2nd ILC Accelerator Workshop
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Study of Space Charge Effects in the ILC-DR's Using MaryLie/Impact

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Space charge matters…

- It is quite unusual to have to worry about direct space-charge (SC) effects in electron storage rings …
- However two factors conspire to make them relevant for the ILC DR's in spite of large (5 GeV) energy.
  - long circumference (3 to 17 km)
  - small vertical bunch size
- Fairly large SC tuneshifts
- We don't expect adverse effects on dynamic aperture / injection efficiency …
- … but SC could cause unacceptable degradation of the 2pm vertical emittance desired at the end of damping cycle.
- Emittance growth due to SC was identified as a potential issue in the TESLA dogbone DR's (W. Decking et al.)
- Studies for TESLA DR pointed at the problem, did not aim at a systematic characterization of the effect. Proposal of using coupling bumps was made.
- A preliminary calculation for TESLA DR's presented at Nov. 2004 ILC workshop by K. Oide showed impact of SC smaller than anticipated by DESY studies

Task Group on space charge effects in DR's: Statement of goals

- Group coordinated by K.Oide and MV
- Task is to study emittance degradation for the main lattice configurations proposed for the DR's
- In particular our goal is to
  - single out the lattice(s) where SC is relevant
  - determine emittance growth at proposed working points and explore tune space
  - study effect of lattice errors
  - investigate effectiveness of coupling bumps (if they apply) including sensitivity to errors in skew quads
  - do scaling with respect to current (energy)
- Lattice evaluation is essentially based on numerical simulations. Tools that have been used include the codes
  - SAD (K. Oide)
  - expanded version of MaryLie/Impact developed for this study (MV)
- Both codes implement a `weak-strong' model of space charge on top of standard single-particle dynamics including lattice nonlinearities to relevant order

Space-charge tuneshifts for ILC DR's can be fairly large

- First-order tuneshift for uncoupled lattice for particle with orbit close to the centroid of gauss bunch (λ is peak part. density):

$$\Delta \nu_x = -\frac{1}{4\pi}\frac{2\lambda r_e}{\beta\gamma^3}\int_0^C \frac{\beta_x}{\sigma_x(\sigma_x+\sigma_y)}ds; \quad \Delta \nu_y = -\frac{1}{4\pi}\frac{2\lambda r_e}{\beta\gamma^3}\int_0^C \frac{\beta_y}{\sigma_y(\sigma_x+\sigma_y)}ds$$

- Range of vertical tuneshift varies from 0.06 to 0.27 from shortest to longest lattice

lattice	C (Km)	ε_x (nm)	ν_{xo}	ν_{yo}	$\Delta\nu_x$	$\Delta\nu_y$
MCH w/o b.	15.9	0.68	75.783	76.413	-0.014	-0.270
MCH w/ b.	15.9	0.68	75.783	76.413	-0.014	-0.089
OCS	6.1	0.56	50.84	40.80	-0.006	-0.127
PPA	2.8	0.43	47.81	47.68	-0.004	-0.064

$\varepsilon_y = 2$ pm, $\sigma_z = 6$ mm (assumed uniform), $N = 2\times 10^{10}$ part./bunch

How space charge can affects the dynamics

- Space charge may cause emittance degradation by
  - moving particles through lattice resonances
  - amplifying effects of skew-quad like lattice errors adding to linear coupling
  - introducing purely space-charge driven nonlinear resonances; e.g. the `Montague resonance'. These have been observed in hadron machines but are not expected to be important here
- Large horizontal/vertical-emittance ratio makes vertical emittance sensitive to x-y coupling – (linear or nonlinear)
- Effect of lattice errors (affecting both linear and nonlinear part of dynamics) can be amplified by presence of space charge.
- Impact of nonlinear resonances can be minimized by suitable choice of working point in tune space but space-charge induced tune-spread may limit the range of choices

Modeling space charge

- Use **splitting-operator** technique
  - lattice elements are cut into a number of slices.
  - space charge force is applied to particles as a kick in the middle of each slice
  - particle propagation between space-charge kicks is done by 3^{rd} order transfer maps or other symplectic integrators.

- **Weak-strong model**:
  - for the purpose of calculating space-charge kick charge density of bunch is assumed to be the 3D-Gauss distribution corresponding to equilibrium in a linear lattice w/o space charge.
  - neglect longitudinal component of space charge forces
  - compute transverse space charge kick as if produced by an infinitely long beam with uniform longitudinal density and transverse density equal to that of the Gaussian bunch at z (location of test particle relative to bunch centroid)

- Option for a **`Quasi Strong-Strong' calculation**. Gauss charge density used for determining space-charge kick dynamically determined at each integration step from 2^{nd} moments of tracked particle distribution

Weak-Strong Model of Space Charge

- Strong beam has 6D gauss distribution with nominal equlilibrium emittances, matched to the (unperturbed) linear lattice

$$f = f_0 \exp\left(-\tfrac{1}{2}\Sigma_{ij}^{-1}(s) x_i x_j\right) \quad \text{where} \quad \Sigma_{ij}(s) = \langle x_i(s) x_j(s) \rangle$$

$$x_i = (x, p_x, y, p_y, \tau, p_\tau)$$

$$\tau = \text{differ. time of flight (scaled to length)}$$

- Ray tracing is done with ‘*s*’ as independent variable
- Propagation of sigma matrices through a lattice element (or sections thereof) is done by **Σ’ = M Σ M$^{\mathbf{T}}$** where **M** is transfer matrix for the lattice element
- Evaluation of space charge force requires snapshot of bunch particles at equal time not equal ‘*s*’
- We need a transformation to ‘unpack’ distribution from distribution of particles having equal ‘*s*’ to particles having equal “*t*”.

\` s – to – t ' transformation

- Suppose we are interested in the space charge force on a test particle $\hat{x}_j(s_0)$ that crosses $s=s_0$ at time

$$t = \hat{\tau}(s_0) + t_r(s_0)$$

- Finding the s-location of all other particles at the same time t, knowing that these particles have coordinates $x_j(s_0)$ at $s=s_0$, requires solving the following equation for s

$$\tau(s) = M_{5j}^{s_0 \to s} x_j(s_0)$$

$$\tau(s) = t$$

- Fortunately, for a ultrarelativistic beam the \`s-to-t' transformation amounts to **simple replacement $\tau \to s$** if

$$\frac{\sigma_z}{\beta_{x,y}} << 1, \qquad \frac{\sigma_z}{\beta_{x,y}} \alpha_{x,y} << 1$$

(reminiscent of condition for hourglass effect)

Rms beam eigensizes, tilt angle

- Charge density is 3D gauss in space. In normal coordinates

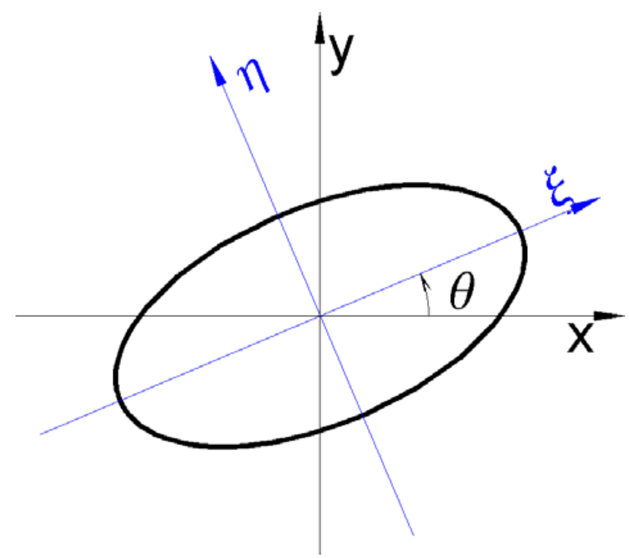
$$f = f_0 \exp\left(-\frac{z^2}{2\sigma_z^2}\right)\exp\left(-\frac{\xi^2}{2\sigma_\xi^2}-\frac{\eta^2}{2\sigma_\eta^2}\right)$$

- The rms `eigen' sizes $c_{\xi},\ c_{\eta},\ c_{z}$ are obtained by diagonalization of the reduced sigma matrix

$$\Sigma_{red} = \begin{bmatrix} \Sigma_{11} & \Sigma_{13} & \Sigma_{15} \\ . & \Sigma_{33} & \Sigma_{35} \\ . & . & \Sigma_{55} \end{bmatrix}$$

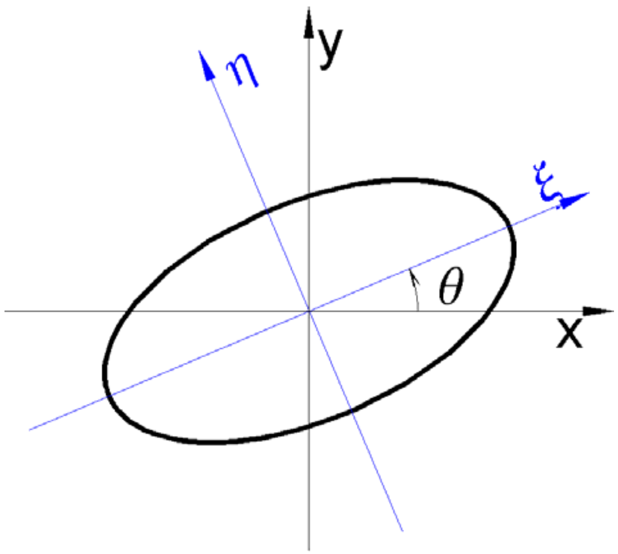
- Diagonalization also provides tilt angle θ

