# Emittance Dilution Due Transverse Mode Coupling

- Injected emittance: $\varepsilon_{x,y}$ = 8e-6, 2e-8 m.rads
- Injection beam energy 5 GeV => O/P 500 GeV c.m. energy
- $\gamma_{i,f}$ ~ 9782, and 489,000
- All simulations are made with the code LIAR.
- 500 bunches in all simulations.
- Initial vertical and horizontal offset ~10 μm and 400μm.
- c.f. initial beam $\sigma_{y,x}$ (= $[\beta\varepsilon_n/\gamma]^{1/2}$) ~ 10.1 μm, 270 μm.
- $\sigma_{x',y'}$ (= $[(1+ \alpha^2)/\beta\ \varepsilon_n/\gamma]^{1/2}$) ~ 5.1, 0.26 μrads
- Initial $\beta_{x,y}$ ~ 89.3, 50.7 m

TESLA 9-Cell Cavity
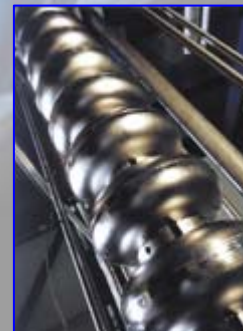
Thanks to *Chris Adolphsen*, Nicoleta Baboi,
Joe Frisch, *Roger Miller*, Tor Raubenheimer, Mark Ross, *Peter Tenenbaum.*

Overview

Make several injection offset simulations to investigate issue of emittance dilution.
**1. Force the angle of all couplers relative to the dipole field to be static.**
2. Allow complete azimuthal randomization of couplers (0 to 2π).
3. Limit the azimuthal spread of the couplers.

➢ **For *detailed* simulations consider a single machine for the above cases.**
➢ **For general emittance dilution consider 200 separate machines.**

Two cases are investigated:
1. 'Properly' damped cavities (all modes damped according to the design)
2. 'Rogue' mode miss-damped. What is the impact on the emittance dilution?

Degenerate Dipole Mode Rotation and E.M. Field Coupling
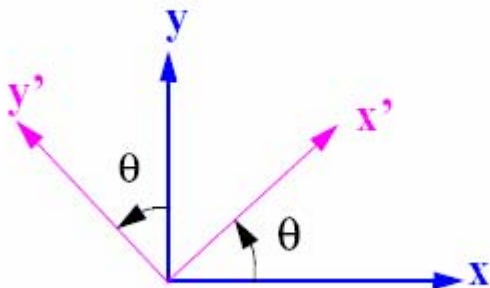
$$V_{x'}(0) = V_x(0) \cos \theta$$
$$V_{y'}(0) = -V_x(0) \sin \theta$$

$$V_y(t) = V_{x'}(t) \sin \theta + V_{y'}(t) \cos \theta$$

$$\Rightarrow V_y(t) = V_x(0) \sin \theta \cos \theta \exp(i\,\omega_{x'} t) - V_x(0) \sin \theta \cos \theta \exp(i\,\omega_{y'} t)$$

$$\Rightarrow V_y(t) = i\, V_x(0) \sin 2\theta \exp(i\langle\omega\rangle t) \sin(\Delta\omega t/2)$$

where the splitting of the degenerate frequency is given by:

$$\Delta\omega = \omega_{x'} - \omega_{y'}$$

and the mean frequency: $\langle\omega\rangle = (\omega_{x'} + \omega_{y'})/2$

$$V_y = iV_x(0) \sin 2\theta \exp(i<\omega>t) \sin(\Delta\omega t/2)$$

Thus for $\theta = 0$ or $\pi/2$ $V_y(0) = 0$ as expected!

The additional kick to the beam is of the form:

TESLA 9-Cell Cavity

$$V(t)= \Sigma K_n \sin(\omega_p t) \exp(-\omega_p t/2Q_p)U(t)y+$$
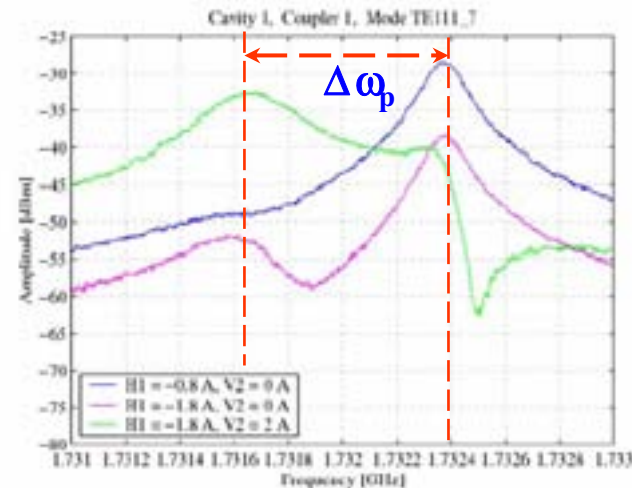$$\Sigma K_n \cos(<\omega_p>t) \sin(\Delta\omega_p t/2) \exp(-<\omega_p>t/2Q_p)U(t)x$$

Where the sum is taken over all modes of interest and U(t) is the unit step function

=> Recent TTF experiments (Ross, Napoly & Baboi) suggest that this frequency splitting varies by ~ 400 kHz to 800 kHz.   In all simulations presented here we use 600 kHz frequency splitting for all modes.

International Linear Collider
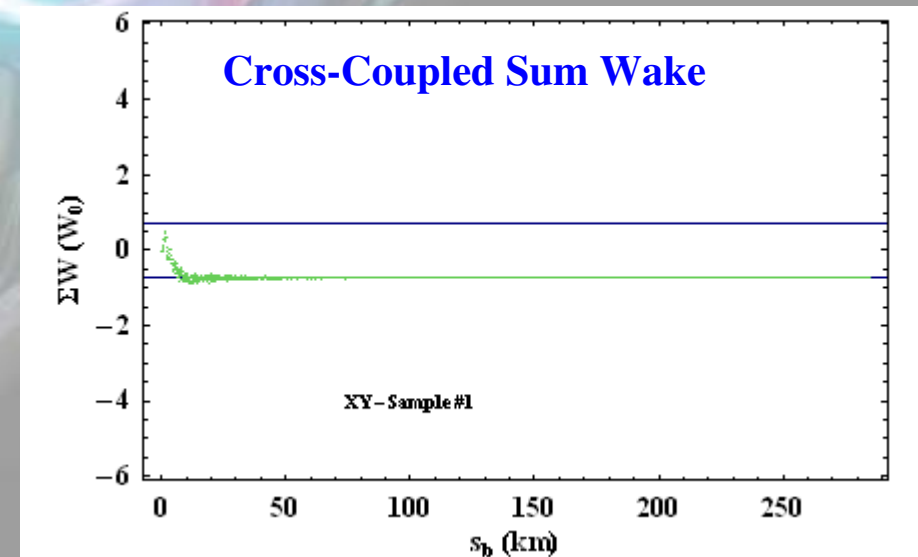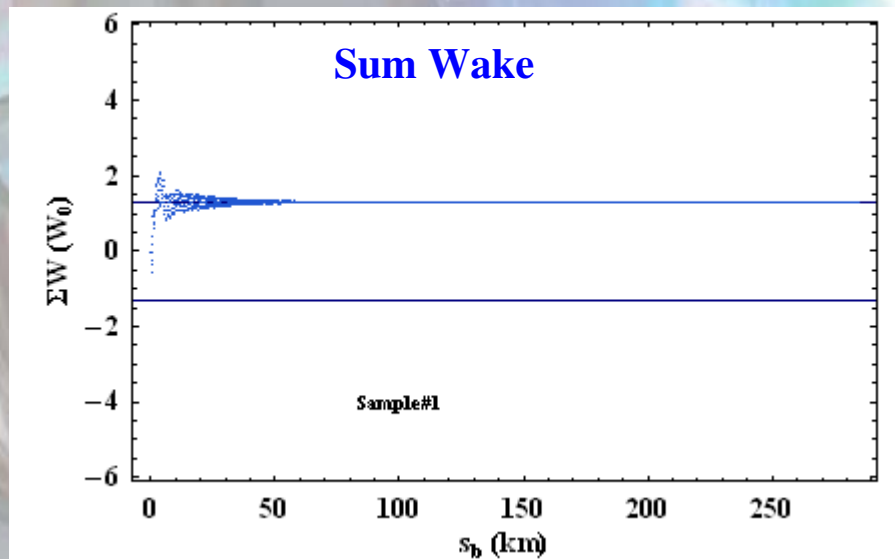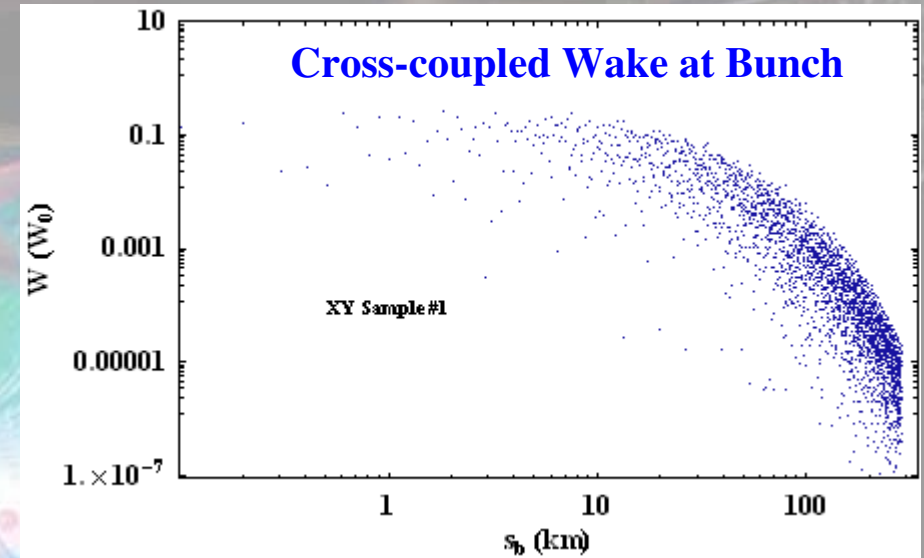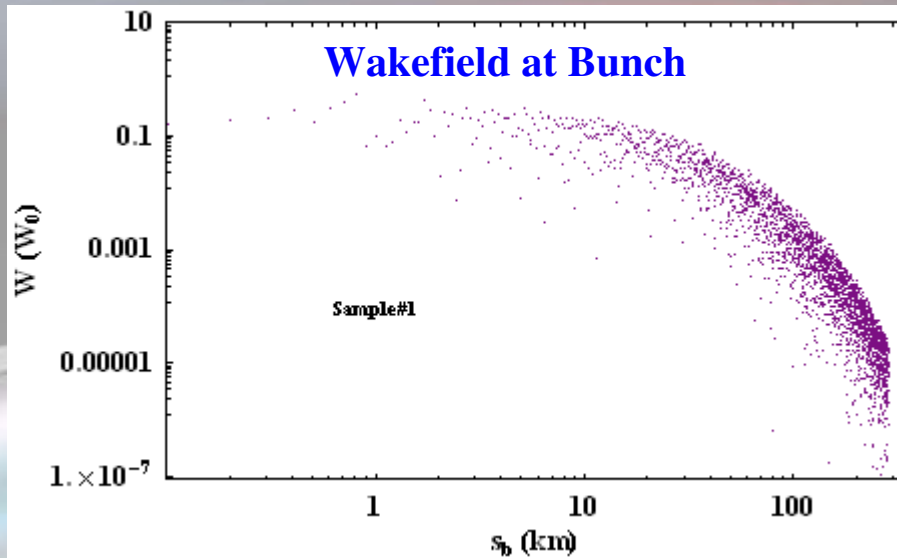at Stanford Linear Accelerator Center

rf gun   steerers   ACC1

4.5MeV

12 MV/m

pick up
flange

HOM coupler
flange

HOM coupler
flange
(rotated by 65°)   115.4 mm

power coupler
flange

1061 mm

1276 mm

view signal with
spectrum analyzer

Cavity 1, Coupler 1, Mode TM110_4

Cavity 1, Coupler 1, Mode TE111_6

~ 800 kHz

Cavity 1, Coupler 1, Mode TE111_7

$\Delta\omega_p$
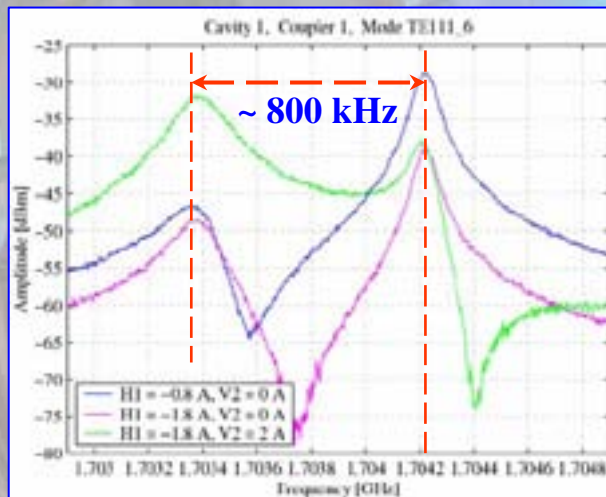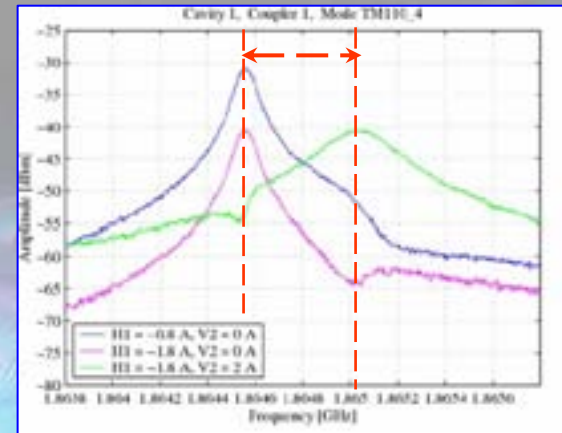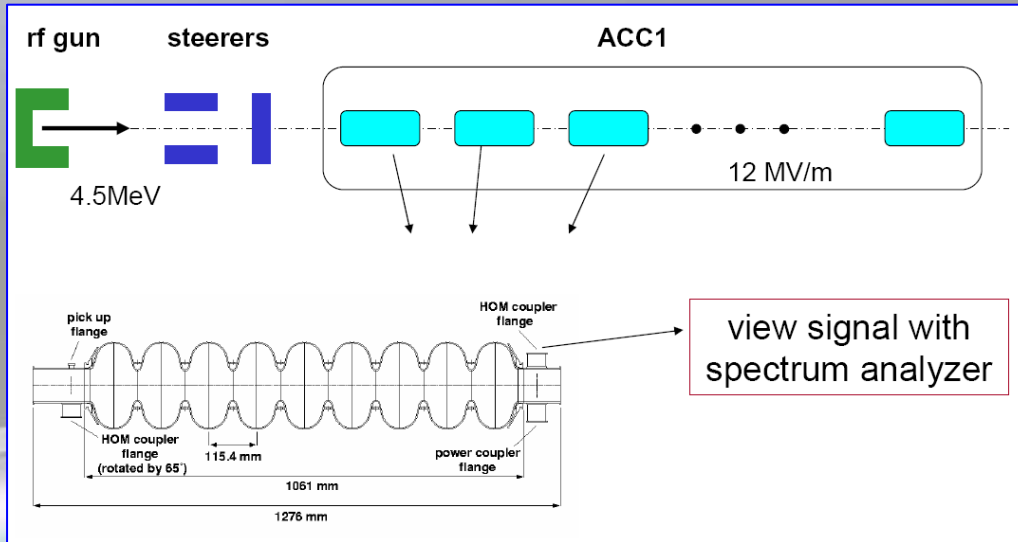
Taken from Ross, Baboi, Napoly et al. See Linac04 paper & SLAC-PUB 11190.

