On $t\bar{t}H$ Production at the ILC

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more to come . . .

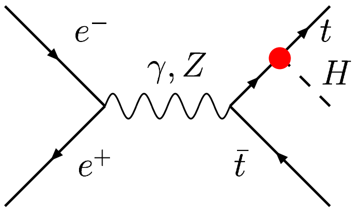
Outline

- Introduction & Status of $\sigma(e^+e^- \to t\bar{t}H)$
- Region of large Higgs energie $\Leftrightarrow$ $t\bar{t}$ threshold
  - Effective Theory $\quad \to$ vNRQCD
  - $\left(\frac{d\sigma}{dE_H}\right)_{E_H \approx E_{H,max}}$ at NLL order
- Preliminary: $\sigma_{\mathrm{tot}}(e^+e^- \to t\bar{t}H)$ at NLL order for the ILC$_{\mathrm{phase\ I}}$
- Summary & Outlook

Top Yukawa Coupling

- Massgeneration $\Leftrightarrow$ SSB $\Leftrightarrow$ Higgsmechanism

  $m_i = \lambda_i \langle H \rangle$ $\longrightarrow$ top quark physics



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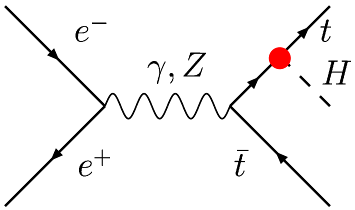
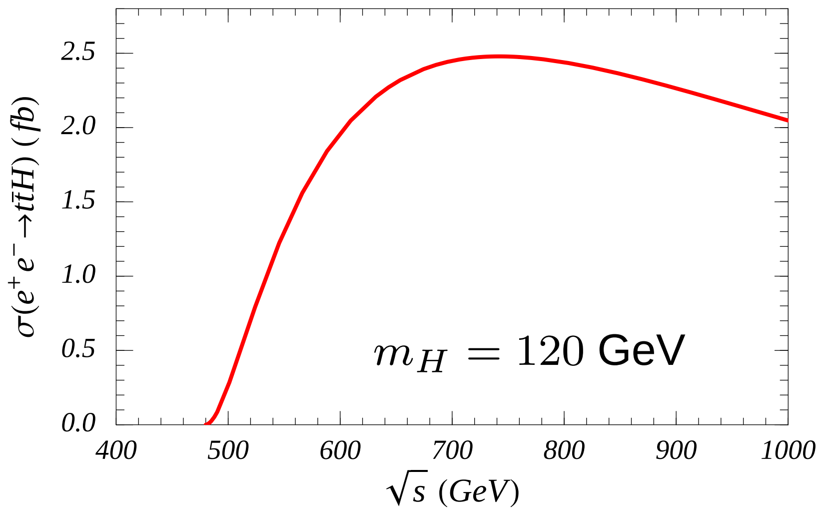
- ILC: $\quad e^+e^- \to t\bar{t}H$





$\longrightarrow \quad \delta\lambda_t/\lambda_t \simeq 5\%$ ($m_H = 120$ GeV)

[Gay; Besson; Winter]

$\longrightarrow \quad$ 2nd phase ($\sqrt{s} > 500$ GeV)

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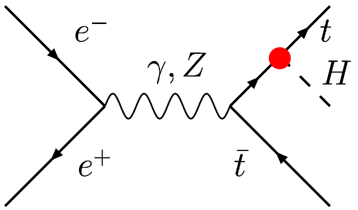

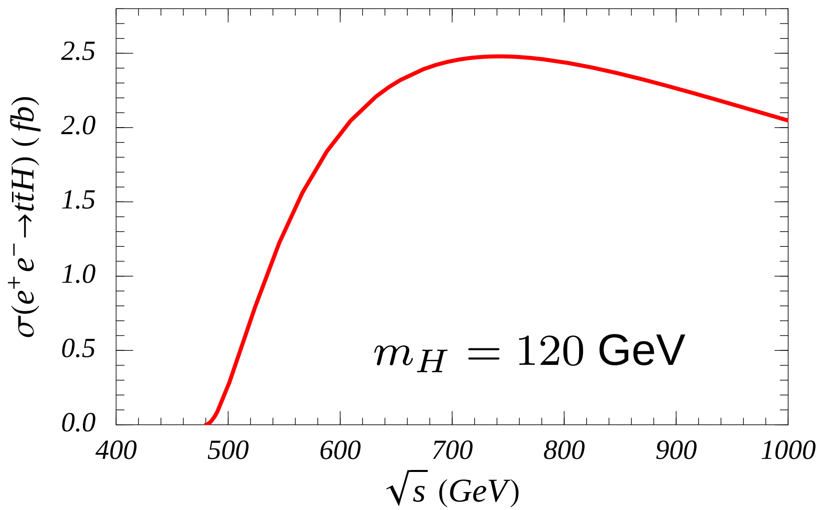

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[Gay; Besson; Winter]

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- Theory Status: $\sigma_{\rm tot}(e^+e^- \to t\bar{t}H)$

  Born ✓ [Gaemers et al., Djouadi et al.]

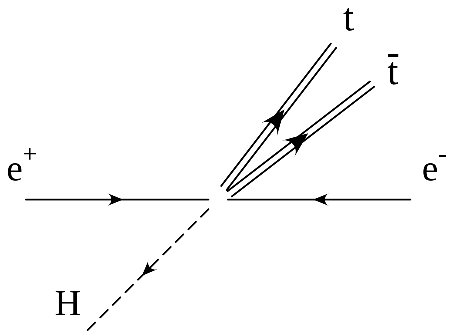
  $\mathcal{O}(\alpha_s)$ ✓ [Dittmaier et al., Dawson et al.]

  1-loop ew. ✓ [Denner et al., Belanger et al., You et al.]

  fully differential partonic $\mathcal{O}(\alpha_s)$: ✓ [Denner et al.]

Large Higge Energy Region

$\longrightarrow$ region of large Higgs energy



$\rightarrow t\bar{t}$ collinear

$\rightarrow$ QCD effects localized in $t\bar{t}$ system

$\Rightarrow t\bar{t}$ dynamics non-relativistic

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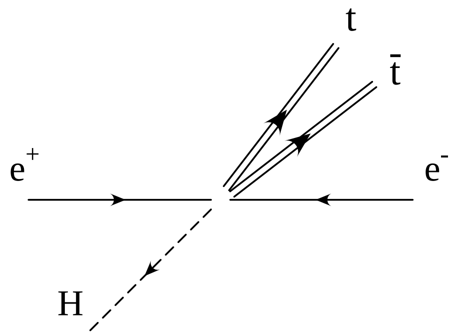

