Monte Carlo Studies of the Neutralino Relic Density

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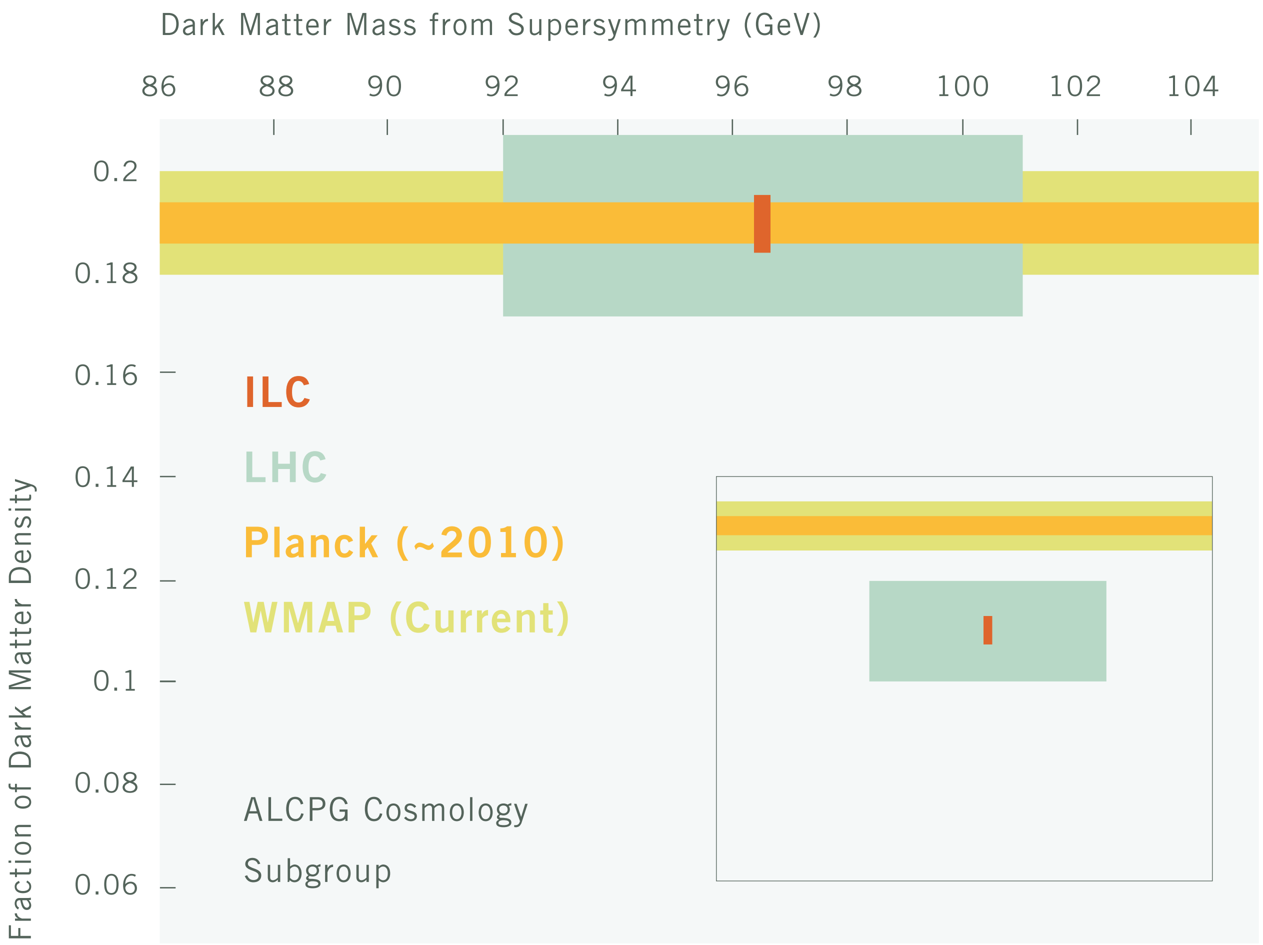
In the HEPAP subpanel report on future particle colliders, you saw this figure:



In this talk, I would like to address the following questions:

What methodology should be used to make such a figure ?

Did the HEPAP subpanel get it right ?

Is the situation they showed the generic one ?

The figure is made for the mSUGRA point SPS1a. In principle, we could make similar plots for other supersymmetry points, and for model points in other models.

It is by now standard to consider SPS1a first, so I will do that.

