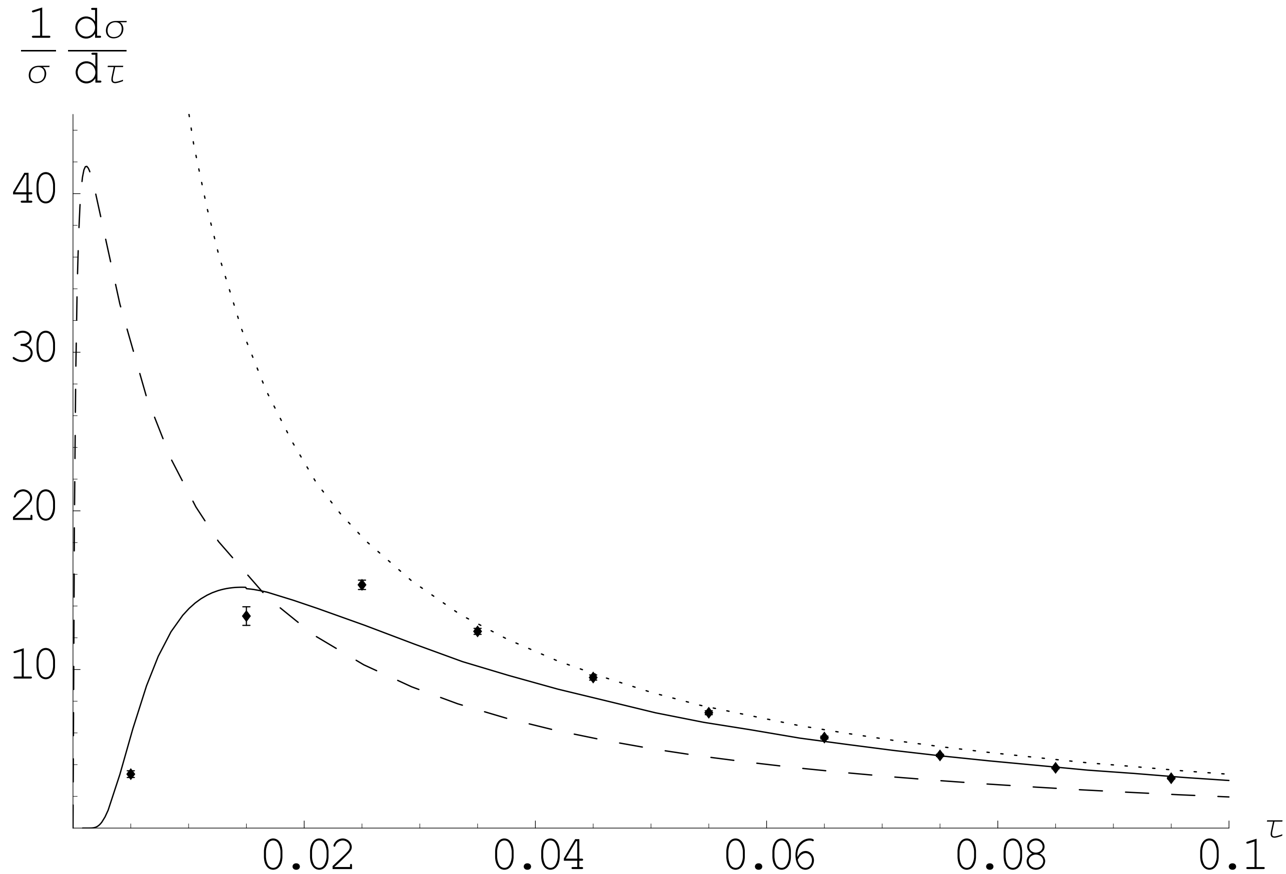
$$\rho_{\mathsf{LL}}(x) = \beta(a)\frac{d\mathcal{R}_{\mathsf{LL}}}{da} = -ba^2\frac{d\mathcal{R}_{\mathsf{LL}}}{da}$$

with a chosen so that $\mathcal{R}_{LL}(a) = x$. The same relation with $\beta(a) = -ba^2(1+ca)$ suffices for $\rho_{NLL}(\mathcal{R})$.

