International Linear Collider Physics and Detector Workshop

Snowmass — August 2005

Unstable particle production

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An effective theory approach

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Fermilab

☛ In collaboration with

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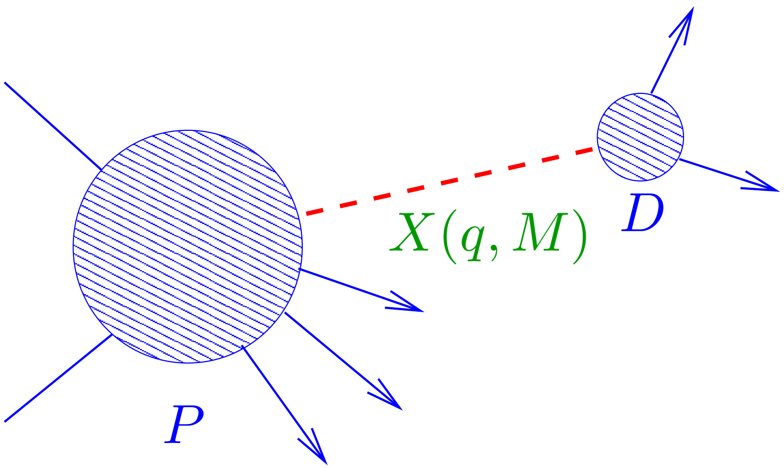
Study of unstable particles $X \in \{W^{\pm}, Z, t, H(?) \ldots\}$ close to resonance

Physical picture: separation of production, propagation and decay

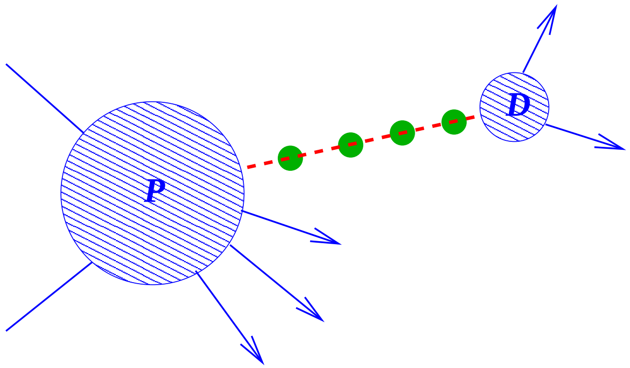


Amplitude:

$$\mathcal{A}^{(\text{tree})}(q^2) = \mathcal{P}(q^2) \frac{ig^2}{q^2 - M^2} \mathcal{D}(q^2)$$

► non-integrable singularity for resonant unstable particle, i.e. $q^2 \sim M^2$

The problem



Dyson summation of self-energy Π

$$\mathcal{A}^{(\text{“tree”})} = P(q^2)\frac{i}{q^2 - M^2 - \Pi(q^2)} D(q^2)$$

- $\mathrm{Im}(\Pi) \neq 0$ (finite width) $\rightsquigarrow$ pole *off* the real axis
- resummation: divergence $\rightsquigarrow$ resonance

⚠ However: not a strict order-by-order expansion

☛ The selection of only some *arbitrary* higher order corrections spoils properties valid order by order in PT ($\Rightarrow$ gauge invariance)! ☹

Various “standard” approaches

Theoretical approaches	Problems/drawbacks
✗ fixed width scheme	✗ ad-hoc, no physical justification
✗ running width scheme	✗ predictions violate unitary
✗ overall-factor scheme	✗ complex mass and weak mixing angles
✗ complex mass scheme	✗ unphysical effects off resonance
✗ fermion loop scheme	✗ no hope to improve accuracy
✗ pole approximation	

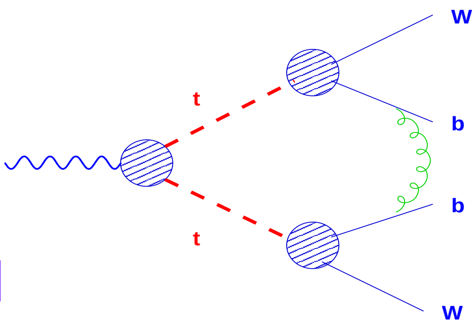
$$\Downarrow$$

- ☞ not clear how to extend these beyond NLO in α and Γ/M
- ☞ need rules for a *systematic* double expansion in α and Γ/M

Beyond

✗ At Linear Collider need to go beyond DPA, e.g:
$\Delta m_t \lesssim 100$ MeV $\quad \Delta m_W \lesssim 10$ MeV

✗ Problem in Quantum Field Theory ! [Veltman 1963]



Two ways to go beyond

- higher order in α $\Longrightarrow$ beyond one loop
  standard PT expansion
- higher order in Γ/M $\Longrightarrow$ beyond the pole approximation
  how to expand?

