Towards One-Loop MHV Techniques

Stanford Linear Accelerator Center

Carola F. Berger

Snowmass - August 22, 2005

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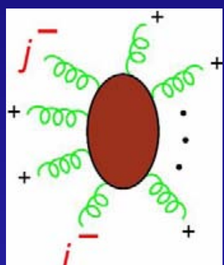
Tree Level Techniques

Parke-Taylor (1986) $\Rightarrow$ Twistor space – Witten (2003)



$$\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}\delta\left(\sum_i k_i\right)$$

$$\langle ij\rangle = \bar{u}_-(k_i)u_+(k_j)$$

$$u_+(k_i) = (\lambda_i)_\alpha;\; u_-(k_i) = (\tilde{\lambda}_i)_{\dot{\alpha}}$$

$$\Rightarrow \int d\tilde{\lambda}\, e^{i\mu\tilde{\lambda}}$$

In general: curve in twistor space of genus $g \leq l$ and of degree

$$d = q - 1 + l$$

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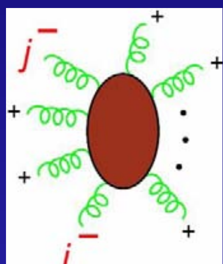

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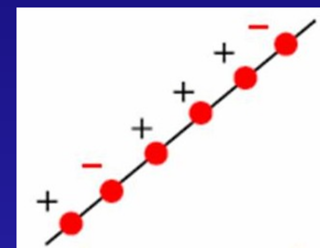

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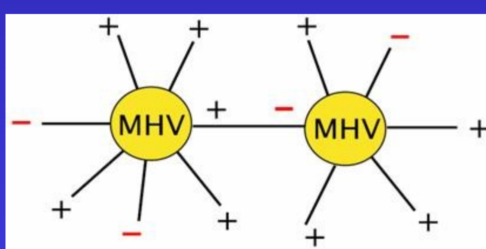
MHV rules – Cachazo, Svrcek, Witten (2004)



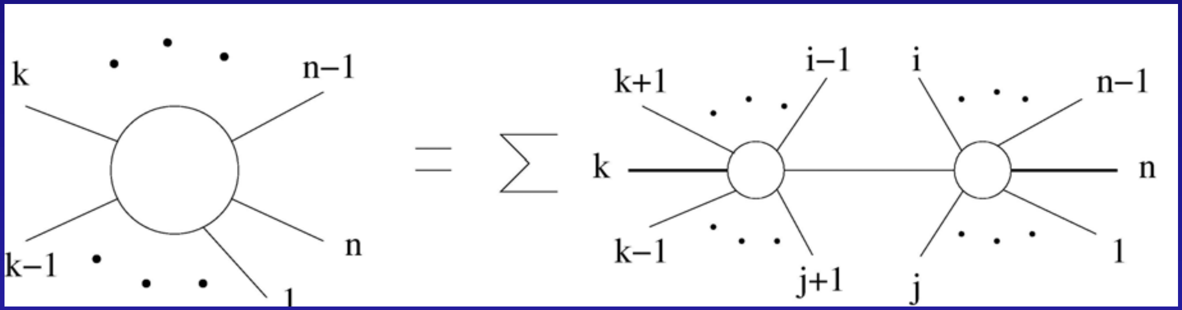
Off-shell Parke-Taylor amplitudes connected by **scalar propagator**

More efficient than Feynman rules for QCD (and SUSY) tree amplitudes.

Lagrangian connection ?

Tree Level Techniques contd.

On-shell recursion relations – Britto, Cachazo, Feng, and Witten (2004-05)



$$A_n = \sum_{r,s} \sum_{\lambda=\pm} A_L^{\lambda} \frac{1}{P_{r,s}^2} A_R^{-\lambda} \tag{1}$$

Proof: Shift gluons i, j

$$\begin{aligned} p_i(z) &= \lambda_i(\tilde{\lambda}_i - z\tilde{\lambda}_j) \\ p_j(z) &= (\lambda_j + z\lambda_i)\tilde{\lambda}_j \end{aligned} \tag{2}$$

$\Rightarrow$ amplitude

$$A_n(z) = A_n(p_1, \ldots, p_i(z), \ldots, p_j(z), \ldots) \tag{3}$$

Tree Level Techniques contd.

Amplitude is on-shell for all z, momentum is conserved.
Internal momentum $1/P_{kl}^2 \to 1/P_{kl}^2(z)$
$A_n(z)$ has only **simple poles**.