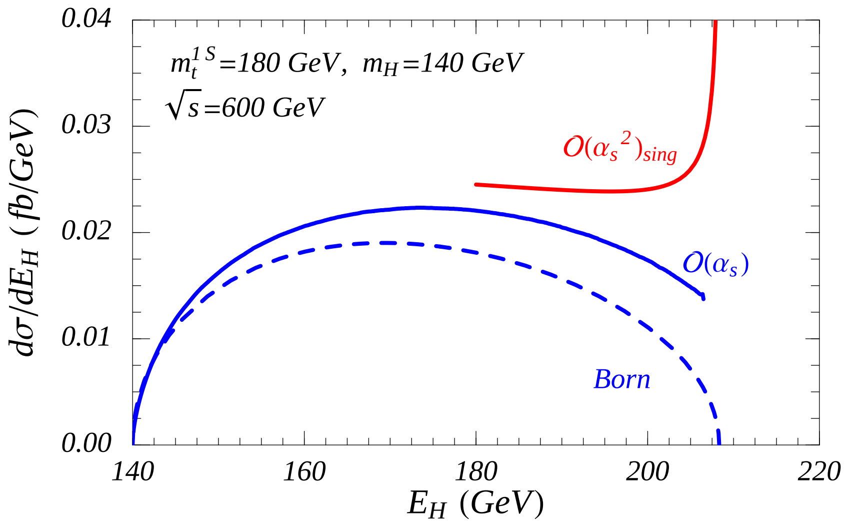

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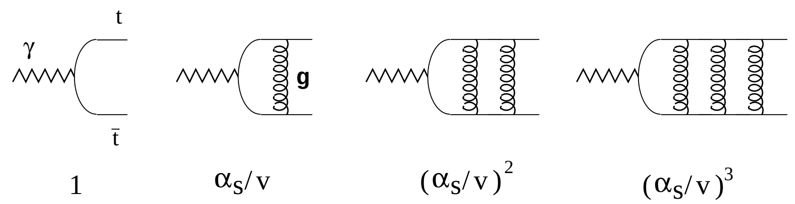

$\rightarrow$ singularities: $\sim (\alpha_s/v)^n$,

$$\sim (\alpha_s \ln v)^n$$

$\rightarrow$ fixed order expansion breaks down

$\Rightarrow$ summation of singular terms

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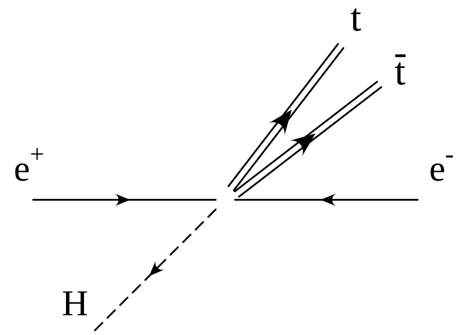

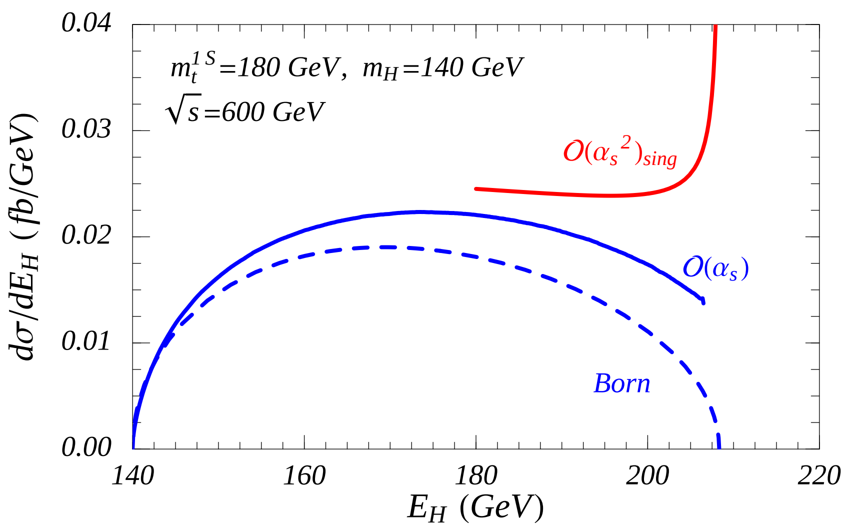

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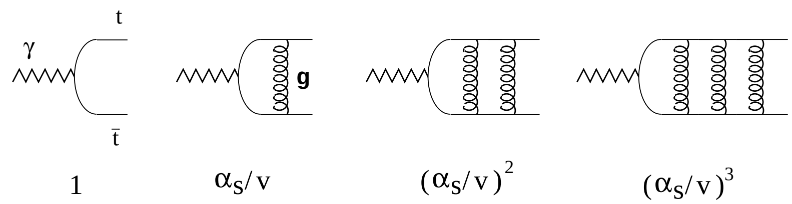

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(A) impose cut: $E_H > E_H^{\max}$

(B) summation via vNRQCD

$\Rightarrow$ control of singular QCD effects desired

$\longrightarrow$ computation at NLL order

vNRQCD in a Nutshell

$$\boxed{m_t \quad \gg \quad \mathbf{p} \sim m_t v \quad \gg \quad E \sim m_t v^2 \sim \Gamma_t \quad > \quad \Lambda_{\mathrm{QCD}}}$$

(hard) (soft) (ultrasoft)

- scale hierarchy $\longrightarrow$ factorization
- non-perturbative effects suppressed

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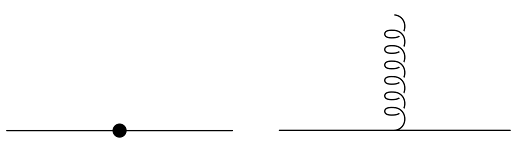
$\underline{\mathcal{L} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm potential} + \mathcal{L}_{\rm soft}}$

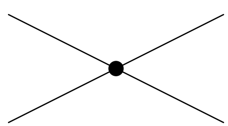
Luke, Manohar, Rothstein, Stewart, A.H.

$\mathcal{L}_{\rm usoft}$:



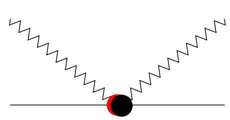
$$\psi^\dagger_{\mathbf{p}}(x)\left\{iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m_t} + i\frac{\Gamma_t}{2} - \delta m_t\right\}\psi_{\mathbf{p}}(x)$$

$\mathcal{L}_{\rm potential}$:



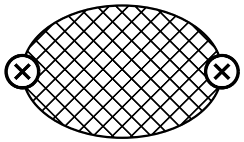
$$\left\{\frac{V_c(\nu)}{(\mathbf{p}-\mathbf{p}')^2} + \cdots\right\}\psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}}$$

$\mathcal{L}_{\rm soft}$:



vNRQCD in a Nutshell

Currents: $\rightarrow$ production & annihilation of $t\bar{t}$ pairs



$$\mathbf{O}_{\mathbf{p}}^{\rm trip} = C_1(\mu)\cdot(\psi_{\mathbf{p}}^{\dagger}\,\boldsymbol{\sigma}\,\tilde{\chi}^{*}_{-\mathbf{p}}) + \cdots \qquad ({}^3S_1)$$

$$\mathbf{O}_{\mathbf{p}}^{\rm sing} = C_0(\mu)\cdot(\psi_{\mathbf{p}}^{\dagger}\,\tilde{\chi}^{*}_{-\mathbf{p}}) + \cdots \qquad ({}^1S_0)$$

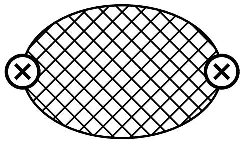
$\rightarrow$ $t\bar{t}$ production rate:

$$R_{t\bar{t}} \propto \mathrm{Im}\left[\int \mathrm{d}^4\mathrm{x}\,\mathrm{e}^{-\mathrm{i}\hat{\mathrm{q}}.\mathrm{x}}\left\langle 0\left|\mathrm{T}\,\mathbf{O}_{\mathbf{p}}^{\dagger}(0)\,\mathbf{O}_{\mathbf{p}'}(\mathrm{x})\right|0\right\rangle\right]$$

$$\propto C(\mu)^2\,\mathrm{Im}\left[\mathrm{G}(0,0,\mathrm{v},\mu)\right]$$

vNRQCD in a Nutshell

Currents: $\rightarrow$ production & annihilation of $t\bar{t}$ pairs



$$\mathbf{O}_{\mathbf{p}}^{\mathrm{trip}} = C_1(\mu) \cdot (\psi_{\mathbf{p}}^{\dagger}\, \boldsymbol{\sigma}\, \tilde{\chi}_{-\mathbf{p}}^{*}) + \cdots \qquad ({}^3S_1)$$