SPS1a is a point in the “bulk region”, where the neutralino relic density is determined by the annihilation processes:

$$NN \to \ell^+ \ell^-$$

through t-channel slepton exchange. To find the right density, we need light sleptons:

$$m(\widetilde{\ell}) < 200 \text{ GeV}$$

In fact, this is a good reason to bet on light sleptons.

The best thing about SPS1a is that the authors of the LHC/ILC report (G. Weiglein et al.) spent years quantifying the capabilities of ILC and LHC for SUSY spectrum measurements at this point.

I would like to apply their results in the following way:

I will ask what distribution of results for the neutralino dark matter density Ωh^2 is consistent with a set of measurements to the accuracies they find.

The analysis I will describe is being done in collaboration with Ted Baltz, Marco Battaglia, and Tommer Wizansky. Results I will show today are preliminary.

First, we have to agree on a model. This model should have enough degrees of freedom that one can claim some measure of “model-independence” in its context.

mSUGRA is not appropriate. Four precision measurements fix the model.

The general MSSM with flavor, CP, R-parity conservation has 24 parameters: 3 gaugino masses, 15 sfermion masses, μ, $\tan\beta$, m_A, 3 A parameters.

This parameter space encodes most of the important alternatives for SUSY phenomenology.

Let K denote a point in the model space, and let R denote the reference point SPS1a. For a set of measurements

$$\{m_j(K)\}$$

(masses, cross sections), define the likelihood of K as

$$\mathcal{L}(K) = \prod_j \exp\left\{ -\frac{(m_j(K) - m_j(R))^2}{2\sigma_j^2} \right\}$$

where the σ_j are the measurements errors computed by Weiglein's study.

If we interpret $\mathcal{L}(K)$ as the probability of K, we can translate this into probabilities of values of the neutralino relic density.

To explore the large-dimension parameter space, we use Markov Chain Monte Carlo:

from K, consider a jump to K+1

if $\mathcal{L}(K+1) > \mathcal{L}(K)$, jump.

if $\mathcal{L}(K+1) < \mathcal{L}(K)$,
jump with probability $\mathcal{L}(K+1)/\mathcal{L}(K)$

This implements detailed balance to generate a distribution of points populated with probability $\mathcal{L}(K)$.

The method can ‘tunnel’ through regions of low probability to find multiple preferred regions.

Now we need a table of measurement errors. For the LHC at the point SPS1a, this table is surprisingly complete:

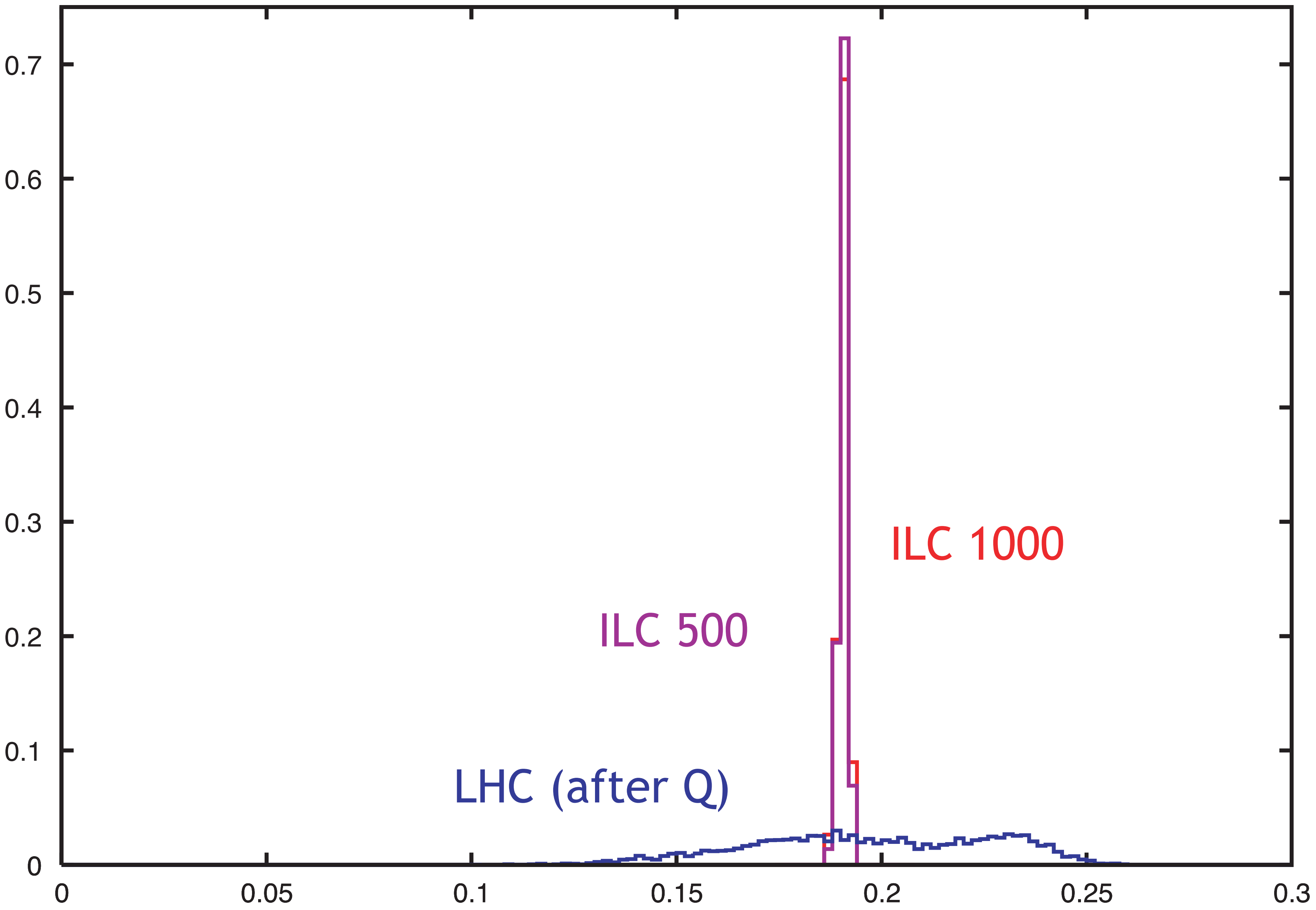
$m(N_1)$	± 4.8	$m(\widetilde{e}_R) - m(N_1)$	± 1.0
$m(N_2) - m(N_1)$	± 1.2	$m(\widetilde{\mu}_R) - m(N_1)$	± 1.0
$m(N_3)/m(N_4) - m(N_1)$	± 2.2	$m(\widetilde{\tau}_R) - m(N_1)$	± 5.0
$m(\widetilde{q}_R)$	± 19.0	$m(\widetilde{e}_L) - m(N_1)$	± 1.2
$m(\widetilde{q}_L)$	± 17.4	$m(\widetilde{\mu}_L) - m(N_1)$	± 1.2
$m(\widetilde{b}_1)$	± 7.5		
$m(\widetilde{b}_2)$	± 7.9		
$m(\widetilde{g})$	± 8.0		

also - this is quite important - $m_A > 200$ and $m_A/\tan\beta > 30.$ Otherwise, the A would be observed at LHC.

Of course, to play the game, we must know that the model is the MSSM. This requires data from the ILC, e.g., spin determinations. So I will label the LHC curves “LHC (after Q)”.

