

Towards One-Loop MHV Techniques



Carola F. Berger

Snowmass - August 22, 2005

Towards One-Loop MHV Techniques

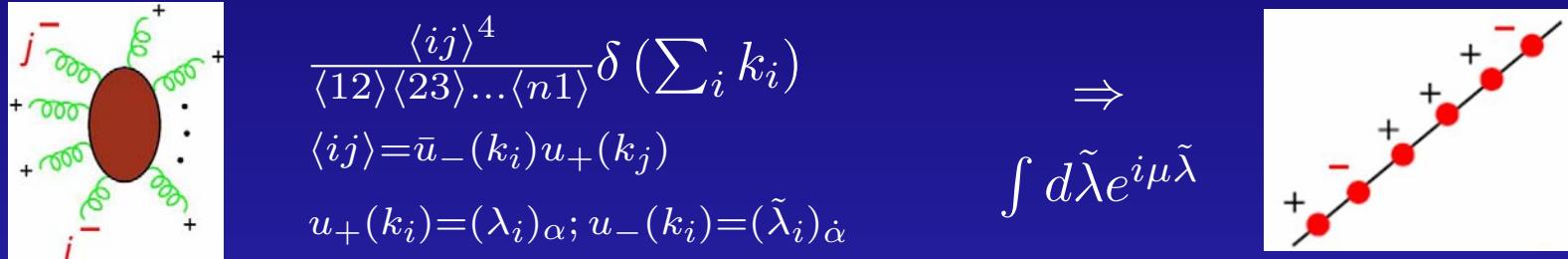


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Tree Level Techniques

Parke-Taylor (1986) \Rightarrow Twistor space – Witten (2003)



The diagram illustrates the mapping from a Feynman diagram to twistor space. On the left, a Feynman diagram shows a red oval vertex with four external lines. Three lines are labeled with green wavy lines and a '+' sign, while one line is labeled with a red wavy line and a '-' sign. Below the diagram, the index i is shown with a red minus sign. To the right, the mapping is given by the equation:

$$\frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle} \delta \left(\sum_i k_i \right) \Rightarrow \int d\tilde{\lambda} e^{i\mu\tilde{\lambda}}$$

Below this, two definitions are provided:

$$\langle ij \rangle = \bar{u}_-(k_i) u_+(k_j)$$
$$u_+(k_i) = (\lambda_i)_\alpha; u_-(k_i) = (\tilde{\lambda}_i)_{\dot{\alpha}}$$

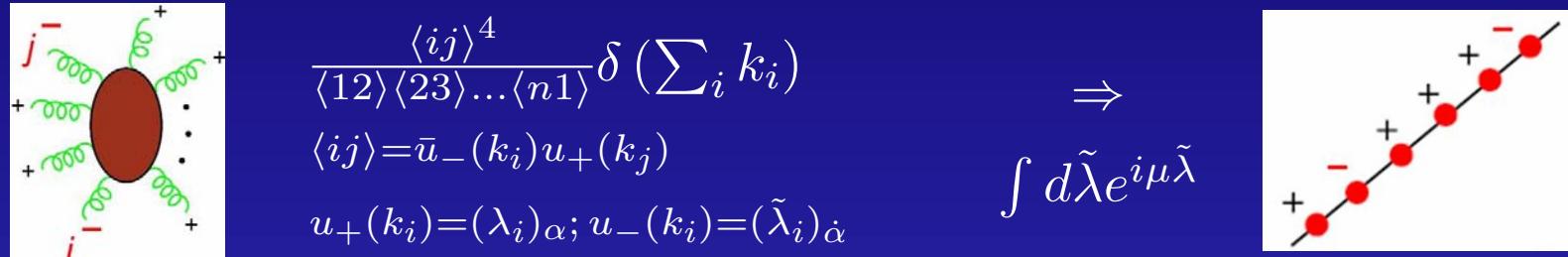
The twistor space representation on the right shows a straight line with red dots representing points. Each point is marked with a '+' or '-' sign, corresponding to the wavy lines in the Feynman diagram.

