DISTRIBUTED FEEDBACK SURFACE-WAVE SMITH-PURCELL FEL

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Abstract

Smith-Purcell radiation is the emission of electromagnetic radiation by an electron beam passing next to an optical grating. Recent measurement of relatively intense power of such radiation was observed in the THz-regime [1]. To explain the high intensity and the super-linear dependence on current beyond the threshold it was suggested that the radiating device operated in the high gain regime, amplifying spontaneous emission (ASE)[1,2].

We contest this interpretation and suggest an alternative mechanism. According to our interpretation the device operates as a distributed feedback (DFB) laser oscillator, in which a forward going surface wave, excited by the beam on the grating surface, is coupled to a backward going surface wave by a second order Bragg reflection process. This feedback process produces a saturated oscillator.

Another plausible mechanism for explaining the experiment [3,4] is Backward Wave Oscillation (BWO) with a surface wave, accompanied by second harmonic bunching of the e-beam at saturation, and consequent coherent (superradiant) Smith-Purcell radiation.

We present analytical models for the two proposed mechanisms, and compare them. The models predict different emission frequencies and angles. The first model matches better the experimental findings.

MODEL FOR CALCULATING DFB INTERACTION OF SURFACE WAVES NEAR BRAGG CONDITION

Our analysis is based on the field excitation equations, in the region above a periodic structure. The field is a periodic function of z, described according to the Floquet theorem:

$$\tilde{\mathbf{A}}(\mathbf{r}) = \sum_{m=-\infty}^{\infty} \tilde{\mathbf{A}}_m(x,y)\exp(ik_{z_m}z)$$

Where $k_{z_m}(\omega) = k_{z_0}(\omega) + m\cdot k_w$, $k_w = 2\cdot\pi/\lambda_w$, and λ_w is the period of the grating.

Assuming $\frac{\partial}{\partial x} = 0$, we are looking for a solution of a Floquet mode for which all space harmonics are evanescent (a surface wave). This will happen if for all *m:*

$$|k_{z_m}| \equiv |k_{z_0} + m\cdot k_w| > \frac{\omega}{c}$$

so that all $\alpha_m = \sqrt{k_{z_m}^2 - k_0^2}$ are real.

The solution is in the form:

$$\tilde{E}_y = i\sum_m \frac{A_m k_{z_m}}{\alpha_m}\varepsilon_{z_m}(y)\exp(ik_{z_m}z)$$

$$\tilde{E}_z = \sum_m A_m\varepsilon_{z_m}(y)\exp(ik_{z_m}z)$$

$$\tilde{H}_x = i\omega\varepsilon_0\sum_m \frac{A_m}{\alpha_m}\varepsilon_{z_m}(y)\exp(ik_{z_m}z)$$

Where: $\varepsilon_{z_m}(y) = \exp(-\alpha_m y)$, and the amplitudes of the space harmonics have to be calculated numerically [7].

INTERACTION WITH ELECTRON BEAM - COUPLED MODES MODEL

In this model there are two modes, propagating in different directions, as indicated by the figure below. The forward propagating mode is described by: af(z)=cf(z)exp(ikzz), and the backward propagating mode – by: ab(z)=cb(z)exp(-ikzz).

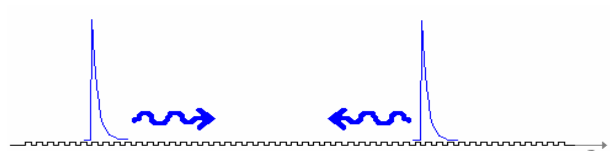

Figure 1: Coupled modes model.

Two kinds of wave interaction are considered in this model:

a. Interaction between the electron beam and a forward propagating mode. This coupling is modeled by the excitation equation for the amplitude $a_f(z)$, according to the familiar theory of TWT.
b. Interaction between a forward propagating mode and a backward mode. The two modes are coupled through the grating. This coupling is modeled through a coupling coefficient parameter κ_G [5].