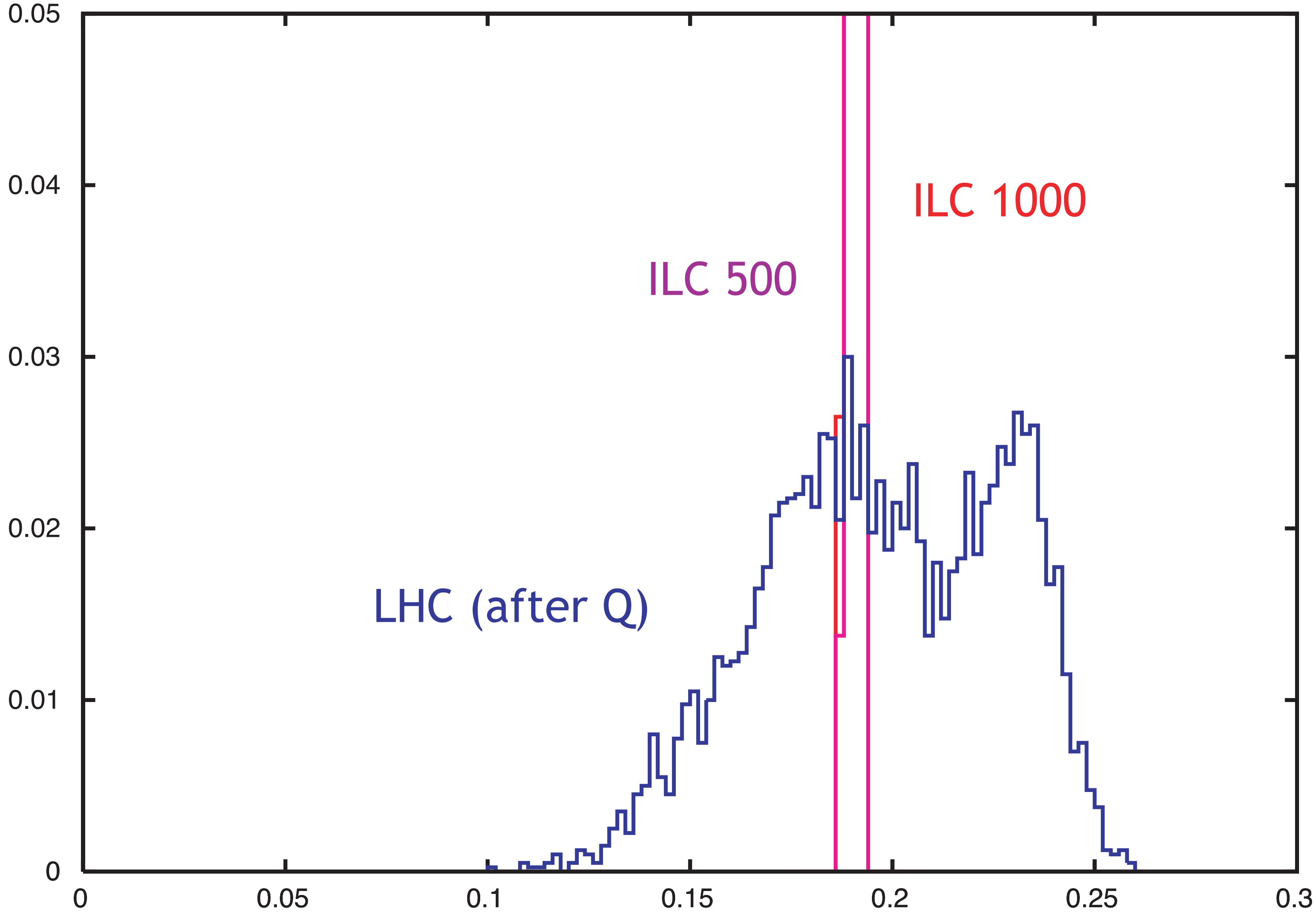
ILC is hardly worth discussing. All of the sleptons, charginos, neutralinos are pinned down. A number of polarized cross sections can be measured to precisions of a few percent.

So, run the scan, here is the result:



ILC 1000
ILC 500
LHC (after Q)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
0.05
0.1
0.15
0.2
0.25
0.3

blowing up the vertical scale:



Our error estimates for ILC and LHC are 1% and 20%. The HEPAP plot has 4% and 10%. Not bad for government work.

It is difficult to predict the neutralino relic density from LHC information alone for three reasons:

The LSP mass is not known precisely. Ωh^2 is very sensitive to this quantity.

The LHC data determines masses, but it is difficult to extract mixing angles. These angles enter into the error.

The LHC often cannot exclude alternative mechanisms for NN annihilation, for example, annihilation through the A pole. (At SPS1a, however, we can exclude this region.)

- Birkedal + Matchev

SPS1a is special in many ways. It is useful to contrast this point with another one that is harder for both LHC and ILC.

So, consider LCC2. This is a point in the 'focus point' region of mSUGRA. It resemble many generic MSSM points in that NN annihilation is dominated by

$$NN \to W^+W^- \ , \quad NN \to ZZ \ , \quad NN \to Zh$$

The point is chosen so that 3 neutralinos can be seen at 500 GeV. Also m(g) = 850 GeV, so the model has cascade decays

$$\widetilde{g} \to q\overline{q} \ N_{2,3} \qquad N_{2,3} \to \ell^+\ell^- N_1$$

that allow for precision measurements at the LHC.

All squarks, sleptons, and heavy Higgs bosons are extremely heavy, at about 3 TeV.

The ILC measurements at 500 GeV have been worked out in a simulation study by Gray et al (Cornell):

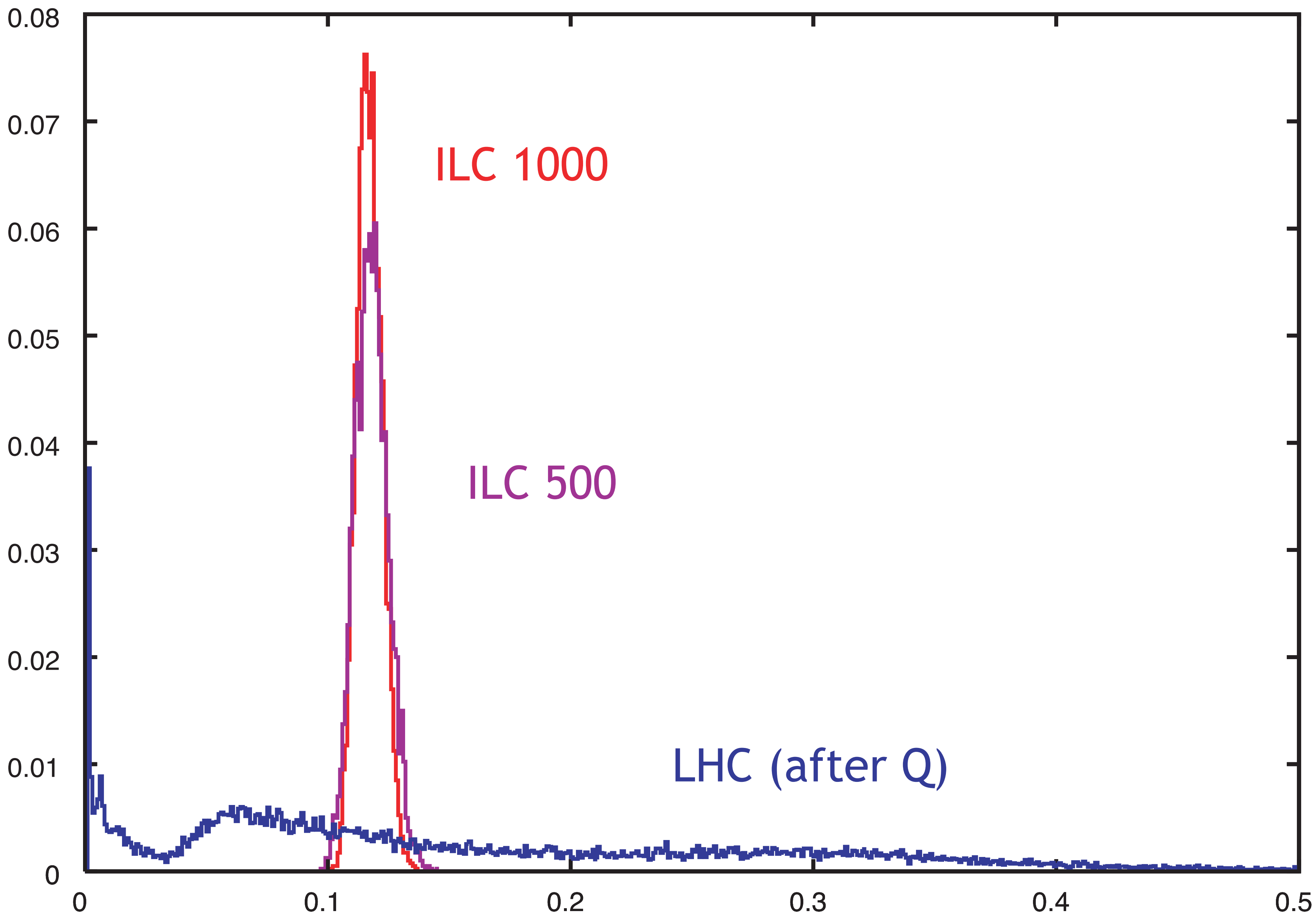
$$\begin{array}{rr|lr} m(N_1) & \pm 1.0 & \sigma(e^+e^- \to N_2 N_3) & \pm\ 4\%\ /\ 5\% \\ m(N_2) - m(N_1) & \pm 0.3 & \sigma(e^+e^- \to C_1^+ C_1^-) & \pm\ 8\%/11\% \\ m(N_3) - m(N_1) & \pm 0.2 & & \\ m(C_1) - m(N_1) & \pm 0.3 & & \end{array}$$

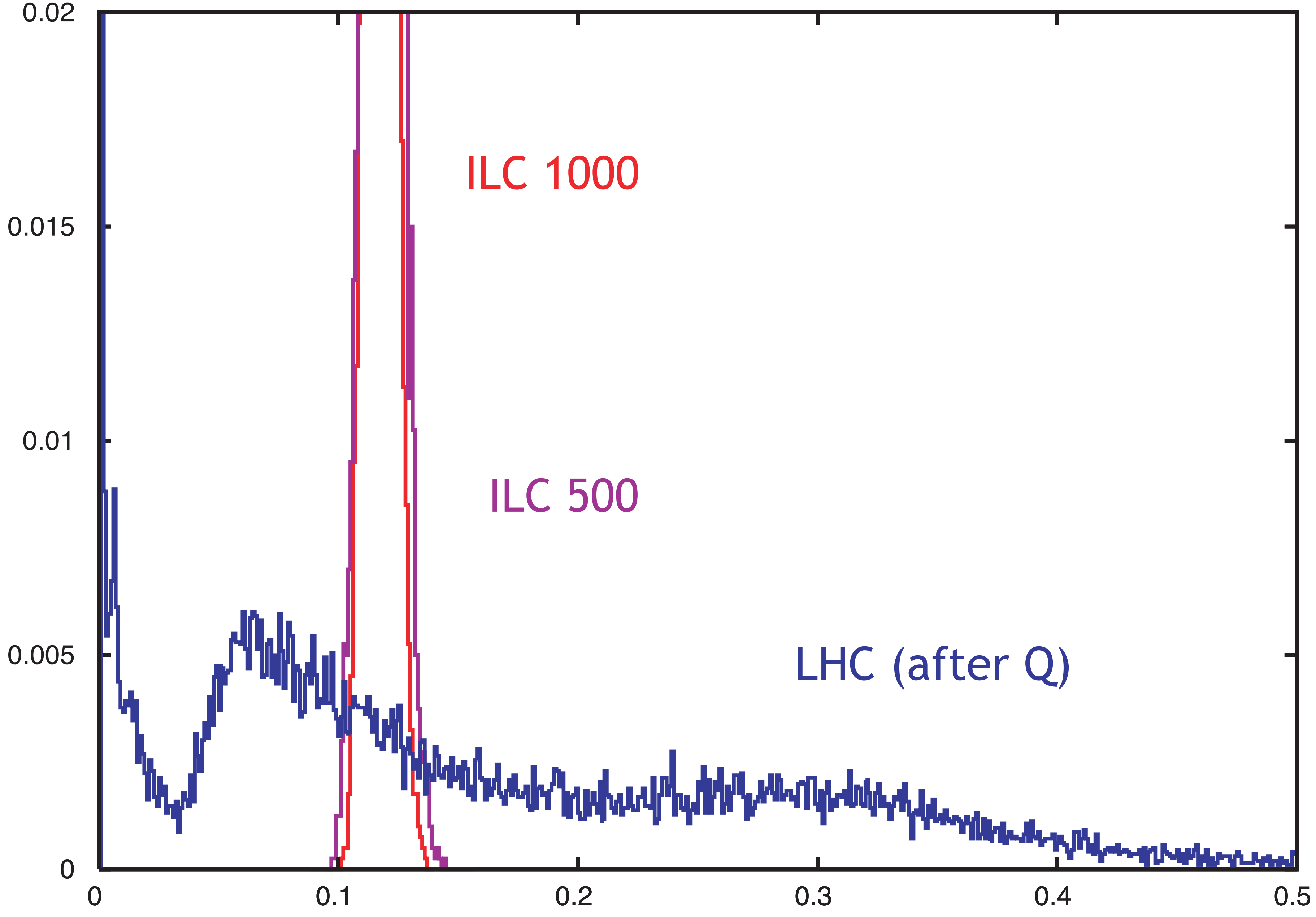
(The chargino analysis is still preliminary.)

The expected accuracies of LHC spectrum measurements can be guessed from the study of SPS1a.

$$\begin{array}{rr|lr} m(N_1) & \pm\ 10. & m(\widetilde{g}) & \pm\ 10\% \\ m(N_2) - m(N_1) & \pm 1.0 & & \\ m(N_3) - m(N_1) & \pm 1.0 & & \end{array}$$

Putting these accuracies into the scanner, we find:


0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.1
0.2
0.3
0.4
0.5
ILC 1000
ILC 500
LHC (after Q)



ILC 1000
ILC 500
LHC (after Q)
0.02
0.015
0.01
0.005
0
0
0.1
0.2
0.3
0.4
0.5

The annihilation cross sections at LCC2 depend on the gaugino-higgsino mixing angles. It is not so easy to measure these angles from the mass spectrum alone, and even adding the ILC cross section information gives only a small improvement. For ILC, we find an accuracy of about 5%.