In general: curve in twistor space of genus $g \leq l$ and of degree

$$d = q - 1 + l$$

Tree Level Techniques

Parke-Taylor (1986) \Rightarrow Twistor space – Witten (2003)

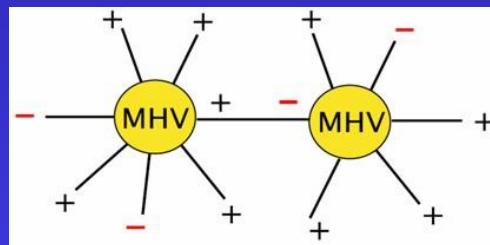


The diagram illustrates the mapping between a Feynman diagram and a curve in twistor space. On the left, a Feynman diagram shows a red oval loop with external lines labeled i^- and j^- . Green wavy lines with '+' signs represent gluons. In the center, the loop is replaced by a curve in twistor space. The equation $\langle ij \rangle^4 / \langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle \delta(\sum_i k_i)$ relates the loop integral to the twistor space curve. Below it, the twistor space representation is given as $\langle ij \rangle = \bar{u}_-(k_i) u_+(k_j)$, with the condition $u_+(k_i) = (\lambda_i)_\alpha; u_-(k_i) = (\tilde{\lambda}_i)_{\dot{\alpha}}$. The right side shows a curve with red dots and '+' signs.

In general: curve in twistor space of genus $g \leq l$ and of degree

$$d = q - 1 + l$$

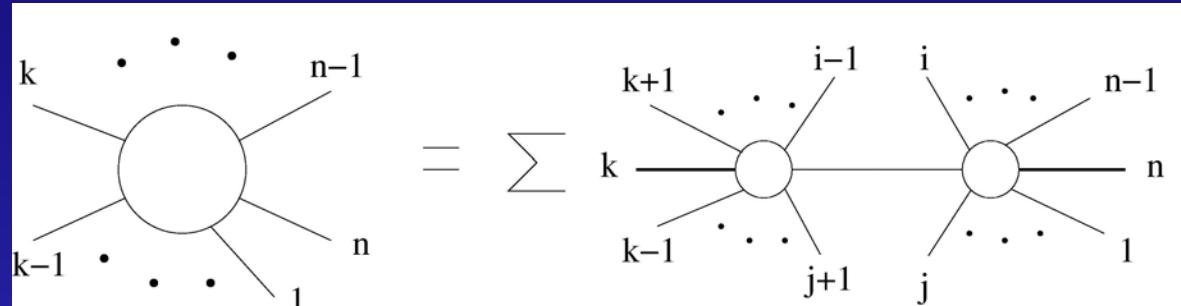
MHV rules – Cachazo, Svrcek, Witten (2004)



Off-shell Parke-Taylor amplitudes connected by
scalar propagator
 More efficient than Feynman rules for QCD (and
 SUSY) tree amplitudes.
 Lagrangian connection ?

Tree Level Techniques contd.

On-shell recursion relations – Britto, Cachazo, Feng, and Witten (2004-05)



$$A_n = \sum_{r,s} \sum_{\lambda=\pm} A_L^\lambda \frac{1}{P_{r,s}^2} A_R^{-\lambda} \quad (1)$$

Proof: Shift gluons i, j

$$\begin{aligned} p_i(z) &= \lambda_i(\tilde{\lambda}_i - z\tilde{\lambda}_j) \\ p_j(z) &= (\lambda_j + z\lambda_i)\tilde{\lambda}_j \end{aligned} \quad (2)$$

⇒ amplitude

$$A_n(z) = A_n(p_1, \dots, p_i(z), \dots, p_j(z), \dots) \quad (3)$$

Tree Level Techniques contd.

Amplitude is on-shell for all z , momentum is conserved.

Internal momentum $1/P_{kl}^2 \rightarrow 1/P_{kl}^2(z)$

$A_n(z)$ has only **simple poles**.

