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# **ILC Damping Rings Stability Study**

## **(work in progress)**

G. Stupakov, K. Bane and S. Heifets, SLAC

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Our goal was to have a setup which allows a quick estimate of beam stability for various designs of the ILC DRs.

The current setup is based on a collection of Mathematica notebooks.

The advantage of using Mathematica:

- Compact code
- Powerful symbolic and graphical capabilities
- Easy documentation of algorithms used in the code
- Transparent handling of the system of units and dimensional variables

We plan to have a tool that is similar to (and better than) the old ZAP computer program.

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Mathematica notebooks:

- Formulate an impedance model for each ring (resistive wall, HOMs, a broadband impedance)
- Compute transverse mode coupling instability (Sato-Chin model)
- Compute transverse and longitudinal multibunch instabilities

For microwave instability, we plan to use the Oide code (or its modification) including Haissinki equilibrium into analysis.

We also have available a Vlasov-Fokker-Planck solver (developed by R. Warnock) that allows to track nonlinear evolution of the single bunch longitudinal instability.

At this point we use input parameters from DRMegaTable (of 7/5/05) and A. Wolski's paper LBNL-57045-CBP Tech Note-331.

# Impedance – resistive wall

We assume aluminum wall of round cross section. The longitudinal impedance and the wake are:

$$Z_{RW, long}(f) = L \frac{1 - i}{cb} \sqrt{\frac{f}{\sigma}}$$

$$W_{RW, long}(z) = -L \frac{1}{2\pi b} \sqrt{\frac{c}{z^3 \sigma}}$$

We arbitrarily assume  $b = 2$  cm for all machines, except for BRU and MCH. For those rings we find  $b_{arc}$ ,  $b_{wiggler}$ , and  $b_{straight}$  in A. Wolski's paper LBNL-57045-CBP Tech Note-331.

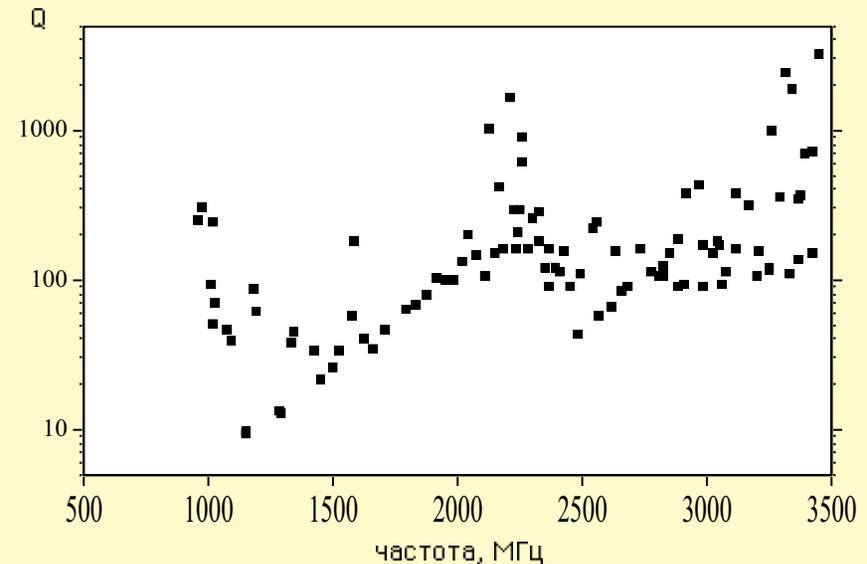
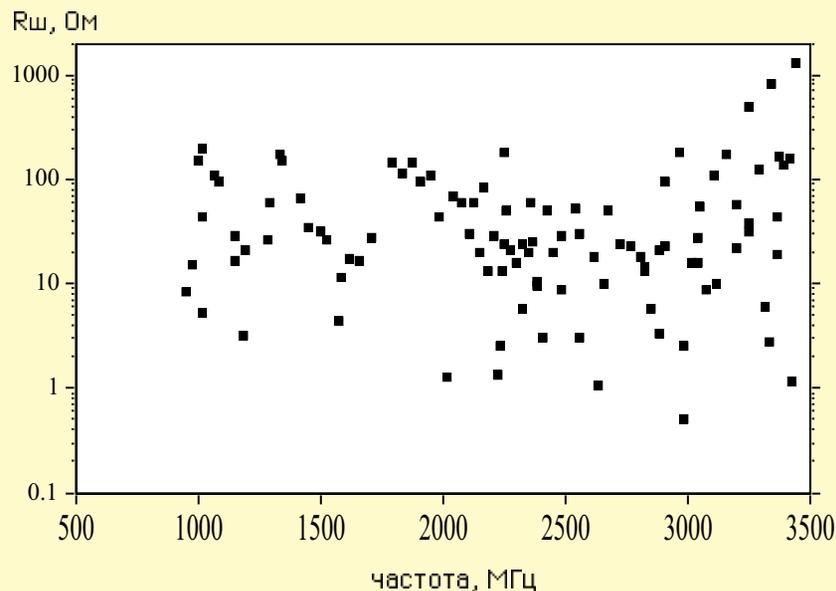
Note singularity of the wake function at  $z = 0$ , which should be properly treated in calculation of the microwave instability.