☛ *Characteristic feature:* two physical scales
formation/decay time $1/M$, lifetime $1/\Gamma \gg 1/M \Longrightarrow$ ***effective theory***

underlying theory

$\mathcal{L}(\phi_h, \phi_{r/c}, \phi_s)$

dynamical modes:

hard, resonant/collinear, soft

integrate out hard modes

effective theory

factorizable corrections

non-factorizable corrections

$\mathcal{L}_{\text{eff}} = \sum_n c_n(\text{h})\, O_n(\phi_{r/c}, \phi_s)$

dynamical modes:

resonant/collinear, soft

Our toy Model

The Lagrangian

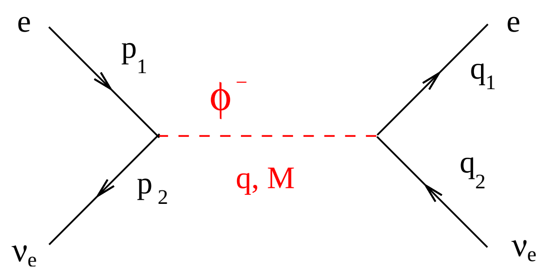
$$\mathcal{L} = (D_\mu\phi)^\dagger D^\mu\phi - M^2\phi^\dagger\phi + \overline{\psi} i \not{D}\psi + \overline{\chi} i \not{\partial}\chi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$+ \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + y\phi\overline{\psi}\chi + y^*\phi^\dagger\overline{\chi}\psi + \frac{\lambda}{4}(\phi^\dagger\phi)^2 + \mathcal{L}_{\rm ct}\ , \qquad D_\mu = \partial_\mu - igA_\mu$$

The process [economic version of $u\bar{d} \to W^- \to e^-\bar{\nu}$]

$$e^-(p_1)\ \bar{\nu}_e(p_2) \quad \to \quad \phi^-(q, M) \to e^-(q_1)\ \bar{\nu}_e(q_2)$$

$$q^2 \equiv (p_1 + p_2)^2$$



close to resonance: $\delta \equiv \dfrac{q^2 - M^2}{M^2} \sim \alpha \sim \Gamma/M \ll 1$

The couplings

$\alpha_g \equiv \dfrac{g^2}{4\pi}\ ,\ \alpha_y \equiv \dfrac{y^2}{4\pi}\ ,\ \alpha_\lambda \equiv \dfrac{\lambda}{4\pi}$ with $\alpha_g \sim \alpha_y \sim \alpha\ ,\ \alpha_\lambda \sim \dfrac{\alpha^2}{4\pi}$

Matching

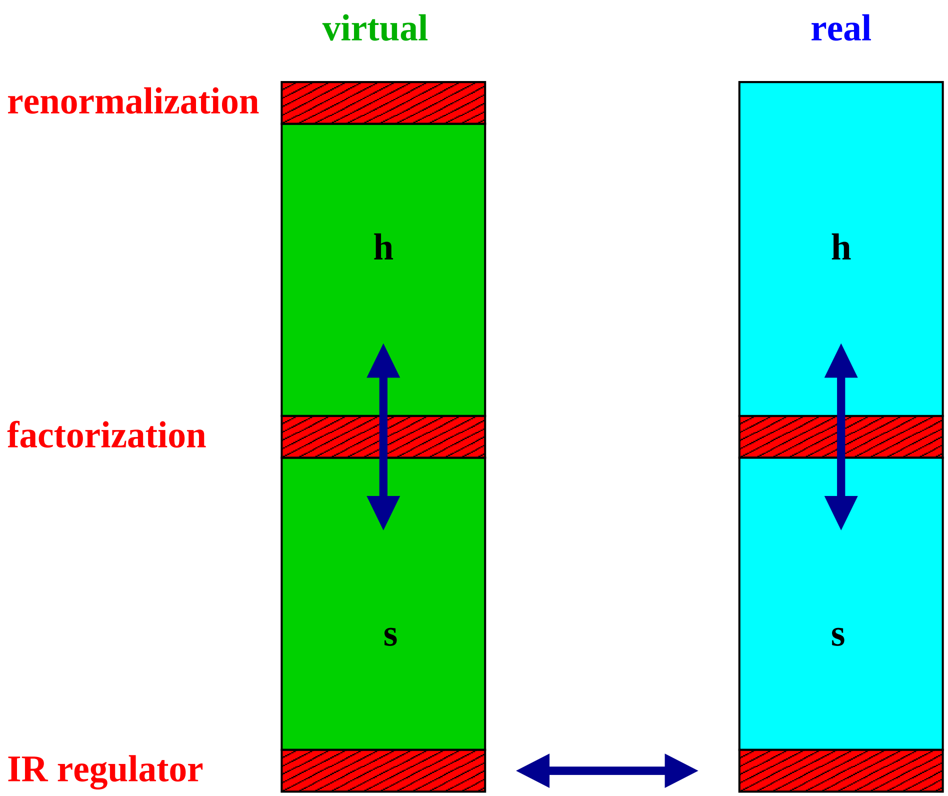
Matching procedure of effective theory to full theory is standard and involves three steps