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$\rightarrow$ $t\bar{t}$ production rate:

$$R_{t\bar{t}} \propto \mathrm{Im}\left[\int \mathrm{d}^4\mathrm{x}\, \mathrm{e}^{-\mathrm{i}\hat{\mathrm{q}}.\mathrm{x}} \left\langle 0 \left| \mathrm{T}\, \mathbf{O}_{\mathbf{p}}^{\dagger}(0)\, \mathbf{O}_{\mathbf{p}'}(\mathrm{x}) \right| 0 \right\rangle\right]$$

$$\propto C(\mu)^2\, \mathrm{Im}\,[\mathrm{G}(0,0,\mathrm{v},\mu)]$$

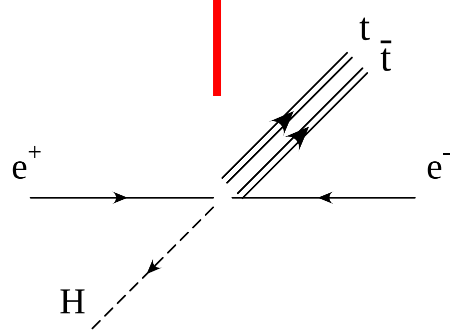
$\rightarrow$ known from $\sigma(e^+e^- \rightarrow t\bar{t})$:

- NLL zero-distance Green function $\checkmark$ [Manohar et al., Teubner, AHH] [Beneke et al., Penin et al.]
- NLL running of $C^1(\mu)$ and $C^0(\mu)$ $\checkmark$ [Luke et al.; Pineda; Manohar et al., AHH]
- matching conditions $C^1(m_t)$ and $C^0(m_t)$ $\rightarrow$ new computation needed

NLL Result

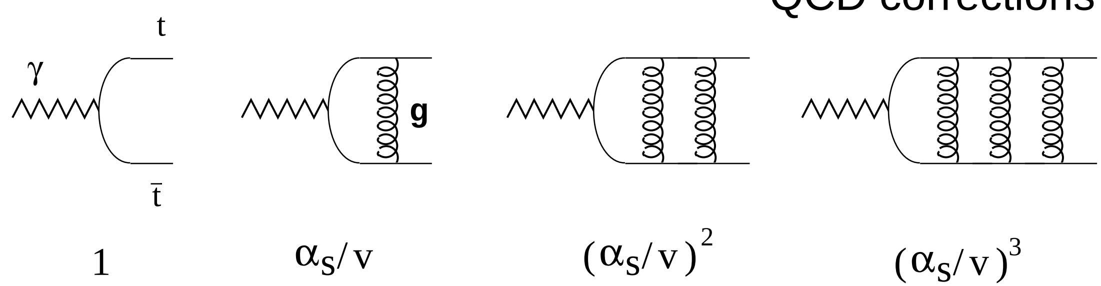
“Factorization Formula”

$$\left(\frac{d\sigma}{dE_H}\right)_{\mathrm{E_H \approx E_H^{max}}} \sim \left[C_0^2(\mu,\sqrt{s},m_t,m_H) + C_1^2(\mu,\sqrt{s},m_t,m_H)\right] \,\mathrm{Im}[G(0,0,v,\mu)]$$



$+$ hard QCD corrections
$E_H = E_H^{\mathrm{max}}$

soft & ultrasoft QCD corrections

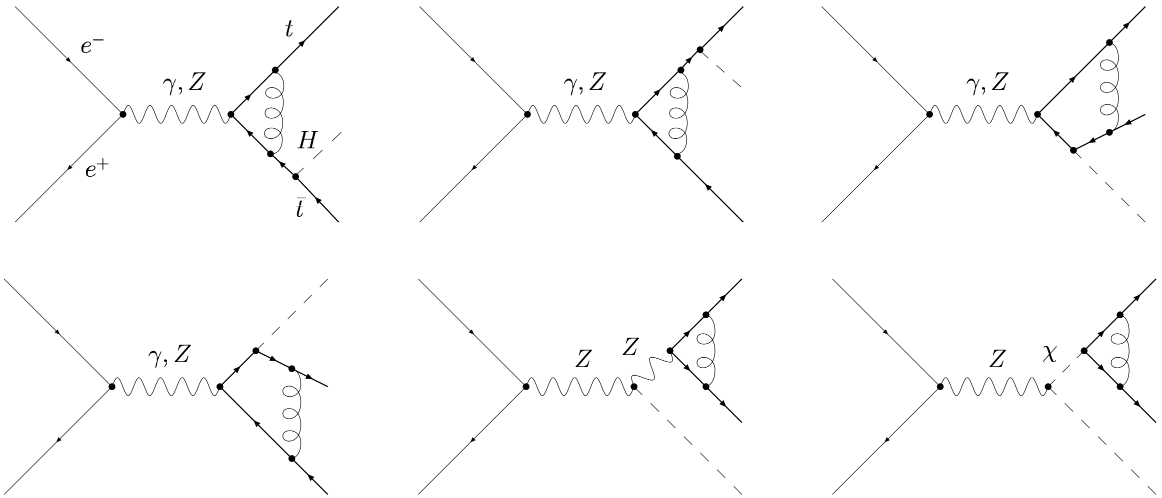


NLL Result

$$\left(\frac{d\sigma}{dE_H}\right)_{\mathrm{E_H \approx E_H^{max}}} \sim \left[C_0^2(\mu, \sqrt{s}, m_t, m_H) + C_1^2(\mu, \sqrt{s}, m_t, m_H) \right] \mathrm{Im}[G(0,0,v,\mu)]$$