# Impedance – HOMs

Impedance of an HOM mode

$$Z_{long,mode}(f) = \frac{R}{1 - iQ(f/f_0 - f_0/f)}$$

HOM modes at CESR SC cavity,  $f_{fund. mode} = 500$  MHz, computed with CLANS (from S. Belomestnykh thesis, 1998).



Typical  $Q$  are from 100 to 1000.

# Impedance – HOMs

The empirical value for the total loss factor of all HOMs is (from S. Belomestnykh, "On the BB1 Cryomodule Loss Factor Calculations", SRF 990714-08)

$$K_{\text{loss}} = 7.73 \left( \frac{\sigma_z}{\text{mm}} \right)^{-1.118} \frac{V}{\text{pC}}$$

We assume the voltage 2 MV per cavity ( CESR - 1.8 MV; KEKB - 1.6/2.0 MV; LHC - 2.0 MV) and compute the number of cavities in the ring from the total voltage.

# Broadband impedance in the ring

The broadband impedance describes a contribution from many small elements in the ring, such as BPMs, transitions, flanges, bellows, etc. It also has a resistive component in it. We use the model proposed by Heifets and Chao (SLAC-PUB-8398, 2000):

$$Z_{induc,long}(f) = -\frac{Z_0}{4\pi} \frac{i\omega\mathcal{L}/c}{(1 - i\omega T)^{3/2}}$$

where  $\mathcal{L}$  is the inductance, and  $T$  is a parameter with the dimension of time. For  $\omega \gg 1/T$ ,  $Z_{induc,long} \propto \omega^{-3/2}$  which is the diffraction limit for a high frequency impedance.

For PEP-II  $\mathcal{L} \approx 100$  nH. We assume that this impedance is distributed uniformly over the ring and get a value per cell:  $100/146 = 0.68$  nH. We are trying to figure out the number of cells in each ring using the averaged beta functions. We do not rescale the value of  $\mathcal{L}$  to account for various pipe radius (?).

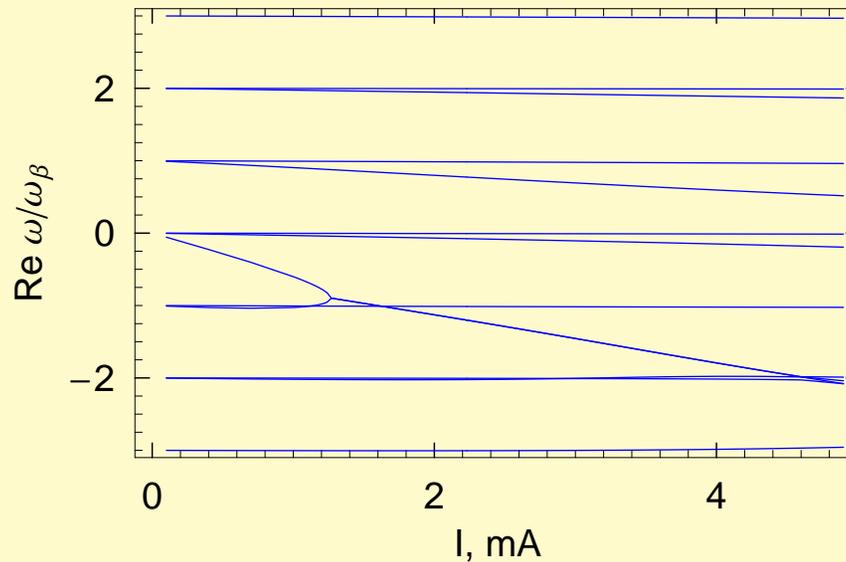
# Mode Coupling

We solve Satch-Chin equations.

$$\alpha_h = -i \frac{Nc^3 r_e}{4\pi C \gamma \omega_\beta \omega_s \sigma_z} \beta_h(\lambda) \sum_{h'=0}^{\infty} \alpha_{h'} \int d\chi Z_\perp(\chi) F_{h'}(\chi - \chi_\xi) F_h(\chi - \chi_\xi)$$

where  $\chi = \omega \sigma_z / c$ ,  $\chi_\xi = \omega_\beta \xi \sigma_z / c \eta$ ,  $\lambda = (\Omega - \omega_\beta) / \omega_s$ ,

$$F_h(\omega) = \frac{\omega^h}{\sqrt{2^h h!}} e^{-\omega^2/2}, \quad \beta_0 = \frac{1}{\lambda}, \quad \beta_1 = \frac{2\lambda}{\lambda^2 - 1}, \quad \beta_2 = \frac{2\lambda}{\lambda^2 - 4} + \frac{2}{\lambda}, \dots$$



Result for the BRU ring for zero chromaticity

# Coupled Bunch Instabilities

We use formulation in terms of wakes rather than impedances. Those series converge faster. We assume uniform distribution of bunches over the ring.

Longitudinal CBI:

$$\frac{\delta\omega}{\omega_s} = -\frac{N_{\text{part}} r_0 \eta c}{2\omega_s^2 \gamma T} \sum_{k=1}^{\infty} w'(k s_b) \left[ e^{ik(2\pi l/M + \omega_s s_b/c)} - 1 \right]$$

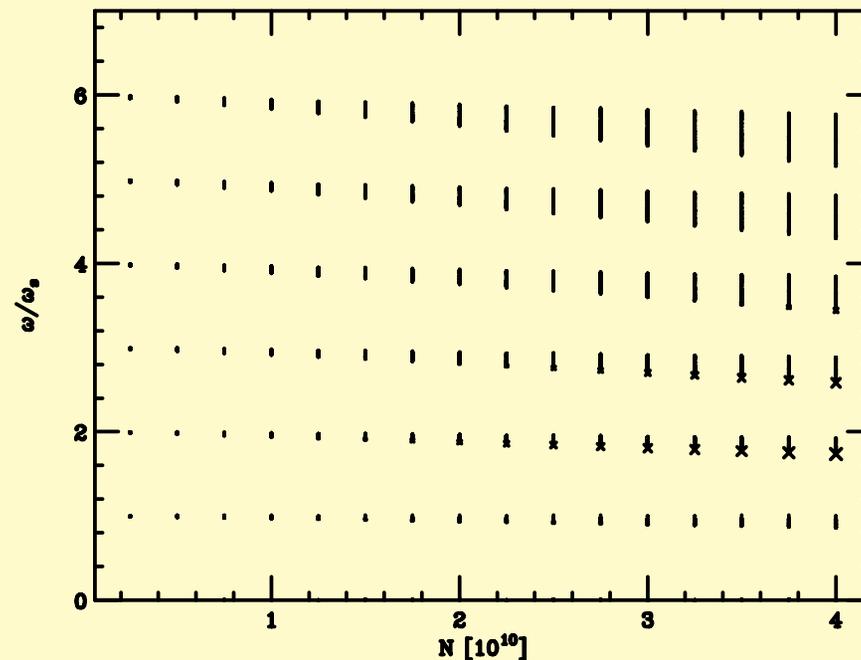
where  $s_b = C/N_b$  is the distance between the bunches.

Transverse CBI:

$$\frac{\delta\omega}{\omega_s} = -\frac{\beta N_{\text{part}} r_0}{2\gamma T \omega_\beta} \sum_{k=1}^{\infty} w_\perp(k s_b) e^{ik(2\pi l/M + \omega_\beta s_b/c)}$$

# Microwave Instability

The simplest approach is to compute the Keill-Schnell-Boussard criterion which gives  $Z/n$ . We plan to use the Oide code that solves eigenmode equations and finds the threshold of the instabilities.



Calculation of microwave instability threshold for PPA. The growth rate equals  $\tau_l$  at  $N_p = 1.75 \times 10^{10}$  with  $\omega = 1.89 \times \omega_s$ .

- The existing model of impedance is incomplete—we need more input for vacuum pipe radii, cavities HOMs, etc.
- We need a better understanding of the multibunch instabilities in the situation when we do not have detailed information about HOMs
- More work is needed with the microwave instability
- We plan to add estimations of the CSR instability to the code