Why multi-jet studies?

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Outline

1. Introduction
   - Why event-shape variables?
   - Power corrections and Feynman tube model
2. Three-jet event shapes
   - PQCD and NP corrections
   - $K_{\rm out}$ and D-parameter in e^+e^- annihilation
   - $K_{\rm out}$ in DY and underlying event

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Event-shape variables

- Event-shape variables $V(p_1, \ldots, p_n)$ are continuous measures of the geometrical properties of hadron energy-momentum flow.
- Thrust: longitudinal particle alignment

$$T \equiv \frac{1}{Q} \max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T| \qquad t \equiv 1 - T$$

Pencil-like event: $t \gtrsim 0$ Planar event: $t \simeq 1/3$

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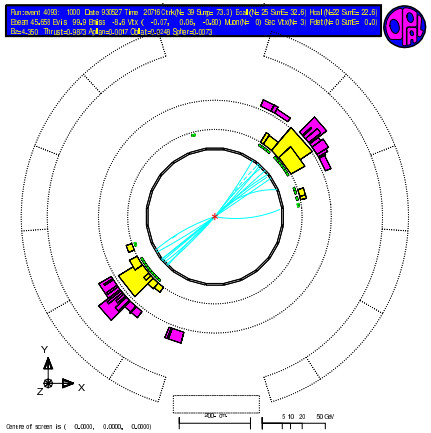

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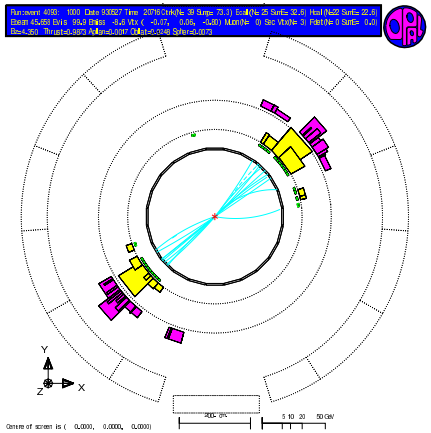

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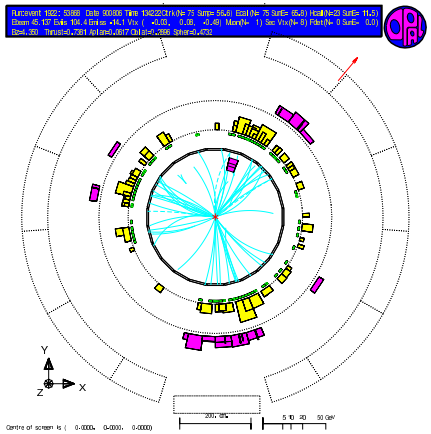
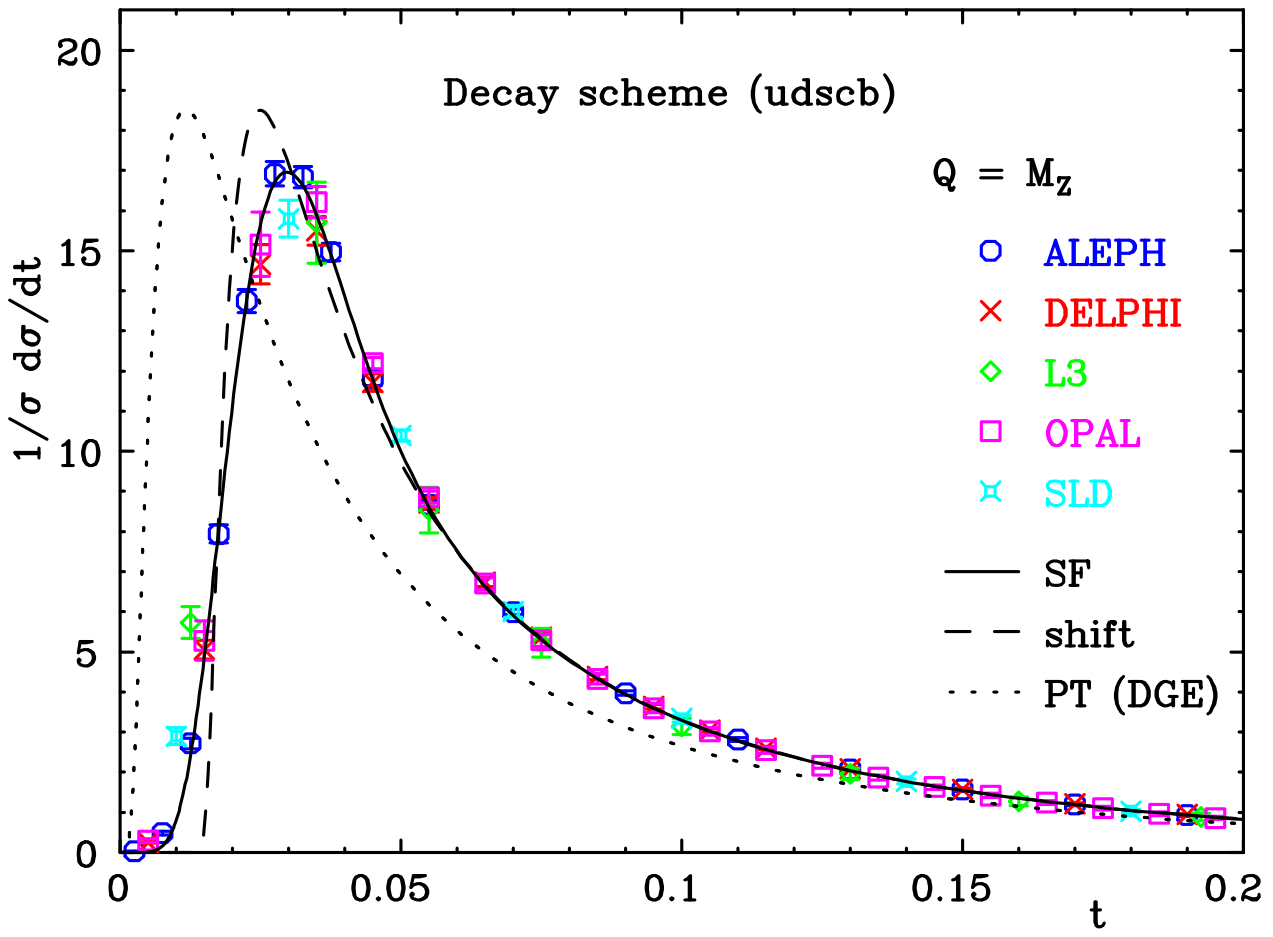