$\longrightarrow$ matching computation: $(\mu = m_t)$

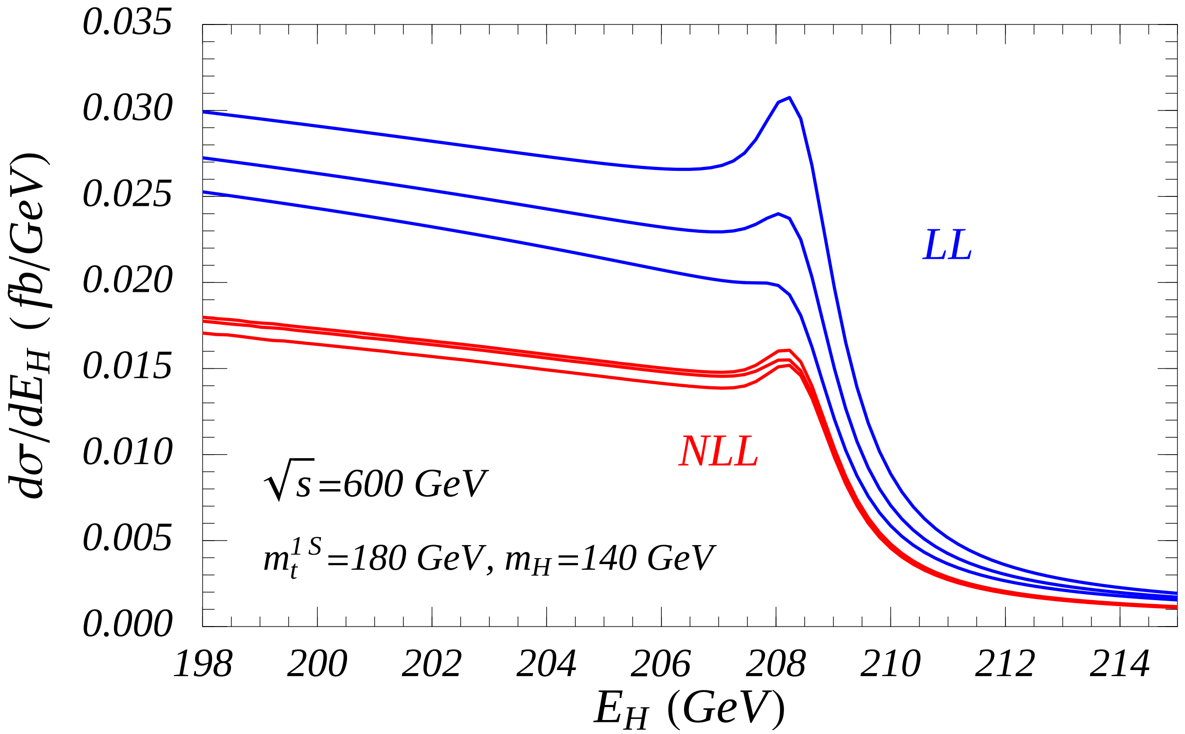
$$\left.\begin{array}{l} \text{EFT result at NLL (expanded to } \mathcal{O}(\alpha_s)) \\ \overset{!}{=} \mathcal{O}(\alpha_s) \text{ result (full theory) for } E_H \approx E_H^{max} \end{array}\right\} \Rightarrow C_{0,1}(m_t, \sqrt{s}, m_t, m_H)$$



$\longrightarrow$ virtual corrections sufficient

Dittmaier, Roth, et al.
[Dawson et al.]

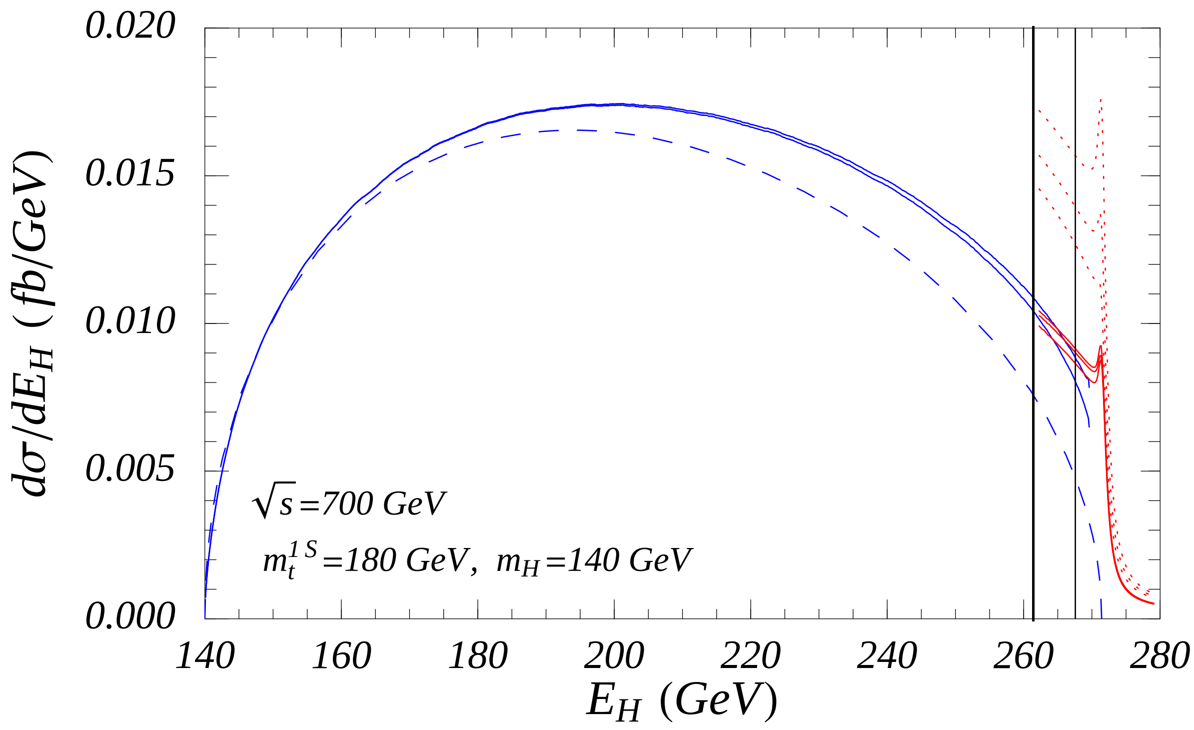
Numerics



$\mu_{\rm soft} = 18 - 72$ GeV

- signifi cant reduction of scale variation: LL $\longrightarrow$ NLL
- generic feature for all cases

Numerics



- good matching of NLL vNRQCD and $\mathcal{O}(\alpha_s)$ full theory results
- matching at $v \approx 0.2$

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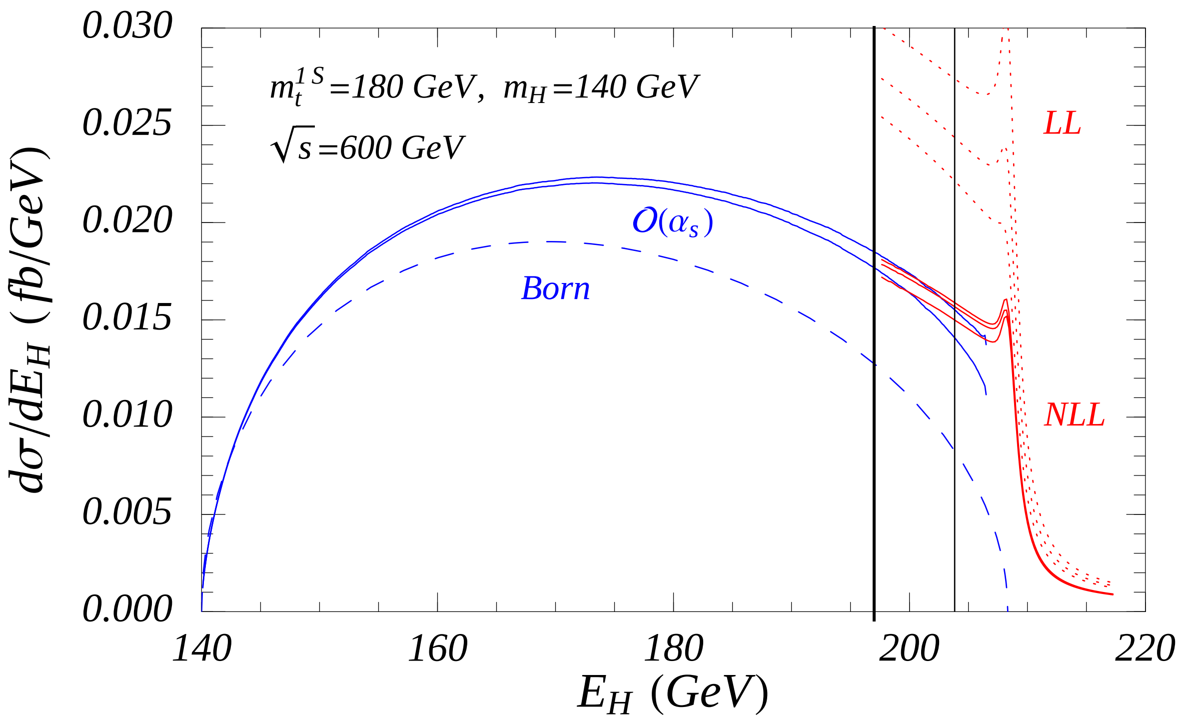