Beam density transverse isodensity curve with a tilt angle θ



Space-charge force is in terms of error function of complex argument

- Space charge is purely transverse (beam is ultrarelativistic)
- 2D Poisson equation with gauss charge density in the transverse variables has closed-form solution in terms of *w*, error function of complex argument [first introduced into beam-physics in connection with modelling of beam-beam (Bassetti-Erskine)].
- Space charge force at s is $F = (f_\xi, f_\eta)$



$$f_\eta + i f_\xi = 2\sqrt{\pi}\,\frac{r_e}{\gamma^3}\frac{\lambda(z)}{S}\left[w(a_1) - w(a_2)\exp\left(-\frac{\xi^2}{2\sigma_\xi^2} - \frac{\eta^2}{2\sigma_\eta^2}\right)\right]$$

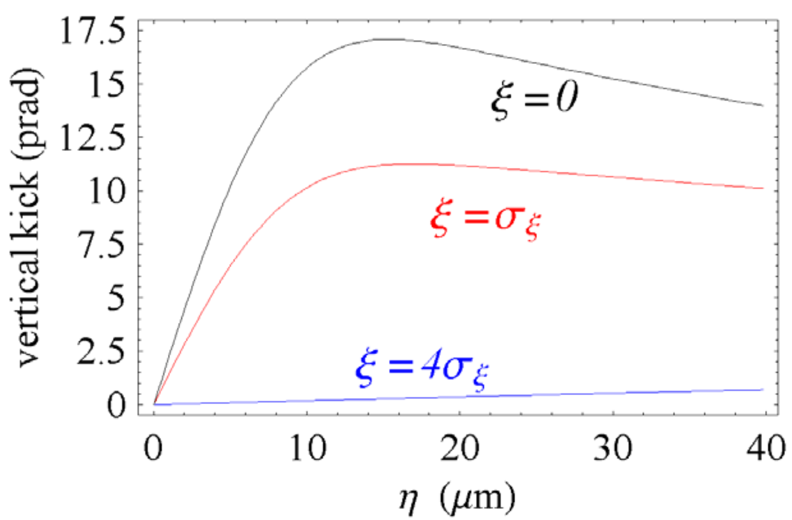
$$S = \sqrt{2(\sigma_\xi^2 - \sigma_\eta^2)}; \quad a_1 = (\xi + i\eta)/S; \quad a_2 = (\xi\frac{\sigma_\eta}{\sigma_\xi} + \eta\frac{\sigma_\xi}{\sigma_\eta})/S$$

- Difference from expression for beam-beam is a factor $1/2\gamma^2$ due to test particle traveling along with source instead of going in the opposite direction.

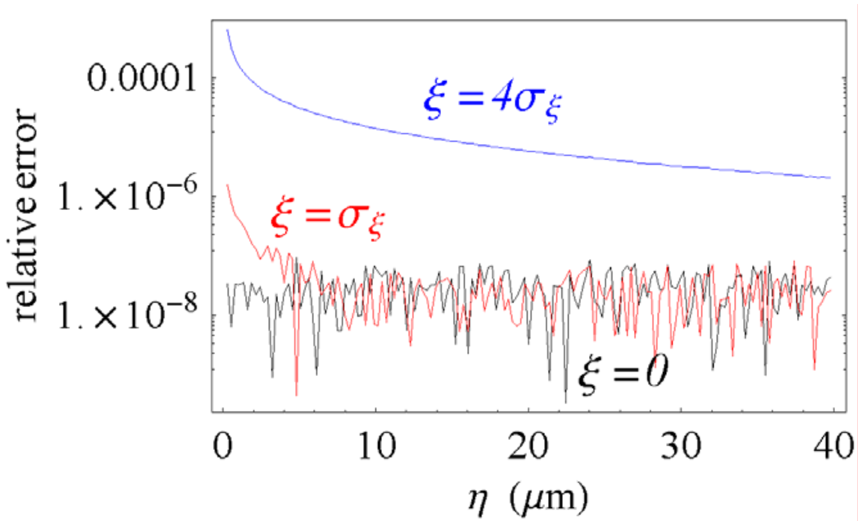
Fast evaluation of error function of c. argument done by Pade' approximants is accurate

- Routine for evaluation of w-function comes from Talman via Ziemann via Furman
- Numerical checks show a relative error of 10^{-4} or smaller in beam core

Vertical kick vs. vertical distance from beam center in 3 horizontal planes



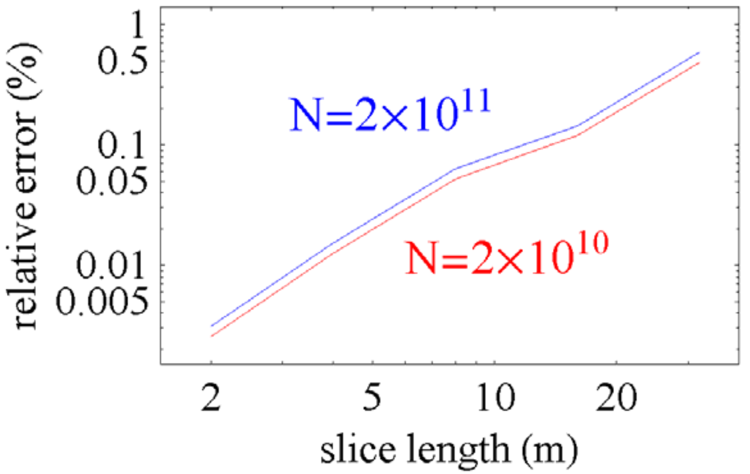
Numerical error of kick against evaluation by *Mathematica*



Remarks on simulations:

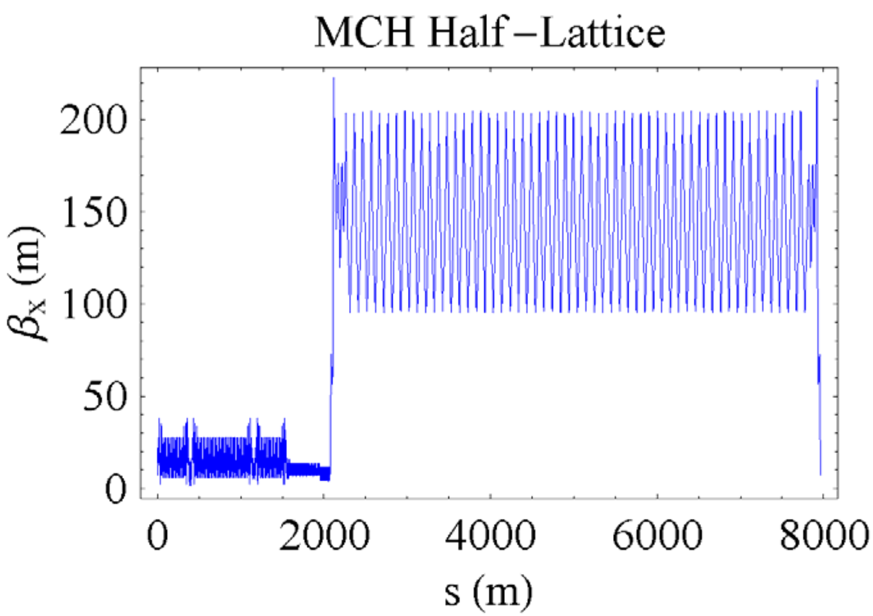
- The initial particle distribution is matched to 3rd order lattice using normal form techniques. This was found to yield smaller amplitude oscillation in the evolution of emittance. More consistent with equilibrium distribution from radiation damping?
- The vertical rms ‘eigen-emittance’ is monitored – i.e. the rms emittance of `mode II’ in the linearly normal coordinate system. This is usually smaller than the rms y-emittance when significant coupling and nonlinearities are present
- In the following, random lattice errors are not included

Scaling of error in tuneshift for a single FODO cell in long straight sections



- Space charge kick is applied at least once in the middle of each thick element. In some long drifts the space charge kick is applied multiple times. But max. separation of 20 m between kicks still gives errors to tuneshift smaller than 1%

