PROPERTIES OF THE THIRD HARMONIC OF THE SASE FEL RADIATION

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Abstract

Recent theoretical and experimental studies have shown that SASE FEL with a planar undulator holds a potential for generation of relatively strong coherent radiation at the third harmonic of the fundamental frequency. Here we present detailed study of the nonlinear harmonic generation in SASE FEL obtained with time-dependent FEL simulation code FAST. Using similarity techniques we present universal dependencies for temporal, spectral, and statistical properties of the third harmonic radiation from SASE FEL. In particular, we derived universal formulae for radiation power of higher harmonics at saturation. It was also found that coherence time at saturation falls inversely proportional to harmonic number, and relative spectrum bandwidth remains constant with harmonic number.

INTRODUCTION

During last years a significant efforts of researchers have been devoted for studying the process of the higher harmonic generation in the high-gain free electron lasers [1]-[11]. Such an interest has been mainly driven by practical needs for prediction of the properties of X-ray free electron lasers. Analytical techniques have been used to predict properties of the higher harmonics for FEL amplifier operating in the linear mode of operation [8, 9]. However, the most fraction of the radiation power is produced in the nonlinear regime, and a set of assumptions needs to be accepted in order to estimate saturation power of higher harmonics on the base of extrapolation of analytical results. A lot of studies has been performed with numerical simulation codes. These studies developed in two directions. The first direction is investigations of higher harmonic phenomena by means of steady-state codes [4, 5, 6, 7]. Despite the results of these studies are applicable to externally seeded FEL amplifiers only, it is relevant to appreciate that they gave the first predictions for high radiation power in higher harmonics of SASE FEL [2, 4]. Another direction was an extraction of time structure for the beam bunching from time-dependent simulation code with subsequent use of analytical formulae of the linear theory [8]. Giving an estimate for the power, such an approach does not allow to describe statistical properties of the output radiation.

In this paper we perform comprehensive study of the statistical properties of the odd harmonic radiation from SASE FEL. The study is performed in the framework of one-dimensional model with time-dependent simulation code FAST [12, 13] upgraded for simulation of higher harmonic generation. We restrict our study with odd harmonics produced in the SASE FEL. We omit from consideration an effect of self-consistent amplification of the higher harmonics. In other words, we solve only electrodynamic problem assuming that particle motion is governed by the fundamental harmonic. The latter approximation is valid when power in higher harmonics is much less than in the fundamental. This does not limit practical applicability of the results: it has been shown in earlier papers that the growth rate of higher harmonics is too small to produce visible increase of the coherent amplification above shot noise in X-ray FELs [8]. Under this approximation and using similarity techniques we derive universal relations describing general properties of the odd harmonics in the SASE FEL: power, statistical and spectral properties. The results are illustrated for the 3rd and 5th harmonic having practical importance for X-ray FELs.

BASIC RELATIONS

The one-dimensional model describes the amplification of the plane electromagnetic wave by the electron beam in the undulator. When space charge and energy spread effects can be neglected, operation of an FEL amplifier is described in terms of the gain parameter $\Gamma = \left[\pi j_0 K_1^2/(I_A \lambda_{\mathrm{w}} \gamma^3)\right]^{1/3}$, efficiency parameter $\rho = \lambda_{\mathrm{w}} \Gamma/(4\pi)$, and detuning parameter $\hat{C} = [2\pi/\lambda_{\mathrm{w}} - \omega(1+K^2/2)/(2c\gamma^2)]/\Gamma$ (see,e.g. [13]). Here K_1 is coupling factor of the radiation to the first harmonic $h = 1$, $K_h = K(-1)^{(h-1)/2}[J_{(h-1)/2}(Q) - J_{(h+1)/2}(Q)]$, and $Q = K^2/[2(1+K^2)]$. Other parameters of the electron beam, undulator and radiation are: λ_{w} is undulator period, $K = e\lambda_{\mathrm{w}} H_{\mathrm{w}}/2\sqrt{2}\pi mc^2$ is rms undulator parameter, γ is relativistic factor, H_{w} is undulator field, j_0 is the beam current density, $(-e)$ and m are charge and mass of electron, $I_A = mc^3/e \simeq 17$ kA, and ω is frequency of electromagnetic wave. When describing start-up from shot noise, one more parameters of the theory appears – number of particles in coherence volume, $N_{\mathrm{c}} = I/(e\rho\omega)$, where I is beam current.