1. evaluate renormalized on-shell Green functions in full theory
2. evaluate of the same quantity in the effective theory
3. determine the hard matching coefficient so that the two calculations agree within the specified accuracy

☞ Simplification: use dimensional regularization

Since the matching is onshell all effective theory loops vanish (scaleless integrals) $\Rightarrow$ need only tree level terms in the effective theory

Singularities



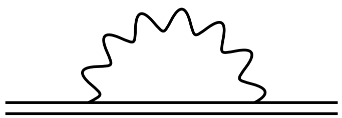
- use strategy of regions to separate hard/soft
- split into hard and soft modes introduces additional singularities
- matching coefficients will have $1/\epsilon$ *divergences*
- they correspond to *UV divergences* in the effective theory

$\Rightarrow$ a choice a *renormalization scheme* in the effective theory amounts to a choice of *factorization scheme*

The NLO effective Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm eff} &= \mathcal{L}_{\rm HSET} + \mathcal{L}_{\rm SCET} + \mathcal{L}_{\rm int} \\
&= 2\hat{M}\,\phi_v^\dagger \left(iv\cdot D_s - \frac{\Delta}{2} \right) \phi_v + 2\hat{M}\phi_v^\dagger \left(\frac{(iD_{s,\top})^2}{2\hat{M}} + \frac{\Delta^2}{8\hat{M}} \right) \phi_v \\
&- \frac{1}{4} F_{s\mu\nu} F_s^{\mu\nu} + \bar{\psi}_s i \not{D}_s \psi_s + \bar{\chi}_s i \not{\partial} \chi_s + \bar{\psi}_{n_-} i n_- D_s \frac{\not{n}_+}{2} \psi_{n_-} \\
&+ C[y\phi_v \bar{\psi}_{n_-}\chi_{n_+} + h.\ c.\ ] + \frac{yy^* D}{4\hat{M}^2} \left(\bar{\psi}_{n_-}\chi_{n_+}\right)\left(\bar{\chi}_{n_+}\psi_{n_-}\right) + \ldots
\end{aligned}
$$

Heavy Scalar Effective Theory (HSET)



- propagation of the heavy scalar and it's interaction with soft fields
- $\Delta \equiv \frac{(\bar{s}-\hat{M}^2)}{\hat{M}}$ (pole scheme: $\Delta = -i\Gamma$ with Γ the onshell width)
- unstable particle propagator is $\dfrac{i}{2\hat{M}(v\cdot k - \frac{\Delta^{(1)}}{2})}$

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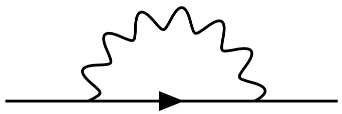
Soft Collinear Effective Theory (SCET)



☛ propagation of energetic fermions and their interaction with SC fields

[soft-collinear fi elds, i.e. fluctuations only around the classical trajectory]

The NLO effective Lagrangian

$$\begin{aligned}
\mathcal{L}_{\rm eff} &= \mathcal{L}_{\rm HSET} + \mathcal{L}_{\rm SCET} + \mathcal{L}_{\rm int} \\
&= 2\hat{M}\,\phi_v^\dagger \left(iv\cdot D_s - \frac{\Delta}{2}\right)\phi_v + 2\hat{M}\phi_v^\dagger \left(\frac{(iD_{s,\top})^2}{2\hat{M}} + \frac{\Delta^2}{8\hat{M}} \right)\phi_v \\
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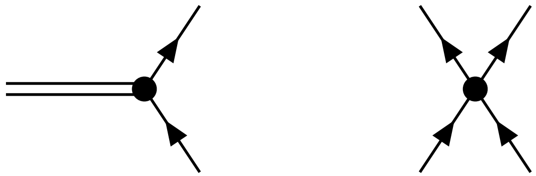
\end{aligned}$$

Interaction: production/decay vertices



☛ interaction terms between heavy scalar and energetic fermions

The NLO effective Lagrangian

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Hard fluctuations in matching coefficients. At NLO need

- Δ to order α^2
- C to order α
- D at tree level

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Hard fluctuations in matching coefficients. At NLO need

- Δ to order α^2
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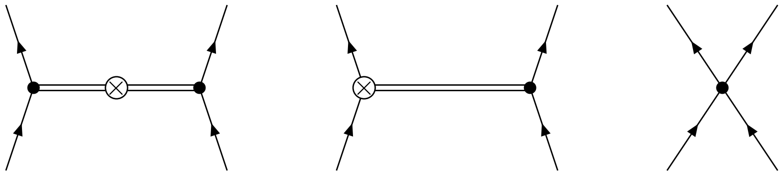
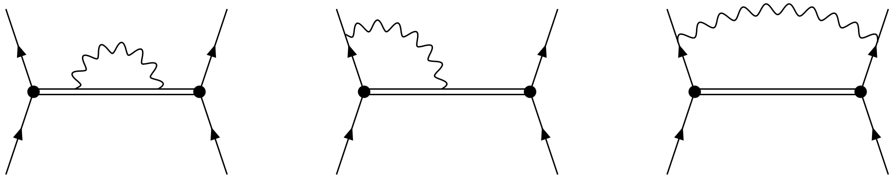
☞ After deriving $\mathcal{L}_{\rm eff}$ to the required accuracy by matching calculations, compute amplitude in the effective theory using *conventional PT*

NLO line shape



$$i\,\mathcal{T}_h^{(1)} = i\,\mathcal{T}^{(0)} \times \left[2\,C^{(1)} - \frac{[\Delta^{(1)}]^2}{8\mathcal{D}\hat{M}} + \frac{\Delta^{(2)}}{2\mathcal{D}} \right]$$

with $\mathcal{D} \equiv \sqrt{s} - \hat{M} - \dfrac{\Delta^{(1)}}{2}$



$$i\,\mathcal{T}_s^{(1)} = i\,\mathcal{T}^{(0)} \times a_g \left[4L^2 - 4L + \frac{5\pi^2}{6} \right]$$

with $L \equiv \ln\left(\dfrac{-2\mathcal{D}}{\mu} \right)$

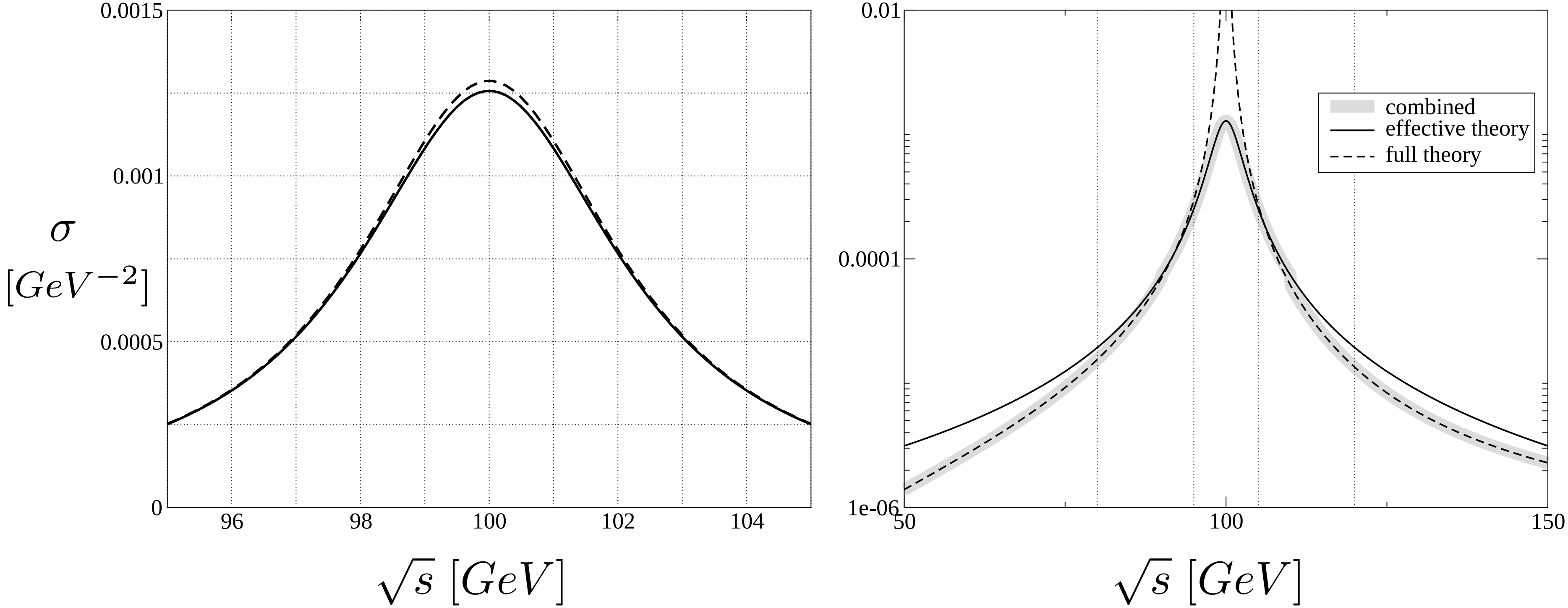
- LO amplitude $\mathcal{T}^{(0)}$ is a Breit Wigner
- poles cancel when adding hard and soft contributions, up to initial state collinear singularity [standard]