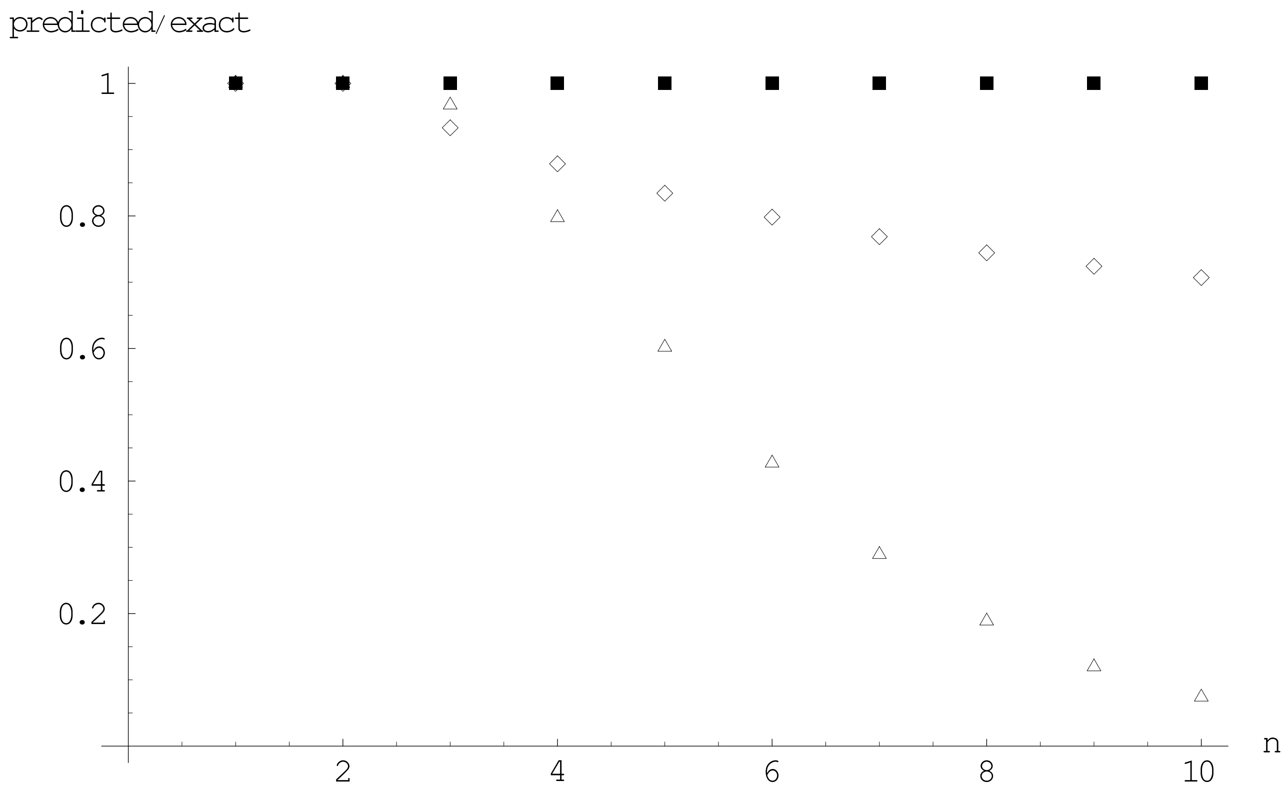
A crucial feature of the effective charge approach is that it resums to all-orders *RG-Predictable* pieces of the higher-order coefficients, thus the NLO ECH result (assuming $c=0$ for simplicity) corresponds to an RS-invariant resummation

$$a + r_1 a^2 + r_1^2 a^3 + \cdots + r_1^n a^{n+1} + \cdots$$

Thus even at fixed-order without any resummation of large logs in $\rho(\mathcal{R})$ a *partial* resummation of large logs is automatically performed. Furthermore one might expect that the LL ECH result contains already NLL pieces of the standard $\overline{MS}$PS result.