International Linear Collider
at Stanford Linear Accelerator Center

$$W_y(t)= \Sigma K_n \sin(\omega_p t) \exp(-\omega_p t/2Q_p)U(t), \; W_{xy}=\Sigma K_n \cos(<\omega_p>t)\sin(\Delta\omega_p t/2)\exp(-<\omega_p>t/2Q_p)U(t).$$
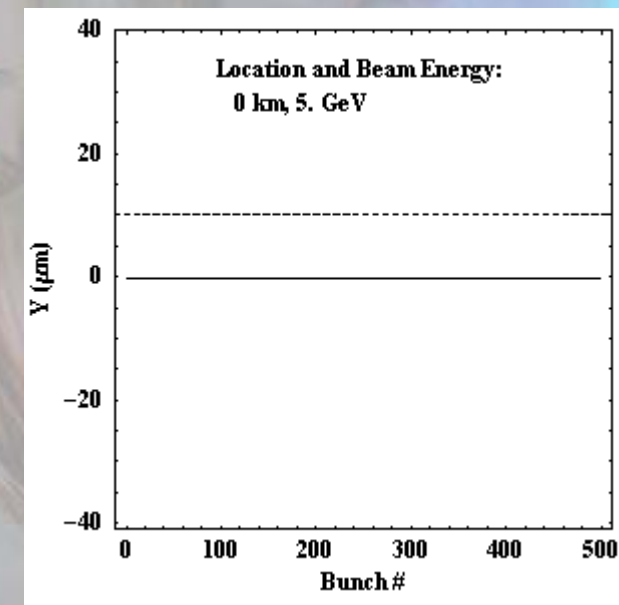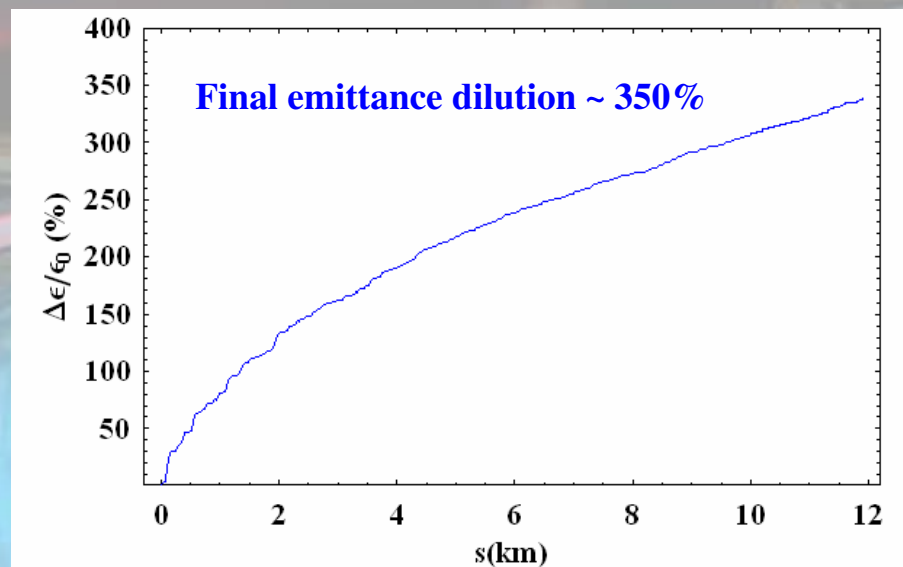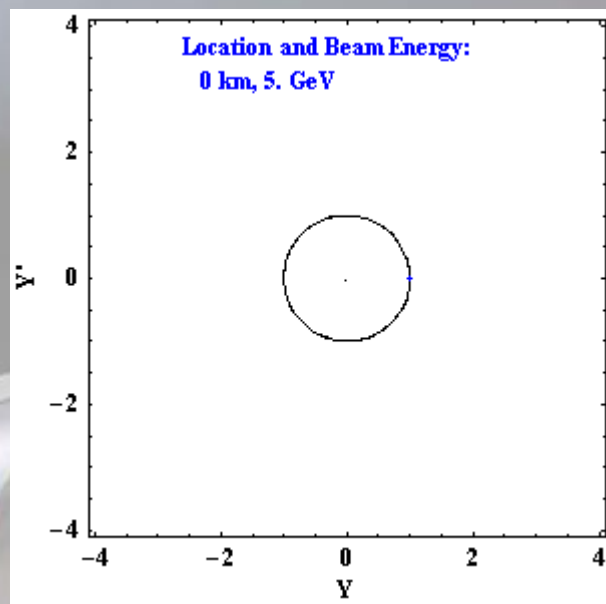
Wakefield at Bunch

Cross-coupled Wake at Bunch

Sum Wake

Cross-Coupled Sum Wake

Final emittance dilution ~ 350%

- •**500 particles in all simulations**
- •**Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**
- •**(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
- •*Fixed azimuthal orientation of couplers.*
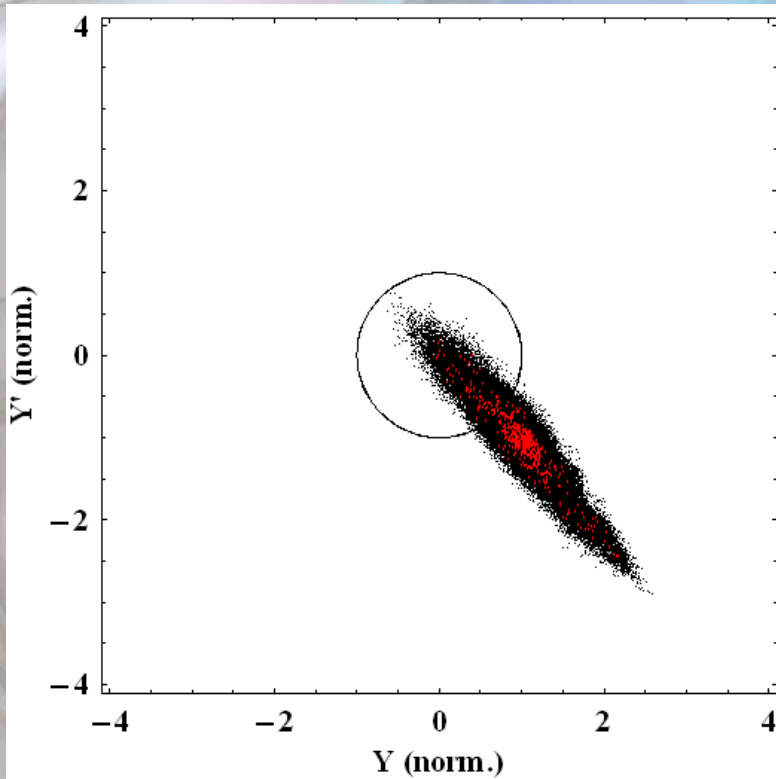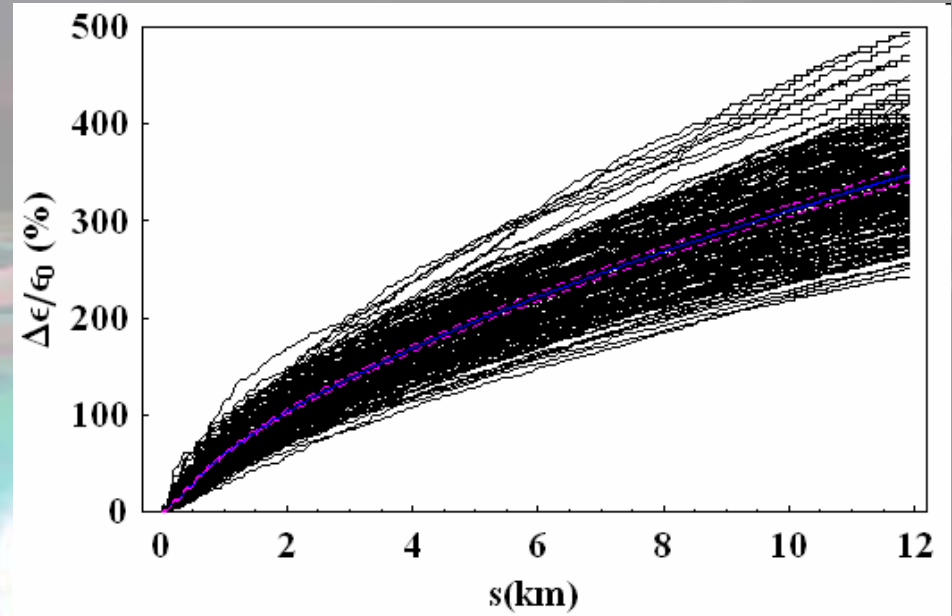- •**Long range wakes included in simulations**

Emittance Dilution From Cross-Coupling Wakes => Yoffset =0

Location and Beam Energy: 0 km, 5. GeV

Location and Beam Energy: 0 km, 5. GeV

Final Emittance Dilution ~ 20.5 %
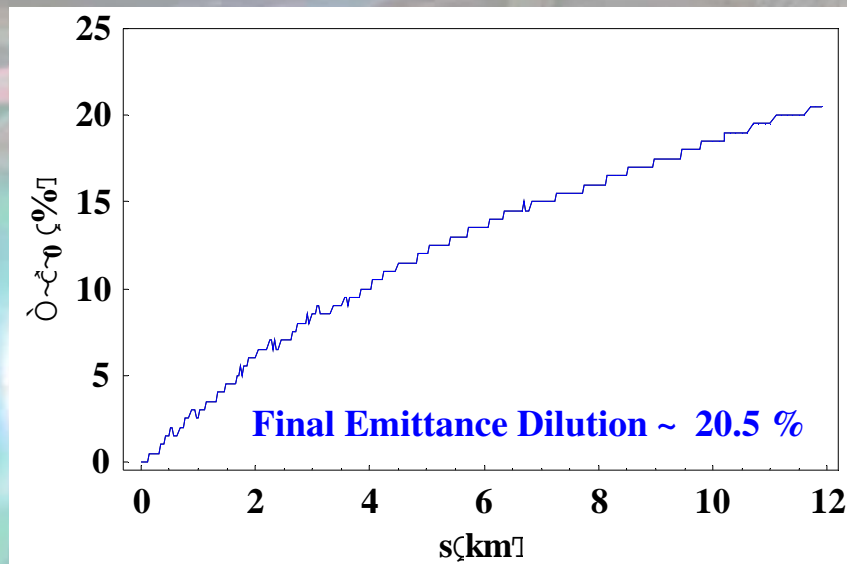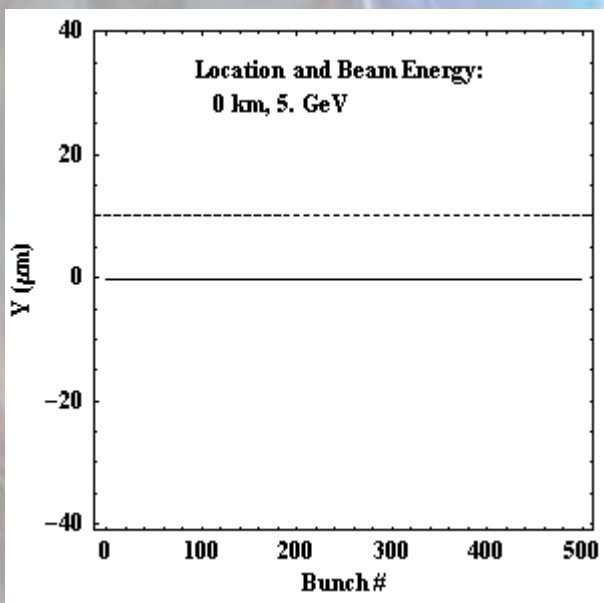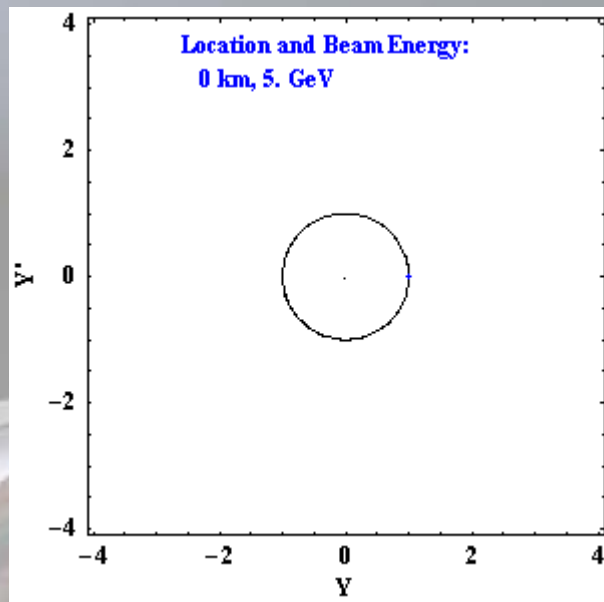
- **500 particles in all simulations.**
- **Initial horizonal offset ~400 μm**
- **No vertical offset!**
- **(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
- *Random azimuthal phase about mean of $\pi/4$*
- **Long + short range wakes included in simulations**

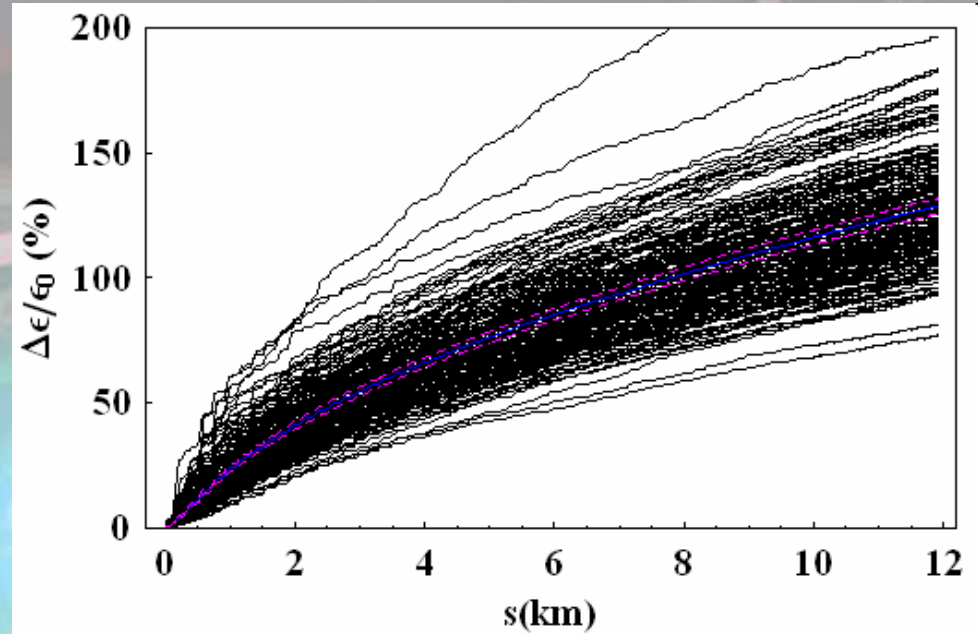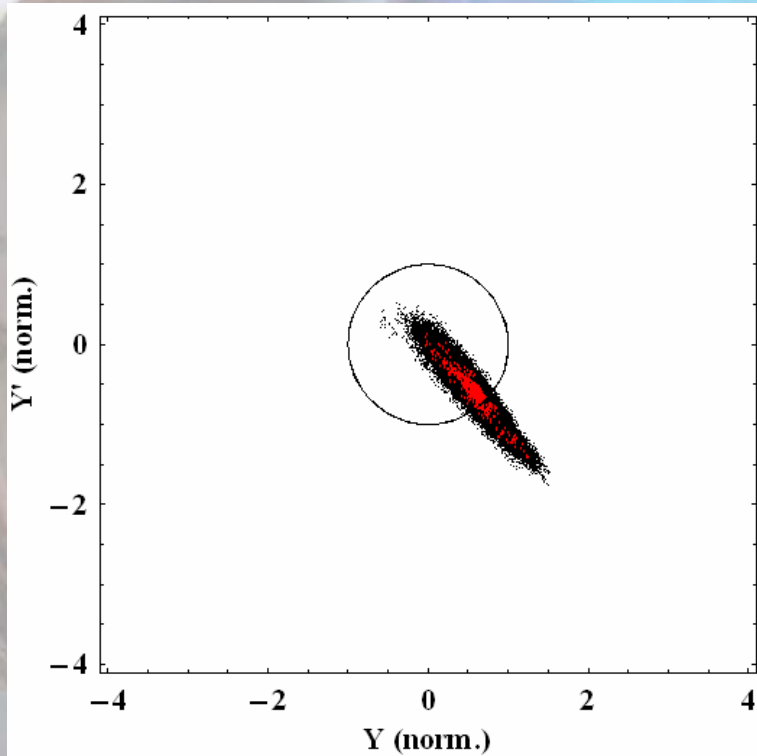- •**500 particles in all simulations and 200 machines shown**
- •**Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**
- •**(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
- •*Fixed azimuthal orientation of couplers.*
- •**Long range wakes included in simulations**

Fixed AzimuthalPhase Distribution (Yoff=0).

- **500 particles in all simulations and 200 machines shown**
- **Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**
- **(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
- *Random azimuthal orientation of couplers (0 to 2π about a mean value of π/4).*
- **Long range wakes included in simulations**

Amelioration of Large Emittance Dilution?

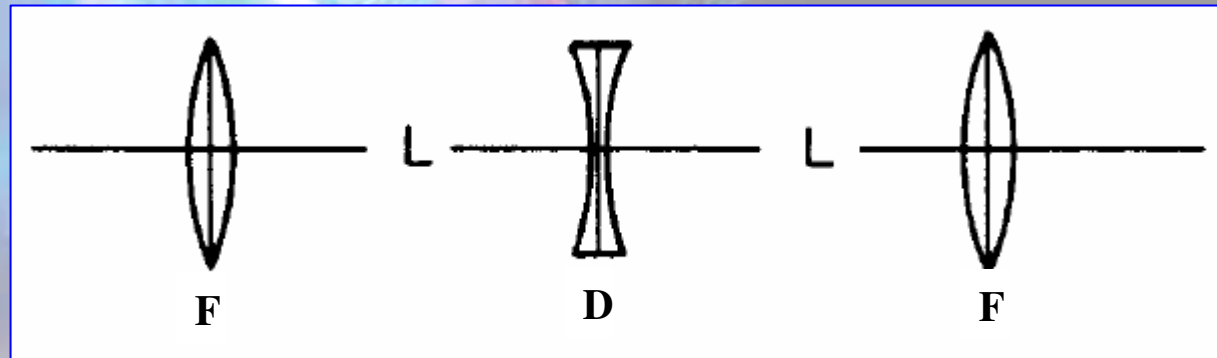•**Increase the damping of the HOM modes? At present the damping is ~10^4 - 10^5. It may be difficult to reduce the damping even further?**

•**The dipole frequency degeneracy's are already split –by ~ 400 kHz to 800 kHz. Increasing the splitting (which can be achieved by making markedly asymmetrical cavities) to say 10 MHz may allow the influence of mode coupling to be minimal over the length of the linac.**

•**Splitting the tune of the linac. Present design is 60/60 – horizontal/vertical.   This may be the most straightforward to implement.   => We investigate splitting the tune in this talk.**

1.3 GHz

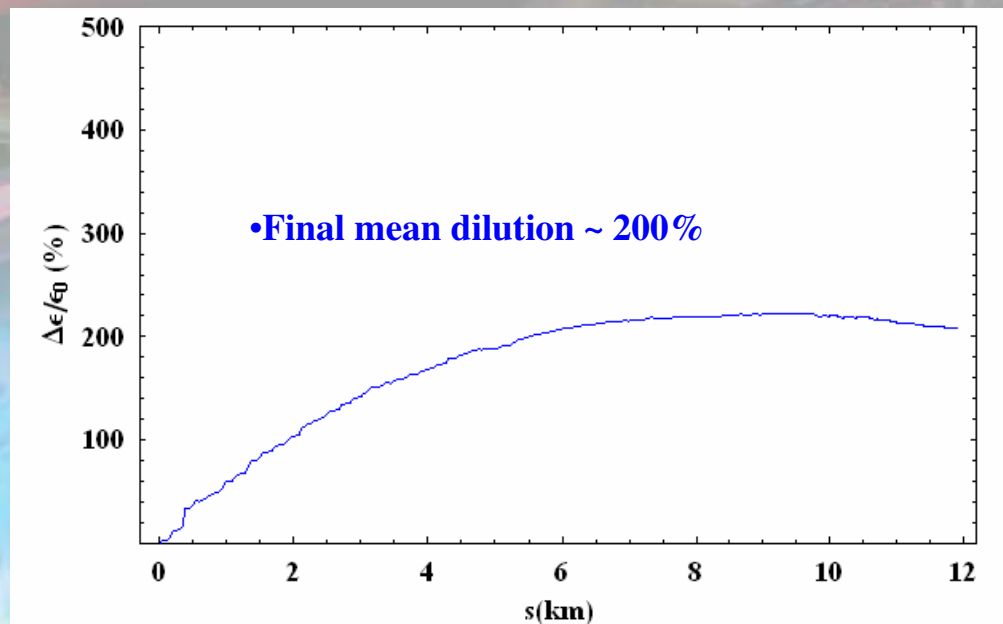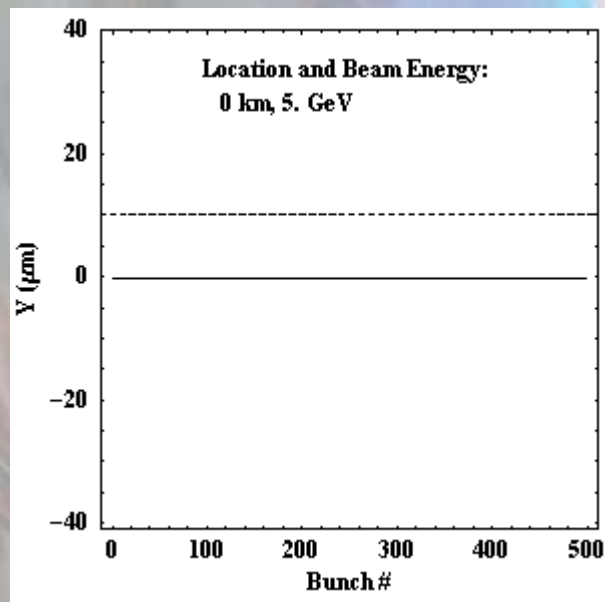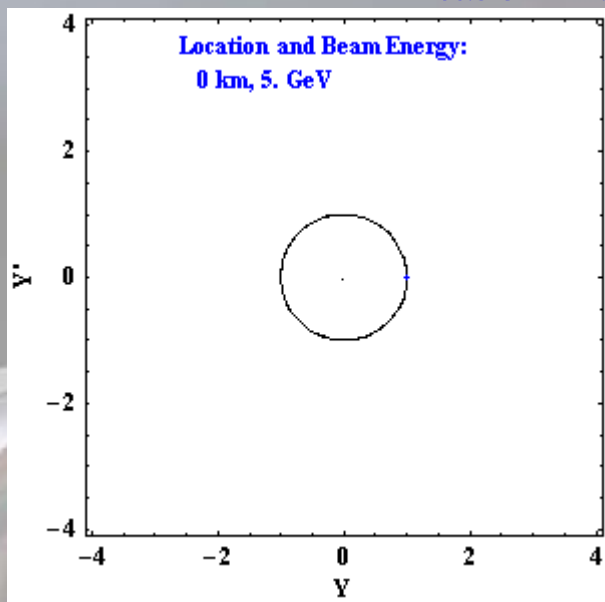**Re-entrant cavity shape is shown. Taken from Rong-Li Geng SRF 2005**

International Linear Collider
at Stanford Linear Accelerator Center

• **Present lattice has phase advance per cell of 60 degrees per cell.**

• **Each lattice cell consists of two groups of:**
quadupole + 2 cryo-module + quadrupole+ 2 cryo-module.
• **There are 12 cavities per cryo-module.**
• **If we split the tune then the coupling should be out of resonance.**

• **We look at 61/60 through 90/60.**

• **Single FODO array shown.**

• **We split the tune by raising the F functions for the focusing quadrupole with respect to the D quadrupole.**

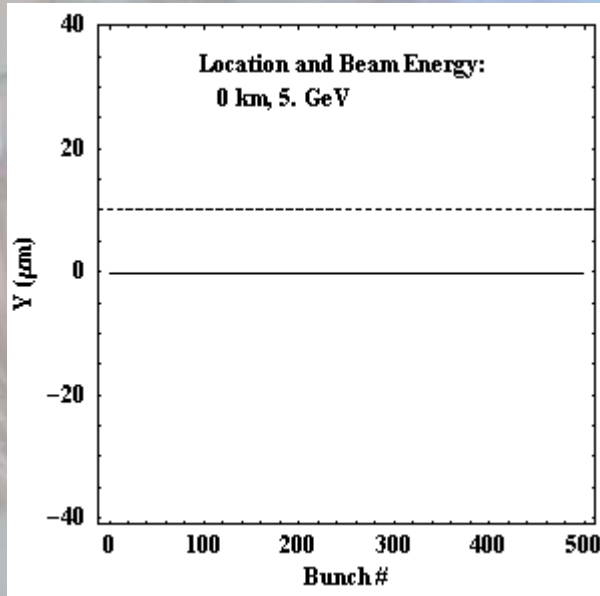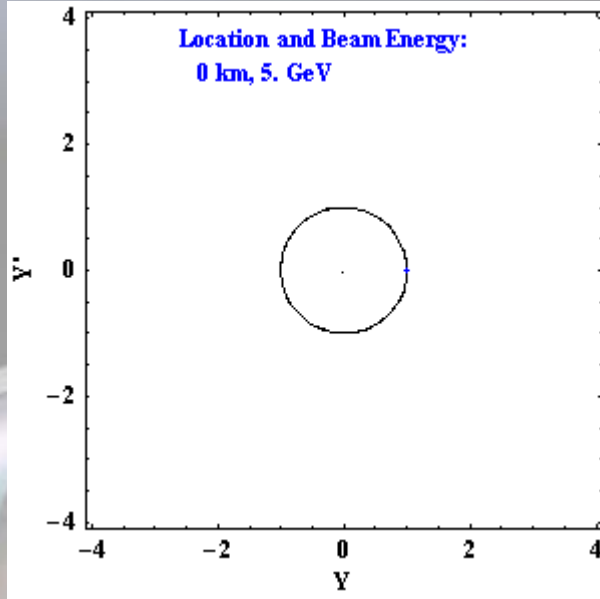• **The D quadrupole is then readjusted in order to achieve 60 in the vertical plane.**

F D F

Location and Beam Energy: 0 km, 5. GeV

•Final mean dilution ~ 200%

Location and Beam Energy: 0 km, 5. GeV

•500 particles in all simulations
•Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
•(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
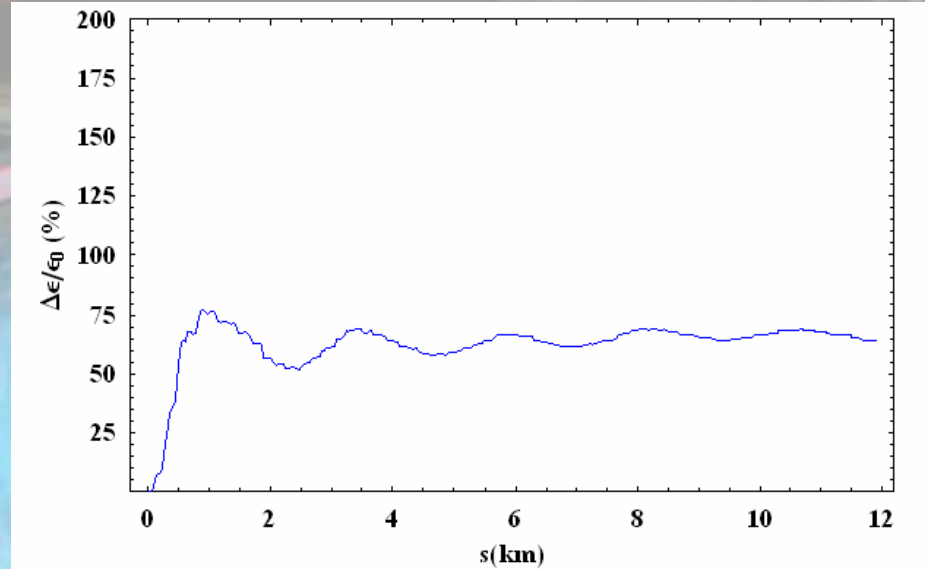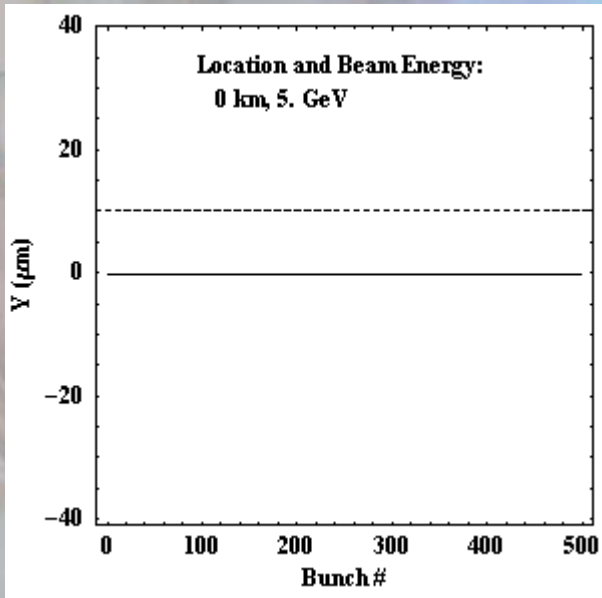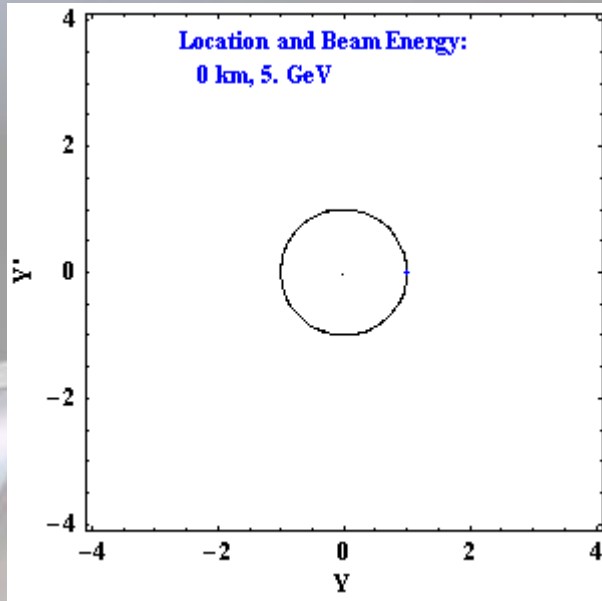•*Fixed azimuthal orientation of couplers.*
•Long range wakes included in simulations

- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Fixed azimuthal orientation of couplers.*
- Long range wakes included in simulations

- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Fixed azimuthal orientation of couplers.*
- **Long + Short range wakes included in simulations**

Location and Beam Energy:
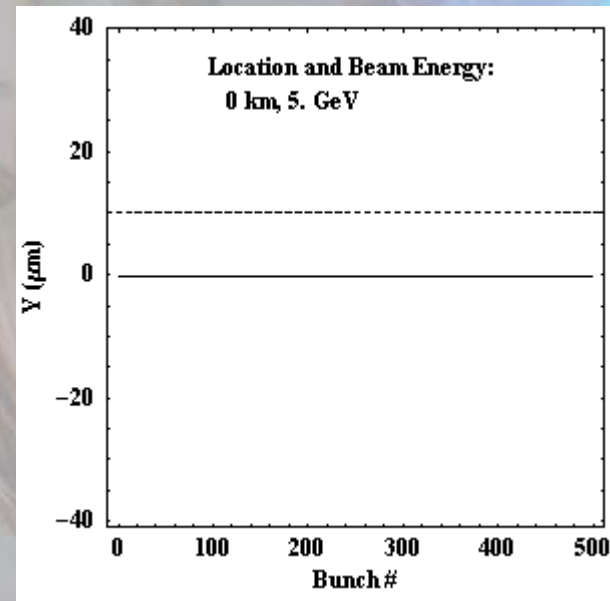0 km, 5. GeV

Location and Beam Energy:
0 km, 5. GeV

- **500 particles in all simulations**
- **Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x),**
- **(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
- *Random azimuthal orientation of couplers (0 to 2π).*
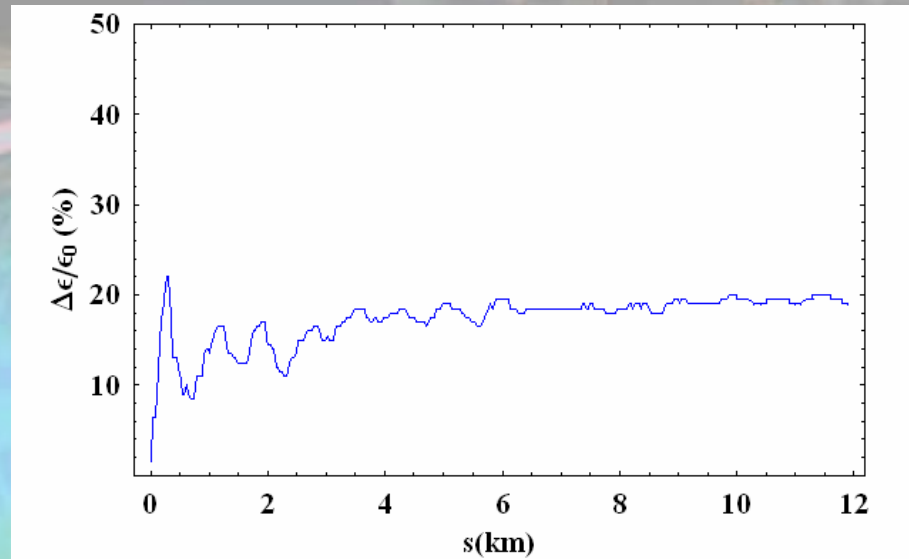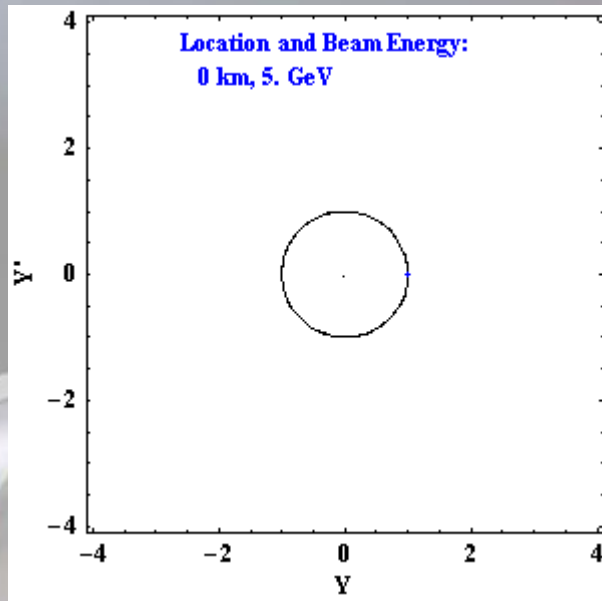- **Long + Short range wakes included in simulations**

Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 90/60*

- •500 particles in all simulations
- •Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- •(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
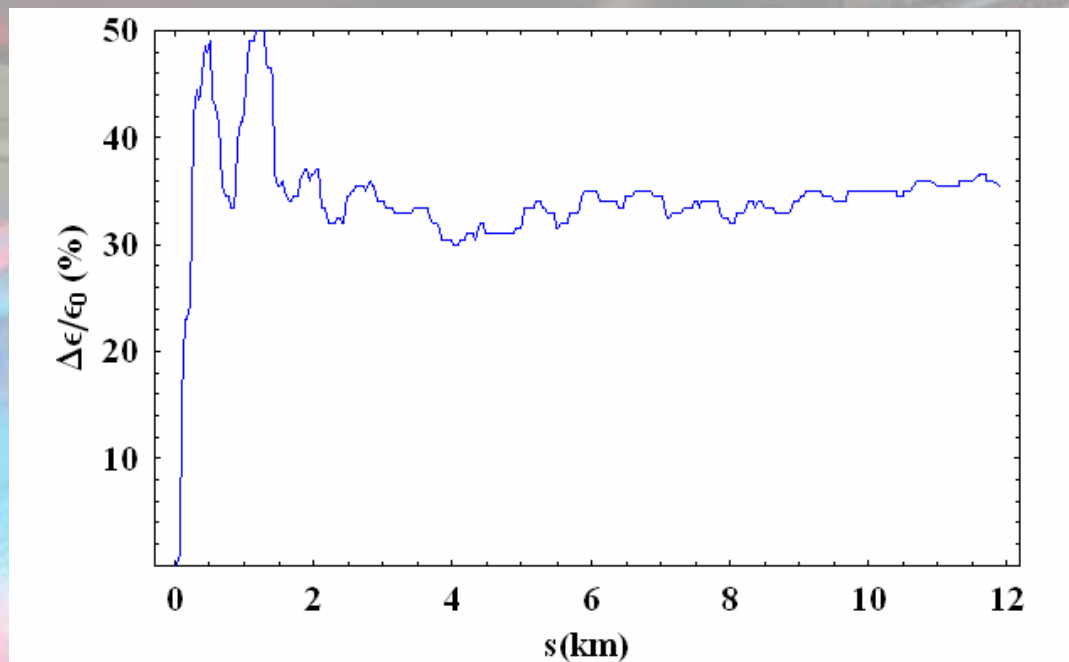- •*Fixed azimuthal orientation of couplers.*
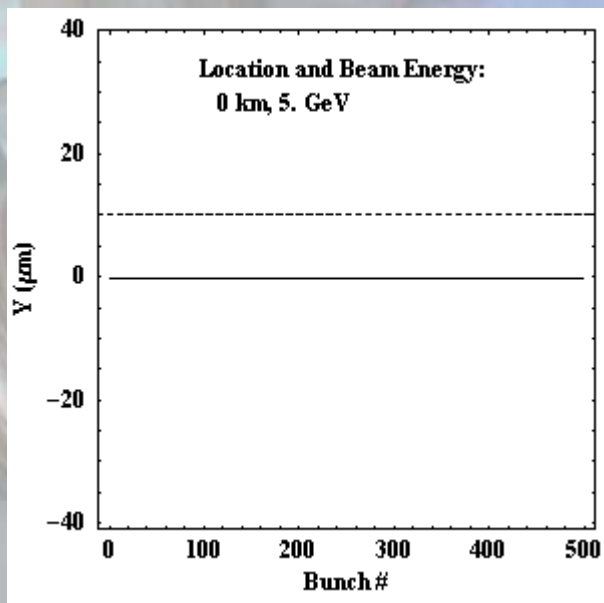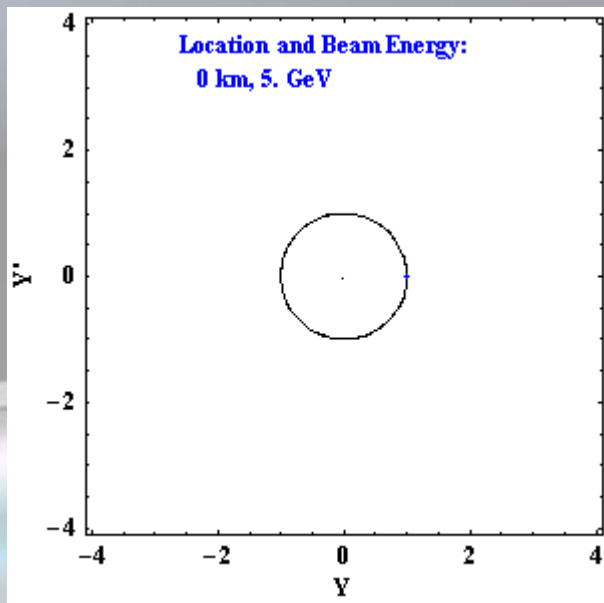- •**Long range wakes included in simulations**

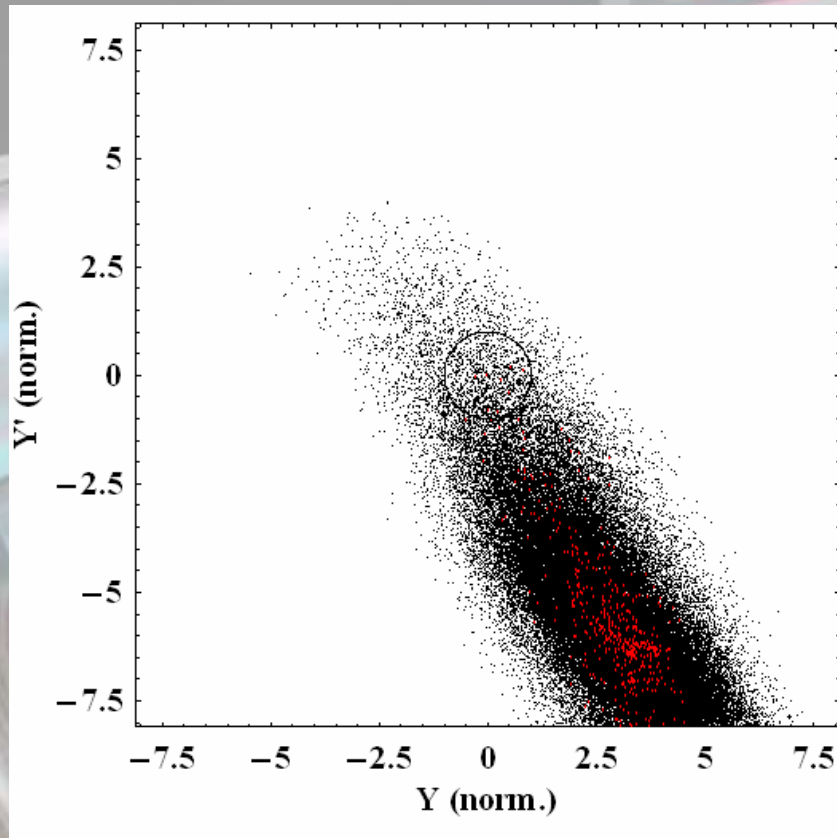# Cross-Coupled Motion *At* Fixed Azimuthal Phase Distribution (Yoff=0). *Split Tune 90/60*

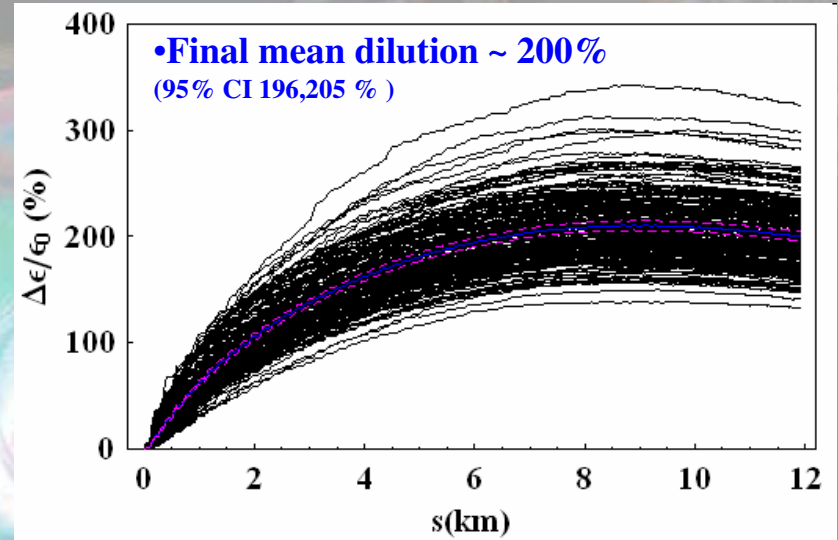

- **500 particles in all simulations**
- **Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**
- **(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
- *Limited randomization of azimuthal phase*
  –Max 10% of mean.
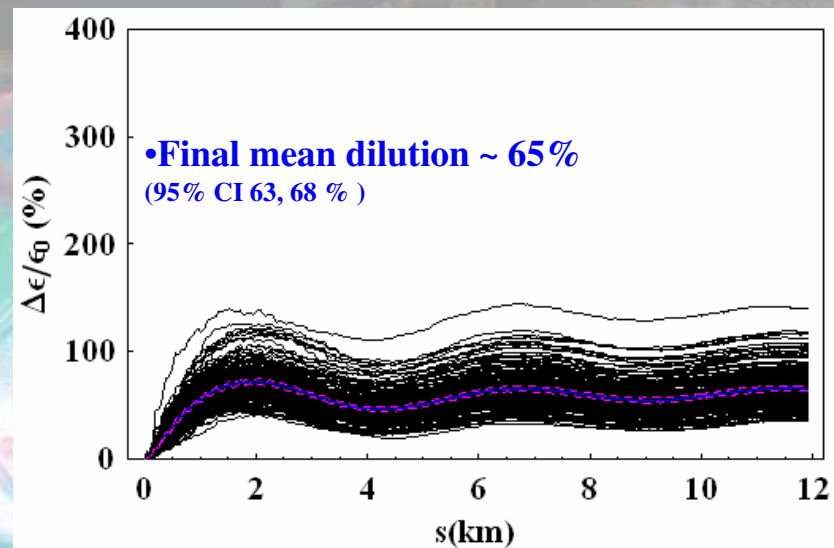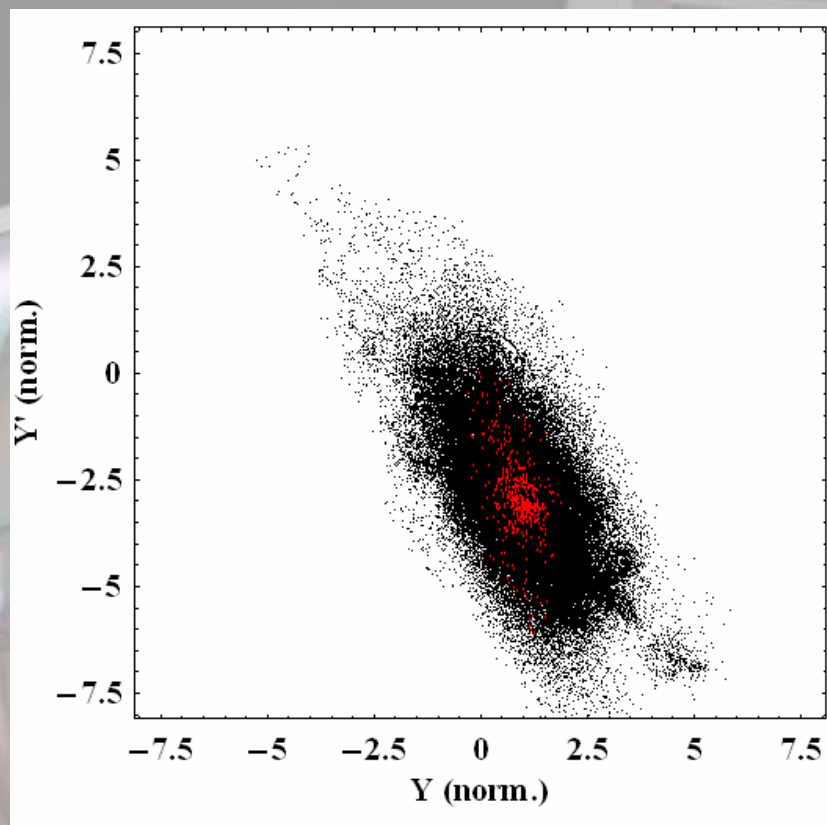- **Long range wakes included in simulations**

Small circle represents scaling w.r.t. energy at *end* of linac.

• **Final mean dilution ~ 200%** (95% CI 196,205 %)

- • **500 particles in all simulations and 200 machines are illustrated**
- • **The dashed line marks the injection offset of all bunches**
- • **Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**
- • **(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
- • *Fixed azimuthal phase.*
- • **Long range wakes included in simulations**

- **Final mean dilution ~ 65%**
  (95% CI 63, 68 %)

- **500 particles in all simulations and 200 machines are illustrated**
- **The dashed line marks the injection offset of all bunches**
- **Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**
- **(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
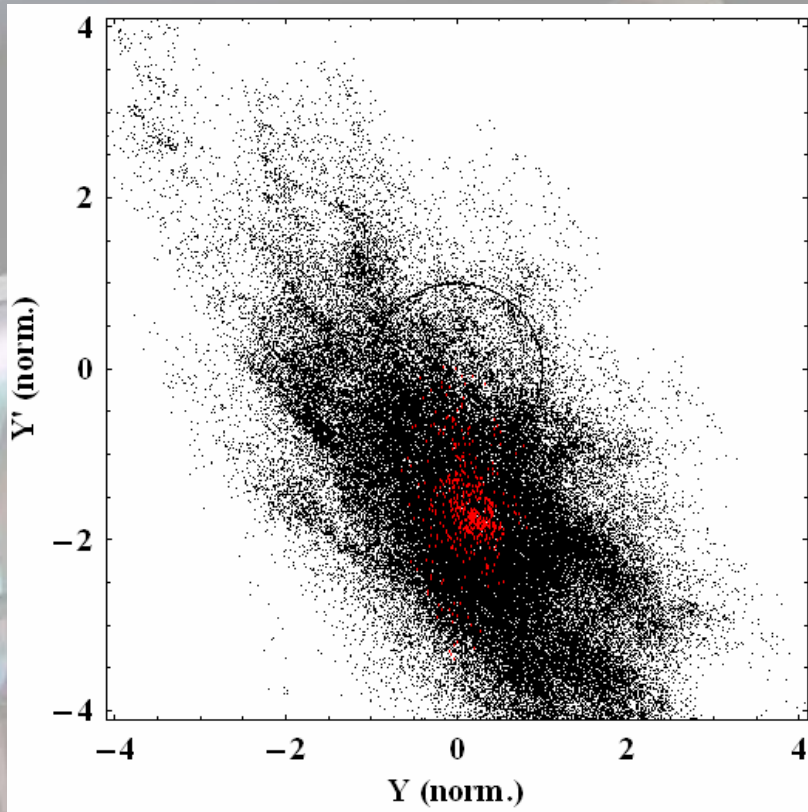- *Fixed azimuthal phase.*
- **Long range wakes included in simulations**

These have been rescaled relative to the previous ones

•**Final mean dilution ~ 35%** (95% CI 33, 37 %)

•**500 particles in all simulations and 200 machines are illustrated**

•**The dashed line marks the injection offset of all bunches**

•**Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**

•**(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**

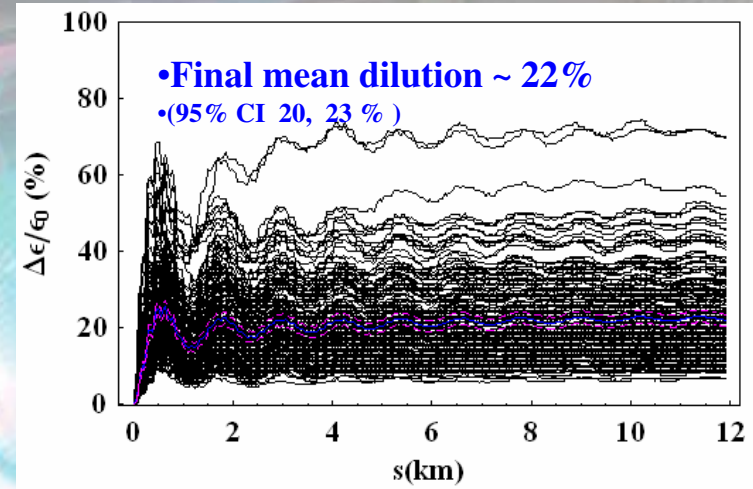•*Fixed azimuthal phase.*

•**Long range wakes included in simulations**

- **Final mean dilution ~ 22%**
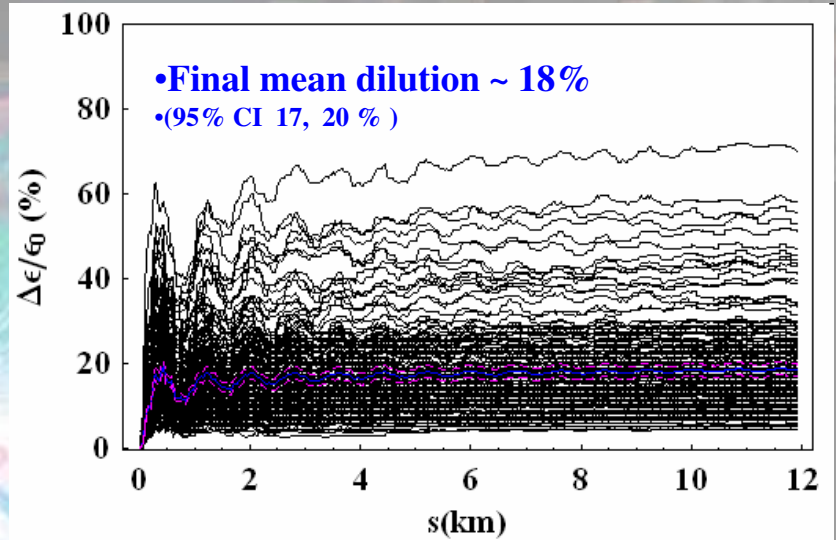- **(95% CI 20, 23 %)**

- **500 particles in all simulations and 200 machines are illustrated**
- **The dashed line marks the injection offset of all bunches**
- **Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**
- **(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
- ***Fixed azimuthal phase.***
- **Long range wakes included in simulations**

- **Final mean dilution ~ 18%**
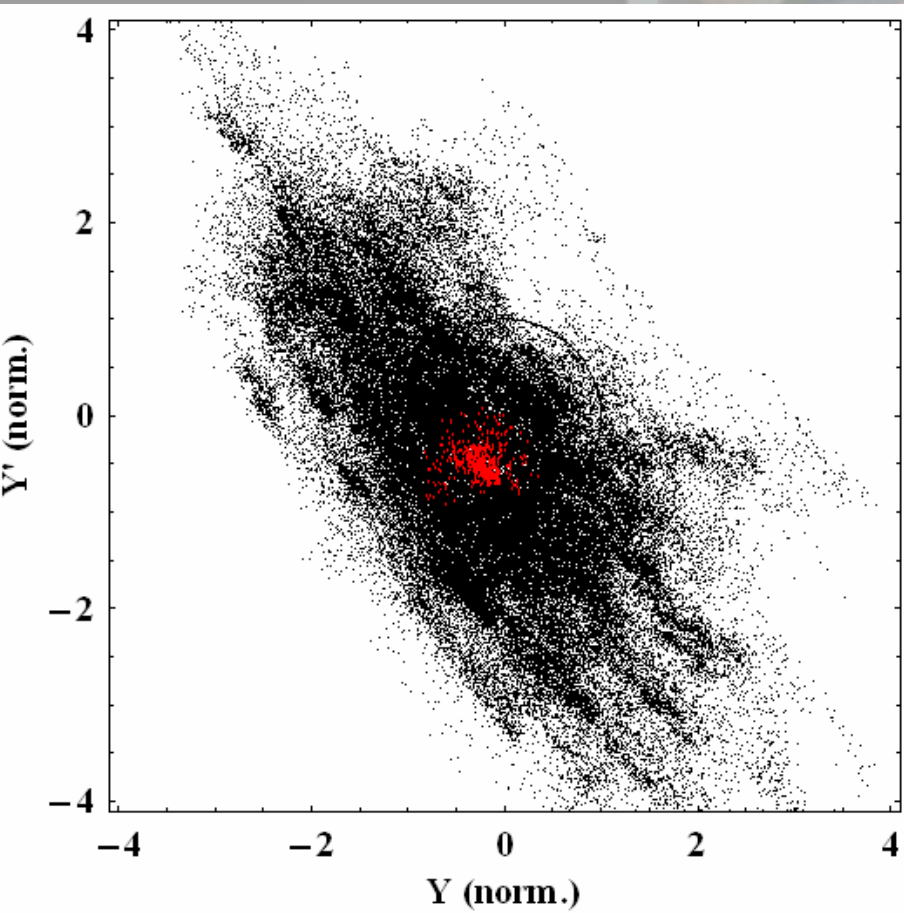- **(95% CI 17, 20 %)**

- **500 particles in all simulations and 200 machines are illustrated**
- **The dashed line marks the injection offset of all bunches**
- **Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**
- **(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
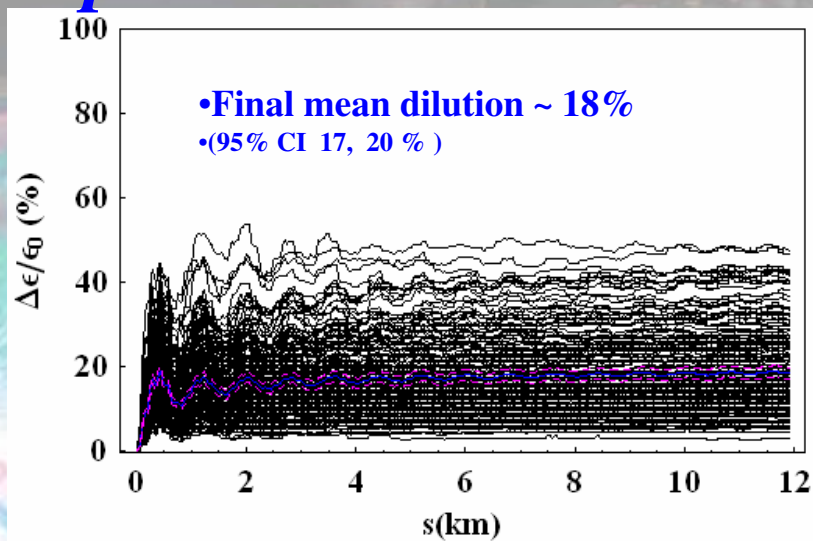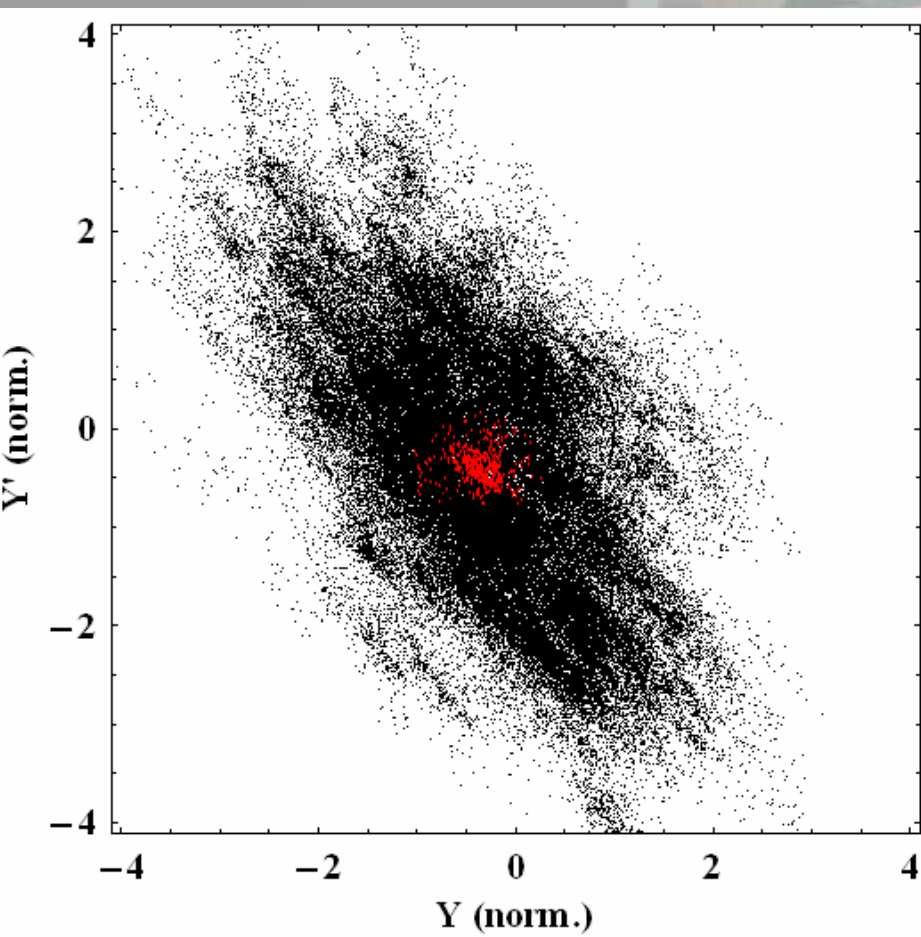- *Fixed azimuthal phase.*
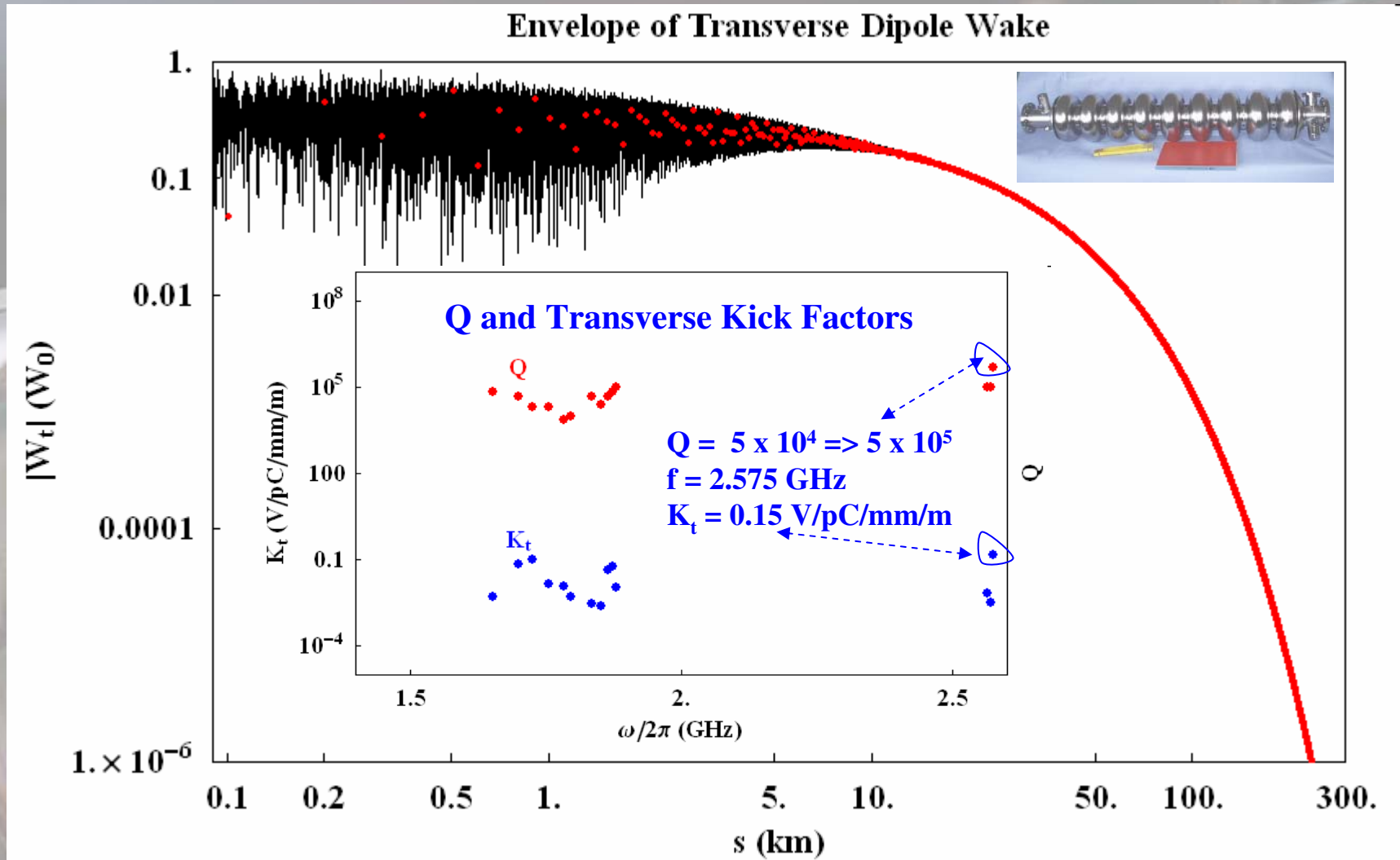- **Long range wakes included in simulations**

Cross-Coupled Motion, Limited Randomization of Azimuthal Phase Distribution of HOM Couplers (Yoff=0). *Split Tune 90/60.*

- •Final mean dilution ~ 18%
- •(95% CI 17, 20 %)

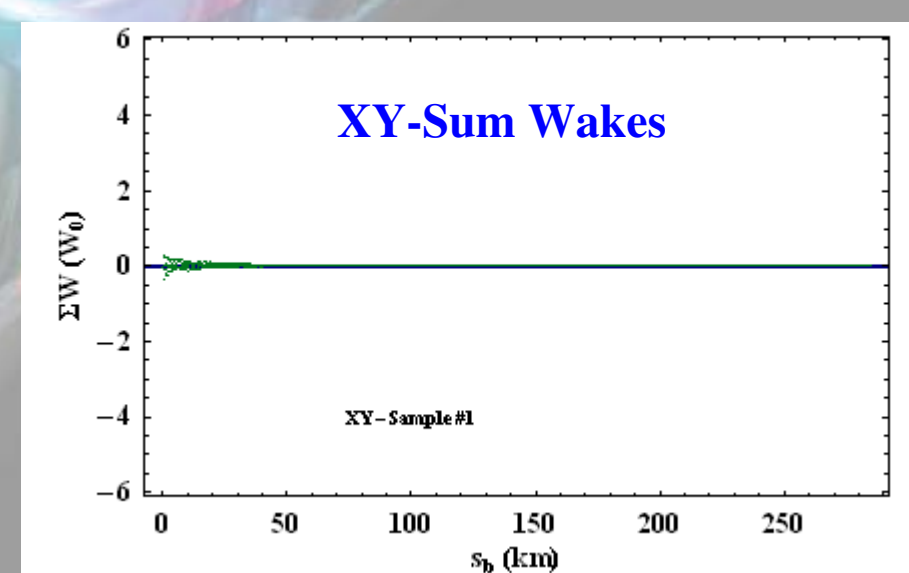- •**500 particles in all simulations and 200 machines are illustrated**
- •**The dashed line marks the injection offset of all bunches**
- •**Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**
- •**(c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)**
- •*Limited randomization of azimuthal phase*
  –Max 10% of mean.
- •**Long range wakes included in simulations**

Envelope of Transverse Dipole Wake

Q and Transverse Kick Factors

Q = 5 x 10^4 => 5 x 10^5
f = 2.575 GHz
K_t = 0.15 V/pC/mm/m

International Linear Collider
at Stanford Linear Accelerator Center

Y-Wake at Bunch

Sample #1

XY-Wake at Bunch

XY Sample #1

Y-Sum Wakes

Sample#1

XY-Sum Wakes

XY–Sample#1

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution + Rogue Mode

Final Mean $\Delta\epsilon/\epsilon_0 \sim$ 658 %

- 500 particles in all simulations.
- The initial position of all bunches in phase space is indicated by the blue dot.
- Initial vertical and horizontal offset ~10 μm and 400μm.
- (c.f. initial beam $\sigma_{y,x} \sim$ 10.1 μm, 270 μm)
- *Azimuthal phase fixed at $\pi/4$*
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- **Long range wakes included in simulations**