Plots: Inclusive line-shape

Inputs: $M_{Pole} = 100$GeV, $\alpha_y(M_{pole}) = 0.1$, $\alpha_g(M_{pole}) = 0.1$, $\alpha_\lambda(M_{pole}) = 0.1^2/(4\pi)$

LO line-shape

Dashed: Pole Scheme, Solid: $\overline{MS}$ Scheme ($M^{(1)}_{\overline{MS}} = 98.8$ GeV)

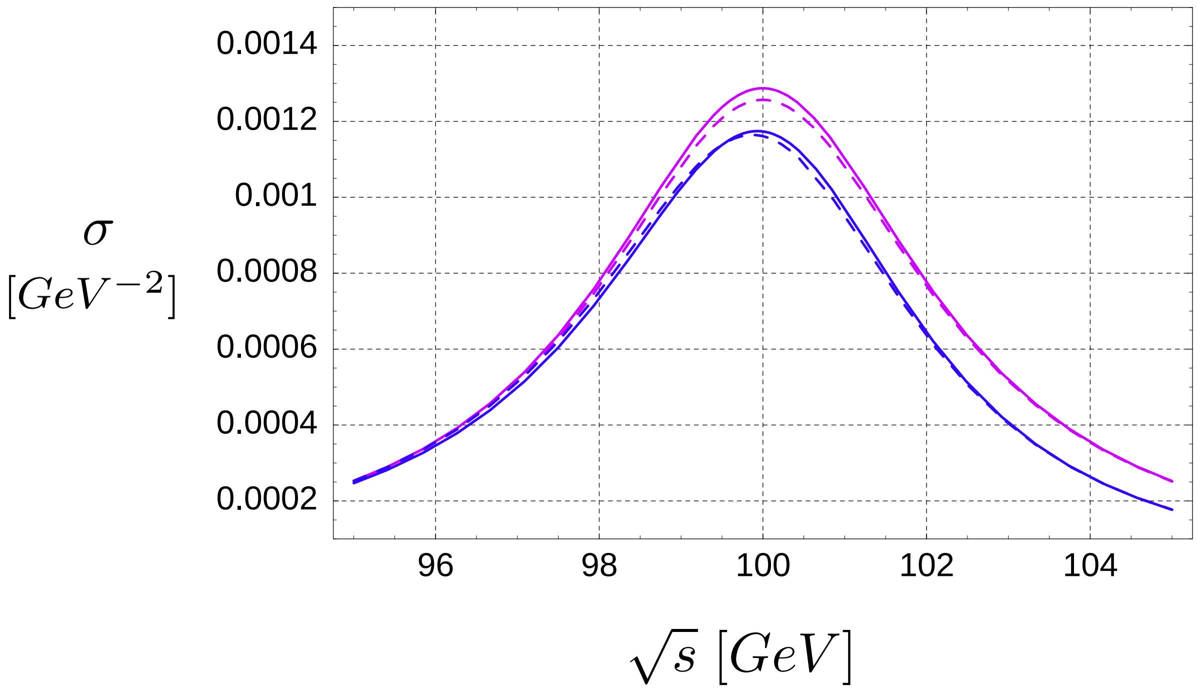


- shown: partonic cross-sections with initial state singularity minimally subtracted
- effective theory result valid where $\sqrt{s} - M \sim \alpha M$
- matching of full and effective theory in intermediate region $\delta \sim \sqrt{s} - M$ needed