Comparison of the 1-thrust distribution using various NLO approximations in the 2-jet region. The solid curve arises from exponentiating the NLO ECH. The dashed curve is obtained by expanding this to NLO in $\overline{MS}$PS. The dotted curve is an unexponentiated NLO ECH fit. DELPHI data at $Q = M_Z$ are plotted. $\Lambda_{\overline{MS}} = 212$ MeV is assumed.



For 1-thrust the ratio of the NLL $\overline{MS}$PS coefficient at O(a^n) "predicted" from the LL ECH result to the exact result (diamonds). The triangles show the "prediction" from the NLO ECH result.

Problems with the Effective Charge resummation

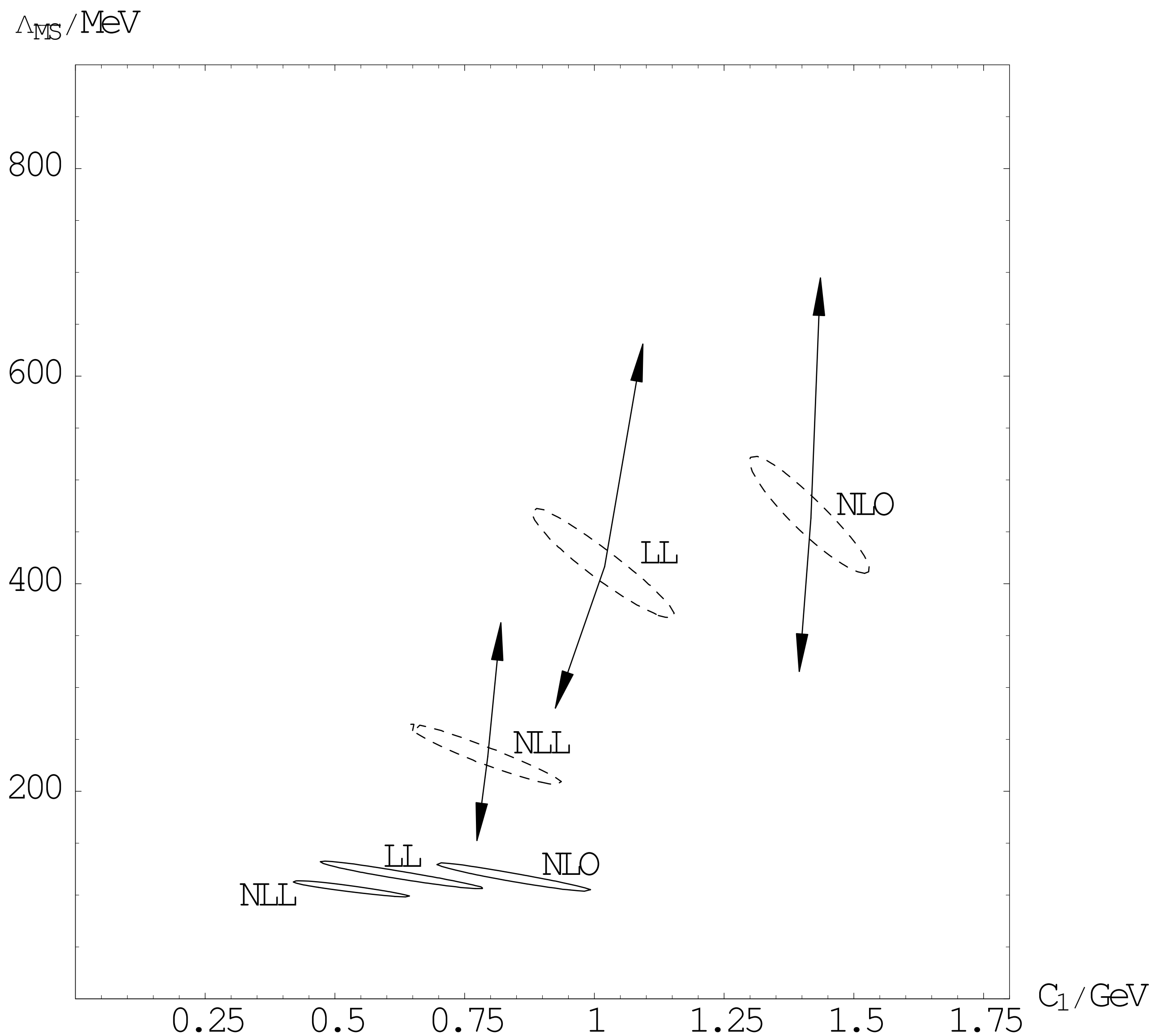
A problem with the Effective Charge resummations is that the $\rho(\mathcal{R})$ function contains a branch cut which limits how far into the 2-jet region one can go. We are limited to $1-T > 0.05 M_Z/Q$ in the fits we have performed. This branch cut mirrors a corresponding branch cut in the resummed $g_1(aL)$ function. An additional problem is that as $1-T$ approaches $2/3$ the leading coefficient $r_0(L)$ vanishes and the Effective Charge formalism breaks down. We need to restrict the fits to $1-T < 0.18$. From the "RG-predictability" arguments we might expect that these difficulties would also become apparent for a NNLL $\overline{MS}$PS resummation.