Cross-Coupled Motion With Random Azimuthal Phase Distribution + Rogue Mode
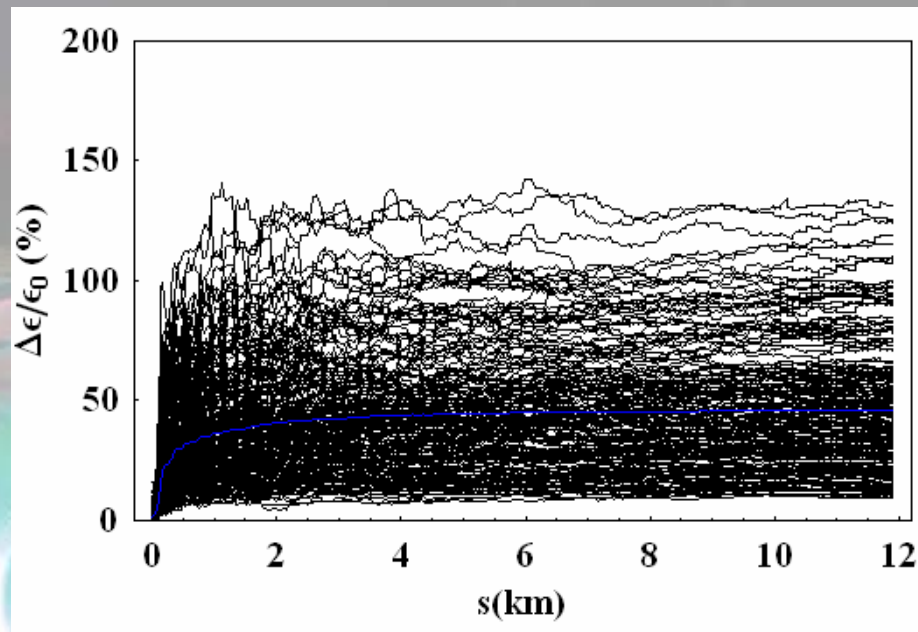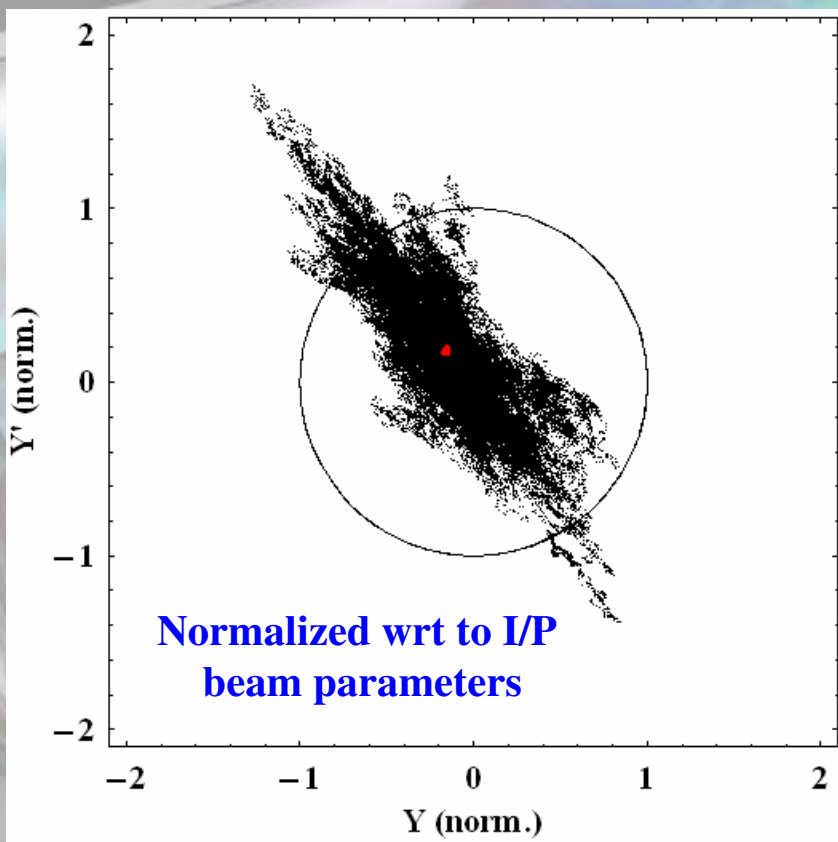
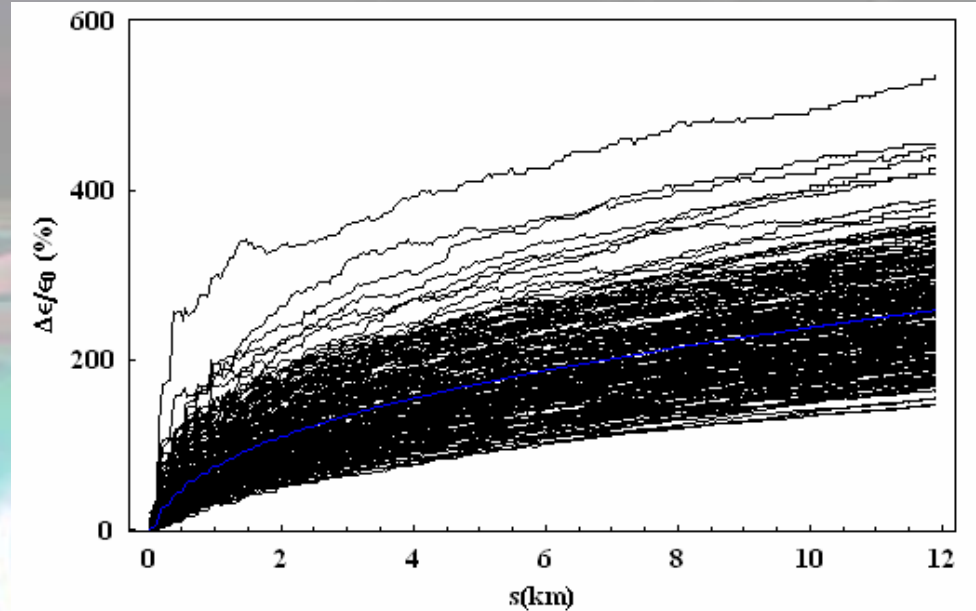
Normalized wrt to I/P beam parameters

- 500 particles in all simulations.
- The initial position of all bunches in phase space is indicated by the blue dot.
- Initial vertical and horizontal offset ~10 μm and 400μm.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Random Azimuthal phase (mean π/4)*
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- **Long range wakes included in simulations**

Cross-Coupled Motion With Limited Random Azimuthal Phase Distribution + Rogue Mode

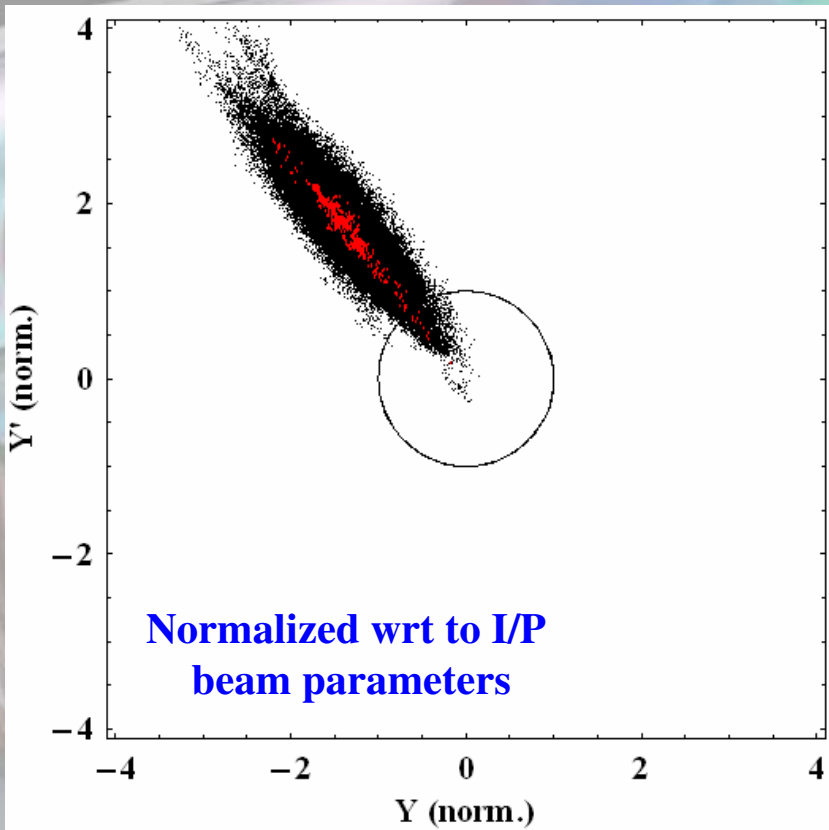**Normalized wrt to I/P beam parameters**

- 500 particles in all simulations.
- The initial position of all bunches in phase space is indicated by the blue dot.
- Initial vertical and horizontal offset ~10 μm and 400μm.
- (c.f. initial beam $\sigma_{y,x}$ ~ 10.1 μm, 270 μm)
- *Azimuthal phase randomly distributed –Max 10% of mean.*
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
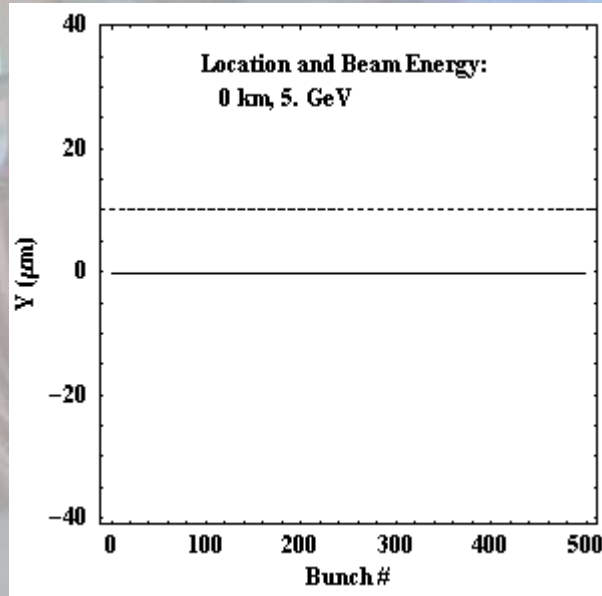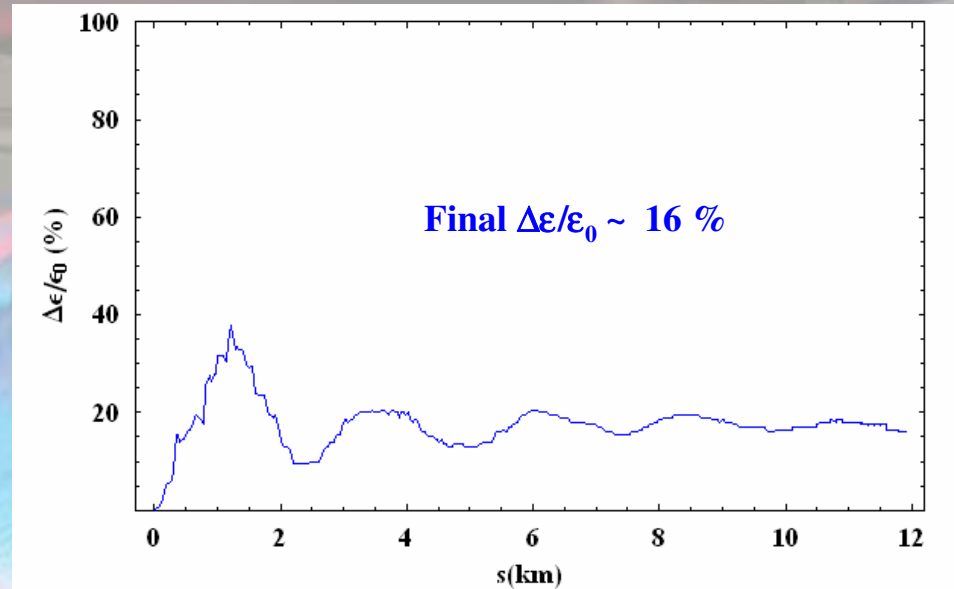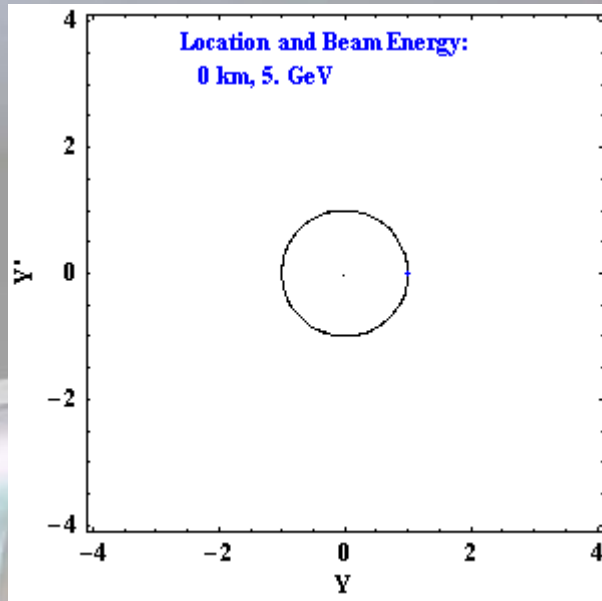- **Long range wakes included in simulations**

Location and Beam Energy: 0 km, 5. GeV

Final $\Delta\varepsilon/\varepsilon_0 \sim$ 16 %

Location and Beam Energy: 0 km, 5. GeV

- Only mode-cross coupling gives rise to emittance dilution
- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim$ 10.1 μm, 270 μm)
- *Fixed azimuthal orientation of couplers.*
- Long range wakes included in simulations

$X_{off} \sim \sigma_x$, $Y_{off} = 0$. *Split Tune 70/60.*

- **500 particles in all simulations.**
- **Initial vertical and horizontal offset =0 and 270 μm.**
- **c.f. initial beam $\sigma_{y,x} \sim 10.1$ μm, 270 μm.**
- **Final mean of the emittance dilution from 200 machines is indicated by the blue line**
- **Long range wakes included**

Split Tune 90/60.