The MCH 15.9 km lattice



Benchmark MLI linear lattice against MAD

	ν_{x0}	ν_{y0}
MAD	75.7830	76.4130
ML/Impact	75.7829	76.4129

Check space-charge calculation against linear theory*

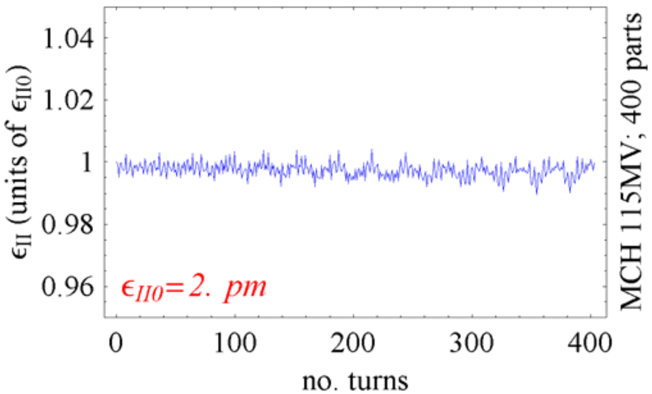
	$\Delta\nu_x$	$\Delta\nu_y$
Linear theory	-0.004304	-0.11645
Orbit FFT	-0.004306	-0.11615

******Tuneshift is for half-ring,* ε_y*=1pm,* σ_z*=1cm*

- **Version of lattice considered for tracking has 115MV RF voltage yielding σ_z~7mm**
- **Damping times:**
  ~500 turns (transverse)
  ~250 truns (longitudinal)

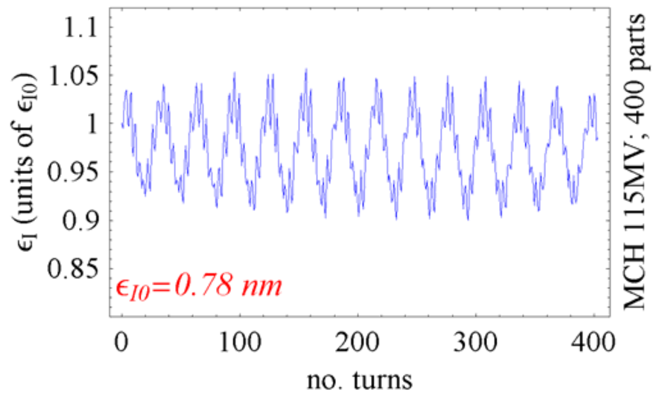
Setting the baseline: MCH ideal lattice at design working point in tune space, with no space charge

Vertical emittance



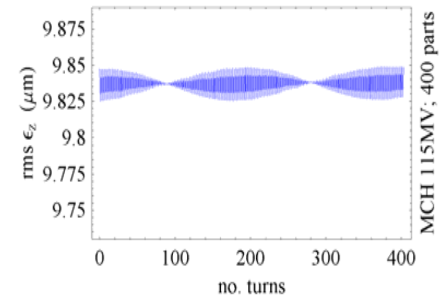
- No evidence of vertical emittance growth over number of turns comparable to damping time.

Horizontal emittance



- Oscillations in horizontal emittances are mostly caused by nonlinear coupling with longitudinal motion – amplitude decreases with smaller longitudinal emittance

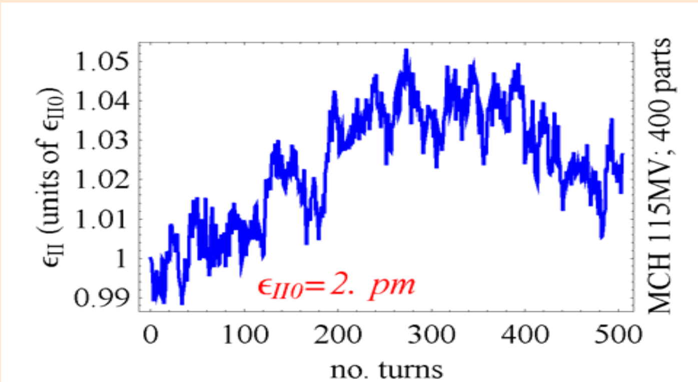
Long. emittance



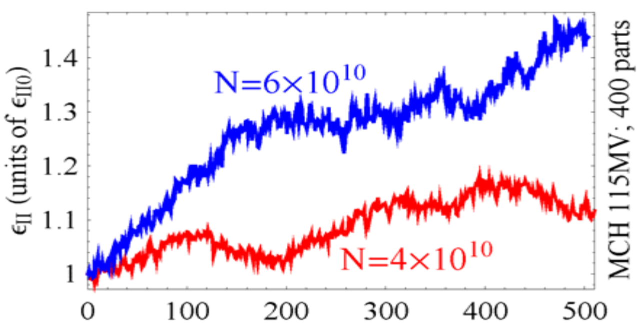
Value of longitudinal emittance is slightly above design value (9.1 μm) because of statistical fluctuations in the realization of particle distribution.

MCH Ideal lattice with space charge, at design working point: there is little emittance growth

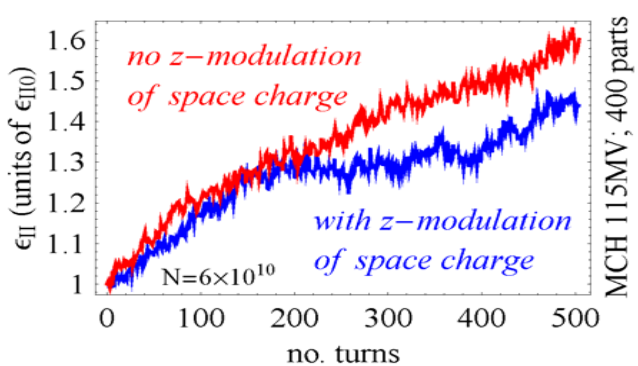
Vertical emittance



At **design** beam current (N = 2×10^{10} particles/bunch) vertical emittance growth stays below modest 5%

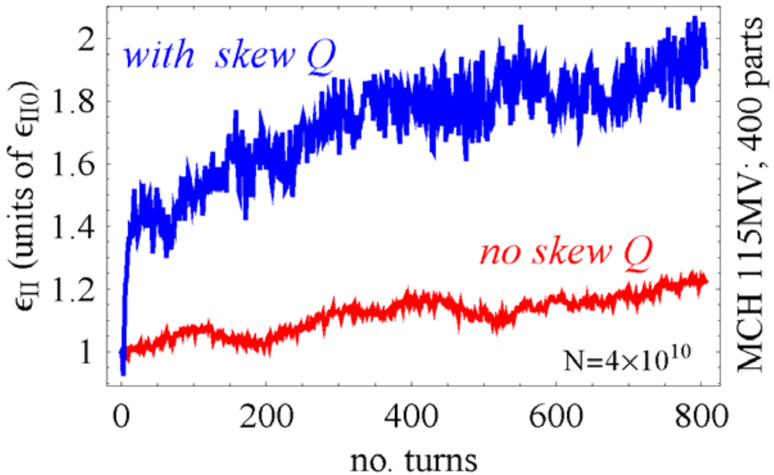


Larger growth occurs at higher current, *e.g.* at N=6×10^{10} part/bunch growth is 50% within 500 turns.

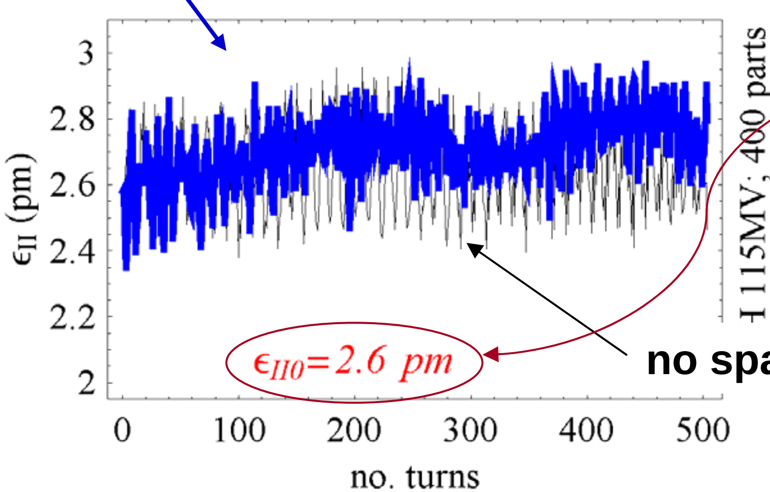


Turning off modulation of space-charge tuneshift caused by longitudinal motion does not affect emittance growth significantly