From short to long distances



NP hadronisation | PT resummation | PT fixed order

NP corrections to mean values

Example: mean value of the thrust

$$\langle t \rangle_{\mathsf{PT}} = \langle t \rangle_1 \; \alpha_{\mathrm{s}}(Q^2) + \langle t \rangle_2 \; \alpha_{\mathrm{s}}^2(Q^2) + \ldots$$

PT theory cannot be the whole story...

$$\langle t \rangle = \langle t \rangle_{\mathsf{PT}} + \langle t \rangle_{\mathsf{NP}} \qquad \langle t \rangle_{\mathsf{NP}} \sim \frac{1\ \mathsf{GeV}}{Q}$$

Sterman's lemma $\Rightarrow 18\alpha_{\mathrm{s}}^3 \simeq 1/Q$

$$\langle t \rangle_{\mathsf{PT}} \simeq \frac{4C_F\alpha_{\mathrm{s}}}{\pi} \sum_{n=0}^{\infty} n! \left(\frac{2\beta_0\alpha_{\mathrm{s}}}{4\pi}\right)^n n^{\beta_1}$$

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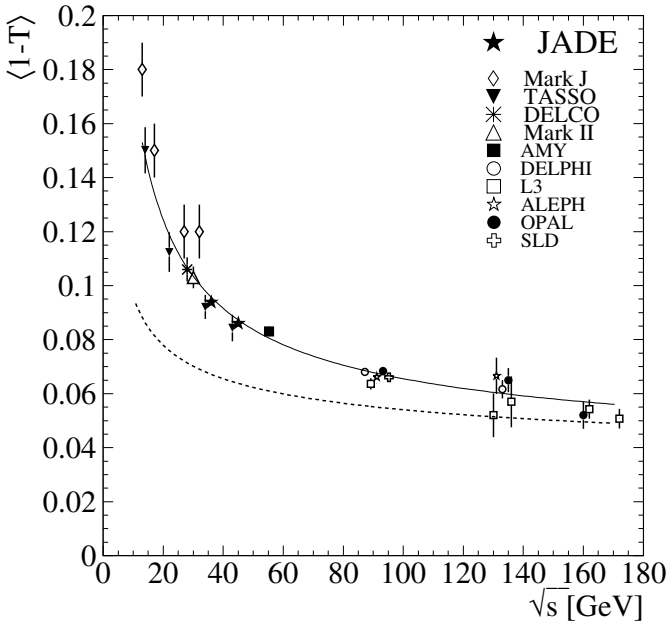

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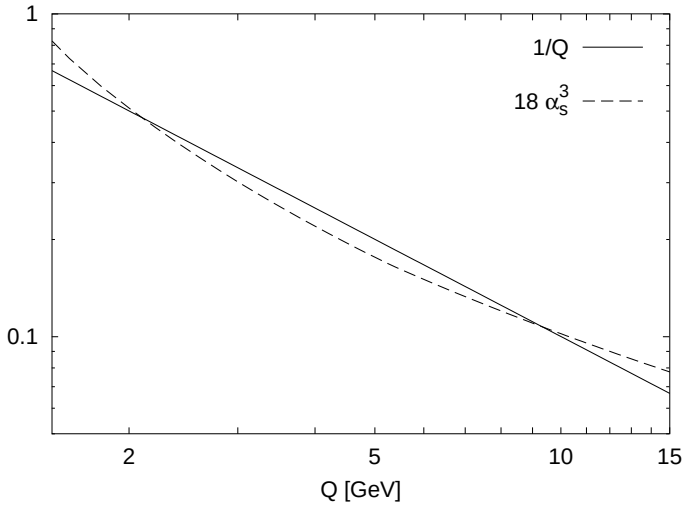

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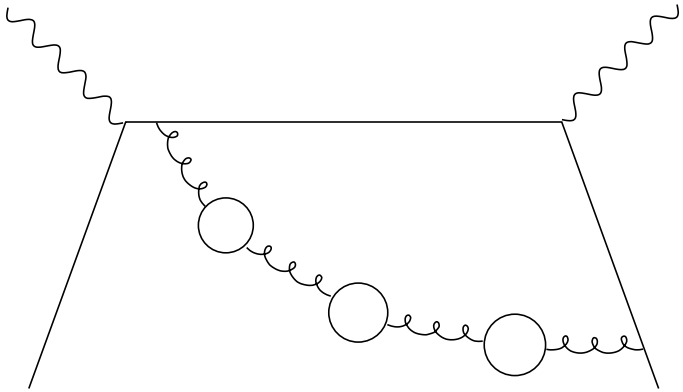

NP shift of PT distributions

- NP emissions give rise to a shift of the PT distribution!
  $t = t_{\mathsf{PT}} + \delta t \simeq t_{\mathsf{PT}} + \frac{\epsilon}{Q}$ $\qquad \epsilon \sim \Lambda_{QCD}$

$$\frac{1}{\sigma}\frac{d\sigma}{dt} = \int_0^{tQ} d\epsilon\, f(\epsilon)\, \frac{d\sigma_{\mathsf{PT}}(t-\frac{\epsilon}{Q})}{\sigma\, dt} \simeq \frac{d\sigma_{\mathsf{PT}}(t-\frac{\langle\epsilon\rangle}{Q})}{\sigma\, dt}$$

- $1/Q$ power correction to $\langle V\rangle \Rightarrow$ full (PT+NP) event shape distributions for $V \gg \Lambda_{QCD}/Q$

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Feynman tube model and leading power corrections

- Hadron multiplicity is uniform in rapidity

$$\frac{dn_h}{d\eta\, d\ln k_t} = \Phi_h(k_t)$$

- Power correction to linear event shapes:

$$\delta V(\{k_i\}) \simeq \sum_i \frac{k_{ti}}{Q} f_V(\eta_i)$$

$$\langle \delta V \rangle = \int \frac{dk_t}{k_t} \frac{k_t}{Q} \sum_h \Phi_h(k_t) \int d\eta f_V(\eta) = \frac{\langle k_t \rangle}{Q} c_V$$

- Leading power corrections are universal, i.e. after factorisation of rapidity dependence, they depend only on $\langle k_t \rangle$

$$\frac{\langle C \rangle_{\text{NP}}}{\langle t \rangle_{\text{NP}}} = \frac{4\ \text{GeV}}{1\ \text{GeV}} \simeq \frac{3\pi}{2} = \frac{c_C}{c_t}$$

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From two- to multi-jet events

- Intra-jet hadrons (collinear to jet direction $\vec{n}_i$)

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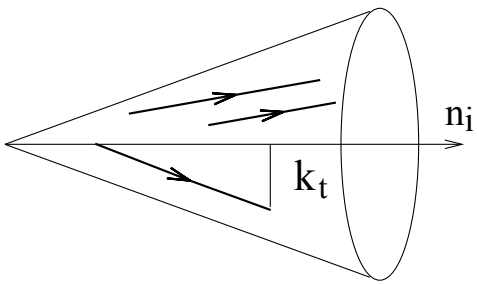

- Inter-jet hadrons (large angles)

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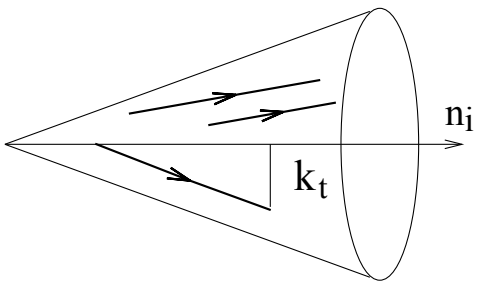

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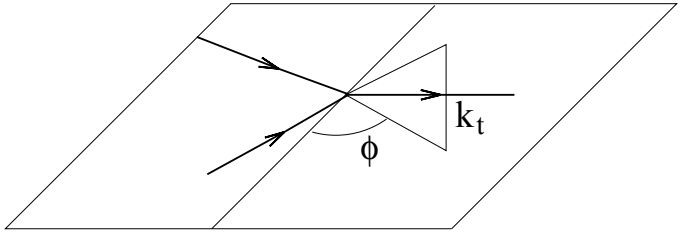

Soft radiation and confi nement

- Soft dressed gluon emission from a $q\bar{q}$ dipole

$$\frac{dw}{d\ln k_t d\eta} = 2A_q[\alpha_s^{\overline{\rm MS}}(k_t)] = 2C_F\frac{\alpha_s^{\rm CMW}(k_t)}{\pi} \to \sum_h \Phi_h(k_t)$$

$$k_t^2 = \frac{(2pk)(2k\bar{p})}{2p\bar{p}} \qquad \eta = \frac{1}{2}\ln\frac{\bar{p}k}{pk}$$

- Soft dressed gluon emission from more dipoles

$$dw = \sum_{i<j}(-2\vec{T}_i\cdot\vec{T}_j)\frac{d\kappa_{ij}}{\kappa_{ij}}d\eta_{ij}\frac{\alpha_s^{\rm CMW}(\kappa_{ij})}{\pi} \to \sum_{i<j}\frac{d\kappa_{ij}}{\kappa_{ij}}d\eta_{ij}\sum_h \Phi_h^{(ij)}(\kappa_{ij})?$$

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Validity of the soft approximation

Power corrections for $V \gg \Lambda_{QCD}$ arise from $\langle k_t \rangle \ll VQ$.

What about hard collinear emissions?

- V damped in rapidity (example: thrust)

$$\delta V(k) \sim k_t \, e^{-|\eta|} \qquad |\eta| \to \infty$$

$\langle k_t \rangle$ receives contributions only from $\eta \sim 0$ (soft large angle)

- V uniform in rapidity (example: broadening)

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Dispersive coupling

- The CMW coupling can be extended at low scales via the dispersion relation

$$\alpha_s(\kappa^2) = \kappa^2 \int_0^\infty \frac{dm^2}{(m^2+\kappa^2)^2} \alpha_{eff}(m^2)$$

- LPHD philosophy $\Rightarrow$ hadron flow $\sim$ parton flow
- Hadronisation corrections are due to extra-soft gluons (gluers) with transverse momenta $\kappa \sim \Lambda_{QCD}$, whose emission probability is ruled by the NP part of the dispersive coupling.

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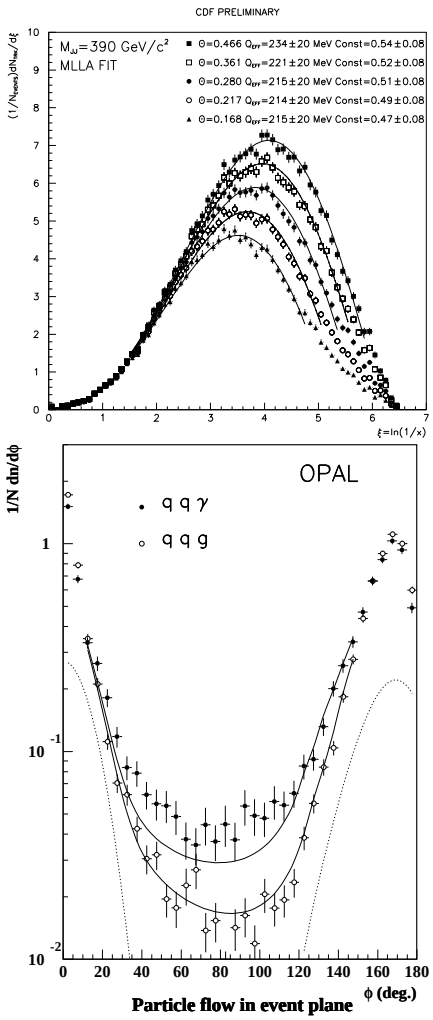

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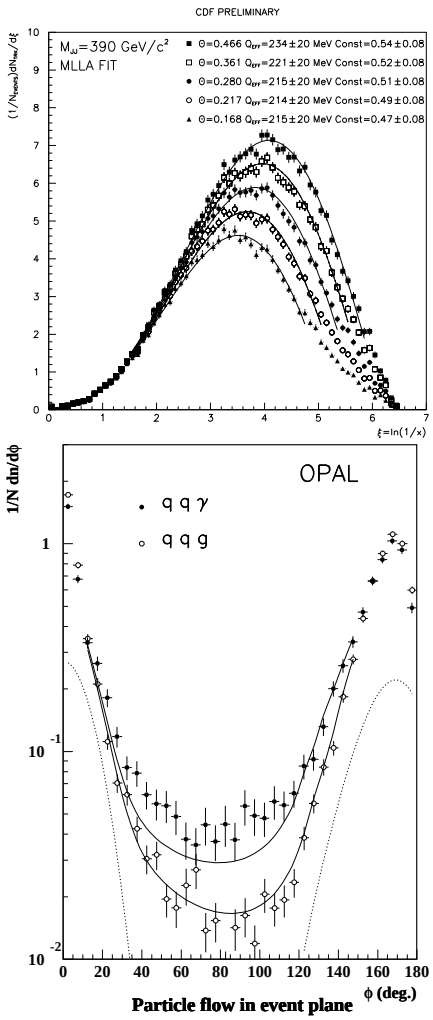

Power corrections in the DMW approach

- Combine real and virtual corrections and obtain

$$\frac{\alpha_s(\kappa^2)}{\kappa^2} \to \int_0^\infty \frac{dm^2}{(m^2+\kappa^2)^2}\alpha_{eff}(m^2)$$

- Allow gluon to decay inclusively $\kappa^2 \to \kappa^2 + m^2$

$$\langle \delta V \rangle_{\rm naive} = \frac{\langle \kappa \rangle_{\rm naive}}{Q} \sum_{i<j} (-2\vec{T}_i \cdot \vec{T}_j) c_V^{(ij)}$$

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- Take into account non-inclusiveness via the Milan factor

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Universality of $1/Q$ corrections $\Rightarrow$ $dn_h/d\ln k_t d\eta$ is uniform in η!

- Multi-jet event shapes $\Rightarrow$ test PQCD-inspired hadronisation

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1. $\langle \kappa \rangle$ same as in two-jet event shapes $\Rightarrow$ Universality
2. dependence on hard parton colour through $\vec{T}_i \cdot \vec{T}_j$
3. dependence on event geometry through $c_V^{(ij)}$

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1. $\langle \kappa \rangle$ same as in two-jet event shapes $\Rightarrow$ Universality
2. dependence on hard parton colour through $\vec{T}_i \cdot \vec{T}_j$
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Power corrections to multi-jet shapes

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$$\langle \delta V \rangle = 2C_F \frac{\langle \kappa \rangle}{Q} c_V = \frac{\langle k_t \rangle}{Q} c_V$$

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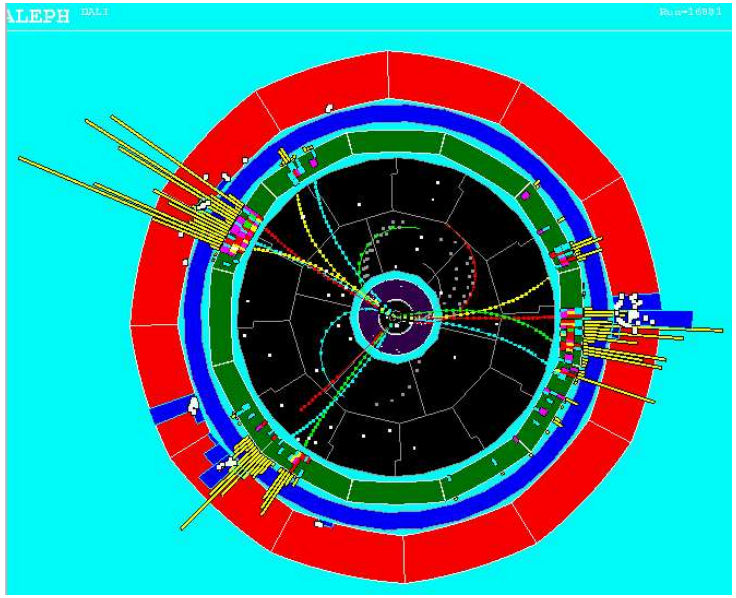
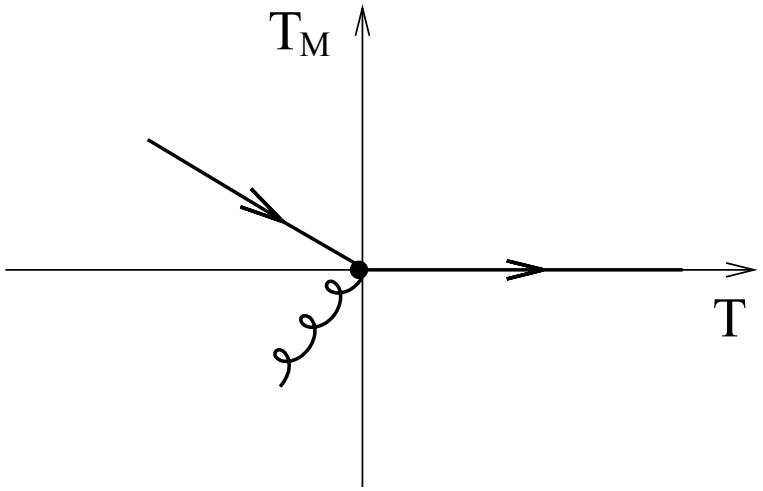
Three-jet events