Physical scattering amplitude $A(0)$

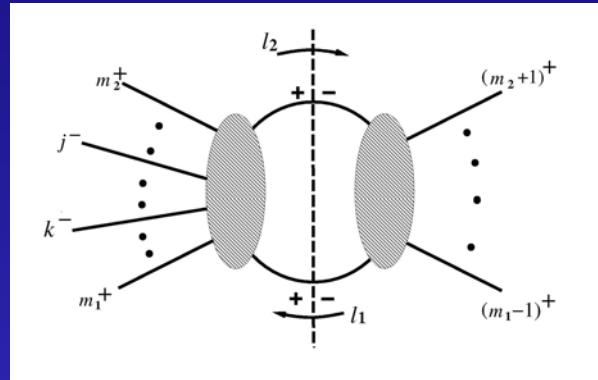
$$0 = \frac{1}{2\pi i} \oint_C \frac{dz}{z} A(z)$$

If $A(z \rightarrow \infty) \rightarrow 0$

$$A(0) = - \sum_{\text{poles } \alpha} \text{Res}_{z=z_\alpha} \frac{A(z)}{z} = \sum_{r,s} \sum_{\lambda=\pm} A_L^\lambda(z=z_\alpha) \frac{1}{P_{r,s}^2} A_R^{-\lambda}(z=z_\alpha) \quad (4)$$

One Loop - Supersymmetric Amplitudes

Supersymmetric amplitudes - cut constructible



$$\int d\text{LIPS}(-l_1, l_2) A^{\text{tree}}(-l_1, m_1, \dots, m_2, l_2) A^{\text{tree}}(-l_2, m_2 + 1, \dots, m_1 - 1, l_1) \quad (5)$$

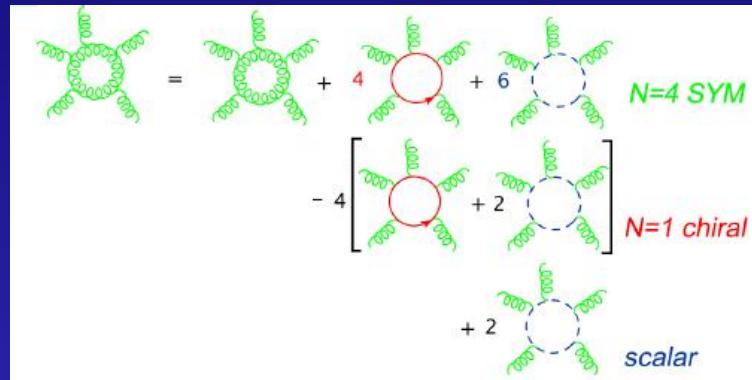
legs in $D = 4$ dimensions (integral in $D = 4 - 2\varepsilon$ dimensions)

also, generalized cuts

\Rightarrow reduction to box, triangle, bubble integrals with coefficients obtainable from tree amplitudes

One Loop - QCD Amplitudes

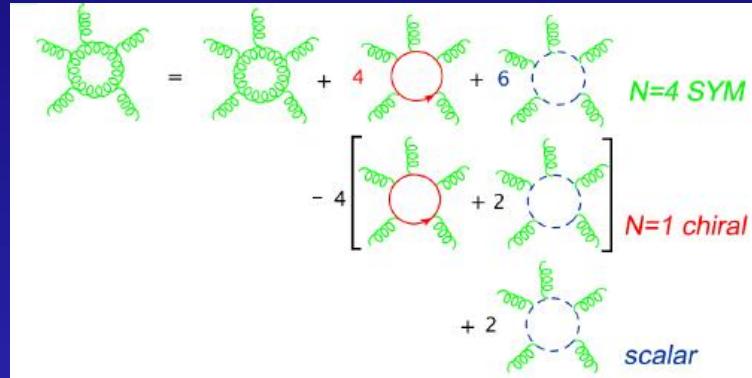
SUSY decomposition \Rightarrow reduction to scalar loop



in SUSY amplitudes rational terms intimately linked to (poly)logarithmic contributions

One Loop - QCD Amplitudes

SUSY decomposition \Rightarrow reduction to scalar loop



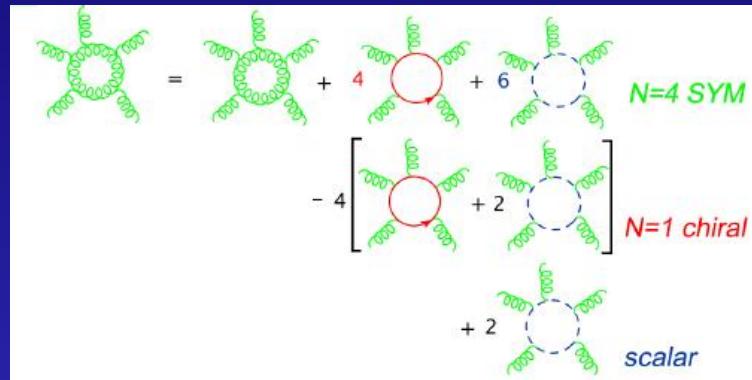
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$\mathcal{N} = 0 - 2$ approaches