Main advantage of accepted approximation (particle's dynamics is governed by the fundamental harmonic) is that we can factorize coupling of the harmonics of the radiation and relevant time-dependent integrals of the harmonic of the beam bunching. Thus, with omission of a common factor, complex amplitude of electric field of harmonic is

$$E_h(z,t) \propto K_h \int\limits_0^z a_h(z', t-(z-z')/c)dz' \ ,$$

where a_h is h-th harmonic of the beam bunching. Subsequent application of similarity techniques allows us to extract universal dependencies from numerical simulations.

RADIATION PROPERTIES

The input parameter of the system is the number of cooperating electrons N_{c}. A typical range of the values of N_{c} is 10^6–10^9 for the SASE FELs of wavelength range from X-ray up to infrared. The numerical results, presented in this paper, are calculated for the value $N_{\mathrm{c}} = 3 \times 10^7$ which is typical for a VUV FEL. Note that the dependence of the output parameters of the SASE FEL on the value of N_{c} is rather weak, in fact logarithmic [13]. Therefore, the obtained results are pretty general and can be used for the estimation of the parameters of actual devices with sufficient accuracy.

A plot for the averaged power of the 1st harmonic is shown in Fig 1 with a solid line (normalized power of h-th harmonic is defined as $\hat{\eta}_h = W_h \times (K_1/K_h)^2/(\rho W_{\mathrm{b}})$).

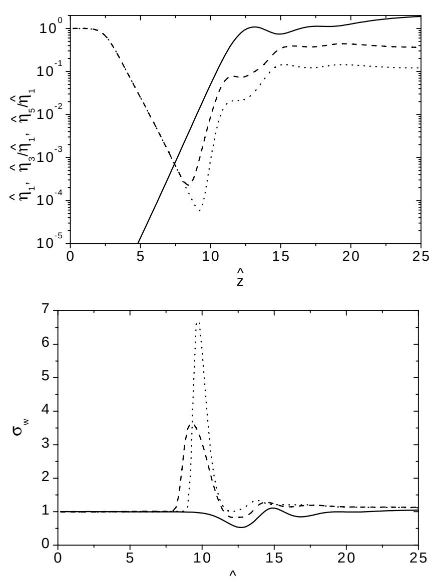

Figure 1: Top: normalized averaged power of a fundamental harmonic of SASE FEL, $\hat{\eta}_1 = P_1/(\rho P_{\mathrm{beam}})$, normalized power ratio, $\hat{\eta}_h/\hat{\eta}_1 = (W_h/W_1) \times (K_1/K_h)^2$, for the 3rd and 5th harmonic. Bottom: Normalized rms deviation of the fluctuations of the instantaneous radiation power. Solid, dashed, and dotted lines correspond to the fundamental, 3rd, and 5th harmonic, respectively

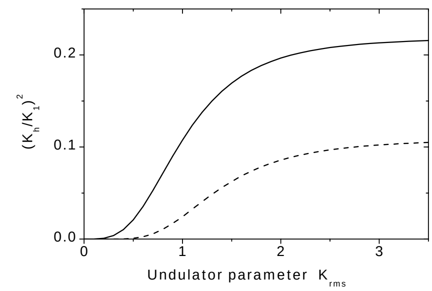

Figure 2: Ratio of coupling factors, $(K_h/K_1)^2$, for the 3rd (solid line) and the 5th (dashed line) harmonics with respect the fundamental harmonic versus rms value of undulator parameter K_{rms}