Location and Beam Energy:
0 km, 5. GeV

Final $\Delta\varepsilon/\varepsilon_0 \sim 40\%$
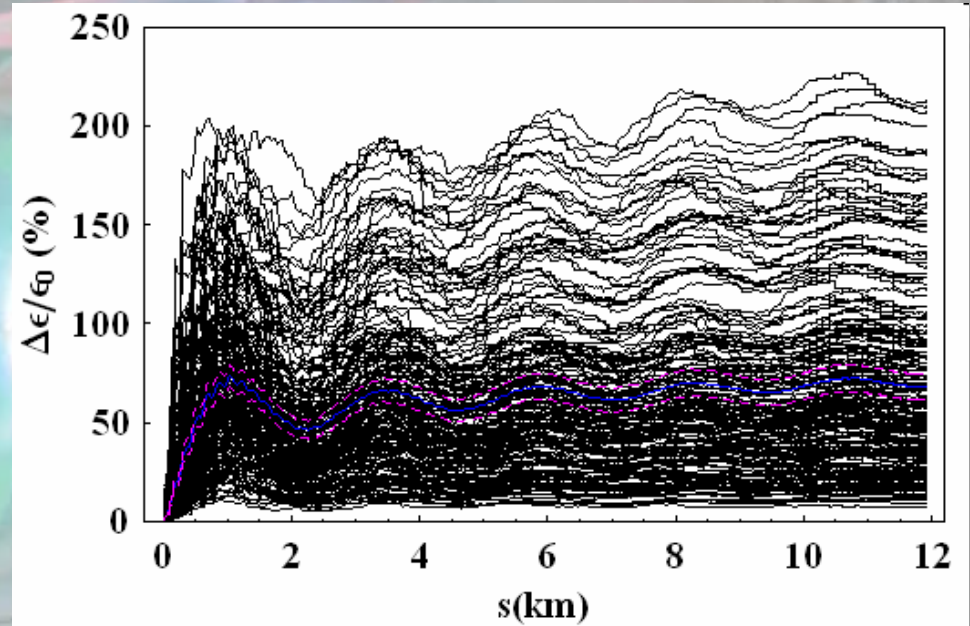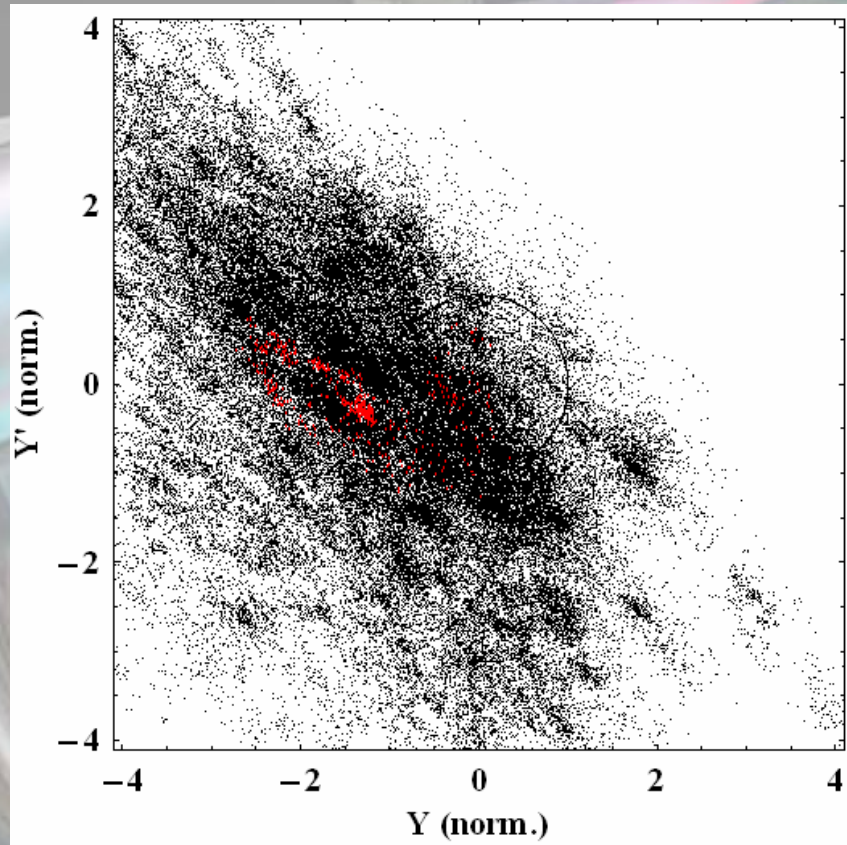

Location and Beam Energy:
0 km, 5. GeV

- **Only mode-cross coupling gives rise to emittance dilution**
- **500 particles in all simulations**
- **Initial vertical and horizontal offset 0 μm and 270μm (~ σ_x), respectively.**
- **(c.f. initial beam $\sigma_{y,x} \sim 10.1$ μm, 270 μm)**
- ***Fixed azimuthal orientation of couplers.***
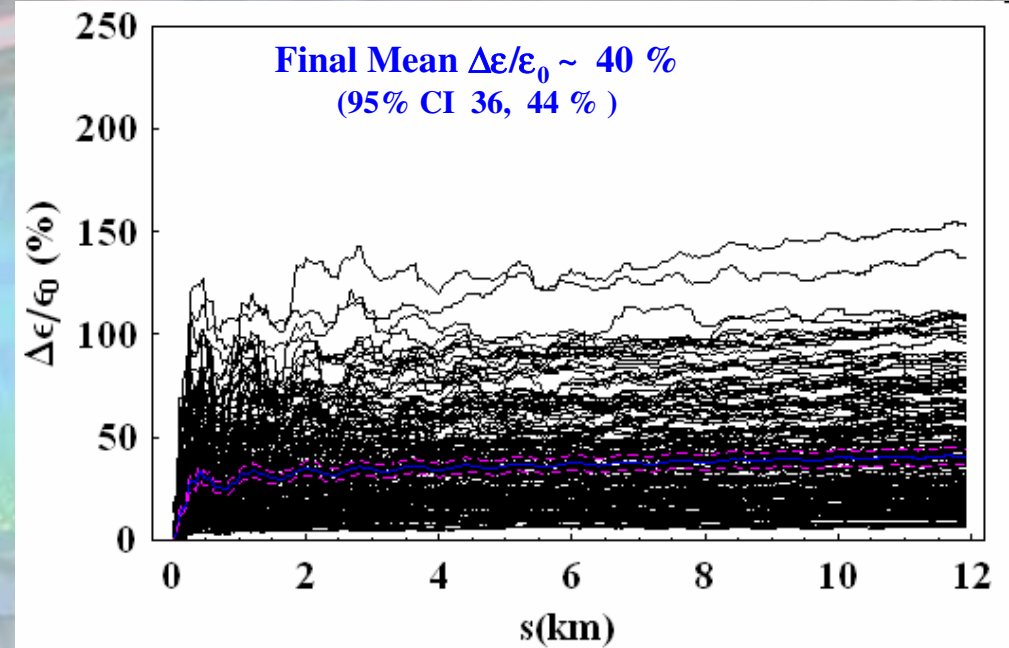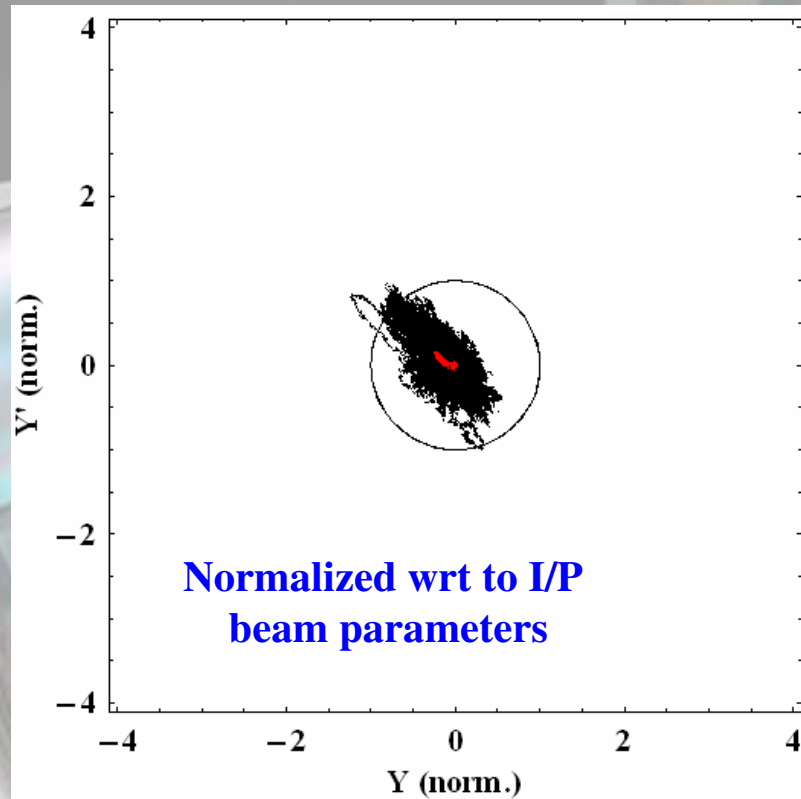- **Long range wakes included in simulations**

$X_{off} \sim \sigma_x$, $Y_{off} = 0$. *Split Tune 90/60.*

Normalized wrt to I/P beam parameters

Final Mean $\Delta\epsilon/\epsilon_0 \sim$ 40 %
(95% CI 36, 44 %)

- 500 particles in all simulations.
- Initial vertical and horizontal offset =0 and 270 μm.
- c.f. initial beam $\sigma_{y,x} \sim$ 10.1 μm, 270 μm.
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- **Long range wakes included**

Mode Coupling Summary

•**Mode-coupling is a significant source of emittance dilution.**

•**Emittance dilution can be ameliorated by splitting the tune of the lattice.**

•**Further experimental data is needed on the actual splitting of the dipole degeneracies in frequencies that is likely to occur in the fabrication of several thousand ILC cavities (experiments at DESY, TTF already in progress will be helpful in this respect –Ross, Napoly, Baboi et al).   In addition, simulations are in progress on the splitting of the tune encountered due to the HOM couplers at SLAC (Z. Li, K. Ko et al).**

•**Analysis of the influence of higher order bands on emittance dilution in the presence of mode coupling is required.**

•**The effect of purposely distorting the shapes of the cavities in order to increase the frequency degeneracy splitting requires further study.**

EXTRA SLIDES
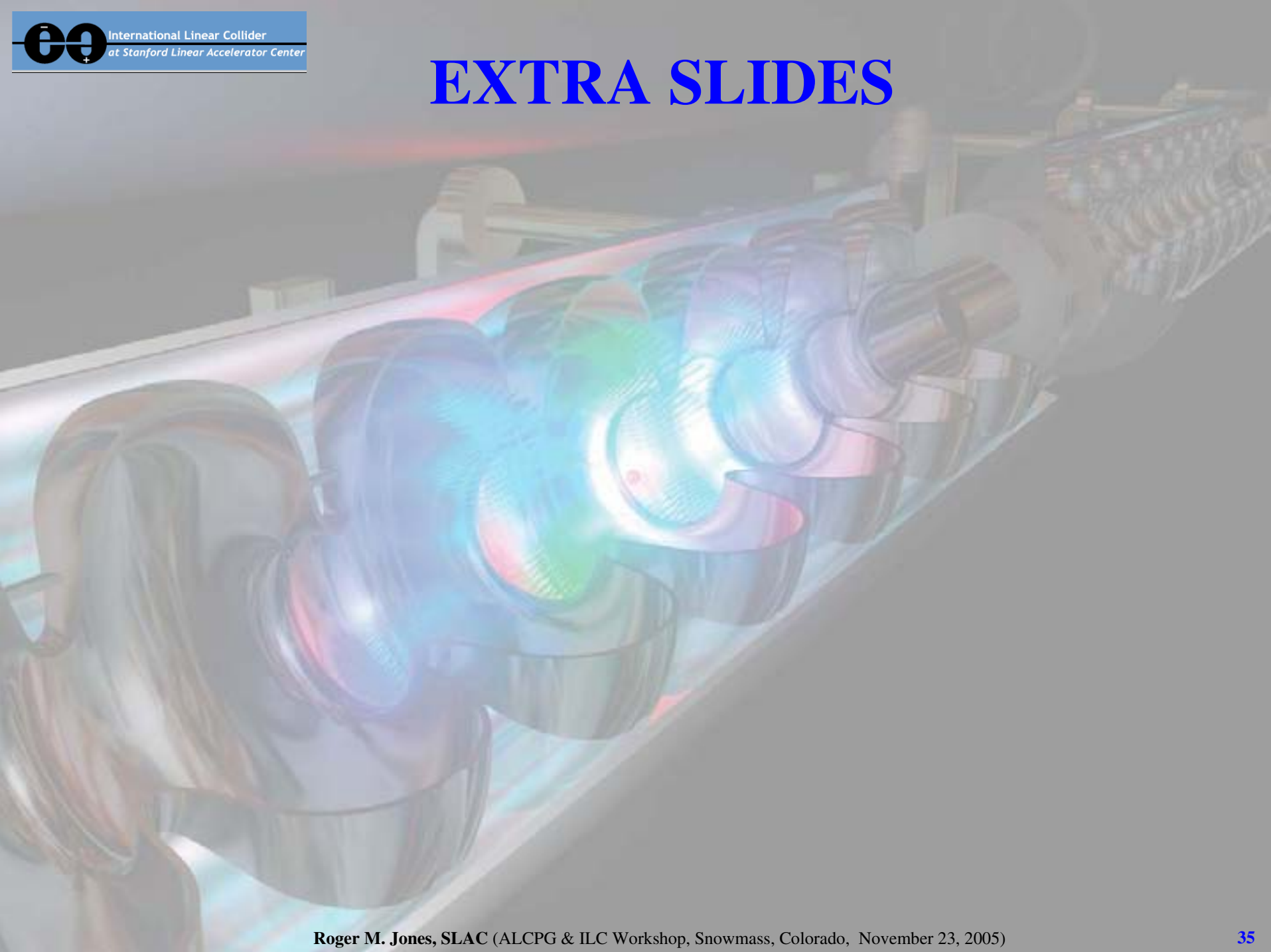
Cross-Coupled Motion
Zero Randomization of Azimuthal Orientation of HOM Couplers + Rogue Mode
$X_{off} \sim \sigma_x$, $Y_{off} = 0$. *Split Tune 90/60.*

Normalized wrt to O/P beam parameters

Final Mean $\Delta\varepsilon/\varepsilon_0 \sim 40\%$
(95% CI 36, 44 %)