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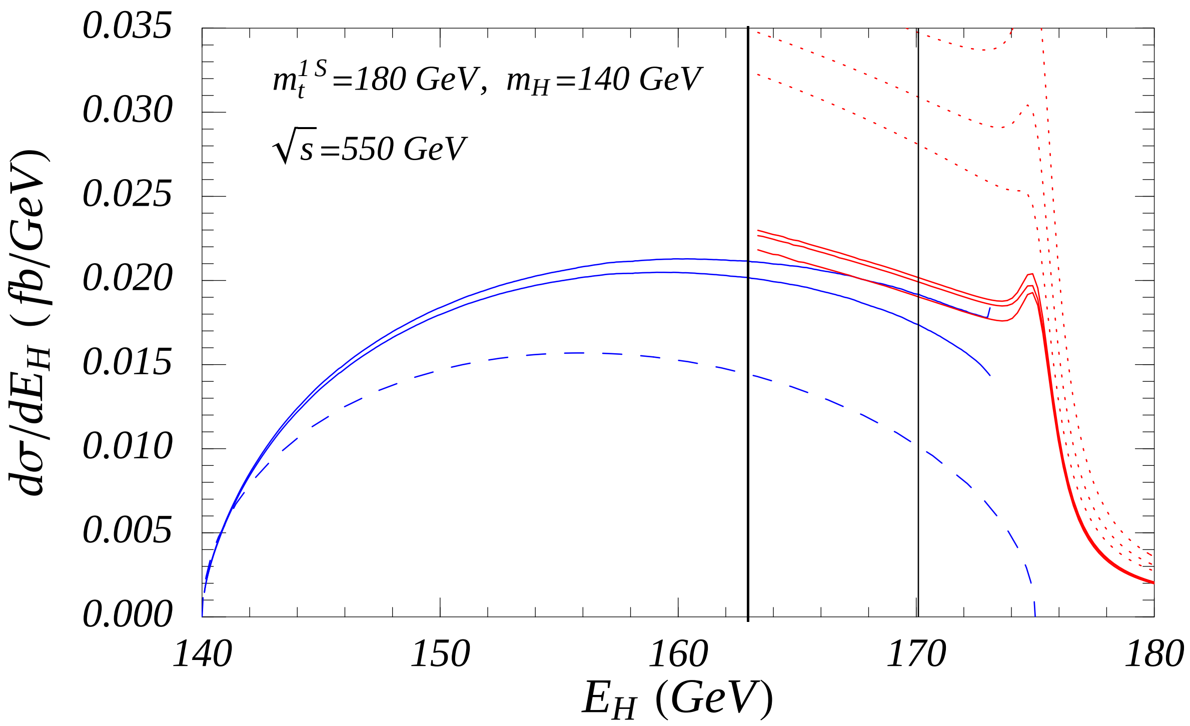

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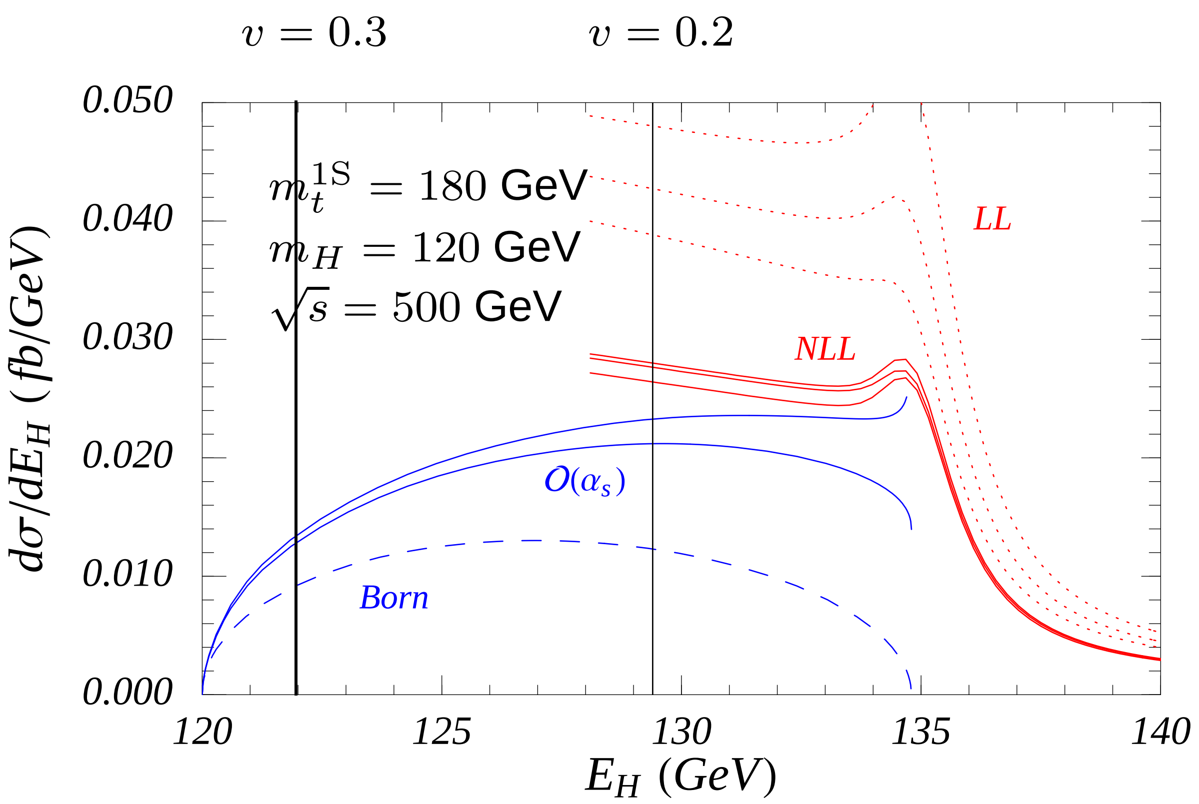


- good matching of NLL vNRQCD and $\mathcal{O}(\alpha_s)$ full theory results
- matching at $v \approx 0.2$
- large E_h endpoint dominates for $\sqrt{s} \leq 500$ GeV !