The amplitudes of the two modes are described according to the following set of coupled differential equations:

$$\frac{d\tilde{a}_f(z)}{dz} - ik_z\tilde{a}_f(z) = -\frac{1}{4P_q}\iint_{A_e}\tilde{J}_z\varepsilon^*_{fz}\,dxdy + \kappa_G\tilde{a}_b(z)e^{ik_w z}$$

$$\frac{d\tilde{a}_b(z)}{dz} + ik_z\tilde{a}_b(z) = \kappa^*_G\tilde{a}_f(z)e^{-ik_w z}$$

In the case when $\kappa_G=0$ (no coupling to the grating), one will expect to get the familiar case of TWT. The other limit, when $J=0$, yields result predicted by coupled mode theory, for a Bragg reflector or DFB laser [5].

The set of coupled equations is solved through a Laplace transform, when the current is modelled according to:

$$J_z(s) = L\{\tilde{J}_z(z)\} = -i\omega \frac{\chi_p(s,a)}{1+\chi_p(s,\omega)/\varepsilon_0}\varepsilon_f(x_e,y_e)a_f(s)$$

$$\chi_p(s,\omega) = -\frac{\omega_p'^2}{(\omega+isv)^2}\varepsilon_0$$

If space-charge effects are neglected, we end up with the following gain-dispersion equation:

$$\frac{\bar{a}_b(s')}{a_b(0)} = \frac{(s'-i\Delta k)(s'-i\theta)^2 - iQ}{(s'+i\Delta k)\left[(s'-i\Delta k)(s'-i\theta)^2 - iQ\right] - (s'-i\theta)^2|\kappa_G|^2}$$

Where:

$$s' \equiv s + i\frac{k_w}{2} \qquad \theta \equiv \frac{\omega}{v} - \frac{k_w}{2}$$

$$Q \equiv \kappa_J \frac{\omega'^2_p}{v^2} \qquad \kappa_J = \frac{1}{2}\frac{Z_q}{Z_0}\frac{A_e}{A_{em}}\frac{\omega}{c}$$

BWO MODEL

As an alternative description, we introduce following [3,4], an analytical model based on interaction of a backward going Floquet wave with the electron beam. This is described by the following excitation equation:

$$\frac{da(z)}{dz} - ik_{z0}a(z) = -\frac{1}{4P}\sum_{m=-\infty}^{\infty} e^{-imk_w z}\iint_{A_e} \tilde{J}_z \cdot \tilde{\varepsilon}_m^* dxdy$$

Where $k_{z0}=-|k_{z0}|$ and $a(z)$ is the fast varying amplitude of the total Floquet mode. Note that in this case, the field-current interaction is through the mth space harmonic, so that the Laplace transform of the current density takes the form:

$$J_z(s) = -i\omega\varepsilon_0 \frac{\frac{\chi_p(s)}{\varepsilon_0}}{1+\frac{\chi_p(s)}{\varepsilon_0}}\varepsilon_m(y)\bar{a}(s-imk_w)$$

Assuming interaction with the $m=1$ space-harmonic, the resulting gain-dispersion equation is:

$$\bar{a}(s) = \frac{(s-ik_{z0}-i\theta)^2+\theta_p^2}{\left[(s-ik_{z0}-i\theta)^2+\theta_p^2\right](s-ik_{z0}) - i\kappa\theta_p^2}a(0)$$

Where:

$$\theta \equiv \frac{\omega}{v} - k_{z0} - k_w, \theta_p^2 \equiv \frac{\omega'^2_p}{v^2}, Q \equiv \kappa\theta_p^2$$

We note that this result is a special case of a more general result obtained in [6]. To explore this result, gain contours of the transfer function $H(L,\theta,Q) = \frac{\tilde{a}(L)}{a(0)}$ were plotted in the figure below. Conditions for backward wave oscillation are the points in the (θ,Q) plane where $|H(L,\theta,Q)|^2=0$.