MCH: Presence of small skew quads may add to vertical emittance growth



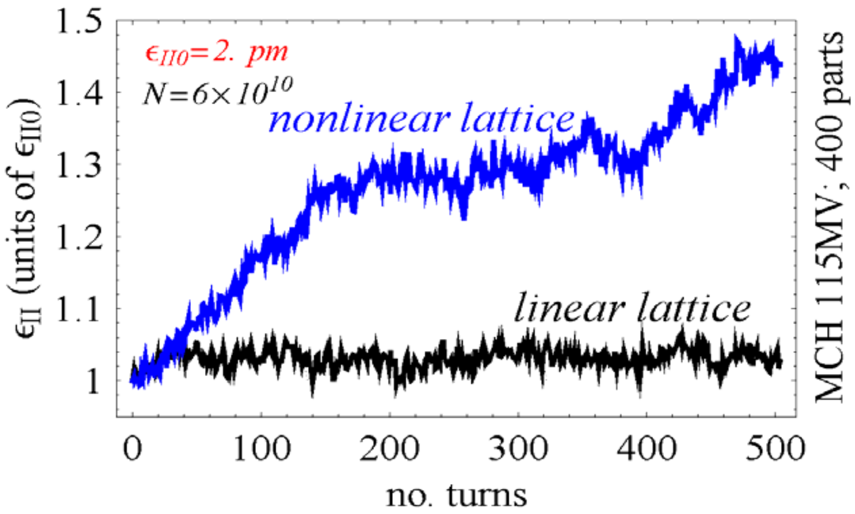
with N= 2×10^10

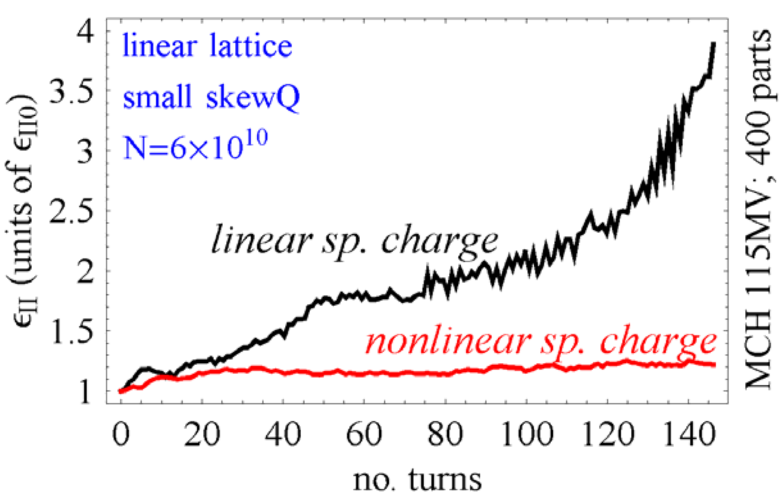


- Only one family of systematic thin skew quads, adjacent to sextupoles in arc cells.
- Strength of the skew is of the order of magnitude required to give design vertical equilibrium emittance (as calculated by MAD)
- Effect of skew quad is dependent on working point, beam current
- If skew quads are relatively large the realization of a macroparticle distribution matched against NL lattices display vertical eigenemittance larger than nominal value (2 pm) obtained by doing matching to linear lattice

MCH: Looking for insight into sources of emittance growth

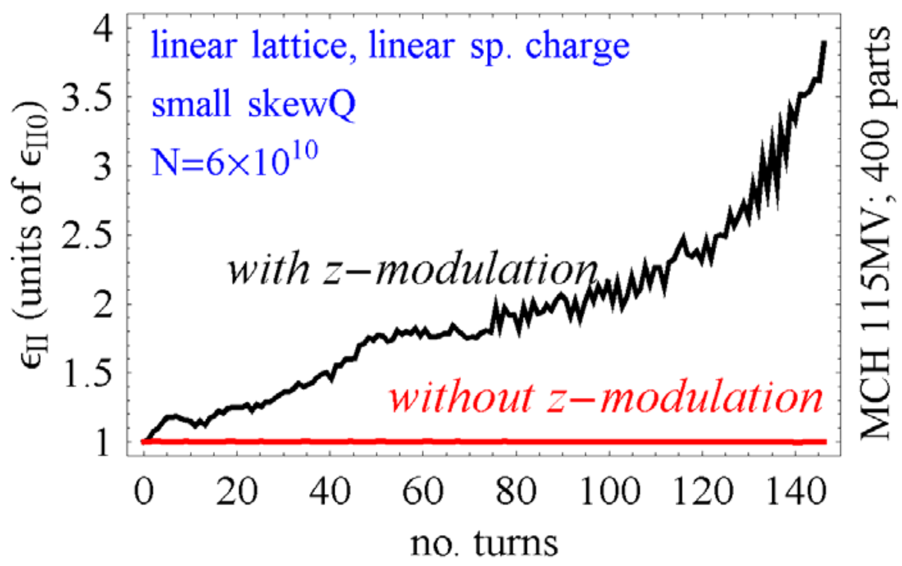
- Lattice nonlinearities are detrimental in combination with space charge
- Space charge nonlinearities can have a stabilizing effect





Q: In a linear lattice with linear space charge: where does the emittance growth come from?

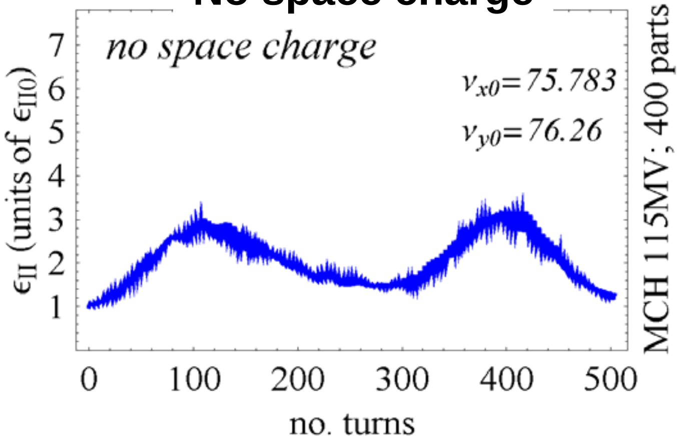
- A: from the modulation of space-charge tuneshift caused by longitudinal motion



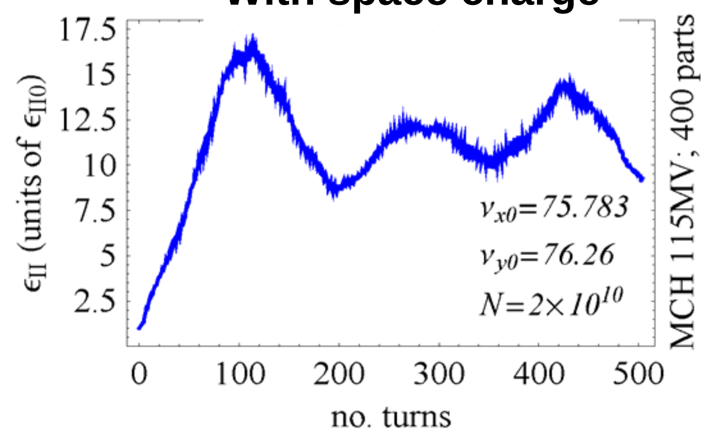
MCH: Exploring different working points

- A bad working point in the absence of space charge may look worse when space charge included …

No space charge

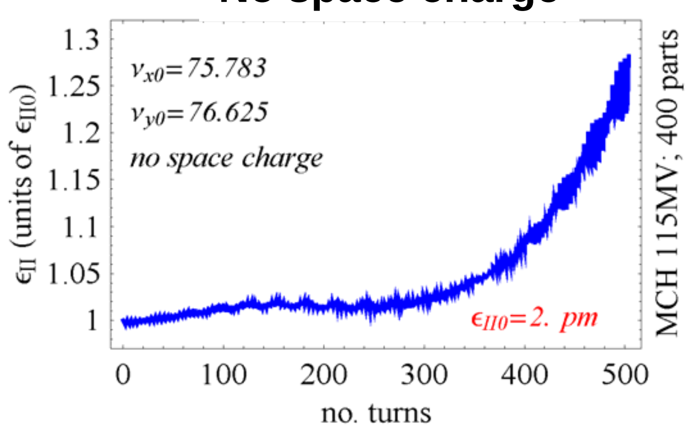


With space charge

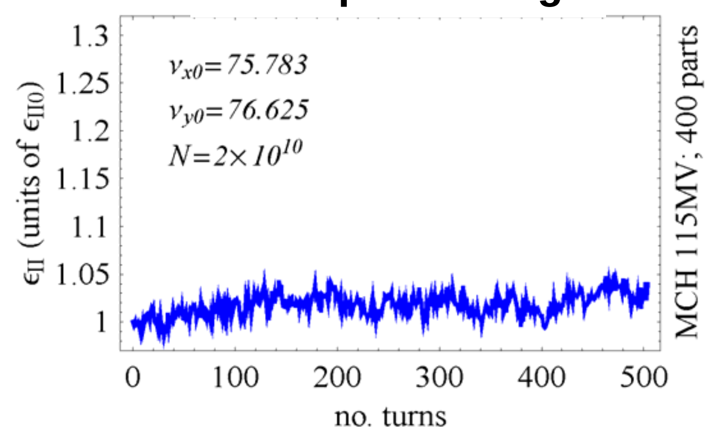


- … in other cases a bad working point may look better (as space charge apparently removes the beam from harmful resonance) …