Fits including power corrections

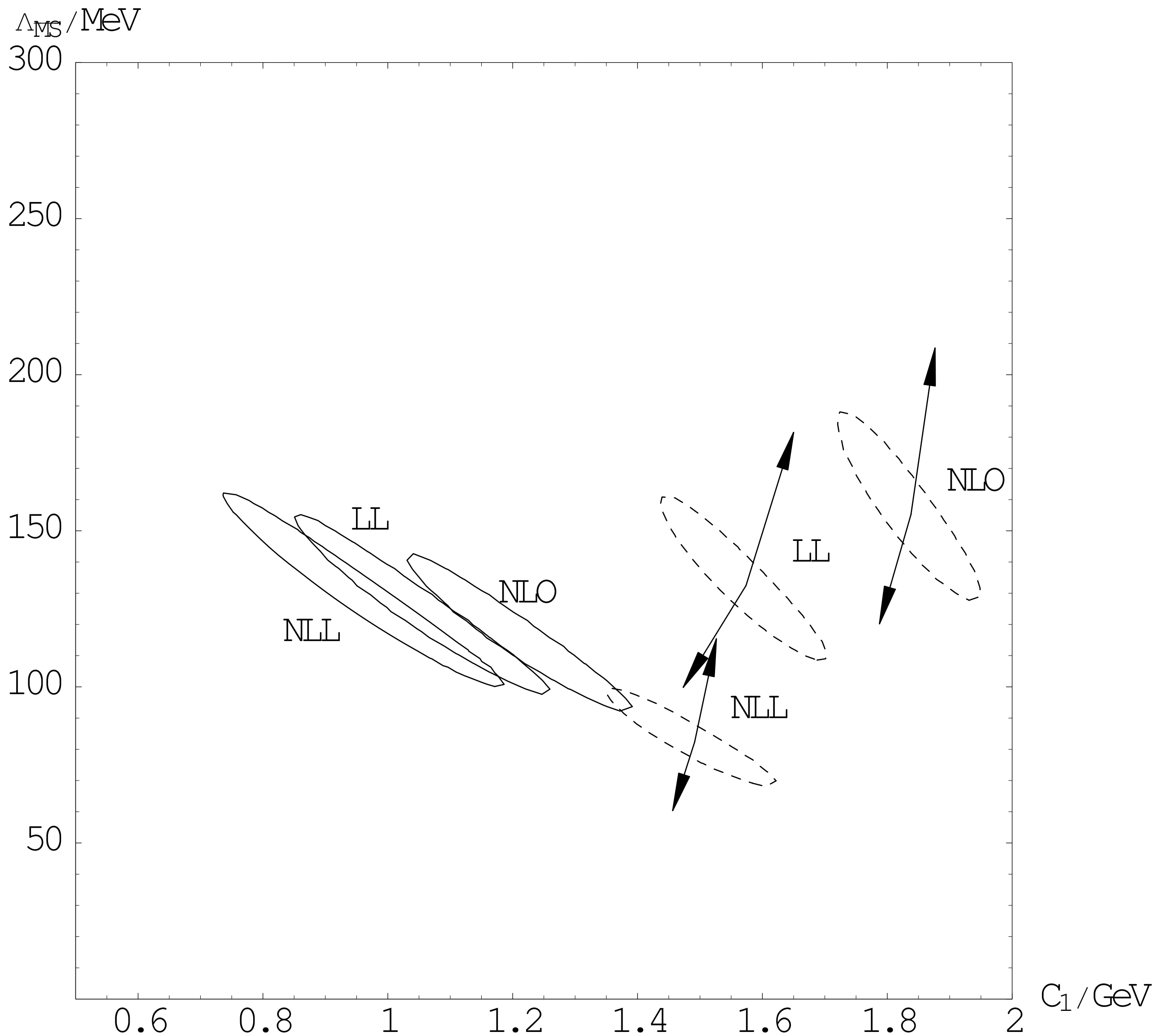
For e^+e^- event shape *means* the DELPHI collaboration have found that the NLO ECH result produces excellent fits to data without including any power corrections. In contrast fits to $\overline{MS}$PS results require additional power corrections C_1/Q with $C_1 \sim 1$ GeV. We wish to perform similar fits to the *distributions*. To this end we define

$$R_{PC}(y) = R_{PT}(y - C_1/Q)$$

This shifted result is then fitted to the data for 1-thrust and heavy jet mass to extract C_1 and $\Lambda_{\overline{MS}}$. .



Fits to 1-thrust for $\Lambda_{\overline{MS}}$ and C_1. Solid 2σ error ellipses are for ECH, dashed are $\overline{MS}$PS.



Fits for heavy-jet mass.

Extension to Event Shape Means at HERA

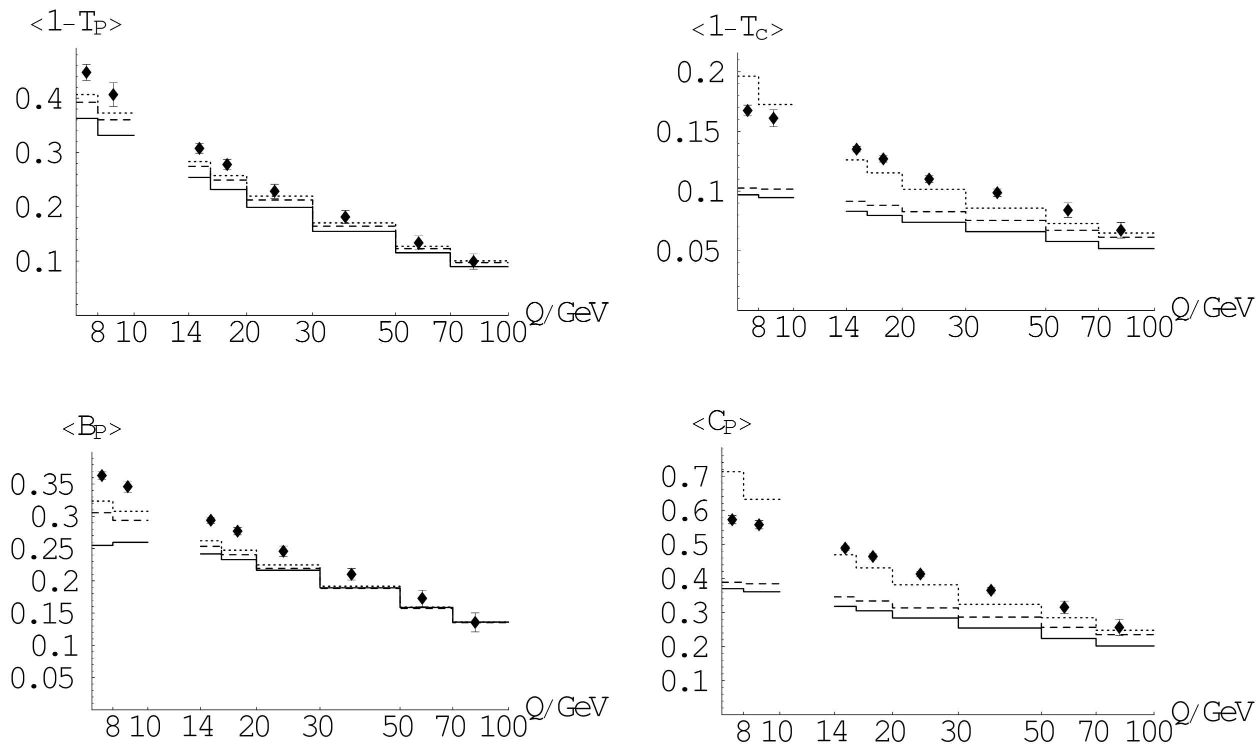
M.J. Dinsdale hep-ph/0512069

For DIS processes at HERA one has a convolution of proton pdf's and hard scattering cross-sections,

$$\frac{d\sigma(ep \to X,Q)}{dX} = \sum_a \int d\xi f_a(\xi,M) \frac{d\hat{\sigma}(ea \to X,Q,M)}{dX}$$

There is no way to directly relate such quantities to effective charges. The DIS cross-sections will depend on a *factorization scale* M, and a renormalization scale μ at NLO. In principle one could identify unphysical scheme-dependent $\ln(M/\tilde{\Lambda}_{\overline{MS}})$ and $\ln(\mu/\tilde{\Lambda}_{\overline{MS}})$, and physical UV Q-logs, and then by all-orders resummation get the M and μ-dependence to "eat itself".

The pattern of logs is far more complicated than the geometrical progression in the Effective Charge case, and a CORGI result for DIS has not been derived so far. Instead one can use the Principle of Minimal Sensitivity (PMS) P.M. Stevenson and for an event shape mean $\langle y \rangle$ look for a stationary saddle point in the (μ, M) plane. It turns that there are large cancellatons between the NLO corrections for quark and gluon initiated subprocesses. One can distinguish between two approaches, PMS_1 where one seeks a saddle point in the (μ, M) plane for the sum of parton subprocesses, and PMS_2 where one introduces two separate scales μ_q and μ_g and finds a saddle point in (μ_q, μ_g, M). PMS_1 gives power corrections fits comparable to $\overline{MS}$PS with $M = \mu = Q$. PMS_2 in contrast gives substantially reduced power corrections.



<1-T_P>
0.4
0.3
0.2
0.1
8 10 14 20 30 50 70 100
Q/GeV
<1-T_C>
0.2
0.15
0.1
0.05
8 10 14 20 30 50 70 100
Q/GeV
<B_P>
0.35
0.3
0.25
0.2
0.15
0.1
0.05
8 10 14 20 30 50 70 100
Q/GeV
<C_P>
0.7
0.6
0.5
0.4
0.3
0.2
0.1
8 10 14 20 30 50 70 100
Q/GeV

Conclusions

Notwithstanding the limited fit range the ECH fits for thrust and heavy jet mass show great stability going from NLO to LL to NLL, presumably because at each stage a partial resummation of higher logs is automatically performed.

The power corrections required with ECH are somewhat smaller than those found with $\overline{MS}$PS, but we do not find as dramatic a reduction as DELPHI find for the means. This may be because their analysis corrects the data for bottom quark mass effects which we have ignored.

The fitted value of $\Lambda_{\overline{MS}}$ for ECH is much smaller than that found with $\overline{MS}$PS, ($\alpha_s(M_Z) = 0.106$ (thrust) and 0.109 (heavy-jet mass)). Similarly small values are found with the Dressed Gluon Exponentiation (DGE) approach.