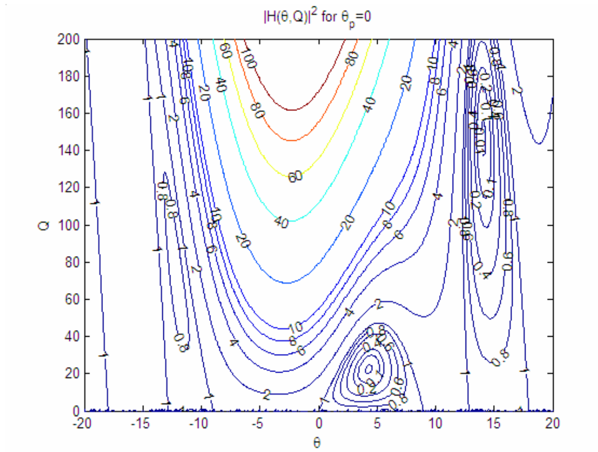

Figure 2: Gain contours for BWO model.

COMPARISION OF THE THREE MODELS

We will now briefly review the the predictions from the three models regarding the Dartmouth college Smith-Purcell FEL experiment.

a. SASE Smith-Purcell (Bakhtyari, Walsh, Brownell [1]). In this model, the radiation mechanism is Self Amplified Spontaneous Emission ("Superradiance"). The emission frequency is determined by:

$$\frac{a_r}{v} - k_w = \frac{a_r}{c}\cos\Theta$$

The radiation is expected at wavelength of $\lambda=500\mu m$, at an angle $\Theta=90^o$. The drawback of this model is that it requires high gain, without a feedback mechanism.

b. Nonlinear Harmonic Bunching in BWO (Andrew, Brau [2], Donohue, Gardelle [4]). This model involves three processes: backward wave oscillation (ω_0), nonlinear bunching at saturation ($2\omega_0$), and 1st order Smith-Purcell superradiance (at $2\omega_0$). The frequency and angle of emission is determined by:

$$\frac{a_0}{v} = k_{z_s}(\omega_0); \omega_r = 2\omega_0$$

$$\cos\Theta = \frac{\omega_r/v - k_w}{\omega_r/c}$$

This model is based on feedback for ω_0, and superradiance for $\omega_r=2\omega_0$. Radiation is expected at wavelength of $\lambda=350\mu m$, at an angle $\Theta=33^o$, which deviates from experiment.

c. Surface Wave DFB (Gover, Kipnis, Dyunin). This model suggests the existence of the three following processes: Surface wave interaction with e-beam (BWO/TWT), 2nd order Bragg coupling to backward surface wave (DFB oscillator), and 1st order grating diffraction to transverse radiation wave. The emission frequency is determined by:

$$k_z(\omega_r) = \frac{\omega}{v}$$

$$k_z(\omega_r) - 2k_w = -k_z(\omega_r)$$

$$\Rightarrow \frac{\omega_r}{v} = k_w$$

Radiation is expected at $\lambda = 500\mu m$, $\Theta = 90^o$.

CONCLUSIONS

Surface wave DFB Smith-Purcell FEL is a possible source for coherent THz-radiation. In the case when the grating surface wave satisfies second order Bragg-condition ($k_z = \omega/v = k_w$) a process of radiating DFB Smith-Purcell Lasing is possible at a wavelength of $\lambda = \lambda_W/\beta$. Another possible mechanism for the Dartmouth College experiment is BWO oscillation accompanied with second harmonic supperradiant emission of $\lambda = \lambda_{BWO}/m$, with $m=2$. The "Amplified Spontaneous Emission (Superradiance)" is not a plausible explanation of the Dartmouth College experiment. The DFB model fits the experiment better.

REFERENCES

[1] A.Bakhtyari, J.E.Walsh, J.H.Brownell, Phys.Rev. E65 006503 (2002).

[2] H.L. Andrews, C.A. Brau, Phys.Rev. ST-AB 7, 070701 (2004).

[3] H.L. Andrews, C.H. Boulware, C.A. Brau, J.D. Jarvis, Phys. Rev. TA-AB 8, 050703 (2005).

[4]] J.T. Donohue, J. Gardelle, Phys. Rev. ST-AB 8, 060702 (2005).

[5] A. Yariv, "Quantum Electronics", *John Wiley & Sons Inc*, 2nd ed. 1975.

[6] B. Steinberg, A. Gover, S. Ruschin, *Phys. Rev. A,* Vol 33, No. 1, (1986).

[7] R.Collin "Foundations for Microwave Engineering", pp.571-577, 1992.