BACKWARD WAVE EXCITATION AND GENERATION OF OSCILLATIONS IN FREE-ELECTRON LASERS IN THE ABSENCE OF FEEDBACK -- BEYOND THE HIGH GAIN APPROXIMATION

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Abstract

Quantum and free-electron lasers (FELs) are based on distributed interactions between electromagnetic radiation and gain media. In an amplifier configuration, a forward wave is amplified while propagating in a polarized medium. Formulating a coupled mode theory for excitation of both forward and backward waves, we identify conditions for phase matching, leading to efficient excitation of backward wave without any mechanism of feedback or resonator assembly. The excitations of incident and reflected waves are described by a set of coupled differential equations expressed in the frequency domain. The induced polarization is given in terms of an electronic susceptibility tensor. In quantum lasers the interaction is described by two first order differential equations. In free-electron lasers, the excitation of the forward and backward modes is described by two coupled third order differential equations. In our previous investigation analytical and numerical solutions of reflectance and transmittance for both quantum lasers and high-gain FELs were presented. In this work we extend the study to a general free-electron laser without restriction of the high-gain approximation. It is found that when the solutions become infinite, the device operates as an oscillator, producing radiation at the output with no field at its input, entirely without any localized or distributed feedback.

INTRODUCTION

Conventional (quantum) lasers, microwave tubes and free-electron lasers (FELs) are based on distributed interactions between electromagnetic radiation and gain media. When such devices are operating in an amplifier configuration, a forward wave is amplified while propagating in a polarized medium, in a stimulated emission process [1]. In an oscillator configuration a resonator [2]-[4] or a distributed feedback in quantum lasers [5] and in free-electron lasers [6, 7] are employed to circulate the radiation, which is excited and amplified by the gain medium. If the single-pass gain is higher than the total losses, the radiation intensity inside the cavity increases and becomes more coherent.

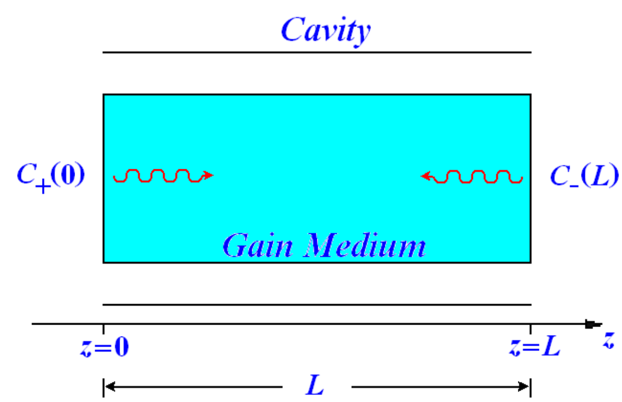

Figure 1: Schematic illustration of incident and reflected waves in a distributed gain medium.

After several round trips, the radiation is built up until arriving at the nonlinear regime and saturation.

In this paper we suggest a mechanism of generation of laser oscillations, without any feedback means. This extends our previous study [8] dealt with free-electron lasers operating in high-gain limit. It is shown that under conditions of phase-matching, both forward and backward waves can be excited in a distributed gain medium as illustrated schematically in Figure 1. The excitation of incident and reflected waves is described by a set of two differential equations coupled by the induced polarization of the gain media. The coupling coefficient is given in terms of the electronic susceptibility tensor. The mechanism was first suggested as a possible explanation for generation of parasitic oscillations near the waveguide cut-off frequency in a waveguide-based FEM configured as an amplifier [9, 10].

Two cases are discussed: In quantum lasers, which are characterized by isotropic, homogeneous gain media, the interaction is described by two first order differential equations. In free-electron lasers [11] where the susceptibility is space dependent, the set includes two differential equations of the third order each. The coupled equations sets are solved analytically for both cases. Oscillation conditions are identified from the derived reflectance and transmittance coefficients.