No space charge



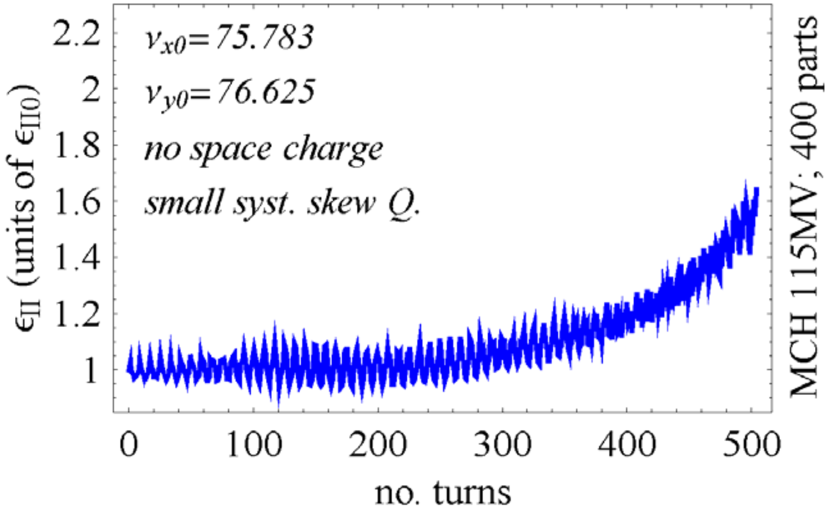
With space charge



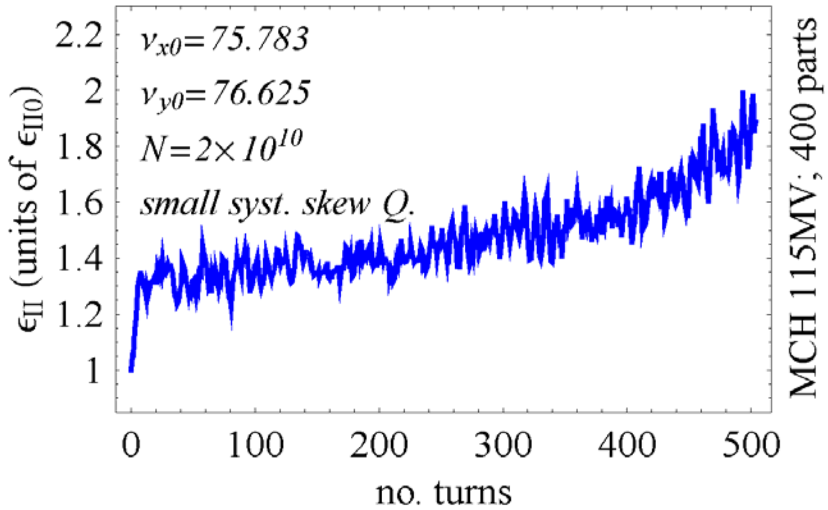
MCH:

- … however, perturbing the same lattice with small systematic skew quads makes once again things worse in the presence of space charge.

No space charge



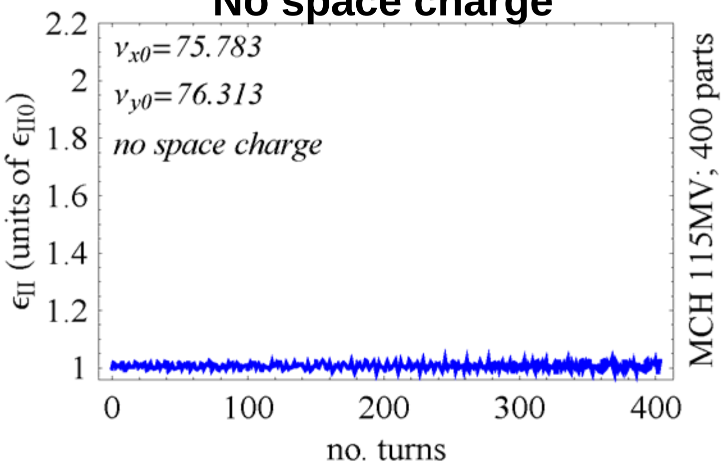
With space charge



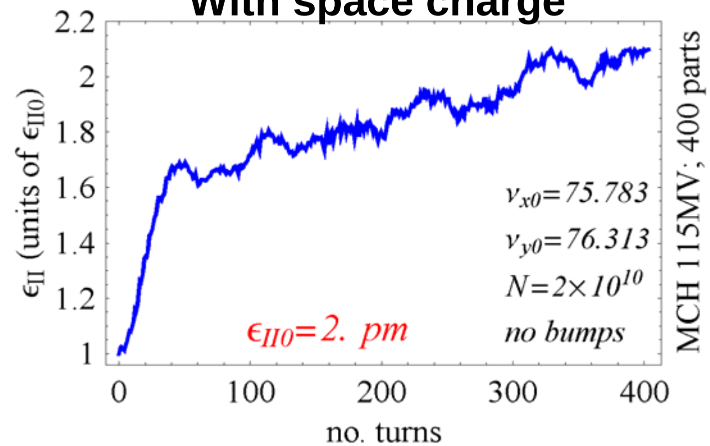
MCH:

- Some working points are made much worse by space charge …

No space charge

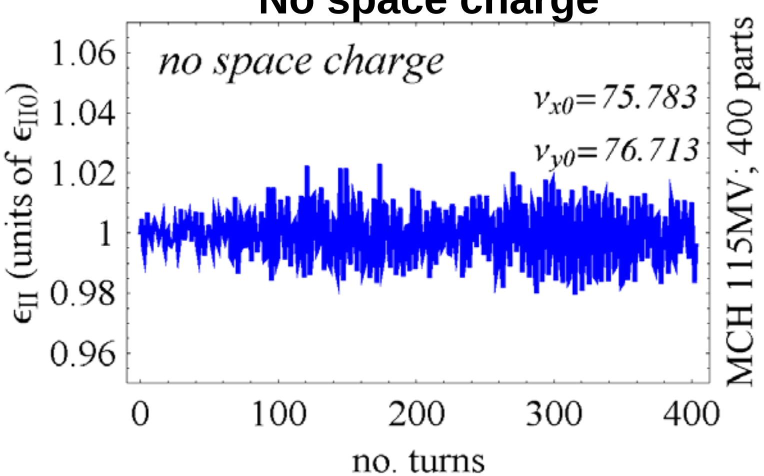


With space charge

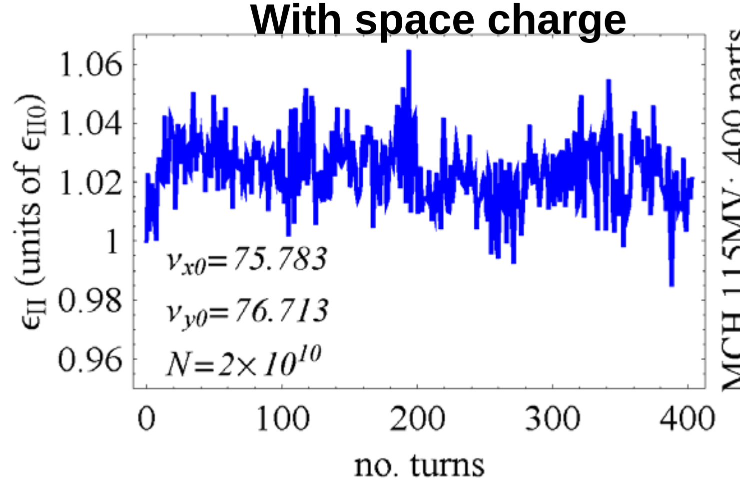


- …others are reasonably robust:

No space charge



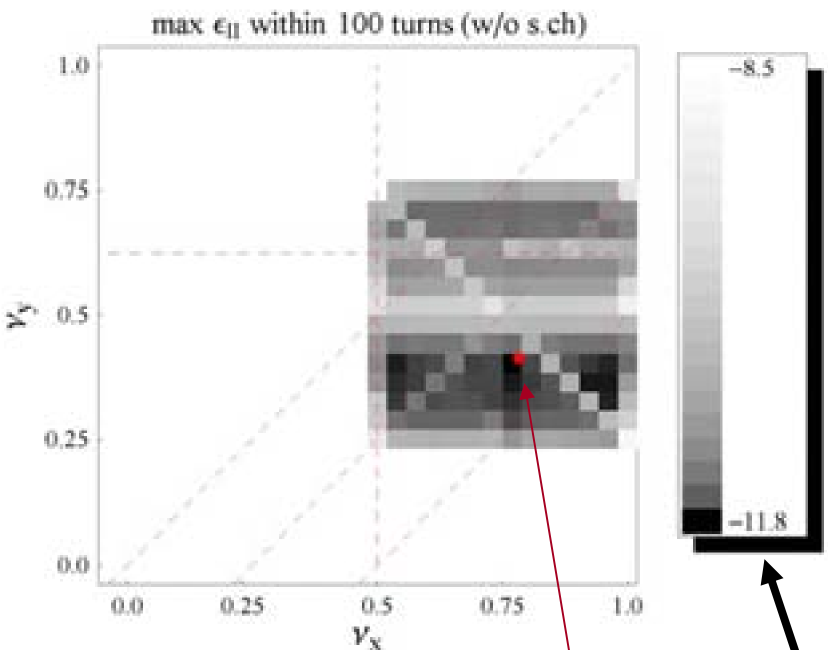
With space charge



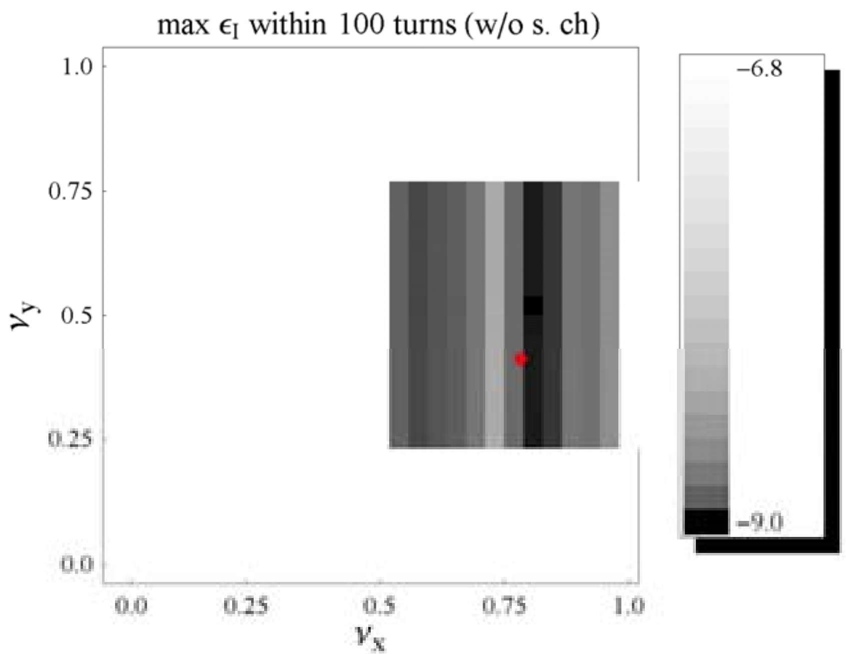
MCH: A more systematic exploration of tune space - no space charge

- Track short term (100 turns) emittance growth
- Lattice detuned by artificial linear transformation applied at end of each turn amounting to pure phase rotation
- One family of small skew quads added to arc cells
- Initial vertical emittance ~1pm

Vertical emittance



Horizontal emittance



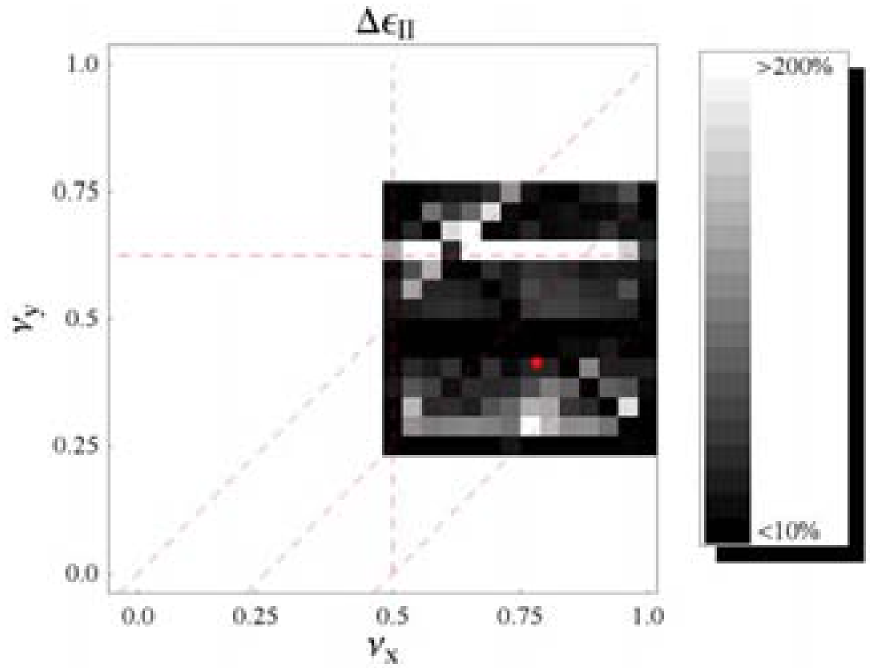
Design working point

Log_{10} scale of emittance

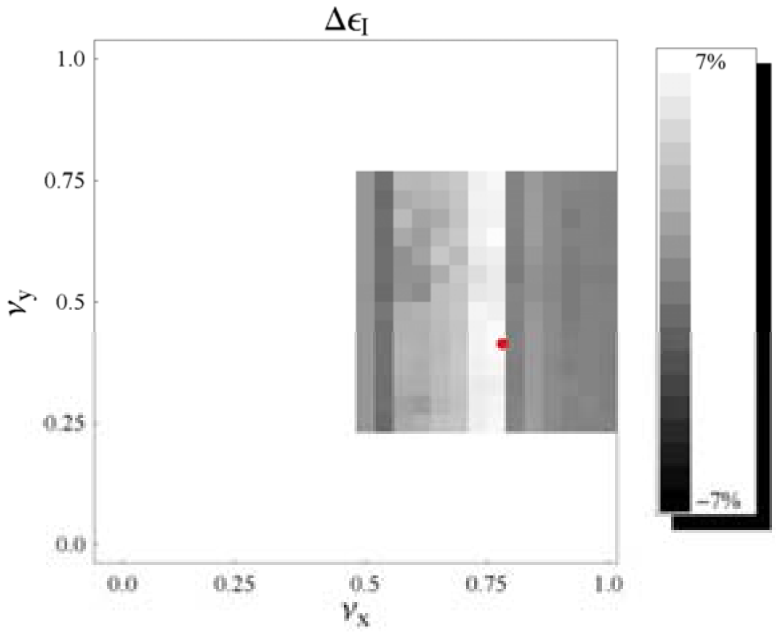
Tune scans with space charge

- Emittance growth with space charge (**N=2×10^{10}**) as a % of emittance growth w/o space charge

Vertical



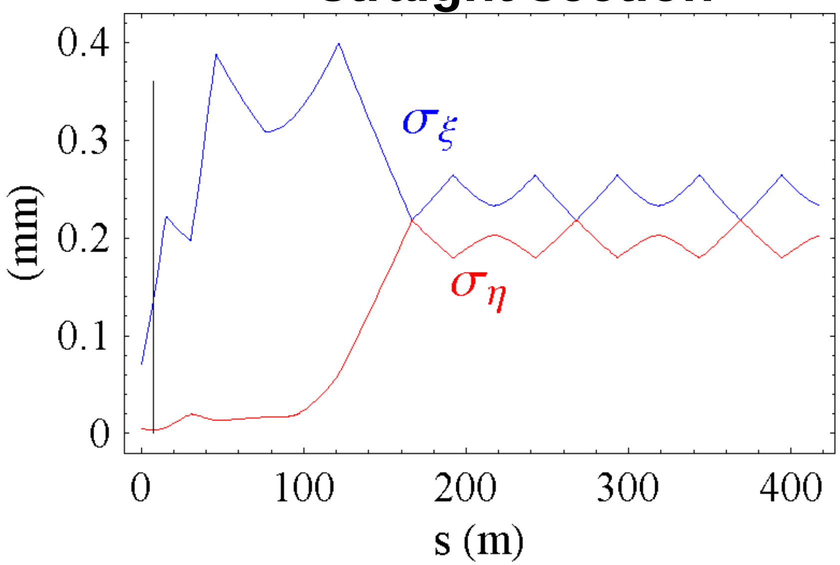
Horizontal



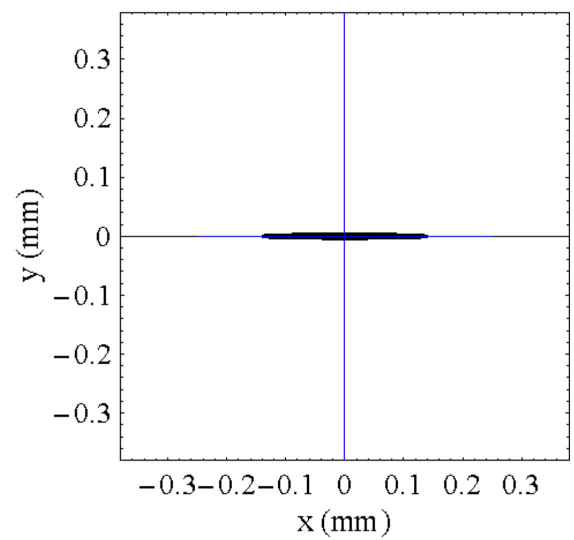
Coupling bumps as a way to tame effect of space-charge

- Introduce linear coupling in the long straights to increase vertical size of beam (at the expense of horizontal size)
- Concept first proposed by N. Walker for the TESLA DR's
- Required coupling can be obtained by set of three skew quads placed at each end of long straights
- Scheme presently implemented in MCH (A. Wolski) uses thin lenses.

First 400 m of 5.9km straight section



Beam cross section



Evaluation of first-order space-charge tuneshift in a coupled lattice

- Closed-form expressions for space-charge tuneshift useful for validating code.
- Formulas derived from perturbation theory for maps. Assume transverse charge-density is uniform and beam cross-section is an ellipse (or assume Gaussian density – with formulas applying to small-amplitude betatron oscillations).

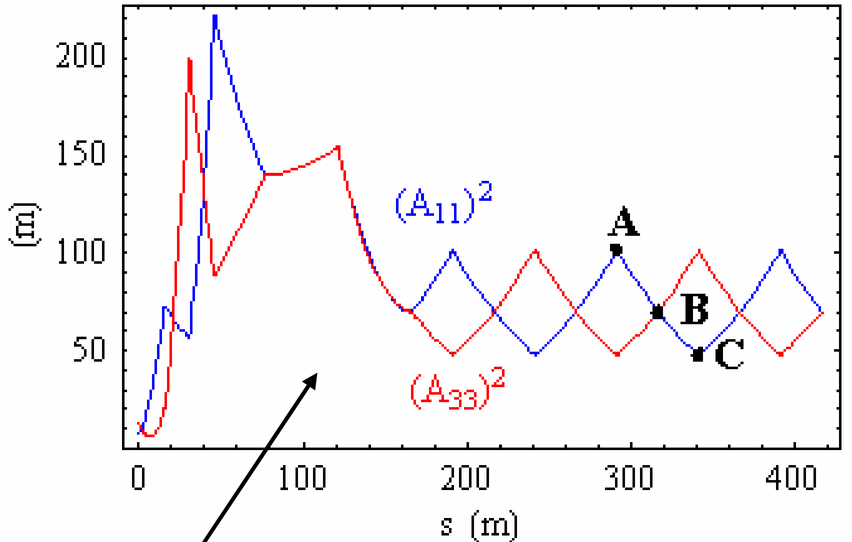
$$\Delta\nu_x = -\frac{1}{4\pi}\int_0^C ds[\, V_{xx}A_{11}^2 + 2V_{xy}A_{11}A_{31} + V_{yy}(A_{31}^2 + A_{32}^2)\,]$$

$$\Delta\nu_y = -\frac{1}{4\pi}\int_0^C ds[\, V_{xx}(A_{13}^2 + A_{14}^2) + 2V_{xy}A_{13}A_{33} + V_{yy}A_{33}^2\,]$$

- Formulas involve:
  - Lattice functions for coupled lattice (essentially, entries of the matrix **A,** normalizing the one-turn matrix $\mathbf{M} = \mathbf{A}\mathbf{R}\mathbf{A}^{-1}$**,** where **R** is a block-diagonal 4×4 rotation matrix)
  - Specification of “space-charge matrix” **V(s)** (which depends on beam transverse rms eigen-sizes, tilt angle)

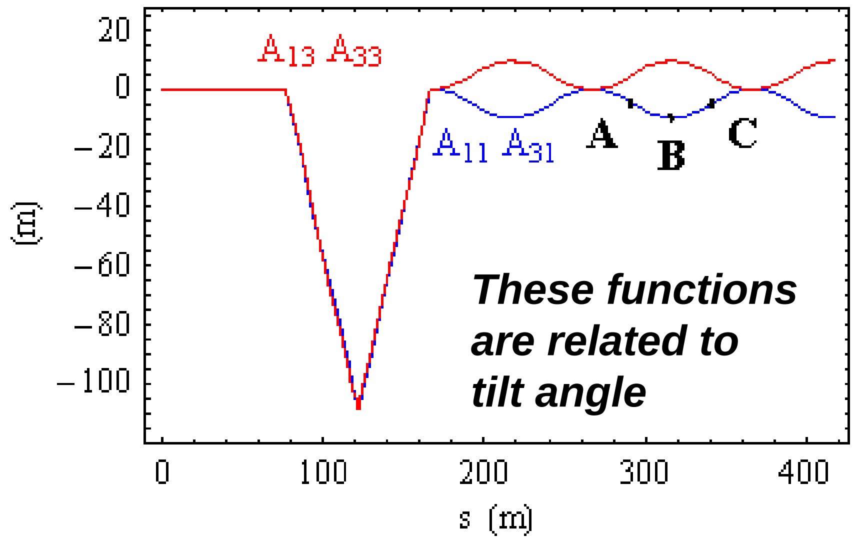
Lattice functions & "space-charge matrix" V(s)

A-matrix entries A_{11} and A_{33}



Correspond to the beta functions
Of an uncoupled lattice

A-matrix entries $A_{13}A_{33}$, $A_{11}A_{31}$



"space charge matrix"

$$\mathbf{V(s)} = \begin{pmatrix} V_{xx} & V_{xy} \\ V_{xy} & V_{yy} \end{pmatrix} = \frac{2\lambda r_e}{\beta^2\gamma^3} \frac{1}{\sigma_\xi \sigma_\eta (\sigma_\xi + \sigma_\eta)} \begin{pmatrix} \sigma_\eta \cos^2\theta + \sigma_\xi \sin^2\theta & (\sigma_\eta - \sigma_\xi)\sin\theta\cos\theta \\ (\sigma_\eta - \sigma_\xi)\sin\theta\cos\theta & \sigma_\xi \cos^2\theta + \sigma_\eta \sin^2\theta \end{pmatrix}$$

FFT of orbits reproduces tuneshift from linear theory

Tuneshift for one long straight with bumps*

	$\Delta\nu_x$ (10^{-3})	$\Delta\nu_y$ (10^{-3})
First-order theory	-5.152	-7.876
FT of particle orbit	-5.152	-7.892
Map analysis	-5.151	-7.903

Vertical tuneshift in straights w/o coupling bumps*:

$\Delta\nu_y$ **= - 0.096**

*Slightly detuned version of MCH lattice

$N = 2\times 10^{10}\ \ part./bnch$

$\varepsilon_x = 0.72\, nm$

$\varepsilon_y = 2.0\ pm$

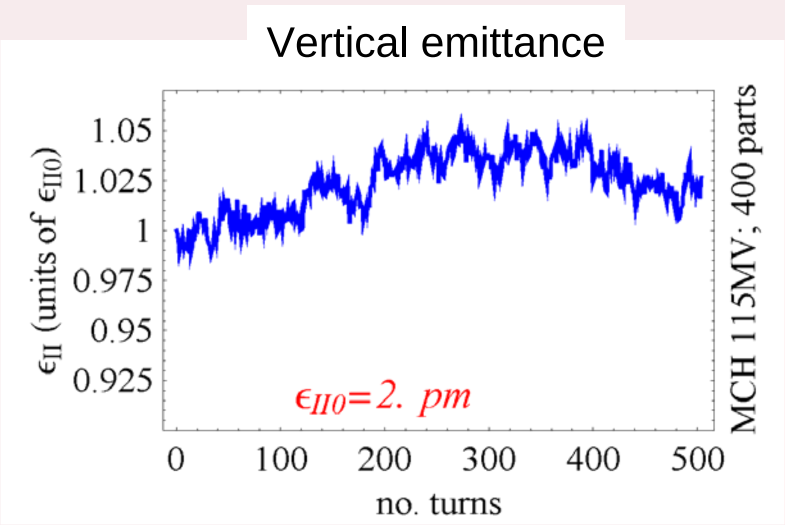
$\sigma_z = 6\ mm$

$\lambda = N/\sqrt{2\pi}\sigma_z$

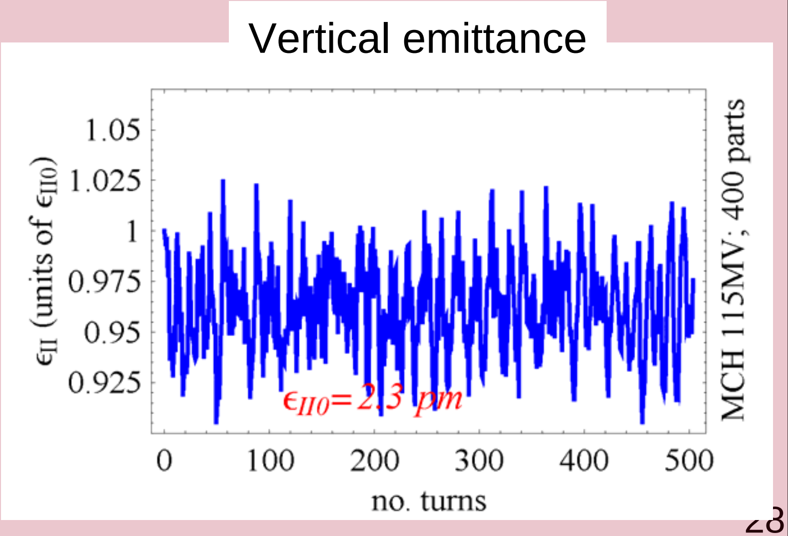
MCH: Effect of coupling bumps

- Vertical emittance has a more noisy profile (apparently due to imperfect cancellation of coupling outside the long straights).
- Vertical (eigen)emittance of initial distribution (with matching to nonlinear lattice) appears larger than nominal value – evidence of some nonlinearities brought in by coupling
- At design current (**$N=2\times10^{10}$**) benefit of coupling bumps is marginal.

No coupling bumps



With coupling bumps

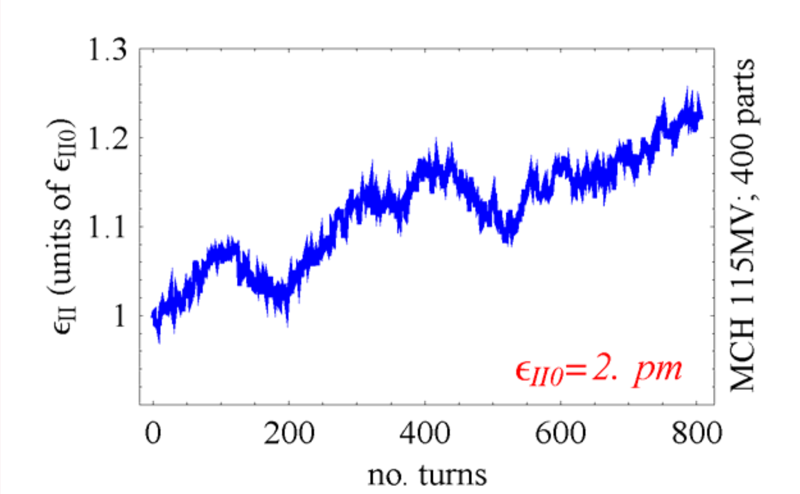


MCH: Effect of coupling bumps at larger current

- At twice the current (**N=4×10^{10}**) beneficial presence of coupling bumps becomes more noticeable

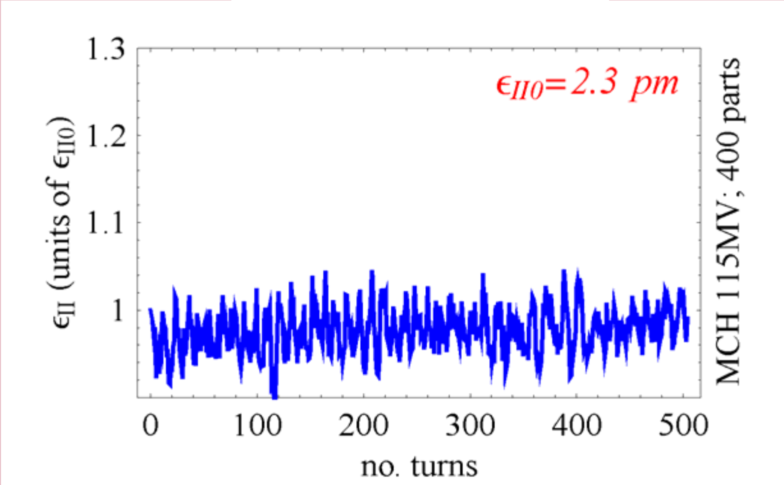
No coupling bumps

Vertical emittance



With coupling bumps

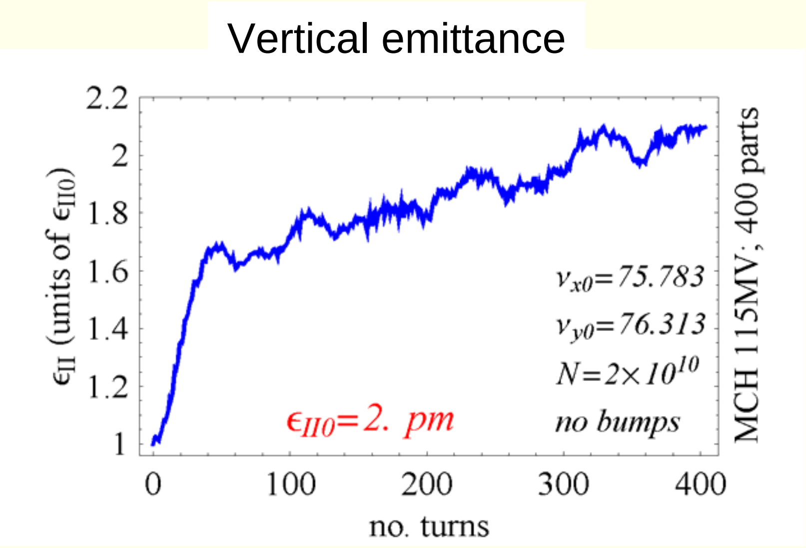
Vertical emittance



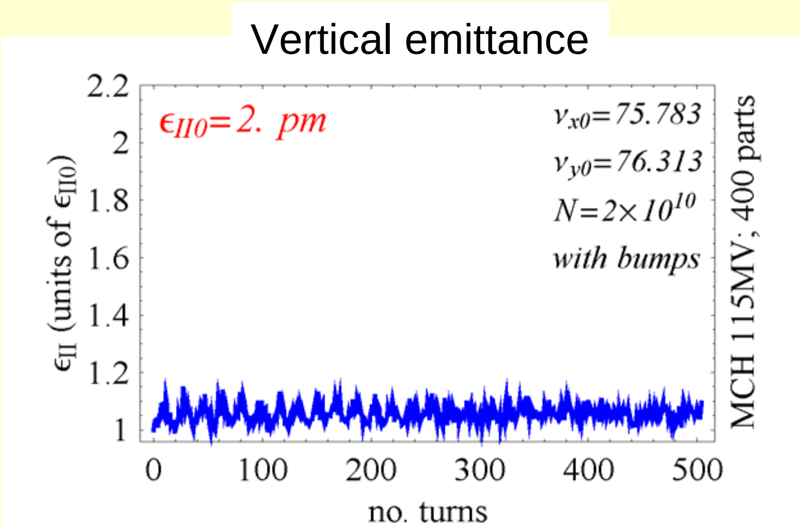
MCH: Effect of bumps at different working point

- Some other working points show benefit of coupling bumps at design current.
- However, tune-space for lattice with coupling bumps on has yet to be systematically explored.

No coupling bumps



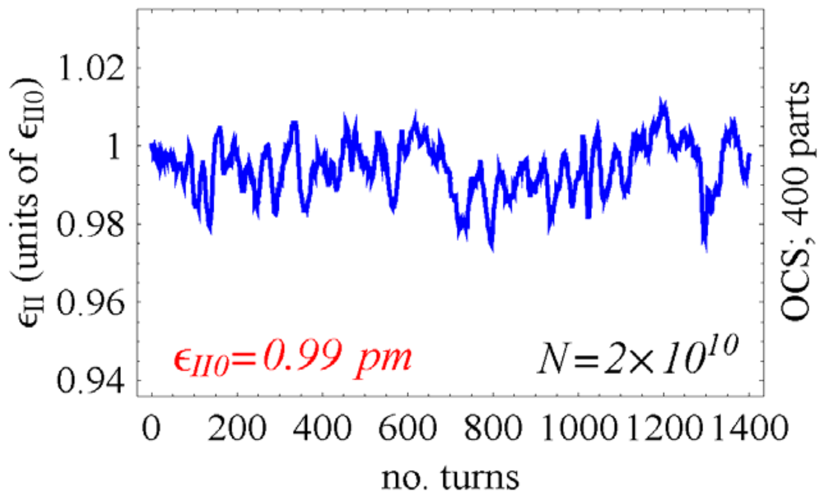
With coupling bumps



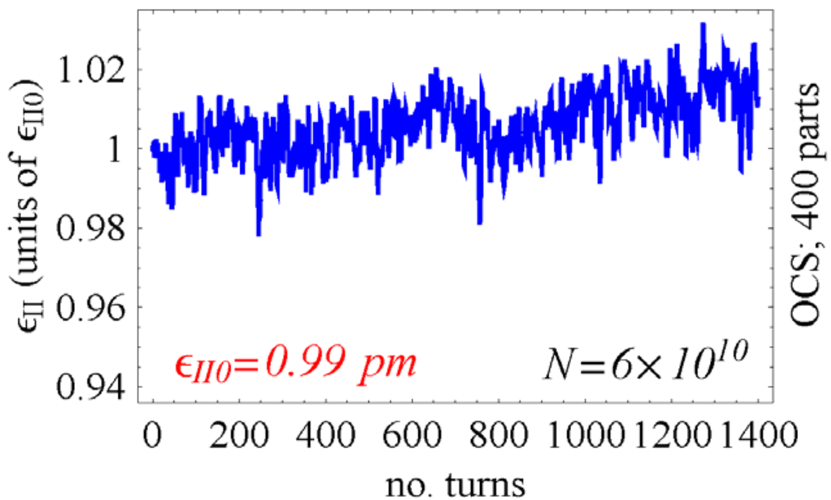
The OCS (6.1 km) lattice

- Preliminary runs show little evidence of emittance growth induced by space charge at design working point (vert. emittance 1pm)

Design current



3× design current



Summary of ML/I tracking

- Modest emittance growth driven by SC at design working point for ideal lattice
- SC effects are dependent on working point in tune space
- Perturbation to lattice in the form of small systematic skew quads may enhance emittance growth with SC
- Random lattice errors, not included so far, could affect outcome significantly
- Coupling bumps would appear to be effective in taming SC; but wait for better assessment until after inclusion of errors
- A number of checks during code development have been carried out yielding a certain amount of confidence in code. Linear dynamics w/ SC agrees well with theory.
- Preliminary comparisons with SAD simulation show differences that will have to be resolved
- Study/interpretation of resonance structure with SC, frequency analysis of orbits still to be carried out
- `Quasi-strong' model will be run with more complete model of lattice including errors using parallel version of code.
- Radiation routines are in place but not completely debugged and radiation effects have not been included so far.