Plots: Inclusive line-shape

Inputs: $M_{Pole} = 100$GeV, $\alpha_y(M_{pole}) = 0.1,\ \alpha_g(M_{pole}) = 0.1,\ \alpha_\lambda(M_{pole}) = 0.1^2/(4\pi)$

NLO line-shape



Dashed: Pole Scheme
Solid: $\overline{MS}$ Scheme
($M^{(2)}_{\overline{MS}} = 99.1$ GeV)

Blue: NLO

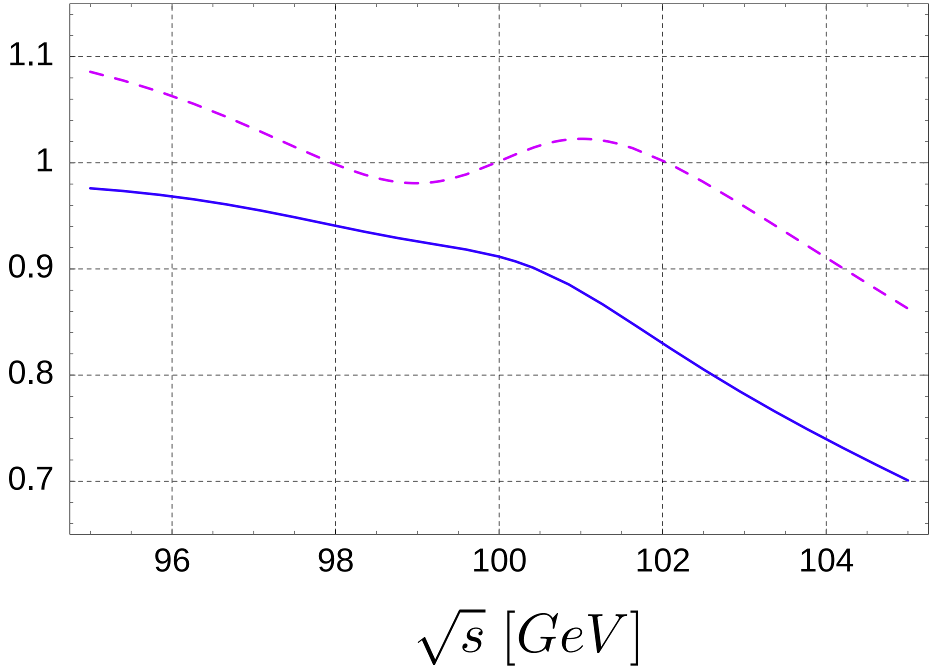
Magenta: LO

- scheme dependence effect very small
- NLO correction $\sim -10\%$ at the peak and up to -30% in the above range

Plots: Inclusive line-shape

Inputs: $M_{Pole} = 100$GeV, $\alpha_y(M_{pole}) = 0.1,\ \alpha_g(M_{pole}) = 0.1,\ \alpha_\lambda(M_{pole}) = 0.1^2/(4\pi)$

Ratio of NLO line-shapes



Solid: σ_{NLO}/σ_{LO}

Dashed: σ_{NLO}/σ_{BW}

- deviation from Breit-Wigner up to 15%
- output mass parameter of the Breit-Wigner fit differs from the input $M = 100$GeV by $\delta M = 160$MeV
  - ➥ data should be fitted to theoretically predicted line-shapes rather than to BW-fits!

At NNLO need LO $\times(\alpha^2, \delta\alpha, \delta^2)$

Notation: $\alpha \Rightarrow \alpha_h, \alpha_s, \alpha_c$

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Contributions at NNLO

loop-order in eff.theory	order of the operator	matching accuracy
α_s, α_c	δ	α_h

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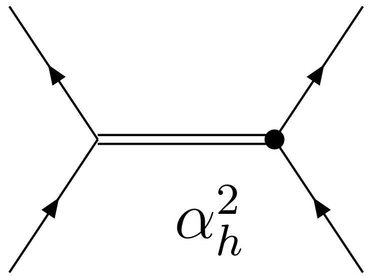
LO	LO	NNLO	α_h^2



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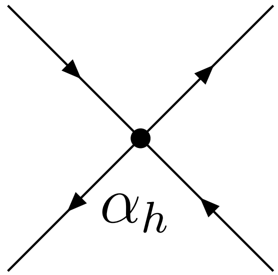
LO	NLO	NLO	$\delta \times \alpha_h$