EXCITATION OF FORWARD AND BACKWARD MODES

The total electromagnetic field is given by the time harmonic wave vector:

$$\mathbf{E}(\mathbf{r},t) = \Re\left\{\widetilde{\mathbf{E}}(\mathbf{r})e^{-j\omega t}\right\} \tag{1}$$

where $\widetilde{\mathbf{E}}(\mathbf{r})$ is the phasor of the wave oscillating at an angular frequency ω. The vector $\mathbf{r}$ stands for the (x, y, z) coordinates, where (x, y) are the transverse coordinates and z is the axis of propagation. In the case of excitation of forward and backward modes, the phasor can be written as the sum [12, 13]:

$$\widetilde{\mathbf{E}}(\mathbf{r}) = \left[C_+(z)\,e^{+jk_z z} \;+\; C_-(z)\,e^{-jk_z z}\right]\,\widetilde{\mathcal{E}}(x,y) \tag{2}$$

$C_+(z)$ and $C_-(z)$ are scalar amplitudes of forward and backward modes respectively, with profile $\widetilde{\mathcal{E}}(x, y)$ and axial wavenumber k_z. The evolution of the amplitudes of the excited modes is described by a set of two coupled differential equations [14]:

$$\frac{d}{dz}C_\pm(z) = \mp\frac{1}{2\mathcal{N}}e^{\mp jk_z z}\int\!\!\int \widetilde{\mathbf{J}}(\mathbf{r})\cdot\widetilde{\mathcal{E}}^*(x,y)\,dx\,dy \tag{3}$$

The normalization of the mode amplitudes is made via the complex Poynting vector power:

$$\mathcal{N} = \int\!\!\int \left[\widetilde{\mathcal{E}}_\perp(x,y)\times\widetilde{\mathcal{H}}^*_\perp(x,y)\right]\cdot\hat{\mathbf{z}}\,dx\,dy \tag{4}$$

The total power carried by the forward and backward (propagating) modes is:

$$\begin{aligned} P(z) &= \frac{1}{2}\Re\int\!\!\int\left[\widetilde{\mathbf{E}}(\mathbf{r})\times\widetilde{\mathbf{H}}^*(\mathbf{r})\right]\cdot\hat{\mathbf{z}}\,dx\,dy \\ &= \frac{1}{2}\left[|C_+(z)|^2 - |C_-(z)|^2\right]\cdot\Re\{\mathcal{N}\} \end{aligned} \tag{5}$$

When the interaction takes place in a polarized gain medium, the driving current density $\widetilde{\mathbf{J}}(\mathbf{r})$ is given in terms of the induced polarization (dipole moment per unit volume) $\widetilde{\mathbf{P}}(\mathbf{r})$. In the time domain, the current density is the time derivative of the induced polarization. Thus, the phasor representation of the driving current density is given by:

$$\widetilde{\mathbf{J}}(\mathbf{r}) = -j\omega\widetilde{\mathbf{P}}(\mathbf{r}) = -j\omega\varepsilon_0\,\chi(\mathbf{r},\omega)\cdot\widetilde{\mathbf{E}}(\mathbf{r}) \tag{6}$$

where $\chi(\mathbf{r}, \omega)$ is the electronic susceptibility tensor at the frequency ω (in a homogeneous isotropic medium it is a scalar). Using (6) in (3) results in:

$$\begin{aligned} &\frac{d}{dz}C_\pm(z) = \\ &\pm j\frac{\omega\varepsilon_0}{2\mathcal{N}}e^{\mp jk_z z}\int\!\!\int\widetilde{\mathbf{E}}(\mathbf{r})\cdot\chi(\mathbf{r},\omega)\cdot\widetilde{\mathcal{E}}^*(x,y)\,dx\,dy \end{aligned} \tag{7}$$

Substitution of the field expansion (2) in the excitation equations (7), the mode amplitudes $C_\pm(z)$ are described by a set of two coupled differential equations, that can be presented in a matrix form:

$$\begin{aligned} &\frac{d}{dz}\begin{bmatrix} C_+(z) \\ C_-(z)\end{bmatrix} = \\ &\begin{bmatrix} +\kappa(z) & +\kappa(z)e^{-j2k_z z} \\ -\kappa(z)e^{+j2k_z z} & -\kappa(z)\end{bmatrix}\begin{bmatrix} C_+(z) \\ C_-(z)\end{bmatrix} \end{aligned} \tag{8}$$

The coupling parameter:

$$\begin{aligned} &\kappa(z,\omega) \equiv \\ &j\frac{\omega\varepsilon_0}{2\mathcal{N}}\int\!\!\int\widetilde{\mathcal{E}}(x,y)\cdot\chi(\mathbf{r},\omega)\cdot\widetilde{\mathcal{E}}^*(x,y)\,dx\,dy \end{aligned} \tag{9}$$

is in general a complex, space-frequency dependent quantity. Since the matrix (8) is singular (the determinant is equal to zero), the following relation is derived:

$$\frac{d}{dz}C_-(z) = -e^{+2jk_z z}\frac{d}{dz}C_+(z) \tag{10}$$

QUANTUM LASER

We relate first to gain media, where the electronic susceptibility does not change along the axis of propagation z. This situation occurs in quantum lasers, where the atomic susceptibility of the gain medium is uniform [1]. In that case the coupling parameter is not yet space (z) dependent and can be presented in the complex form $\kappa(\omega) = \gamma(\omega) + j\beta(\omega)$, where $\gamma(\omega)$ is the field gain factor. Consequently, the set (8) can be written as two coupled first order linear differential equations:

$$\begin{aligned} &\frac{d}{dz}\begin{bmatrix} C_+(z) \\ C_-(z)\end{bmatrix} = \\ &\begin{bmatrix} +\kappa & +\kappa e^{-j2k_z z} \\ -\kappa e^{+j2k_z z} & -\kappa\end{bmatrix}\begin{bmatrix} C_+(z) \\ C_-(z)\end{bmatrix} \end{aligned} \tag{11}$$

Analytical solution of the coupled set (11) for a given forward mode amplitude $C_+(0)$ at the input $z = 0$, while the backward mode amplitude at the exit of the interaction region ($z = L$) is $C_-(L) = 0$, leads to the solution of incident and reflected wave amplitudes:

$$\begin{aligned} \frac{C_+(z)}{C_+(0)} &= \\ &\frac{(\kappa+jk_z)\sinh\left[S(L-z)\right] - S\cosh\left[S(L-z)\right]}{(\kappa+jk_z)\sinh(SL) - S\cosh(SL)}\,e^{-jk_z z} \\ \frac{C_-(z)}{C_+(0)} &= \frac{-\kappa\sinh\left[S(L-z)\right]}{(\kappa+jk_z)\sinh(SL)-S\cosh(SL)}\,e^{+jk_z z} \end{aligned} \tag{12}$$

where $S \equiv \sqrt{(\kappa+jk_z)^2-\kappa^2}$ is a complex parameter. The evolution of incident and reflected wave amplitudes along the gain medium are shown in Fig. 2. It is assumed that the interaction takes place in the vicinity of the resonance frequency, where $\kappa(\omega_0)$ is real

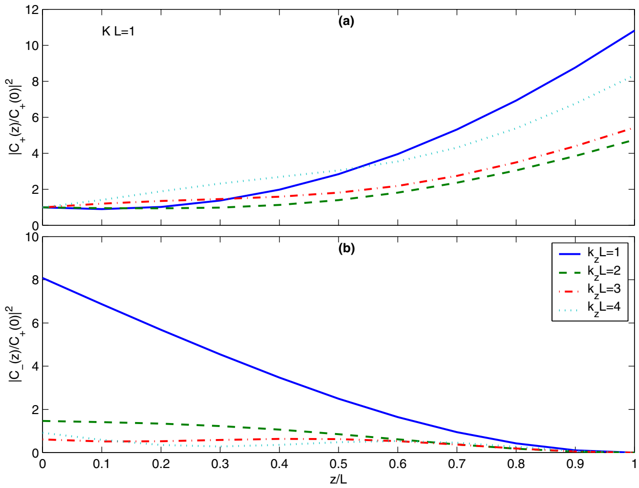

Figure 2: The evolution of (a) incident and (b) reflected wave amplitudes along the gain medium.

The transmission gain is defined by:

$$\frac{C_+(L)}{C_+(0)} = \frac{-SL}{(\kappa + jk_z)L\sinh(SL) - SL\cosh(SL)}\, e^{-jk_zL} \tag{13}$$

Respectively, the reflection gain is:

$$\frac{C_-(0)}{C_+(0)} = \frac{-\kappa L\sinh(SL)}{(\kappa + jk_z)L\sinh(SL) - SL\cosh(SL)} \tag{14}$$

Contour plots of the transmission and reflection power gain in the $(k_zL, \kappa L)$ plane are shown in Figure 3. An infinite gain singularities are inspected when the denominator of the gain dispersion relations given in (13) and (14) vanishes. This happens when:

$$\tanh(SL) = \frac{SL}{\kappa L + jk_zL} \tag{15}$$

In that case the forward and backward modes will be excited in the absence of an input signal, resulting in excitation and buildup of oscillations. Equation (15) expresses the oscillation condition, determining the threshold gain factor required for excitation of oscillations and their resultant frequencies at steady-state.

FREE-ELECTRON LASERS

In free-electron lasers, the accelerated electrons serve as a gain medium and the interaction with the electromagnetic field takes place along the e-beam axis. Coupled mode theory for multi transverse mode excitation was developed previously, deriving an expression for the gain-dispersion relation in the linear regime of the FEL operation [11]. Set of equations (3) for the different modes were solved together with the small-signal moment equations describing the evolution in the driving current modulation.

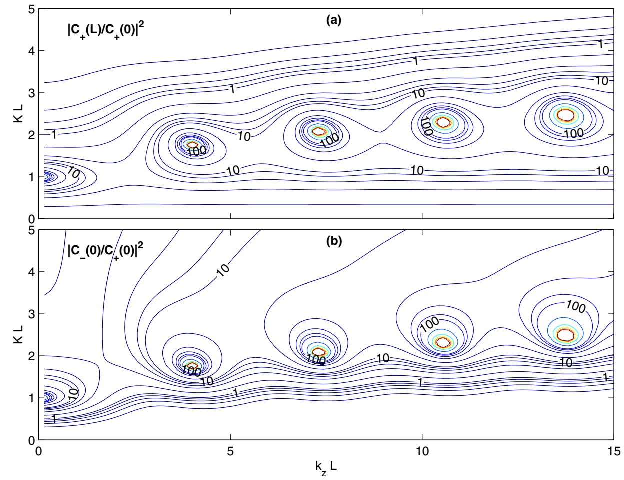

Figure 3: (a) Transmission and (b) reflection contours in the $(k_zL, \kappa L)$ plane for atomic laser. Logarithmic scale in [dB].

In free-electron lasers, the excitation of forward and backward waves is described by two coupled differential equations of the third order each [11]:

$$\begin{aligned}
\frac{d^3}{dz^3}C_+(z) - 2j\Theta_+\frac{d^2}{dz^2}C_+(z) + \left(\Theta_{pr}^2 - \Theta_+^2\right)\frac{d}{dz}C_+(z) \\
= j\kappa C_+(z) + j\kappa C_-(z)e^{-j2k_zz} \\
\frac{d^3}{dz^3}C_-(z) - 2j\Theta_-\frac{d^2}{dz^2}C_-(z) + \left(\Theta_{pr}^2 - \Theta_-^2\right)\frac{d}{dz}C_-(z) \\
= -j\kappa C_+(z)e^{+j2k_zz} - j\kappa C_-(z)
\end{aligned} \tag{16}$$

where the coupling parameter:

$$\begin{aligned}
\kappa &= \frac{\epsilon_0\zeta_q}{4\mathcal{N}}\frac{\omega_p^2}{v_{z0}^2}(k_z + k_w) \\
&\times \int\int f(x,y)\,\tilde{\mathcal{E}}^{pm}(x,y)\,\tilde{\mathcal{V}}_\perp^w \cdot \tilde{\mathcal{E}}_\perp^*(x,y)\,dx\,dy
\end{aligned} \tag{17}$$

here $\tilde{\mathcal{E}}_q^{pm}(x,y)$ is the pondermotive field, $f(x,y)$ is the transverse profile of the e-beam and ω_p is the plasma frequency of a relativistic beam with average axial electron velocity $v_{z0} = \beta_{z0}\,c$ ($c \approx 3\cdot 10^8$ m/s is the speed of light).

In equation (16)

$$\Theta_\pm \equiv \frac{\omega}{v_{z0}} \mp k_z - k_w \tag{18}$$

is the detuning parameter, here $k_w = \frac{2\pi}{\lambda_w}$ (λ_w is the period of the wiggler), and $\Theta_{pr} = \frac{\omega}{v_{z0}}$ is the space-charge parameter.

When the electromagnetic radiation is excited in free-space or in an overmoded waveguide, the detuning parameter can be approximated by

$$\Theta_\pm \cong k_z\left(\frac{1}{\beta_{z0}} \mp 1\right) - \frac{2\pi}{\lambda_w} \tag{19}$$

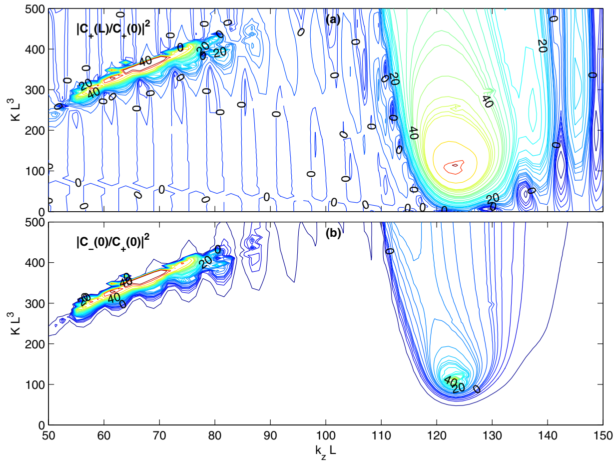

Figure 4: (a) Transmission and (b) reflection contours in the $(k_z L,\ \kappa L^3)$ plane for free-electron laser. Logarithmic scale in [dB].

Figure 4 describes numerical results of the power transmission and reflection coefficients obtained for a mildly relativistic free-electron maser where β_{z0}=0.5 and λ_w=5 cm in Compton regime where space-charge effects are neglected ($\Theta_{pr} = 0$). Considering a case where there is no prebunching in the electron beam, an initial amplitude $C_+(0) \neq 0$ is assumed with derivatives $C'_+(0) = C''_+(0) = 0$ (here $'$ denotes first order derivative $\frac{d}{dz}$). Since no backward wave is propagating at $z = L$, its derivatives up to the second order vanish, that is $C_-(0) = C'_-(0) = C''_-(0) = 0$. Contour plots of the coefficients are described in the $(k_z L,\ \kappa L)$ plane, where infinite peaks express conditions for oscillation excitation. Presenting the plots as a function of the axial wave number k_z enables one to calculate the oscillation frequency using the specific dispersion relation of the cavity or waveguide.

SUMMARY AND CONCLUSIONS

In this paper we presented a coupled-mode theory for excitation of forward and backward modes in distributed gain media. Conventional quantum lasers and free electron lasers were considered. It is shown that under condition of phase matching, the mutual coupling leads to an infinite transmission and reflection gain resulting in self- excitation and oscillations. This effect reveals generation of laser oscillations without the need of feedback mirrors. The frequency of oscillations and the threshold coupling are found using solution of sets of two coupled differential equations describing excitation of forward and backward modes. Analytical and numerical solutions of the coupled equations were carried out, presenting the transmission and reelection as a function of the normalized wave number $k_z L$ and gain parameter κL^3.

ACKNOWLEDGMENTS

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