Physical scattering amplitude $A(0)$

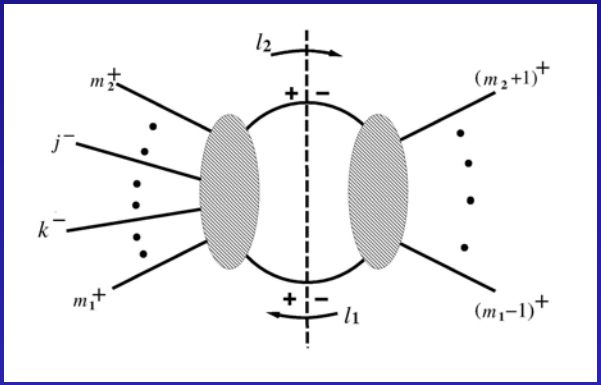
$$0 = \frac{1}{2\pi i} \oint_C \frac{dz}{z} A(z)$$

If $A(z \to \infty) \to 0$

$$A(0) = - \sum_{\text{poles } \alpha} \text{Res}_{\, z=z_\alpha} \frac{A(z)}{z} = \sum_{r,s} \sum_{\lambda=\pm} A_L^{\lambda}(z=z_\alpha) \frac{1}{P_{r,s}^2} A_R^{-\lambda}(z=z_\alpha) \qquad (4)$$

One Loop - Supersymmetric Amplitudes

Supersymmetric amplitudes - cut constructible

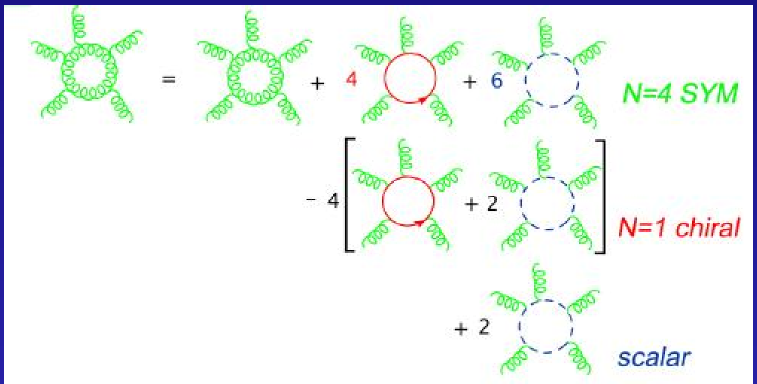


$$\int d\text{LIPS}(-l_1, l_2) A^{\text{tree}}(-l_1, m_1, \ldots, m_2, l_2) A^{\text{tree}}(-l_2, m_2+1, \ldots, m_1-1, l_1) \quad (5)$$

legs in $D = 4$ dimensions (integral in $D = 4 - 2\varepsilon$ dimensions)
also, generalized cuts
$\Rightarrow$ reduction to box, triangle, bubble integrals with coefficients obtainable from tree amplitudes

One Loop - QCD Amplitudes

SUSY decomposition $\Rightarrow$ reduction to scalar loop



in SUSY amplitudes rational terms intimately linked to (poly)logarithmic contributions

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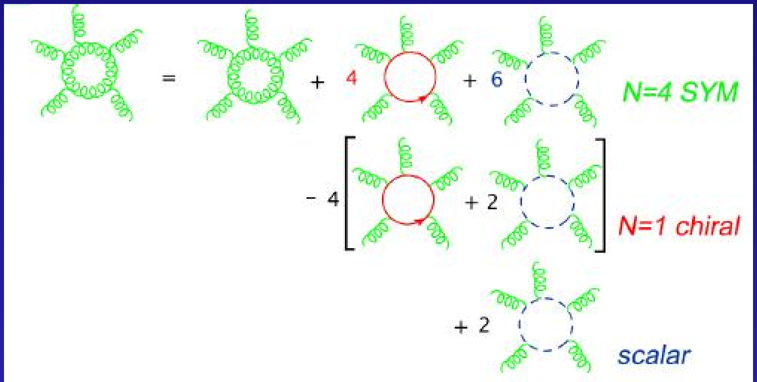

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$\mathcal{N} = 0$ **- 2 approaches**

- Generalized cuts $\Rightarrow$ (poly)logarithmic terms; rational terms require evaluation to higher order in ε (Bedford, Brandhuber, Spence, Travaglini (2004-2005))