- Generalized cuts \Rightarrow (poly)logarithmic terms; rational terms require evaluation to higher order in ε (Bedford, Brandhuber, Spence, Travaglini (2004-2005))

One Loop - QCD Amplitudes

SUSY decomposition \Rightarrow reduction to scalar loop



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$\mathcal{N} = 0 - 2$ approaches

- Generalized cuts \Rightarrow (poly)logarithmic terms; rational terms require evaluation to higher order in ε (Bedford, Brandhuber, Spence, Travaglini (2004-2005))
- Recursion relations (Bern, Dixon, Kosower (2004-05) + in progress, Forde, CFB)

One Loop - QCD Amplitudes contd.

New features at one loop

- branch cuts

One Loop - QCD Amplitudes contd.

New features at one loop

- branch cuts
- double poles:
 - multiparticle factorization \Rightarrow simple poles
 - collinear factorization \Rightarrow one-loop splitting amplitudes for (+++) and (---)
 - helicity configurations $\sim \frac{[ab]}{\langle ab \rangle^2}$

One Loop - QCD Amplitudes contd.

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One Loop - QCD Amplitudes contd.

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helicity configurations $\sim \frac{[ab]}{\langle ab \rangle^2}$
- boundary terms $A(z \rightarrow \infty) \neq 0$
- spurious (unphysical) singularities as $s_1 \rightarrow s_2$

$$\frac{\ln \left(\frac{-s_1}{-s_2} \right)}{(s_1 - s_2)^2}$$

One Loop - QCD Amplitudes contd.

Solutions

- merge unitarity with recursion

$$A_n(0) = \int \frac{dz}{z} \operatorname{Disc} C_n(z) + \sum_{\text{poles}} \operatorname{Res} \frac{C_n(z)}{z} + \sum_{\text{poles}} \operatorname{Res} \frac{\bar{C}_n(z)}{z} \quad (6)$$

where C_n from cuts, R_n rational pieces - can be constructed via recursion relations.

One Loop - QCD Amplitudes contd.

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- double shifts to avoid boundary terms ($j < l < n$)

$$\tilde{\lambda}_j \rightarrow \tilde{\lambda}_j - z\tilde{\lambda}_l - z\frac{\langle nj \rangle}{\langle lj \rangle}\tilde{\lambda}_n \quad \lambda_l \rightarrow \lambda_l + z\lambda_j \quad \lambda_n \rightarrow \lambda_n + z\frac{\langle nj \rangle}{\langle lj \rangle}\lambda_j \quad (7)$$

One Loop - QCD Amplitudes contd.

- double poles - from **soft factors**

$$\sim A_L(\hat{k}, \dots, P_{ij}, l) \mathcal{S}(\hat{k}, P_{ij}, l) \frac{1}{P_{ij}^2} \mathcal{S}(m, -P_{ij}, \hat{n}) A_R(\hat{n}, -P_{ij}, \dots, r) \quad (8)$$

$$\mathcal{S}(a, s^+, b) = \frac{\langle ab \rangle}{\langle as \rangle \langle sb \rangle} \quad \mathcal{S}(a, s^-, b) = -\frac{[ab]}{[as][sb]} \quad (9)$$

independent of helicities of a, b .

One Loop - QCD Amplitudes contd.

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- spurious singularities

$$\frac{\ln \left(\frac{-s_1}{-s_2} \right)}{(s_1 - s_2)^2} \rightarrow \frac{L_1 \left(\frac{-s_1}{-s_2} \right)}{s_2^2} \quad (10)$$

$$L_1(r) = \frac{L_0(r)+1}{1-r}, \quad L_0(r) = \frac{\ln r}{1-r}.$$

Introduction of extra rational terms! \Rightarrow need to subtract **overlap terms** to avoid double counting.

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