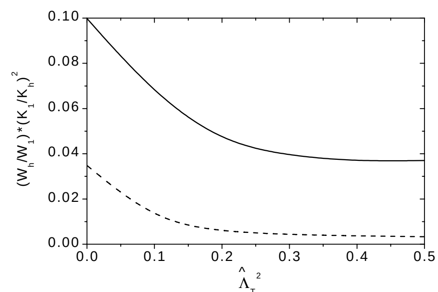

Figure 3: Normalized power ratio at saturation, $(W_h/W_1) \times (K_1/K_h)^2$, for the 3rd (solid line) and 5th (dashed line) harmonic as a function of energy spread parameter $\hat{\Lambda}_{\mathrm{T}}^2$. SASE FEL operates at saturation

Saturation is achieved at the undulator length $\hat{z} = 13$. Dashed and dotted lines show a normalized power ratio, $\hat{\eta}_h/\hat{\eta}_1 = (W_h/W_1) \times (K_1/K_h)^2$, for the 3rd and the 5th harmonic. One can notice that power of the higher harmonics becomes to be above the shot noise level only in the end of linear regime. This becomes clear if one takes into account that the shot noise level of the beam bunching is about $1/\sqrt{N_{\mathrm{c}}}$, and is rather high [8]. For the saturation we find a universal formulae for the power of the 3rd and 5th harmonic:

$$\frac{\langle W_3 \rangle}{\langle W_1 \rangle} = 0.094 \times \frac{K_3^2}{K_1^2} \ , \qquad \frac{\langle W_5 \rangle}{\langle W_1 \rangle} = 0.03 \times \frac{K_5^2}{K_1^2} \ . \quad (1)$$

Universal functions for the ratio $(K_h/K_1)^2$ are plotted in Fig. 2. Asymptotictic values for at large value of undulator parameter are: $(K_3/K_1)^2 \simeq 0.22$, and $(K_5/K_1)^2 \simeq 0.11$. Thus, we can state that contribution of the 3rd harmonic into the total radiation power of SASE FEL at saturation could not exceed a level of 2%. Thus, its influence on the beam dynamics should be small. This result justifies a basic assumption used for derivation of a universal relation (1).

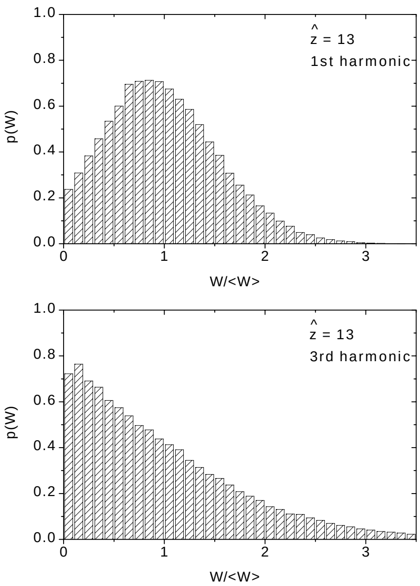

Figure 4: Probability distribution of instantaneous radiation power from SASE FEL operating in saturation regime at $\hat{z} = 13$. Upper and lower plots correspond to the fundamental and 3rd harmonic, respectively

A contribution of the 5th harmonic into the total power at saturation could not exceed the value of 0.3%.

Another important topic is an impact of the electron beam quality on the nonlinear harmonic generation process. In the framework of the one-dimensional theory this effect is described with the energy spread parameter $\hat{\Lambda}_{\mathrm{T}}^2 = \langle (\Delta E)^2 \rangle / (\rho^2 E_0^2)$ where $\langle (\Delta E)^2 \rangle$ is the rms energy spread. Result given by (1) for the case of "cold" electron beam is generalized with the plots presented in Fig. 3. We see that the energy spread in the electron beam suppresses power of the higher harmonics. Within practical range of $\hat{\Lambda}_{\mathrm{T}}^2$ this suppression can be about a factor of 3 for the 3rd harmonic, and about an order of magnitude for the 5th harmonic. Note that typical range of the effective energy spread parameter (taking into account energy spread and emittance, see, e.g., [13]) for X-ray FELs is covered by the plot in Fig. 3. The saturation length at $\hat{\Lambda}_{\mathrm{T}}^2 = 0.5$ is increased by a factor of 1.5 with respect to the "cold" beam case $\hat{\Lambda}_{\mathrm{T}}^2 = 0$.