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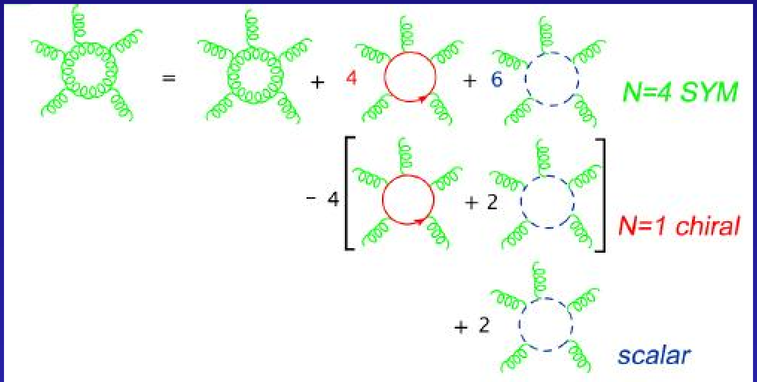

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$\mathcal{N} = 0$ **- 2 approaches**

- Generalized cuts $\Rightarrow$ (poly)logarithmic terms; rational terms require evaluation to higher order in ε (Bedford, Brandhuber, Spence, Travaglini (2004-2005))
- Recursion relations (Bern, Dixon, Kosower (2004-05) + in progress, Forde, CFB)

One Loop - QCD Amplitudes contd.

New features at one loop

- branch cuts

One Loop - QCD Amplitudes contd.

New features at one loop

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- double poles:
  multiparticle factorization $\Rightarrow$ simple poles
  collinear factorization $\Rightarrow$ one-loop splitting amplitudes for $(+++)$ and $(---)$ helicity configurations $\sim \frac{[ab]}{\langle ab\rangle^2}$

One Loop - QCD Amplitudes contd.

New features at one loop

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  multiparticle factorization $\Rightarrow$ simple poles
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  helicity configurations $\sim \frac{[ab]}{\langle ab\rangle^2}$
- boundary terms $A(z \to \infty) \neq 0$

One Loop - QCD Amplitudes contd.

New features at one loop

- branch cuts
- double poles:
  multiparticle factorization $\Rightarrow$ simple poles
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- boundary terms $A(z \to \infty) \neq 0$
- spurious (unphysical) singularities as $s_1 \to s_2$

$$\frac{\ln\left(\frac{-s_1}{-s_2}\right)}{(s_1 - s_2)^2}$$

One Loop - QCD Amplitudes contd.

Solutions

- merge unitarity with recursion

$$A_n(0) = \int \frac{dz}{z} \text{ Disc } C_n(z) + \sum_{\text{poles}} \text{ Res } \frac{C_n(z)}{z} + \sum_{\text{poles}} \text{ Res } \frac{C_n(z)}{z} \quad (6)$$

where C_n from cuts, R_n rational pieces - can be constructed via recursion relations.

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- double shifts to avoid boundary terms $(j < l < n)$

$$\tilde{\lambda}_j \to \tilde{\lambda}_j - z\tilde{\lambda}_l - z\frac{\langle nj \rangle}{\langle lj \rangle}\tilde{\lambda}_n \quad \lambda_l \to \lambda_l + z\lambda_j \quad \lambda_n \to \lambda_n + z\frac{\langle nj \rangle}{\langle lj \rangle}\lambda_j \tag{7}$$

One Loop - QCD Amplitudes contd.

- double poles - from **soft factors**

$$\sim A_L(\hat{k},\ldots,P_{ij},l)\mathcal{S}(\hat{k},P_{ij},l)\frac{1}{P_{ij}^2}\mathcal{S}(m,-P_{ij},\hat{n})A_R(\hat{n},-P_{ij},\ldots,r) \tag{8}$$

$$\mathcal{S}(a,s^+,b)=\frac{\langle ab\rangle}{\langle as\rangle\langle sb\rangle} \qquad \mathcal{S}(a,s^-,b)=-\frac{[ab]}{[as][sb]} \tag{9}$$

independent of helicities of a, b.

One Loop - QCD Amplitudes contd.

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- spurious singularities

$$\frac{\ln\left(\frac{-s_1}{-s_2}\right)}{(s_1-s_2)^2}\rightarrow\frac{L_1\left(\frac{-s_1}{-s_2}\right)}{s_2^2} \tag{10}$$

$L_1(r)=\frac{L_0(r)+1}{1-r}$, $L_0(r)=\frac{\ln r}{1-r}$.
Introduction of extra rational terms! $\Rightarrow$ need to subtract **overlap terms** to avoid double counting.

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