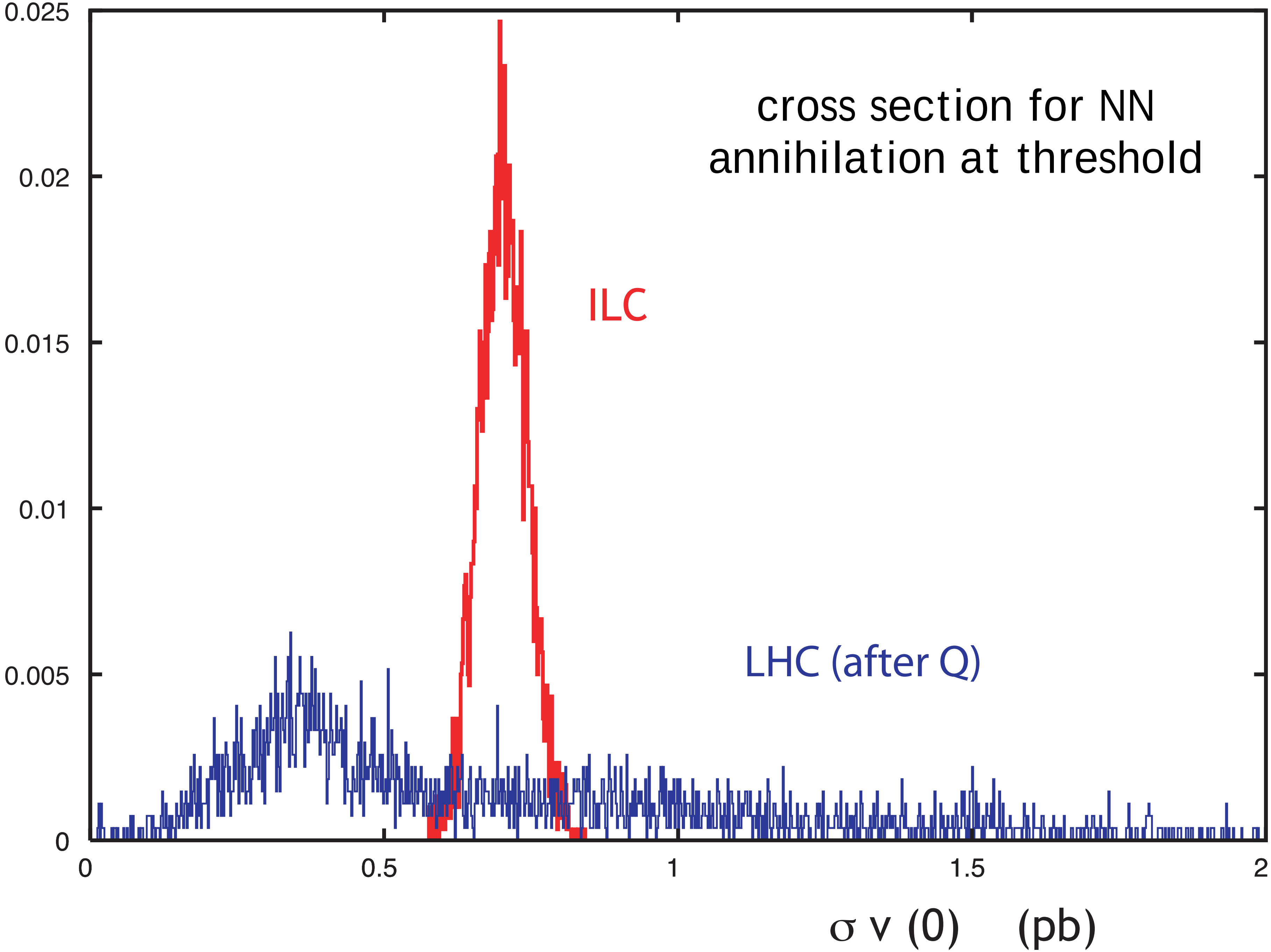
For the determination from LHC data, the problem of accessing the mixing angles is very severe. The LHC data does not give a prediction, only a qualitative indication of the magnitude.

I believe that LCC2 represents the more generic situation for relic density predictions from collider data.

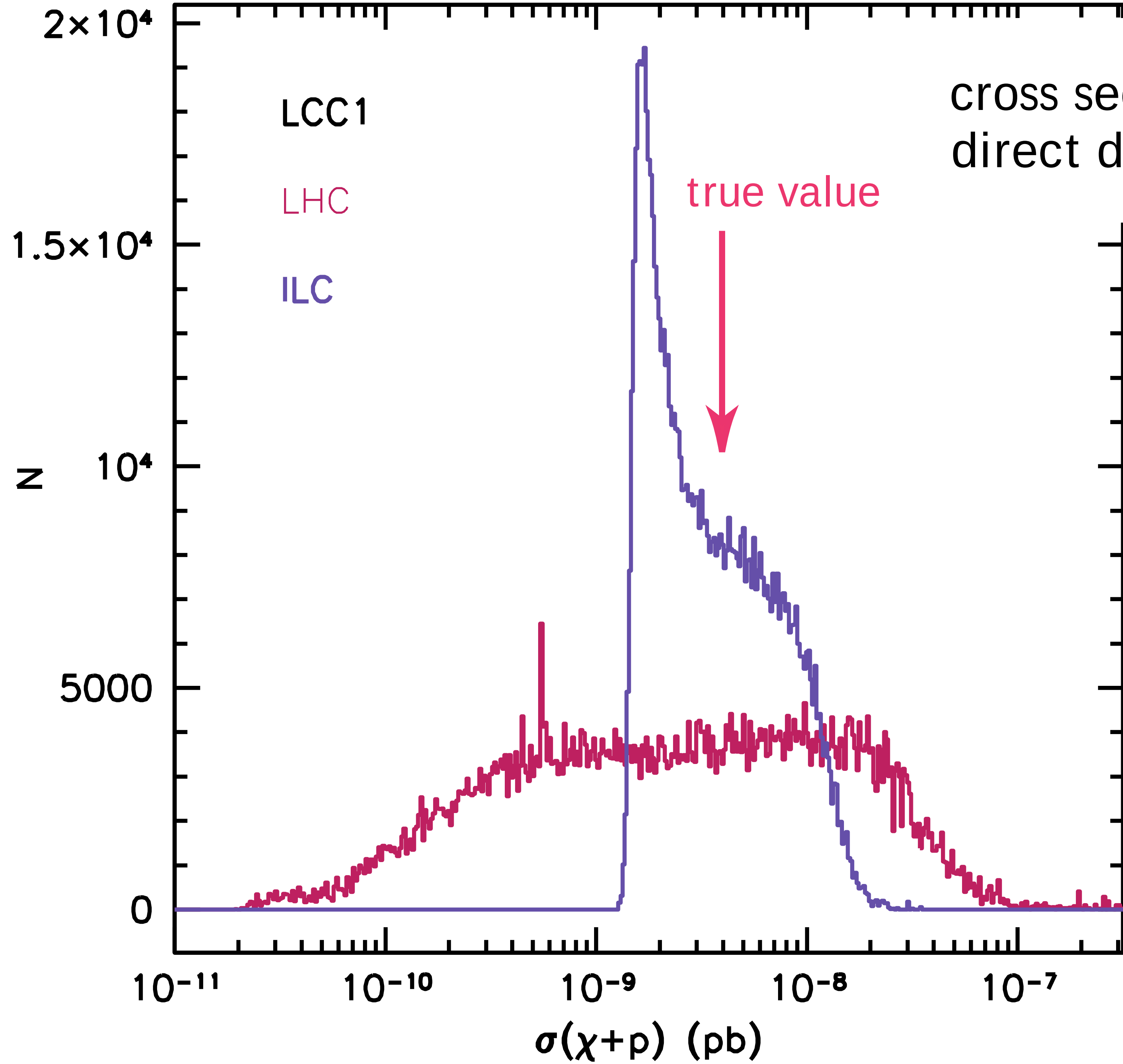
In addition to predicting the dark matter relic density, for comparison to cosmological determinations, this method can be used to predict cross sections for astrophysical detection of dark matter.

It is important to constraint these cross sections at accelerators, so that results on dark matter detection can be used to determine the local density of dark matter at the galactic center, in our local neighborhood, in dwarf companions of the Milky Way, etc.

Here are two preliminary results, one straightforward, one surprising:



cross section for NN
annihilation at threshold
ILC
LHC (after Q)
0.025
0.02
0.015
0.01
0.005
0
0
0.5
1
1.5
2
σ v (0) (pb)



cross section for
direct detection

Baltz

What is the moral ?

In supersymmetry - and probably in other models of new physics - the challenge of providing relic density predictions and cross sections at the accuracies needed by astrophysicists requires very precise determinations of the underlying parameters.

This is where ILC shines.

The discovery of the identity of the dark matter will be a major topic in physics over the next fifteen years. We need to get the word out that e+e- linear colliders will play an essential role in this study.