Numerics

$m_H = 120$ GeV

$$\boxed{\sigma(\text{NLL}) \equiv \sigma(\alpha_s)[v > 0.2] + \sigma(\text{NLL})[v < 0.2]}$$

$\sqrt{s}$ [GeV]	σ(Born) [fb]	$\sigma(\alpha_s)$ [fb]	σ(NLL) [fb]	$\frac{\sigma(\text{NLL})}{\sigma(\alpha_s)}$
500	0.151	0.263	0.357(20)	1.359
550	0.984	1.251	1.342(37)	1.073
600	1.691	1.939	2.005(30)	1.034
700	2.348	2.454	2.485(13)	1.012
800	2.428	2.427	2.442(5)	1.006
900	2.290	2.229	2.237(6)	1.004
1000	2.087	1.997	2.002(8)	1.003

- non-relativistic effects relevant up to $\sqrt{s} \lesssim 700$ GeV
- larger for smaller $\sqrt{s}$ and larger m_H

Cross Section for ILC (Phase I)

- relative velocity of $t\bar{t}$ pair always small
- singularities $(\alpha_s/v)^n$, $(\alpha_s \ln v)^n$ need to be always resummed
- fi xed-order perturbation always inappropriate
- entire process $e^+e^- \to t\bar{t}H$ nonrelativistic

  large Higgs energy endpoint dominates full phase space

$$\left(\frac{d\sigma}{dE_H}\right)_{\mathrm{E_H \approx E_H^{max}}} \sim \left[C_0^2(\mu, \sqrt{s}, m_t, m_H) + C_1^2(\mu, \sqrt{s}, m_t, m_H) \right] \mathrm{Im}[G(0,0,v,\mu)]$$

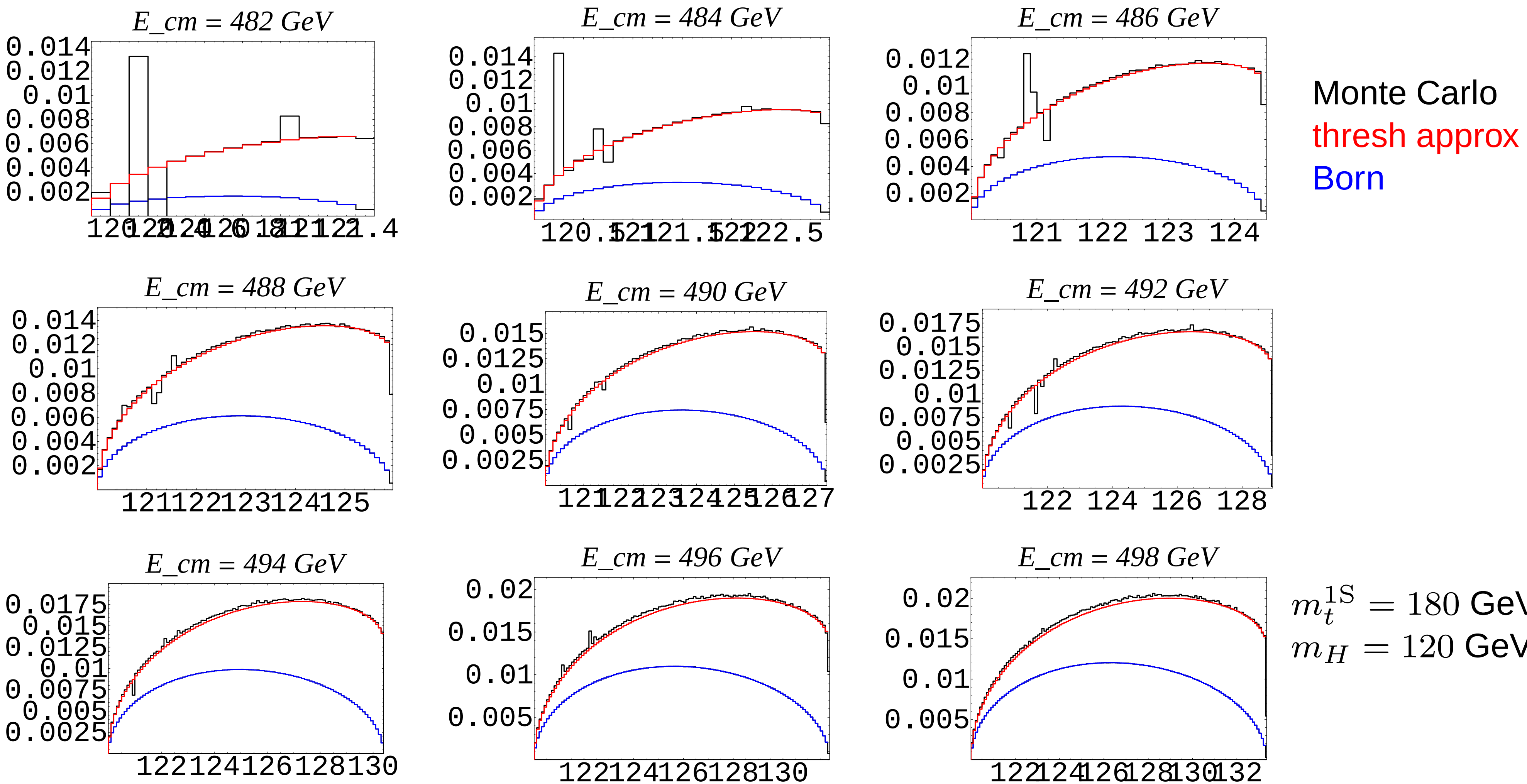
$$\downarrow$$

$$\left(\frac{d\sigma}{dE_H}\right)^{\mathrm{NLL}}_{\mathrm{Phase\ I}} \sim \left(\frac{d\sigma}{dE_H}\right)^{\mathrm{Born}} \Big/ v$$

$$\times \left[C_0^2(\mu, \sqrt{s}, m_t, m_H) + C_1^2(\mu, \sqrt{s}, m_t, m_H) \right] \mathrm{Im}[G(0,0,v,\mu)]$$

Cross Section for ILC (Phase I)

- Cross check: $\left(\frac{d\sigma}{dE_H}\right)^{\mathcal{O}(\alpha_s)\text{ fixed order}}_{\text{Monte Carlo}}$ vs. $\left(\frac{d\sigma}{dE_H}\right)^{\mathcal{O}(\alpha_s)\text{ fixed order}}_{\text{thresh approx}}$