Instantaneous radiation power is subjected to fluctuations because start-up from shot noise. In Fig. 1 we show the normalized rms deviation of the instantaneous radiation power, $\sigma_{\mathrm{w}} = \langle (W - \langle W \rangle)^2 \rangle^{1/2} / \langle W \rangle$, as a function of the undulator length. The next step in our investigation is the behavior of the probability density distribution of the instantaneous power. In the linear stage of amplification the radiation of the fundamental harmonic is described with Gaussian statistics. As a result, the probability distribution of the instantaneous radiation intensity W should be the negative exponential probability density distribution $p(W) = \exp\left(-W/\langle W \rangle\right)/\langle W \rangle$ [13, 14]. The same refers to the higher harmonics when the shot noise dominates above the process of nonlinear harmonic generation. When the latter process becomes to be dominant the statistics of the high-harmonic radiation from the SASE FEL changes significantly with respect to the fundamental harmonic (e.g., with respect to Gaussian statistics). In this case the probability density function $p(W)$ of the fundamental intensity is subjected to a transformation $z = (W)^h$. It can be readily shown that this probability distribution is $p(z) = z^{(1-h)/h} \exp(-z^{1/h}/\langle W \rangle)/(h \langle W \rangle)$ [15]. Using this distribution we get the expression for the mean value: $\langle z \rangle = h! \langle W \rangle^h$. Thus, the hth-harmonic radiation for the SASE FEL has an intensity level roughly $h!$ times larger than the corresponding steady-state case, but with more shot-to-shot fluctuations compared to the fundamental [8]. Note that this regime of nonlinear harmonic generation which can be described with analytical techniques happens only in the end of linear regime. When amplification process in the SASE FEL enters nonlinear regime, statistical properties of the radiation can be found only from numerical simulations. Relevant probability distributions for saturation are shown in Fig. 4. It is seen that the distributions change significantly with respect to the linear regime for both, the fundamental and the 3rd harmonic. An important message is that at the saturation point the 3rd harmonic radiation exhibits much more noisy behavior (nearly negative exponential) while stabilization of the fluctuations of the fundamental harmonics takes place.

Temporal properties of the radiation are described in terms of the first and the second order time correlation functions $g_1(t - t') = \langle \tilde{E}(t) \tilde{E}^*(t') \rangle / \left[\langle |\tilde{E}(t)|^2 \rangle \langle |\tilde{E}(t')|^2 \rangle \right]^{1/2}$, and $g_2(t - t') = \langle |\tilde{E}(t)|^2 |\tilde{E}(t')|^2 \rangle / \left[\langle |\tilde{E}(t)|^2 \rangle \langle |\tilde{E}(t')|^2 \rangle \right]$. In Fig. 5 we show the time correlation functions at saturation. The nontrivial behavior of the second order correlation function reflects the complicated nonlinear evolution of the SASE FEL process. In classical optics, a radiation source with $g_2(0) < 1$ cannot exist but the case of $g_2(0) > 2$ is possible. As one can see from Fig. 5, the latter phenomenon (known as superbunching) occurs for higher harmonics of SASE FEL.

In Fig. 6 we present the dependence on the undulator length of the normalized coherence time $\hat{\tau}_{\mathrm{c}} = \rho \omega_0 \tau_{\mathrm{c}}$, where τ_{c} is $\tau_{\mathrm{c}} = \int_{-\infty}^{\infty} |g_1(\tau)|^2 \, \mathrm{d}\,\tau$. For the fundamental harmonic the coherence time achieves its maximal value near the saturation point and then decreases drastically. The maximal value of $\hat{\tau}_{\mathrm{c}}$ depends on the saturation length and, therefore, on the value of the parameter N_{c}. With logarithmic accuracy we have the following expression for the coherence time of the fundamental harmonic $(\hat{\tau}_{\mathrm{c}})_{\max} \simeq \sqrt{\pi \ln N_{\mathrm{c}}/18}$. The coherence time at saturation for higher harmonics falls approximately inversely proportional to the harmonic number h.