- Thrust-major:

$$T_M\, Q \equiv \max_{\vec{n}_M \cdot \vec{n}_T = 0} \sum_i |\vec{p}_i \cdot \vec{n}_M|$$

- Event plane: $\langle \vec{n}_T ; \vec{n}_M \rangle$

$$\vec{p}_{ti} = p_i^{\rm in}\, \vec{n}_{\rm in} + p_i^{\rm out}\, \vec{n}_{\rm out} \qquad \vec{n}_{\rm out} \equiv \vec{n}_T \times \vec{n}_M$$

D-parameter and Thrust-minor

- D-parameter: determinant of the momentum tensor $\theta_{\alpha\beta}$

$$\theta_{\alpha\beta}\, Q \equiv \sum_h \frac{p_h^\alpha p_h^\beta}{|\vec{p}_h|} \qquad D \equiv 27 \det \theta$$

$$\delta D \simeq 27 \lambda_1 \lambda_2 \sum_i \frac{\kappa_i^2 \sin^2 \phi_i}{\omega_i Q} \qquad \text{damped in rapidity}$$

$$c_D = C_F\, g_q(T, T_M) + C_F\, g_{\bar{q}}(T, T_M) + C_A\, g_g(T, T_M)$$

- Thrust-minor: sum of the momenta out of the event plane

$$K_{\rm out} \equiv \sum_h |p_h^{\rm out}| \quad \delta K_{\rm out} \simeq \sum_i \kappa_i |\sin \phi_i| \qquad \text{uniform in rapidity}$$

$$c_{K_{\rm out}} = \langle |\sin \phi| \rangle \left(C_F \ln \frac{Q_{q\bar{q}}}{|p_q^{\rm out}|} + C_F \ln \frac{Q_{q\bar{q}}}{|p_{\bar{q}}^{\rm out}|} + C_A \ln \frac{Q_{qg} Q_{g\bar{q}}}{|p_g^{\rm out}| Q_{q\bar{q}}} \right)$$

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- Select 3-jet events with $y_3 > y_{\rm cut}$
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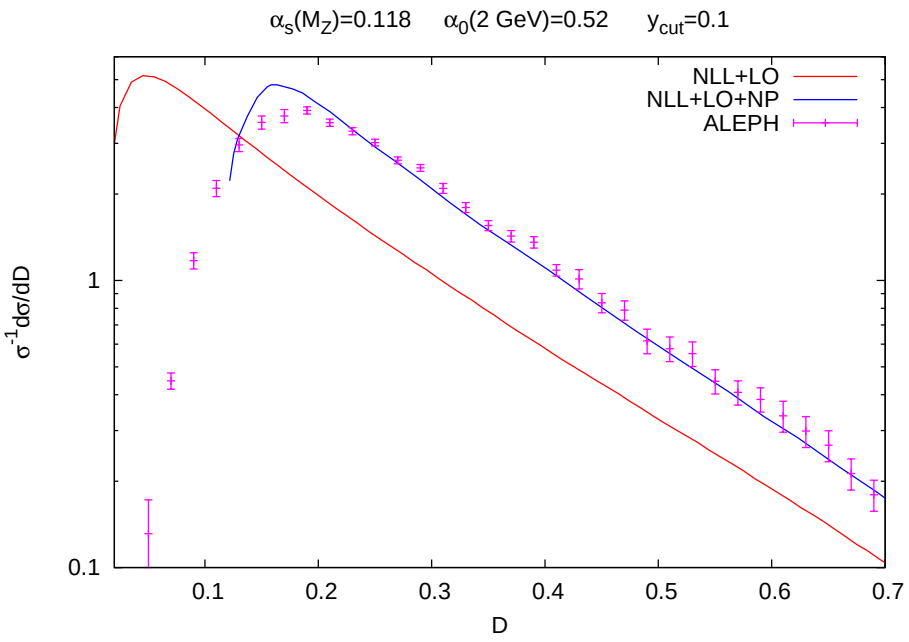

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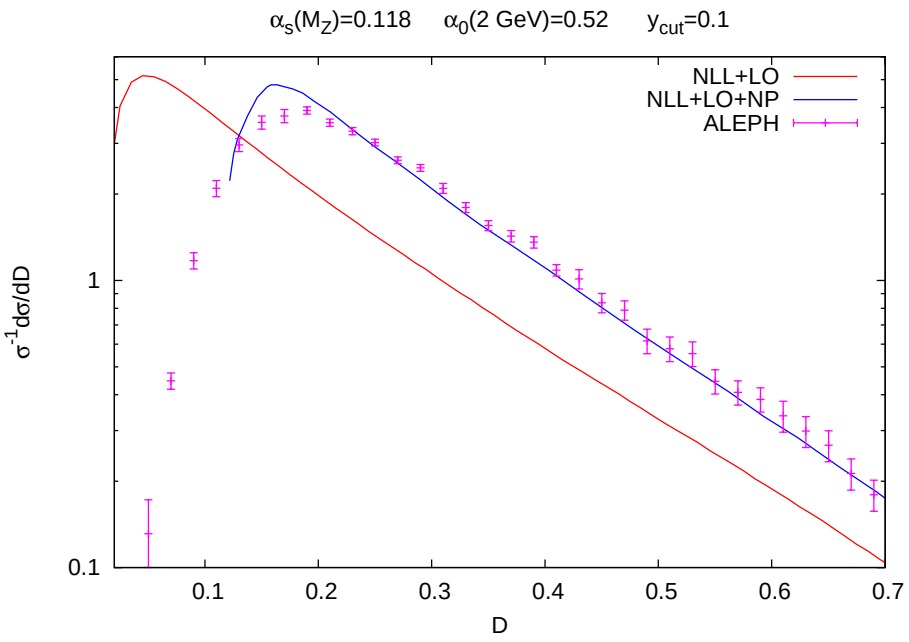

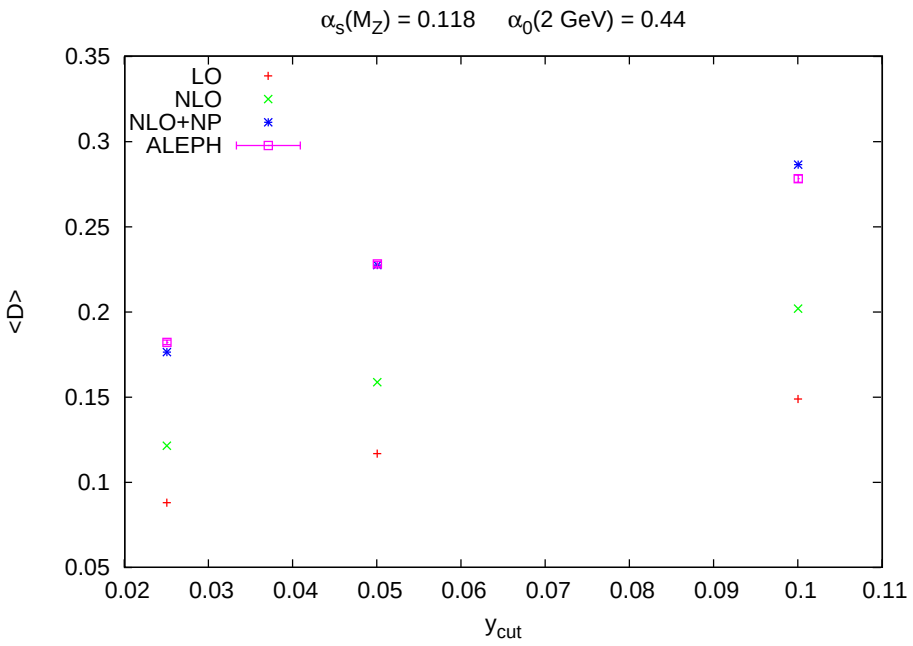

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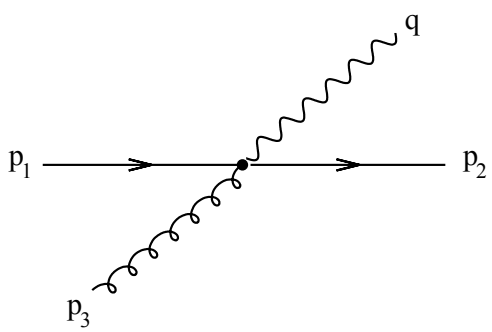
$K_{\rm out}$ in hadronic Z_0 production



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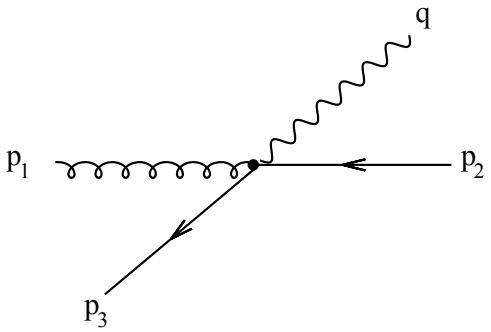

(g)

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rapidity cut around beam pipe

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$$\delta K_{\rm out} \simeq \underbrace{\sum_i \kappa_i |\sin\phi_i|}_{\text{QCD emissions}} + \underbrace{\sum_i k_{ti} |\sin\phi_i|}_{\text{beam remnant}}$$

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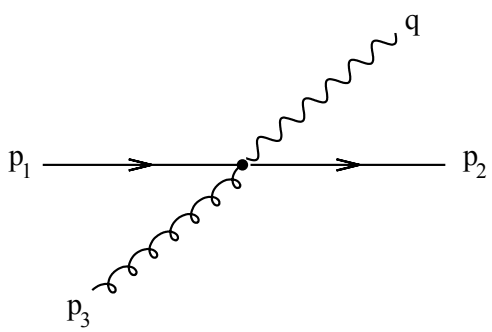

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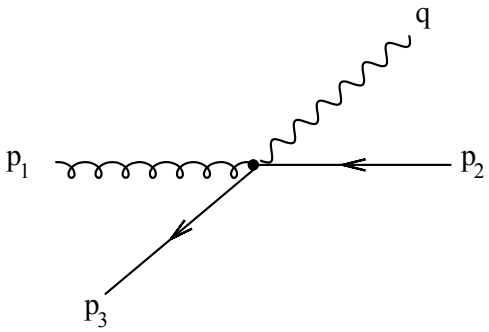

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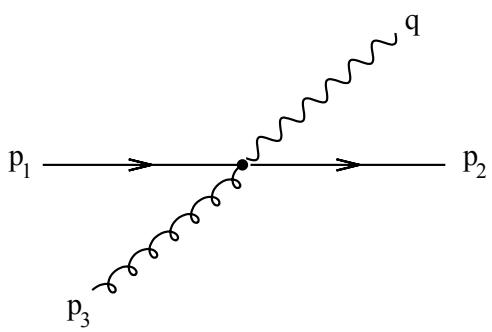

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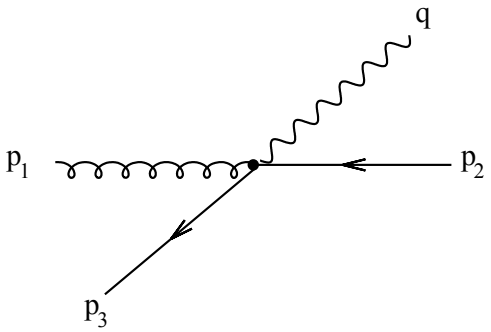

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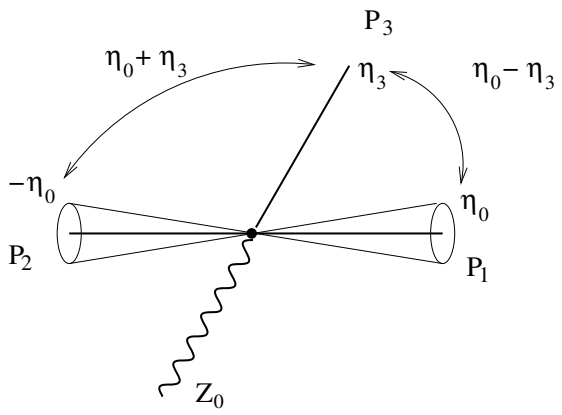
- QCD emissions with $\kappa \sim \Lambda_{QCD}$ (gluers)



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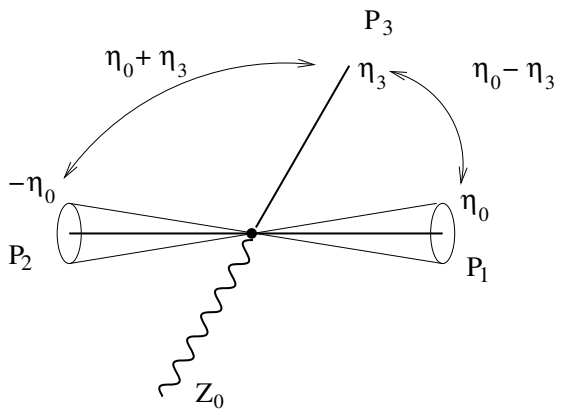

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Conclusions

- Multi-jet event shapes benefits
  1. More stringent tests of universality
  2. $1/Q$ corrections depend on hard parton colour and geometry
- Three-jet event shapes are coming soon
  1. NLO+PC already there for D parameter means
  2. Work in progress for NLL+NLO+PC
- Four jets and more
  1. Quantum evolution of colour?
  2. Soft underlying event (already there in DY)