$m_t^{1S} = 180$ GeV
$m_H = 120$ GeV

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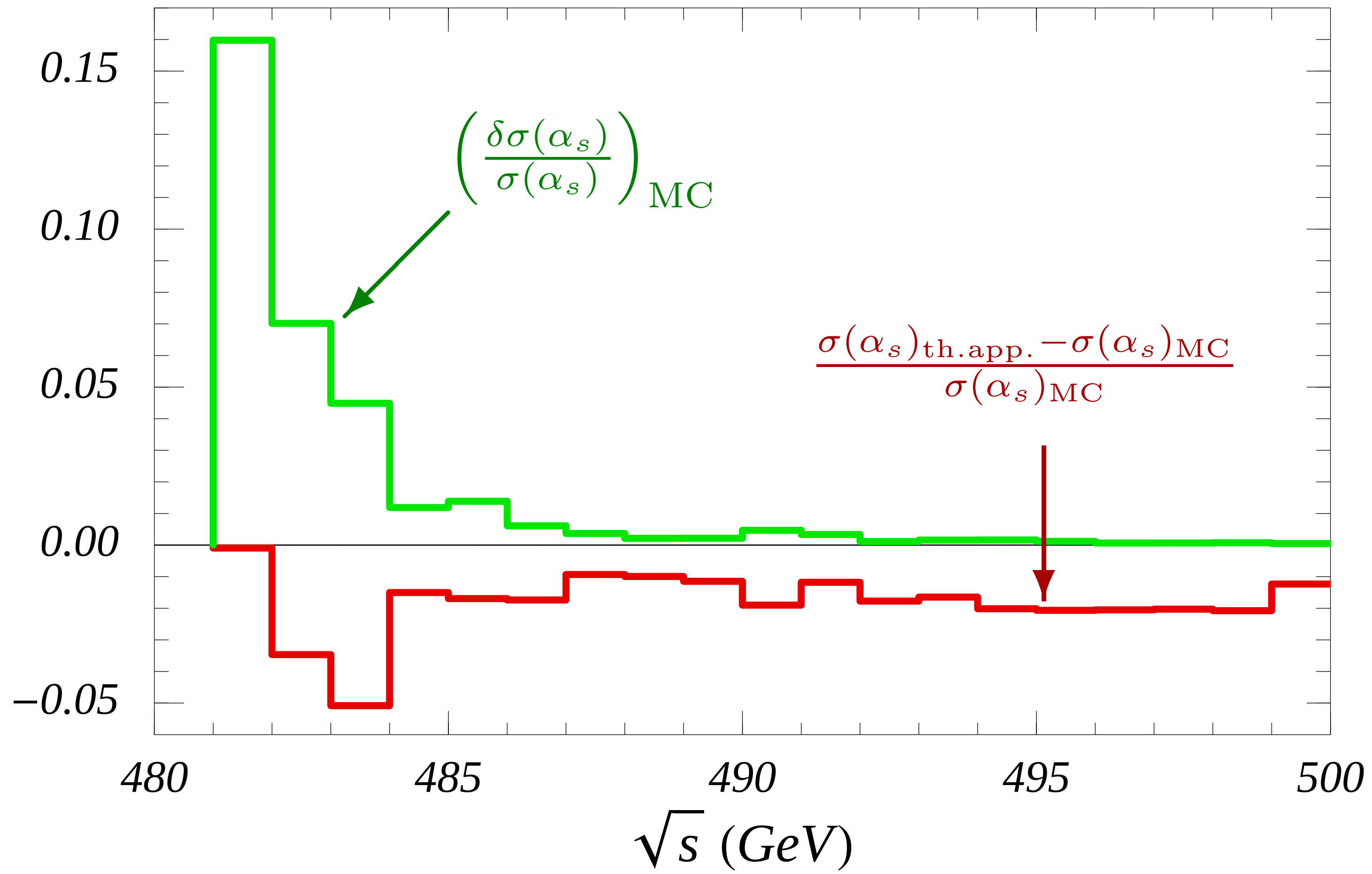

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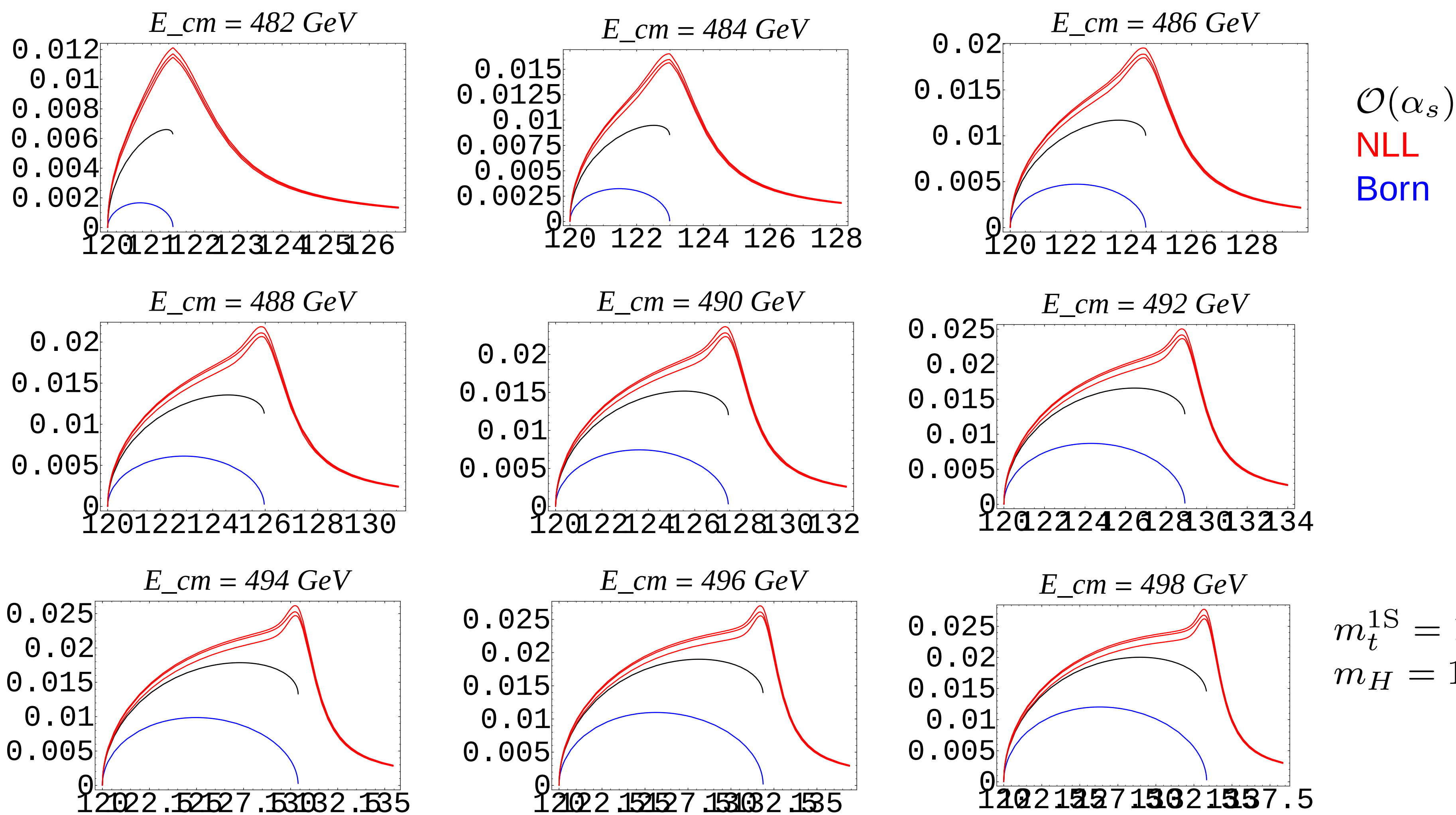
Cross Section for ILC (Phase I)

- Predictions: $\left(\frac{d\sigma}{dE_H}\right)$ $\longrightarrow$ signifi cant enhancement from summation of v-singularities