Radiation spectra are described in terms of the normalized spectral density, $h(\hat{C})$, defined as $\int_{-\infty}^{\infty} \mathrm{d}\,\hat{C} h(\hat{C}) = 1$.

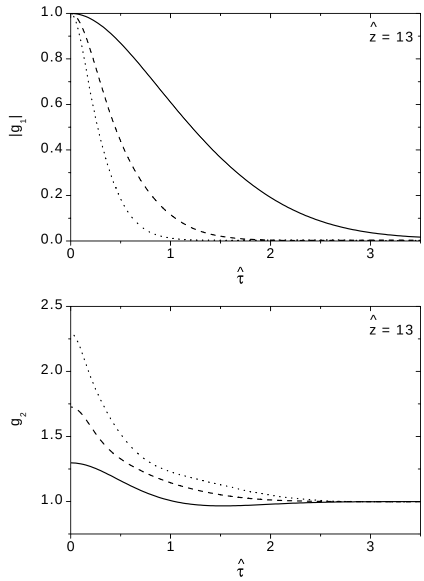

Figure 5: First and second order correlation functions. SASE FEL operates in saturation regime at $\hat{z} = 13$. Solid, dashed, and dotted lines correspond to the fundamental, 3rd and 5th harmonic, respectively. Here $\hat{\tau} = \rho\omega_0(t - t')$

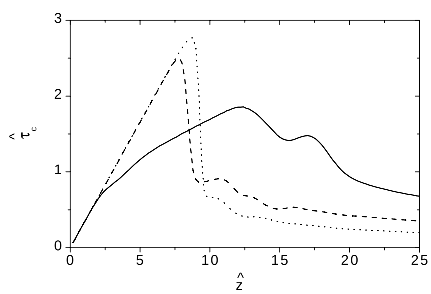

Figure 6: Normalized coherence time of a SASE FEL as a function of normalized undulator length. Solid, dashed, and dotted lines correspond to the fundamental, 3rd, and 5th harmonic, respectively

The frequency deviation, $\Delta\omega$, from the nominal value of ω_h can be recalculated as $\Delta\omega = -2\rho\omega_h\hat{C}$. Normalized envelope of the radiation spectrum and the first order time correlation function are connected by the relation $G(\Delta\omega) = (2\pi)^{-1}\int_{-\infty}^{\infty} d\tau g_1(\tau)\exp(-i\Delta\omega\tau)$ [16]. Figure 7 shows spectra of the SASE FEL radiation at saturation. Note that spectrum width of the higher harmonics from SASE FEL differs significantly from that of incoherent radiation. For the case of incoherent radiation relative spectrum width, $\Delta\omega/\omega_h$ scales inversely proportional to the harmonic number h (see, e.g. [17]). One can see that situation changes dramatically for the case when nonlinear harmonic generation process starts to be dominant. At saturation we find that relative spectrum bandwidth becomes to be nearly the same for all harmonics.

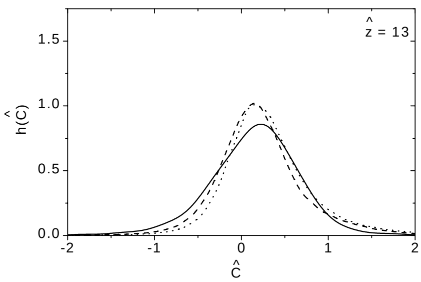

Figure 7: Normalized spectrum from SASE FEL operating in the saturation regime. Solid, dashed, and dotted lines correspond to the fundamental, 3rd and 5th harmonic, respectively

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