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α_s, α_c	δ	α_h	
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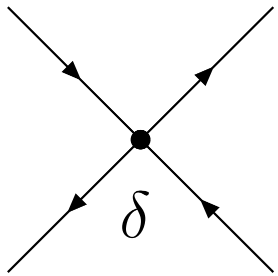
LO	NLO	NLO	$\delta \times \alpha_h$
LO	NNLO	LO	δ^2



Beyond NLO

At NNLO need LO $\times(\alpha^2, \delta\alpha, \delta^2)$

Notation: $\alpha \Rightarrow \alpha_h, \alpha_s, \alpha_c$

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loop-order in eff.theory	order of the operator	matching accuracy	
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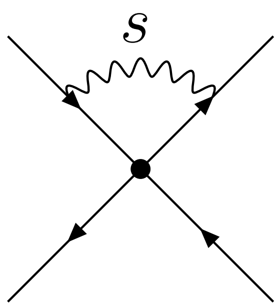

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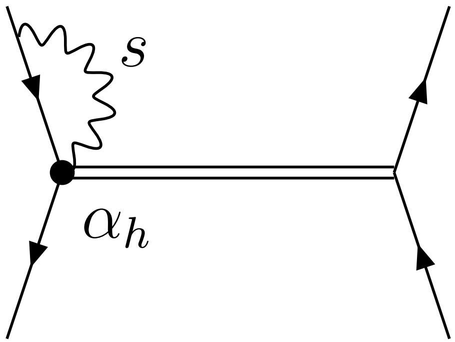
NLO	LO	NLO	$\alpha_h \alpha_{s/c}$



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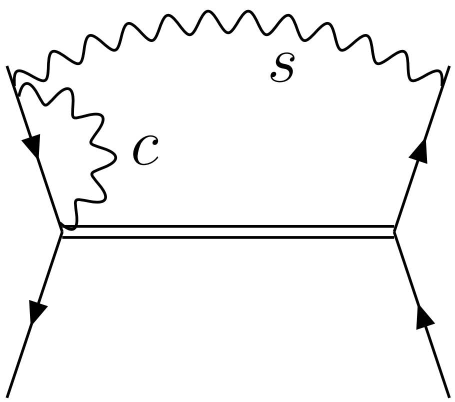
NNLO	LO	LO	$\alpha_{s/c}^2$



Beyond NLO

At NNLO need LO $\times(\alpha^2, \delta\alpha, \delta^2)$

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Contributions at NNLO

loop-order in eff.theory	order of the operator	matching accuracy	
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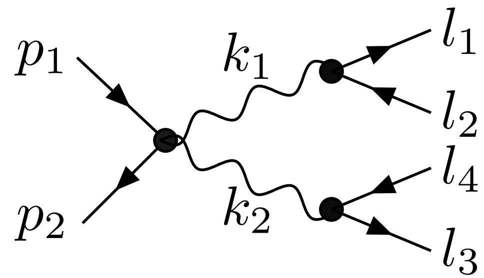
LO	LO	NNLO	α_h^2
LO	NLO	NLO	$\delta \times \alpha_h$
LO	NNLO	LO	δ^2
NLO	NLO	LO	$\delta \times \alpha_{s/c}$
NLO	LO	NLO	$\alpha_h \times \alpha_{s/c}$
NNLO	LO	LO	$\alpha_{s/c}^2$

☛ All contributions *separately gauge invariant*

WW at threshold at e^+e^- collider

Motivation: crucial for the precise determination of M_W

$$e^+(p_1)e^-(p_2) \to W^+(k_1)W^-(k_2) \to \mu(l_1)\bar{\nu}_\mu(l_2)u(l_3)\bar{d}(l_4)$$



counting: $\alpha_{em} \sim \alpha_s^2 \sim v^2 \qquad k_{1/2} \sim \{M(1+v^2), \pm M\vec{v}\}$

Similarly to what has been done in the toy model the procedure is to

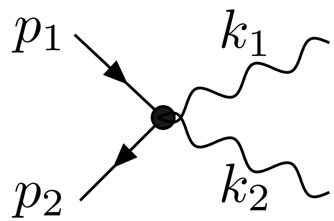
- integrate out hard modes and match to on-shell Greens functions
- construct the Lagrangian for the W-field in terms of the NR vector field Ω ($\rightsquigarrow$ NRQCD Lagrangian)
- systematically expand to the order required

WW at threshold at LO

At LO one needs

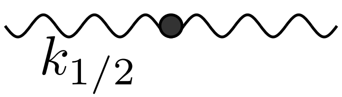
- tree level matching for the production vertex $ee \to WW$

$$\mathcal{L}_{\mathcal{P}}^{(0)} = \frac{2\pi\alpha_{ew}}{M_W^2}\left(\bar{e}_L\gamma^{[i}iD^{j]}e_L\right)\left(\Omega_-^{*i}\Omega_+^{*j}\right)$$



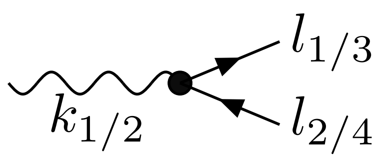
- resummation of $\mathcal{O}\left(\alpha_{ew}\right)$ onshell self-energies in the propagators

$$\mathcal{L}_{\mathcal{NR}}^{(0)} = \sum_{\pm}\Omega_{\pm}^{*i}\left(iD^0 + \frac{\vec{D}^2}{2M_W} - \frac{\Delta_1}{2}\right)\Omega_{\mp}^{i}$$



- tree level matching for the decay vertices $W \to l\bar{l}$

$$\mathcal{L}_{\mathcal{D}}^{(0)} = -\frac{g_{ew}}{\sqrt{2}}\Omega_-^i\bar{\mu}_L\gamma^i\nu_L - \frac{g_{ew}}{\sqrt{2}}\Omega_+^i\bar{u}_L\gamma^i d_L$$

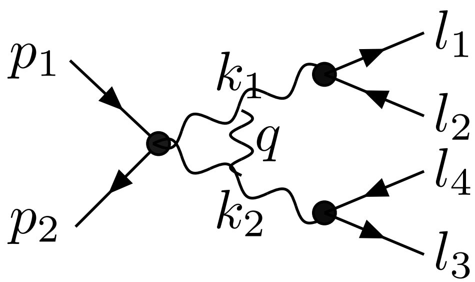


$\Rightarrow$ combining these ingredients one obtains the LO amplitude

WW at threshold at $N^{1/2}LO$

At $N^{1/2}LO$, i. e. $LO \times \mathcal{O}(\alpha_s, v)$

- Production stage: include v-corrections $\Rightarrow \mathcal{L}_P^{(1/2)}$
- Propagation: include $\alpha_s \alpha_{ew}$ corrections $\Rightarrow \mathcal{L}_{\mathcal{NR}}^{(1/2)}$
- Decay stage: include α_s corrections (however they cancel if one is inclusive on hadronic decay products)
- Furthermore: include exchange of one potential photon ($q^0 \sim Mv^2, \vec{q} \sim Mv$), this gives a correction $\mathcal{O}(\alpha_{ew}/v)$



$\Rightarrow$ combining these terms gives the $N^{1/2}LO$ amplitude.

MB,NK,AS,GZ [hep-ph/0411008]

Similarly one obtains NLO, $N^{3/2}LO$ $\ldots$ amplitudes (where at higher orders soft contribution must be also included)

Summary

- ✗ Perturbative treatment of unstable particles requires partial summation of PT series
- ✗ however the guiding principle of resummation was not understood
- ✗ breakdown of weak coupling PT related to the appearance of a second small parameter ($\alpha, \Gamma/M$)
- ✗ we take the attitude that $\Gamma \ll M$ is *the* characteristic feature
- ✗ other issues (resummation, gauge invariance . . .) follow automatically in a theory that formulates the expansion correctly
- ✗ Two-scale problem $\rightsquigarrow$ effective field theory (H”Q”ET + SCET)
- ✗ mode expansion $\Longrightarrow$ strategy of regions

☛ References

M. Beneke, A. Chapovsky, A. Signer, GZ

Phys. Rev. Lett. **93** (2004) 011602 [brief] & Nucl. Phys. **B 686** (2004) 205-247 [details]

Advantages of field theory methods

- split calculations in well-defined pieces (matching, matrix elements, loops)

  $\Rightarrow$ calculation effi cient and transparent

- power counting scheme in the small parameters (α, δ)

  $\Rightarrow$ identifi cation of terms required to achieve a certain accuracy

- Feynman rules to compute the minimal set of terms required

  $\Rightarrow$ since one does not compute “too much”, calculations are as simple as possible

- gauge invariance is automatic

- calculations can be extended to any accuracy in α, δ

  [at the price of performing complicated, but standard loop integrals]

- resummation of $\mathrm{Log}(M/\Gamma)$ standard using R.G. techniques