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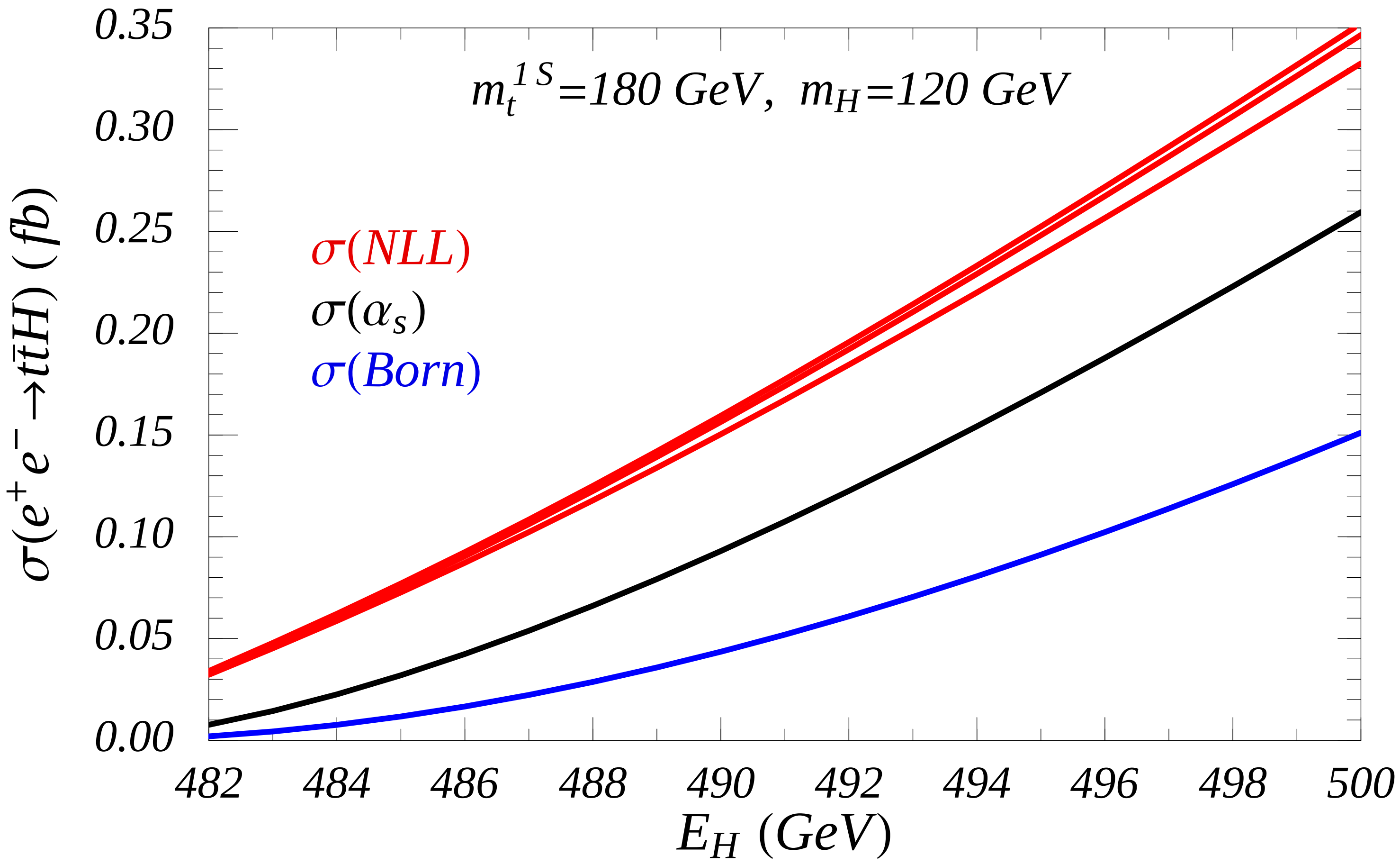

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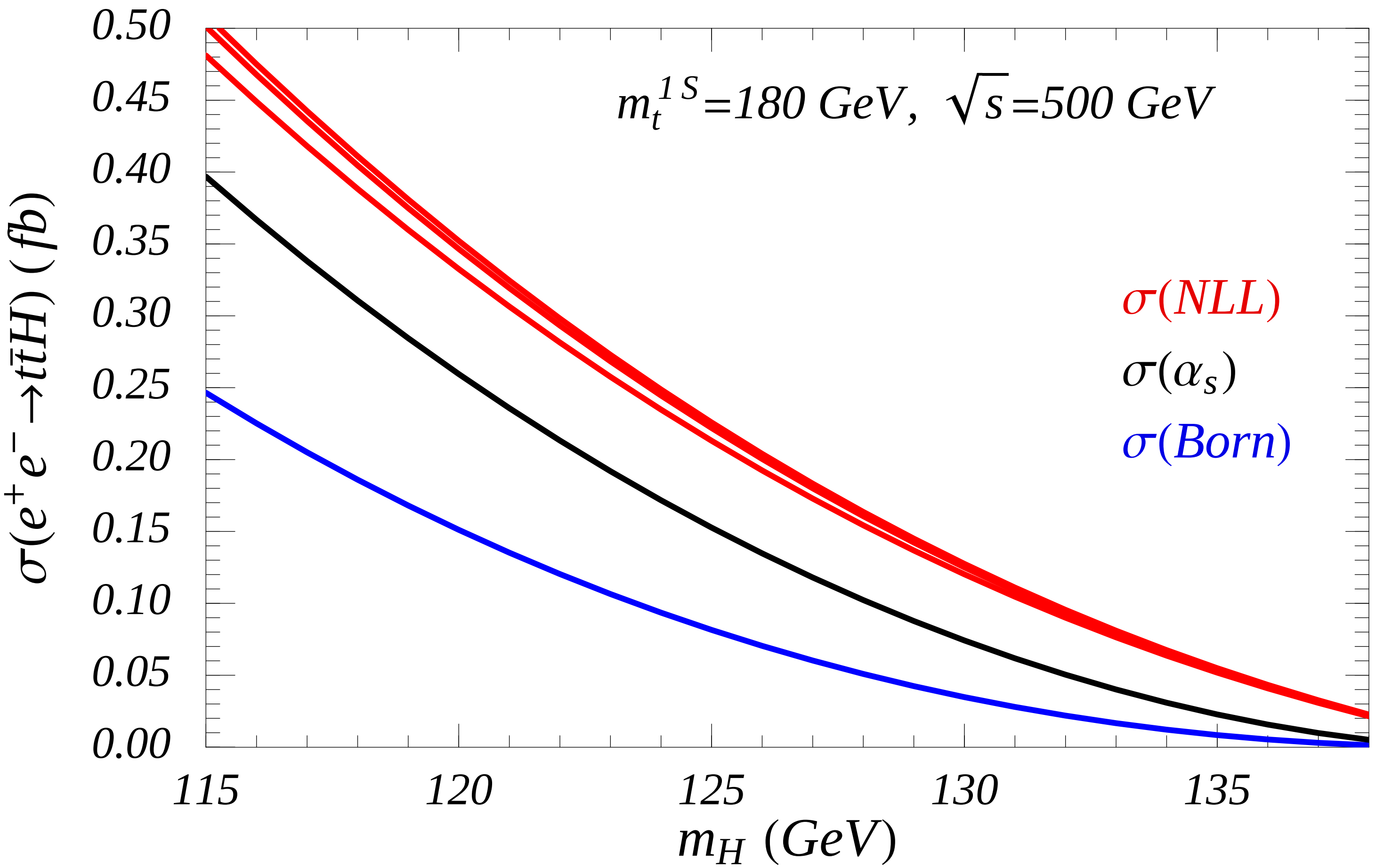

Summary

- large E_{Higgs} region $\Leftrightarrow$ $t\bar{t}$ system non-relativistic
- Summation of $(\alpha_s/v)^n$ and $(\alpha_s \ln v)^m$ singularities relevant for $\sqrt{s} \lesssim 700$ GeV $\longrightarrow$ probably negligible for LHC: $(\sqrt{s})_{\mathrm{eff}} \sim$ TeV
- $t\bar{t}$ are always nonrelativistic for $\sqrt{s} \lesssim 500$ GeV (ILC - phase I)
- NLL order computation using vNRQCD
- Outlook:

$\longrightarrow$ smooth combination: EFT (NLL) & fi xed order $\mathcal{O}(\alpha_s)$ for $\sqrt{s} > 500$ GeV (w.i.p.)

$\longrightarrow$ NLL e.w. corrections and exp. cuts for $\sqrt{s} < 500$ GeV (w.i.p.)

$\longrightarrow$ fully differential treatment ?

. . . still many conceptual problems to solve

Colors

This is blue

This is red

This is brown

This is magenta

This is Dark Green

This is Dark Blue

This is Green

This is Cyan

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks