
Sensitivities to the Scale of Supersymmetry from Precision Observables

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Based on collaboration with J. Ellis, S. Heinemeyer, K. Olive

[hep-ph/0411216](#) + updates for $m_t = 172.7 \pm 2.9$ GeV

1. Introduction
2. Electroweak precision observables
3. Combined sensitivity: present situation and ILC precision
4. Conclusions

1. Introduction

EW precision data:

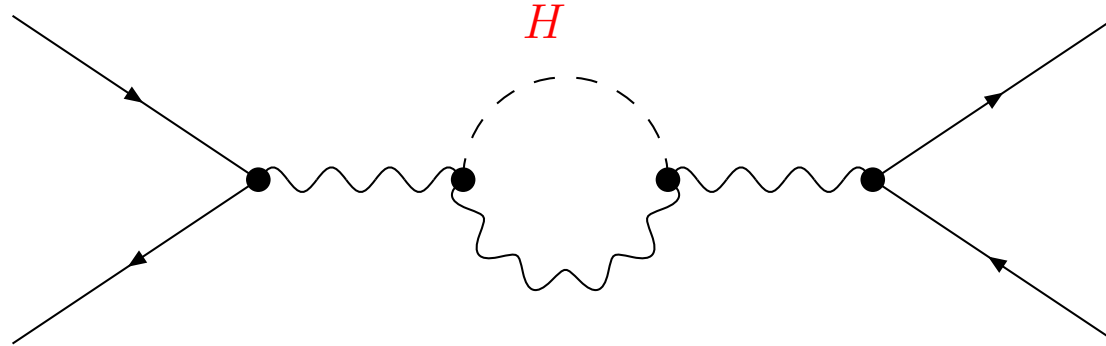
$M_Z, M_W, \sin^2 \theta_{\text{eff}}^{\text{lept}}, \dots$

Theory:

SM, MSSM, ...



Test of theory at quantum level: sensitivity to loop corrections



Indirect constraints on unknown parameters: $M_H, m_{\tilde{t}}, \dots$

Effects of “new physics”?

Constrained MSSM (CMSSM) with restrictions from dark matter relic density

CMSSM characterised by five parameters:

$m_{1/2}$, m_0 , A_0 (GUT scale), $\tan \beta$, $\text{sgn}(\mu)$ (weak scale)

⇒ Low-energy spectrum from renormalisation group running

lightest SUSY particle: $\tilde{\chi}_1^0$

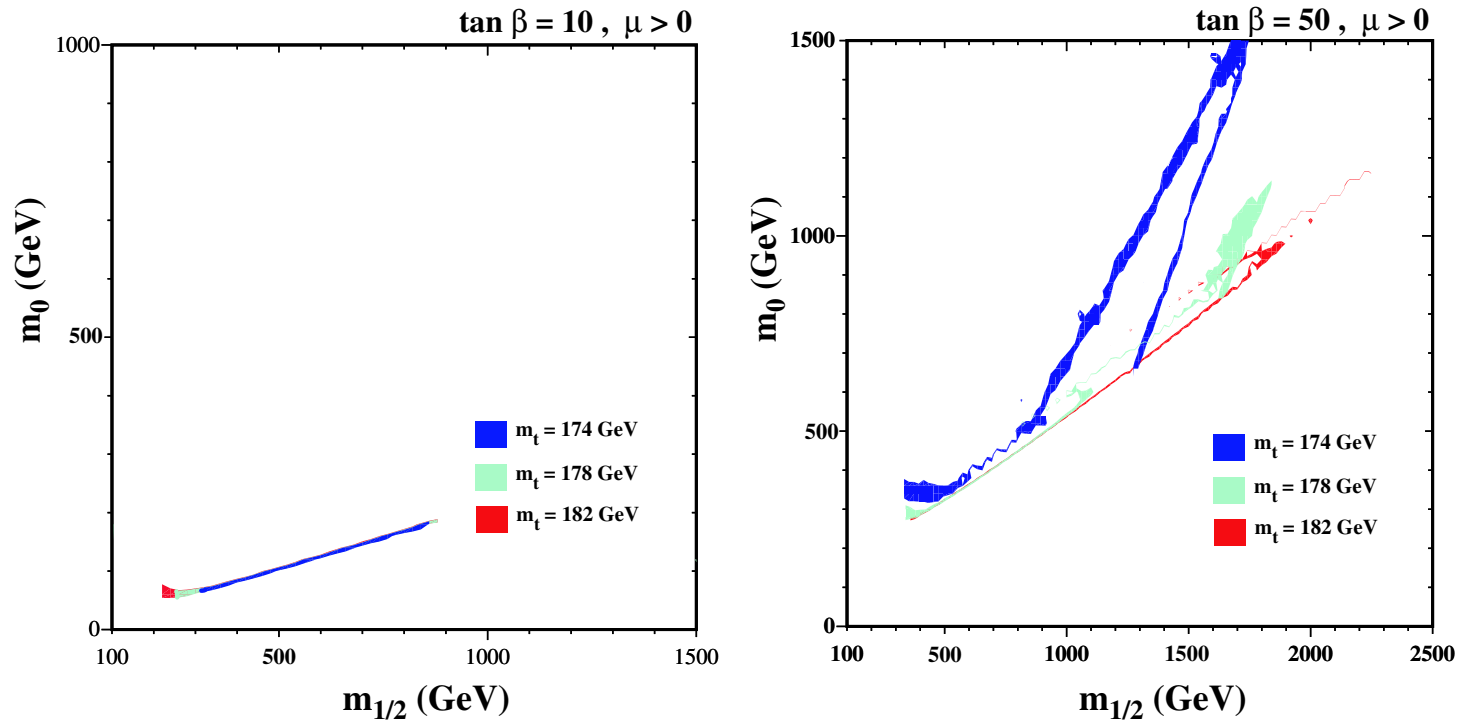
Cold dark matter (CDM) density (WMAP, ...):

$$0.094 < \Omega_{\text{CDM}} h^2 < 0.129$$

⇒ Constraints on SUSY parameter space

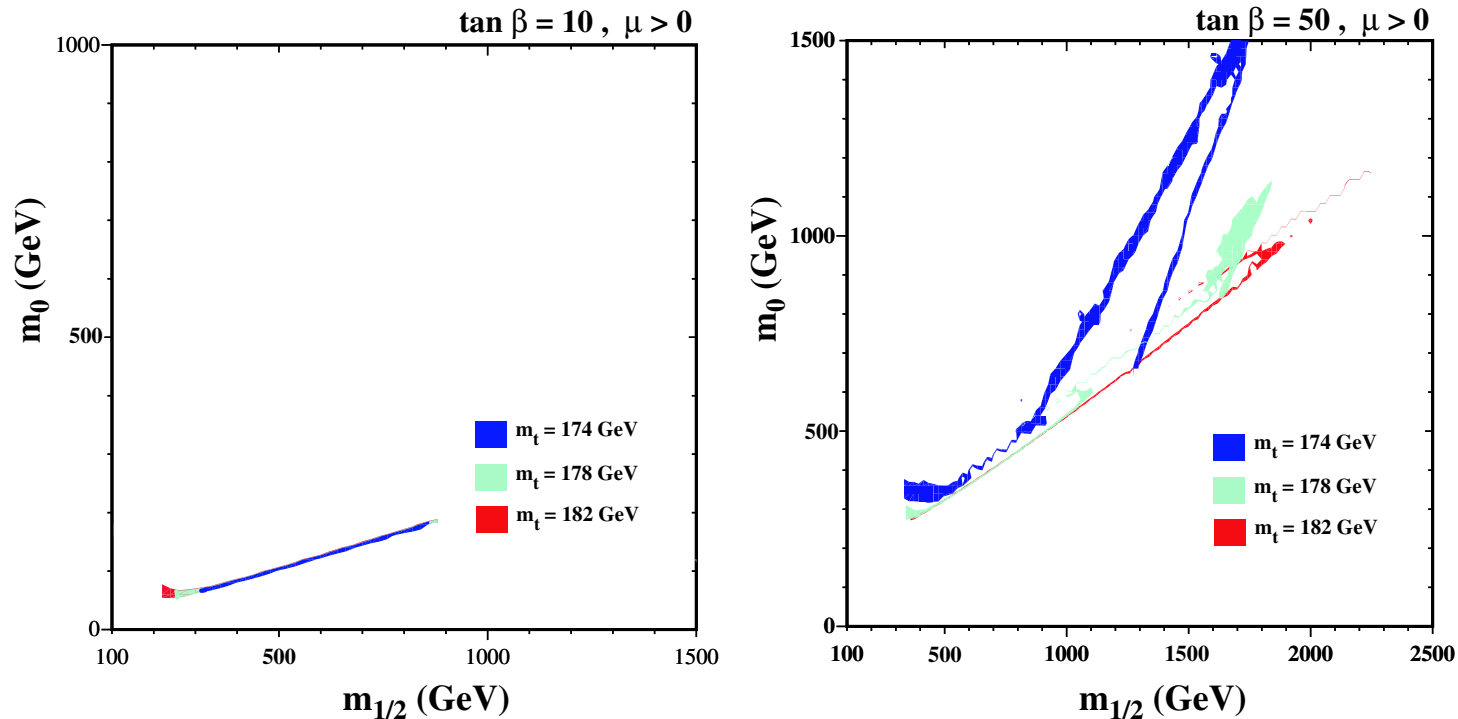
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Allowed region in $(m_{1/2}, m_0)$ plane (fixed A_0 , different m_t):



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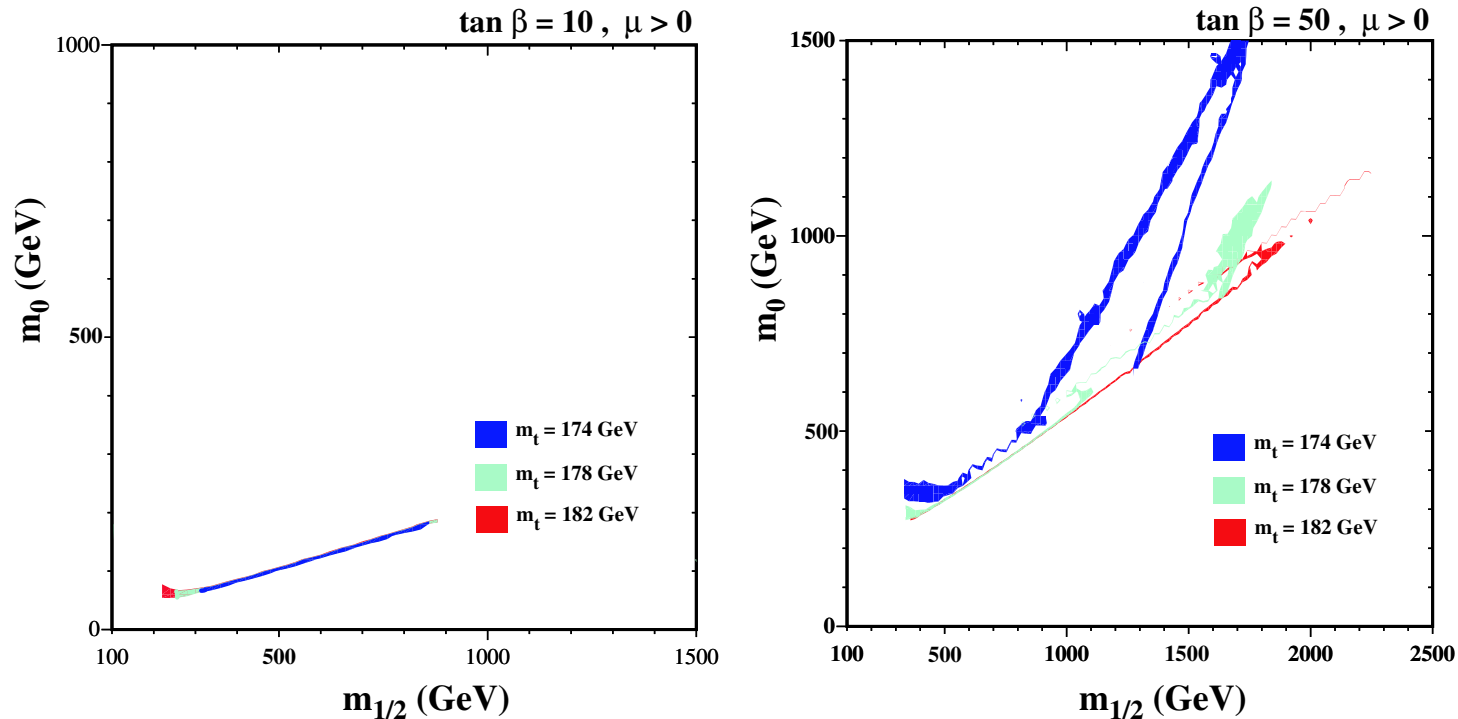


⇒ narrow “WMAP strips”

⇒ effectively reduces dimensionality of parameter space

Constrained MSSM (CMSSM) with restrictions from dark matter relic density

Allowed region in $(m_{1/2}, m_0)$ plane (fixed A_0 , different m_t):



- ⇒ narrow “WMAP strips”
- ⇒ effectively reduces dimensionality of parameter space
- ⇒ analyse CMSSM along WMAP strips

2. Electroweak precision observables

Observables taken into account:

Present:

$$M_W, \sin^2 \theta_{\text{eff}}, (g - 2)_\mu, b \rightarrow s\gamma, B_s \rightarrow \mu^+ \mu^-$$

Current experimental errors + estimate of current theoretical uncertainties (from unknown higher-orders + experimental errors of input parameters)

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Future (ILC):

$$M_W, \sin^2 \theta_{\text{eff}}, (g - 2)_\mu, b \rightarrow s\gamma, B_s \rightarrow \mu^+ \mu^-, \\ M_h, \text{BR}(h \rightarrow b\bar{b})/\text{BR}(h \rightarrow WW^*)$$

Experimental precision at the ILC + estimate of future theoretical uncertainties

Theoretical predictions for M_W , $\sin^2 \theta_{\text{eff}}$:

Comparison of prediction for muon decay with experiment (Fermi constant G_μ)

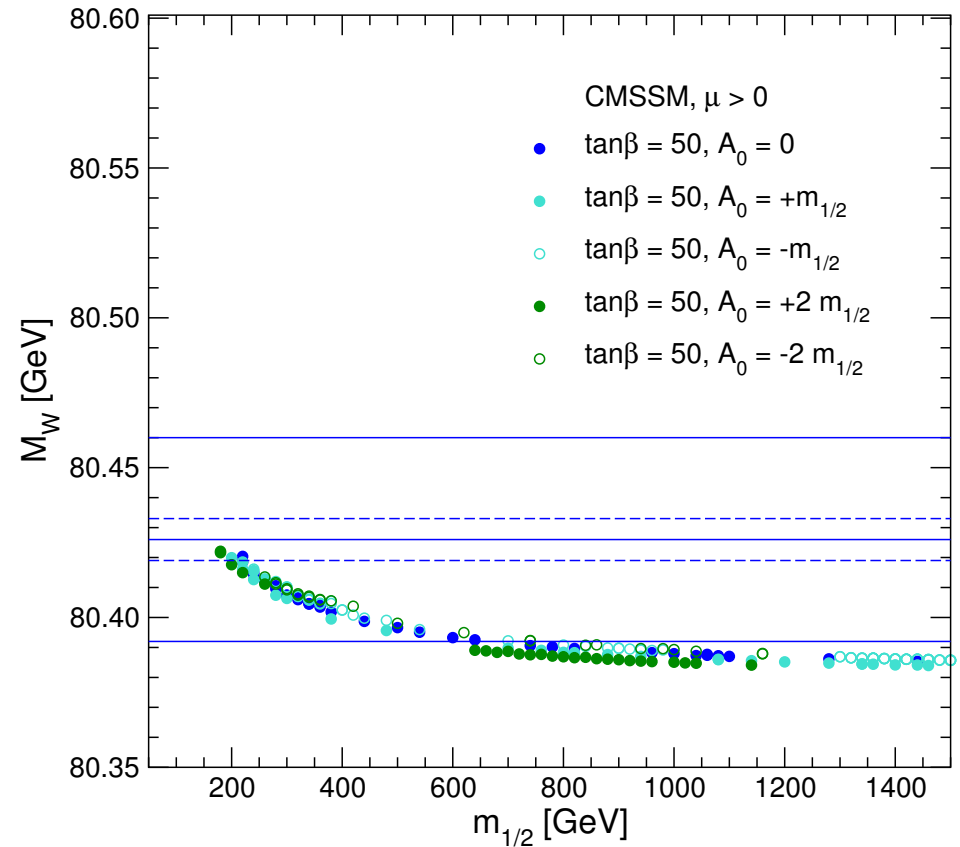
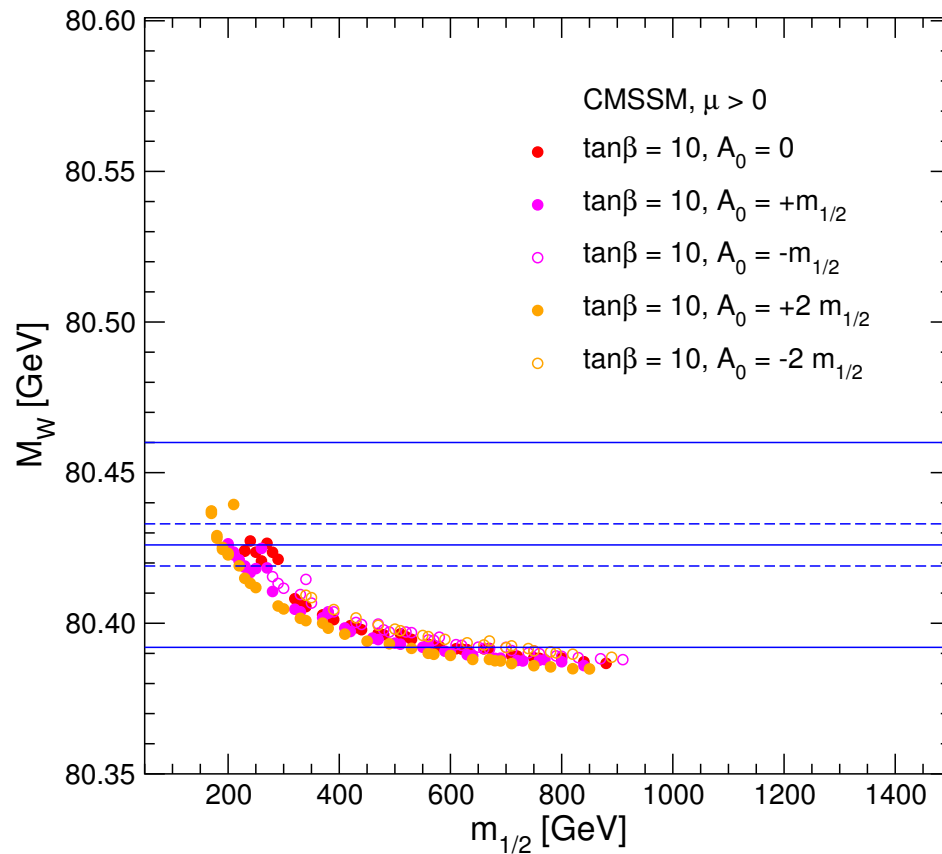
$$\Rightarrow M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \underset{\substack{\updownarrow \\ \text{loop corrections}}}{\Delta r}),$$

$\Rightarrow$ Theo. prediction for M_W in terms of M_Z , α , G_μ , $\Delta r(m_t, m_{\tilde{t}}, \dots)$

Effective couplings at the Z resonance:

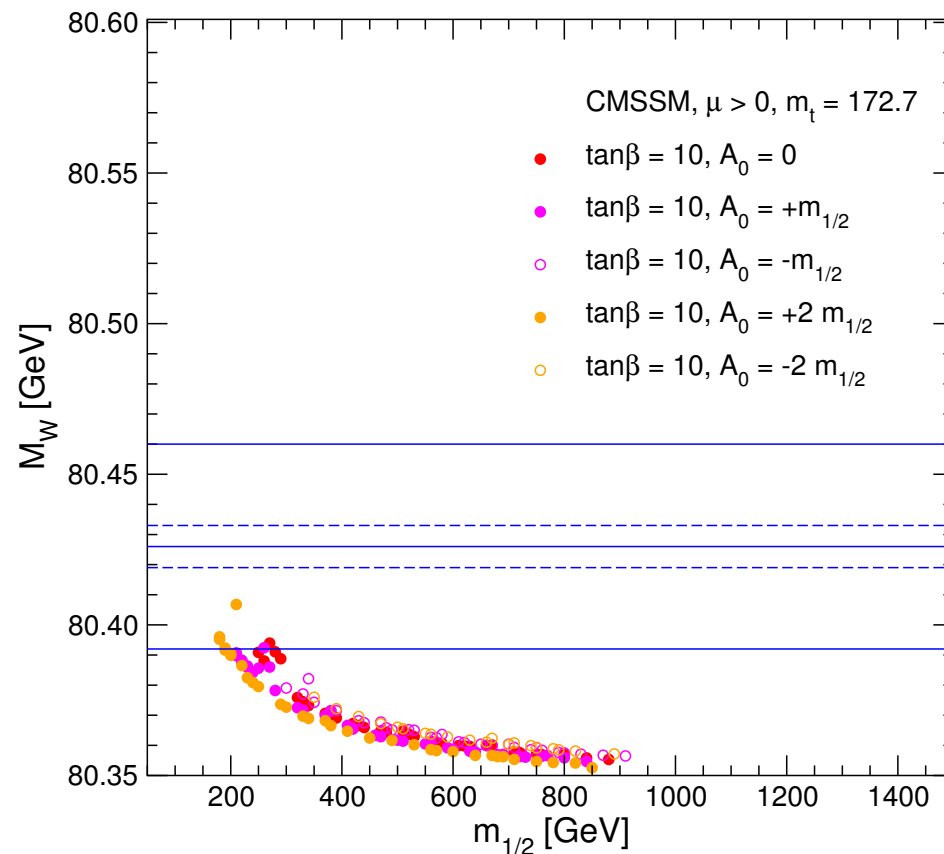
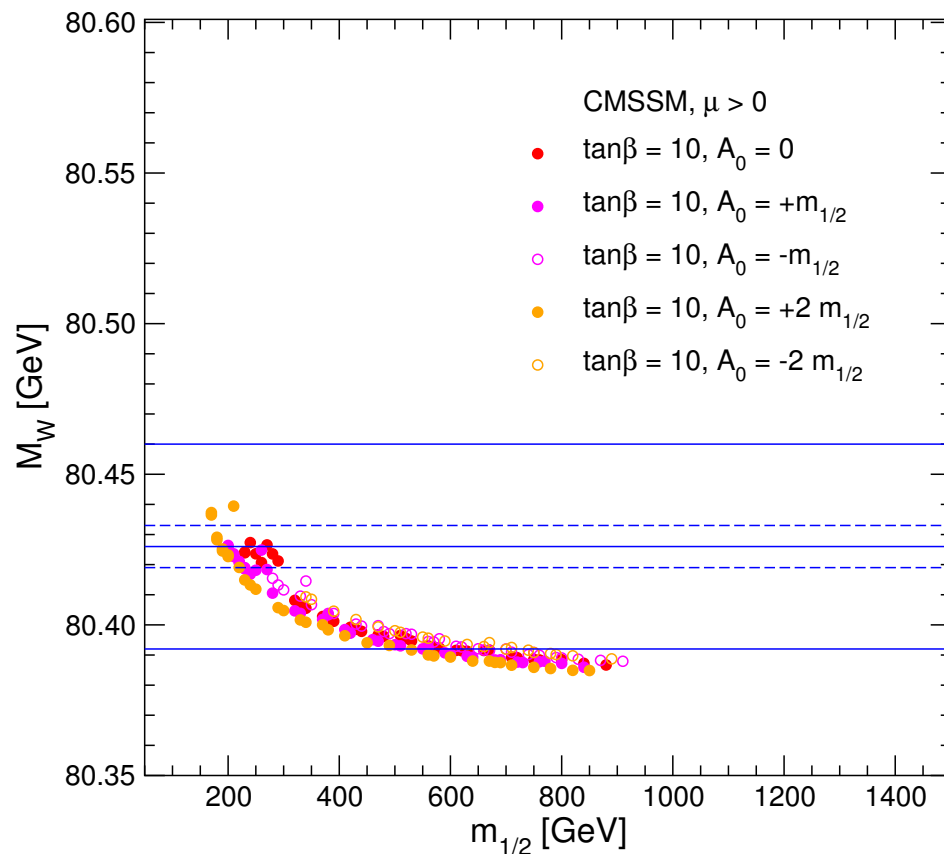
$$\Rightarrow \sin^2 \theta_{\text{eff}} = \frac{1}{4} \left(1 - \text{Re} \frac{g_V}{g_A} \right) = \left(1 - \frac{M_W^2}{M_Z^2} \right) \text{Re} \kappa_l(s = M_Z^2)$$

CMSSM prediction for M_W vs. current precision and ILC (MegaW) for $m_t = 178.0$ GeV



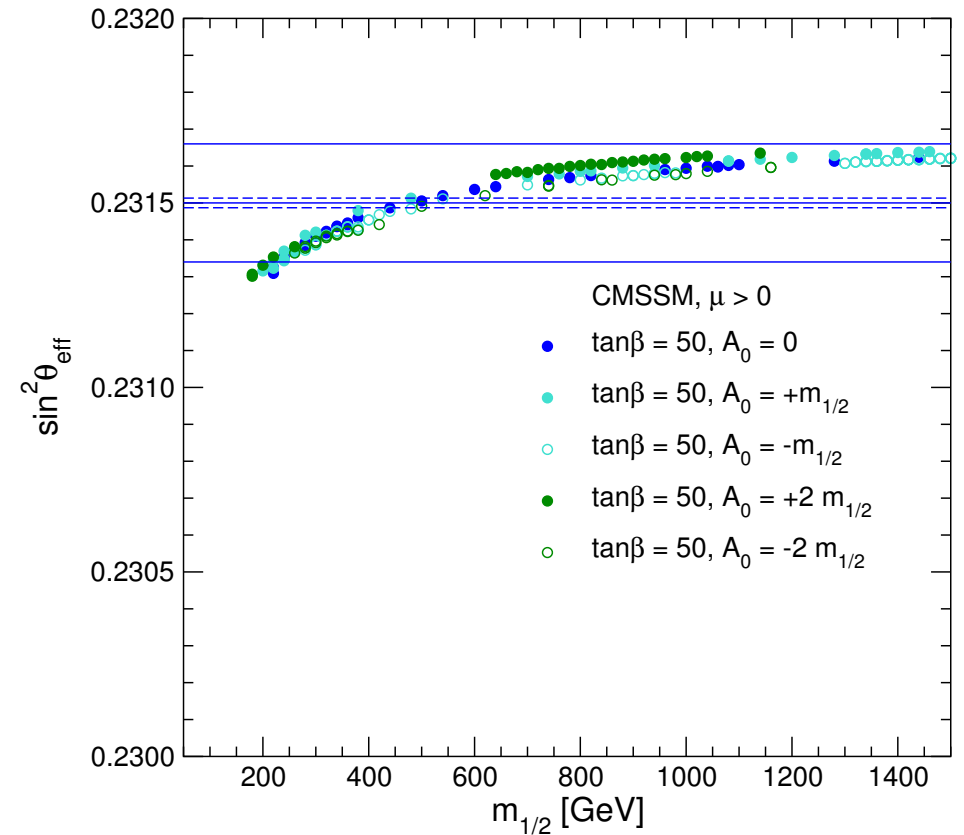
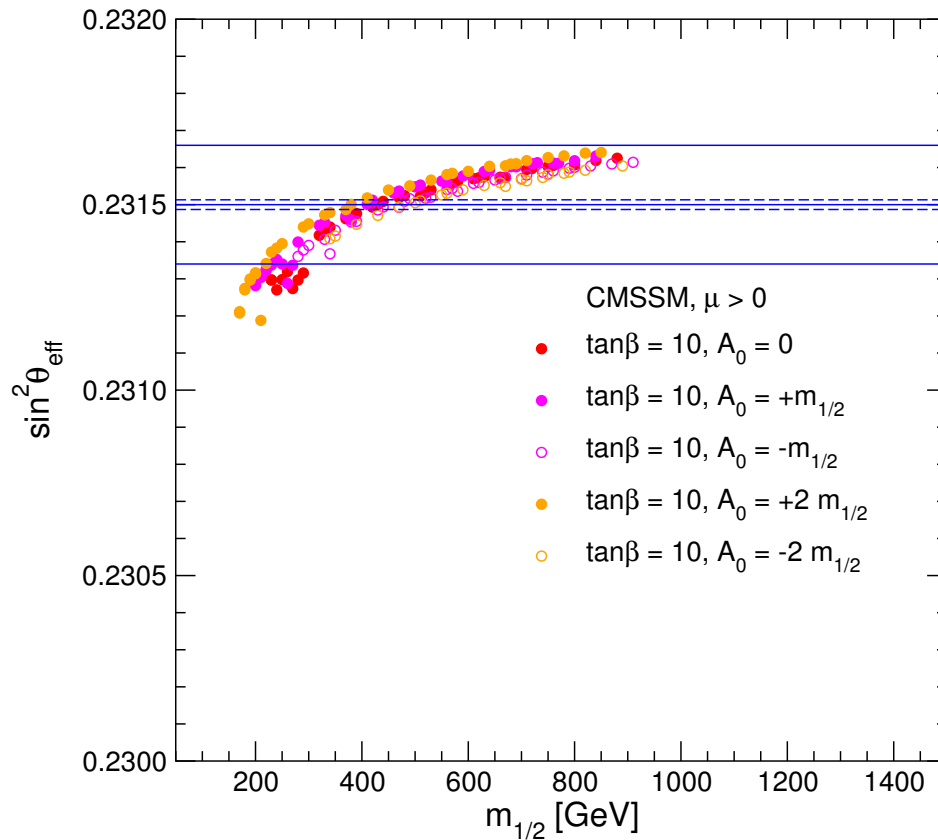
⇒ Relatively small values of $m_{1/2}$ favoured
great improvement at the ILC

Comparison: $m_t = 178.0$ GeV (left) vs. $m_t = 172.7$ GeV (right)



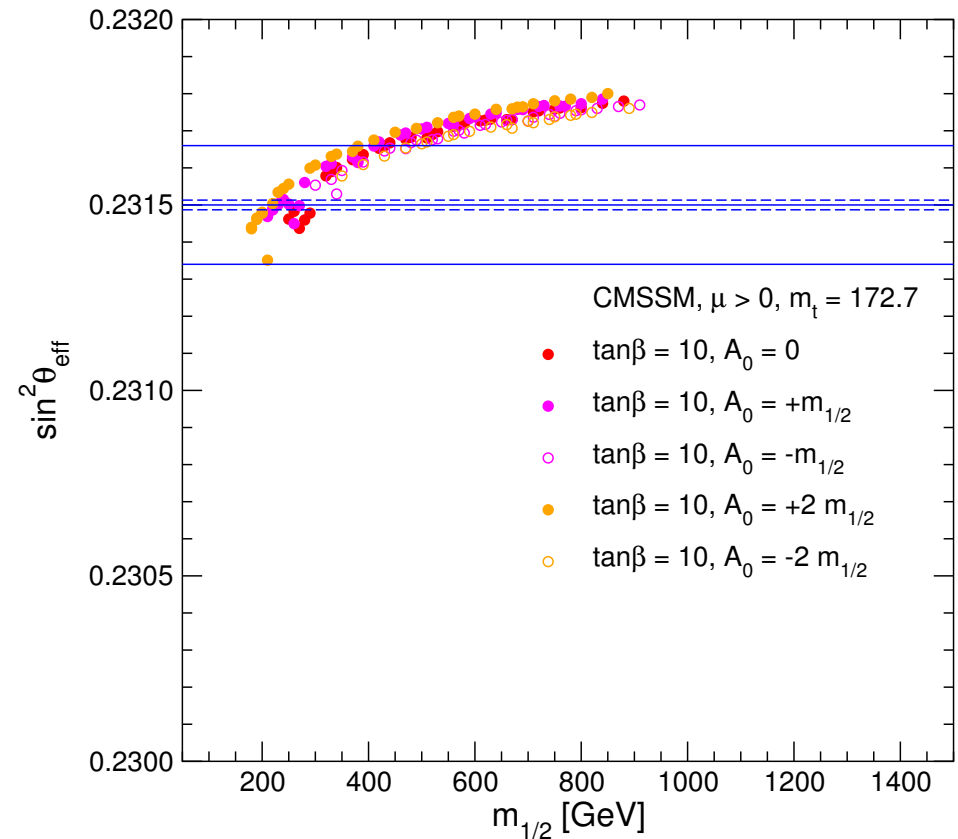
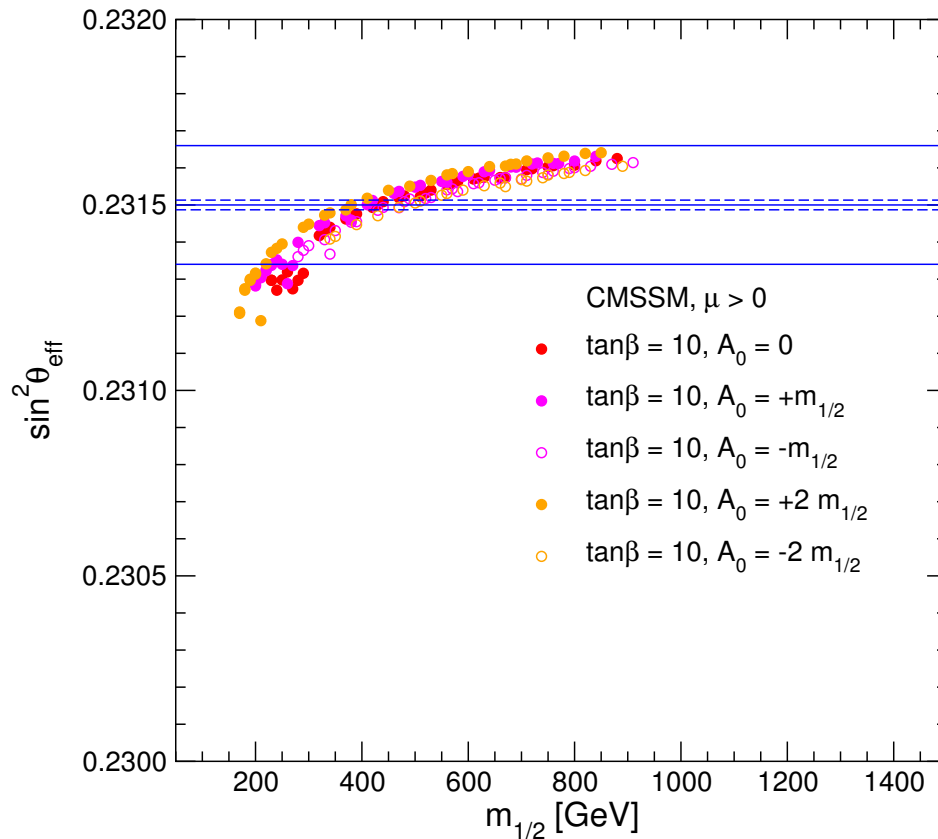
$\Rightarrow$ Lower m_t yields lower M_W
preference for small values of $m_{1/2}$ becomes more pronounced

CMSSM prediction for $\sin^2 \theta_{\text{eff}}$ vs. current precision and ILC (GigaZ)



⇒ Relatively small values of $m_{1/2}$ favoured
great improvement at the ILC

Comparison: $m_t = 178.0$ GeV (left) vs. $m_t = 172.7$ GeV (right)



$\Rightarrow$ Lower m_t yields higher $\sin^2 \theta_{\text{eff}}$
preference for small values of $m_{1/2}$ becomes more pronounced

The anomalous magnetic moment of the muon:

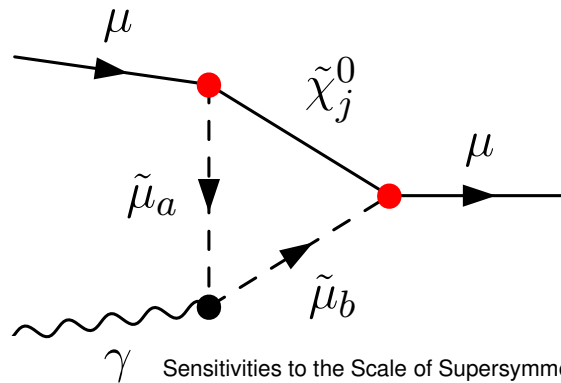
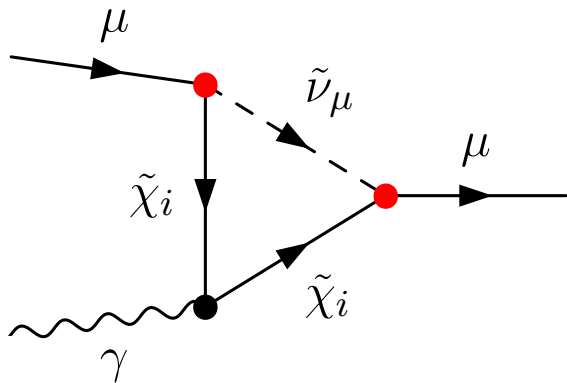
$$(g - 2)_\mu \equiv 2a_\mu$$

Experimental result for a_μ vs. SM prediction (using e^+e^- data for hadronic vacuum polarization contributions):

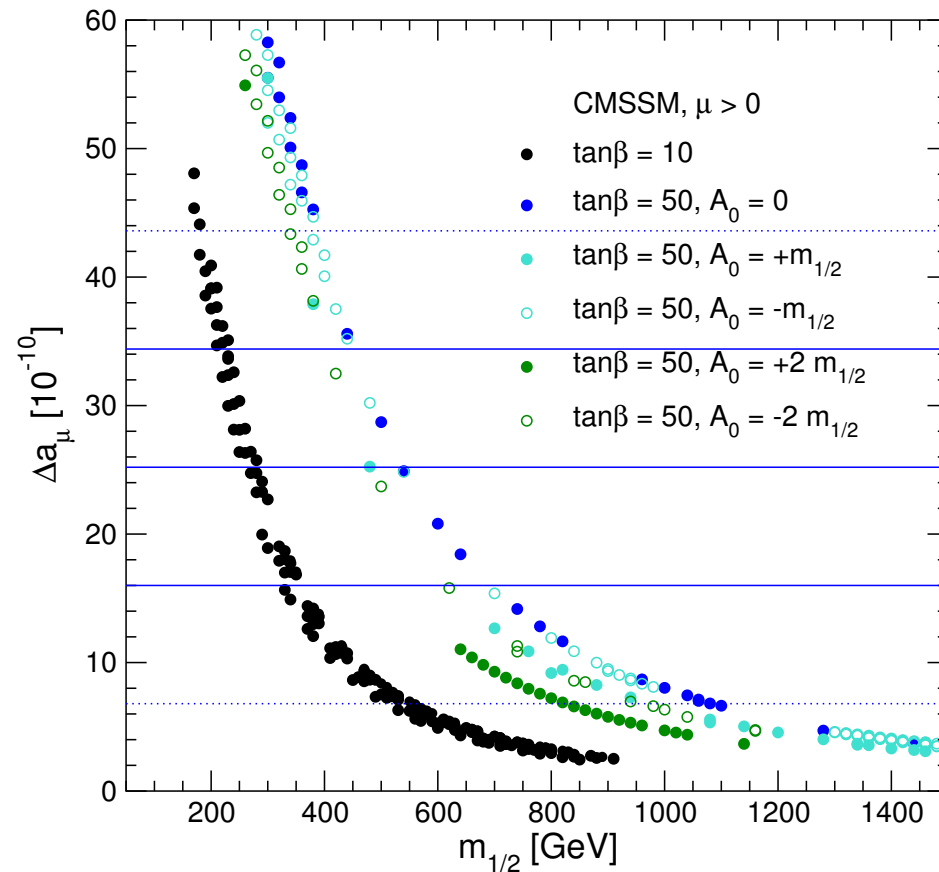
$$a_\mu^{\text{exp}} - a_\mu^{\text{theo}} = (25.2 \pm 9.2) \times 10^{-10} : 2.7 \sigma .$$

Better agreement between theory and experiment possible in models of physics beyond the SM

Example: one-loop contributions of superpartners of fermions and gauge bosons

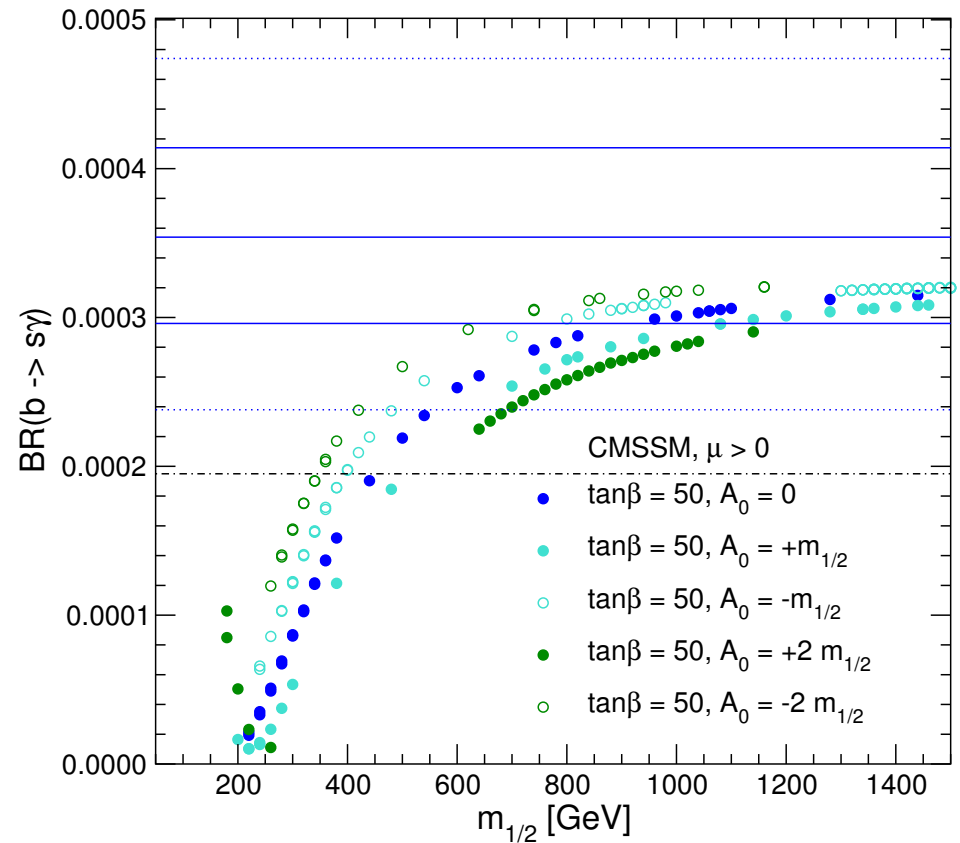
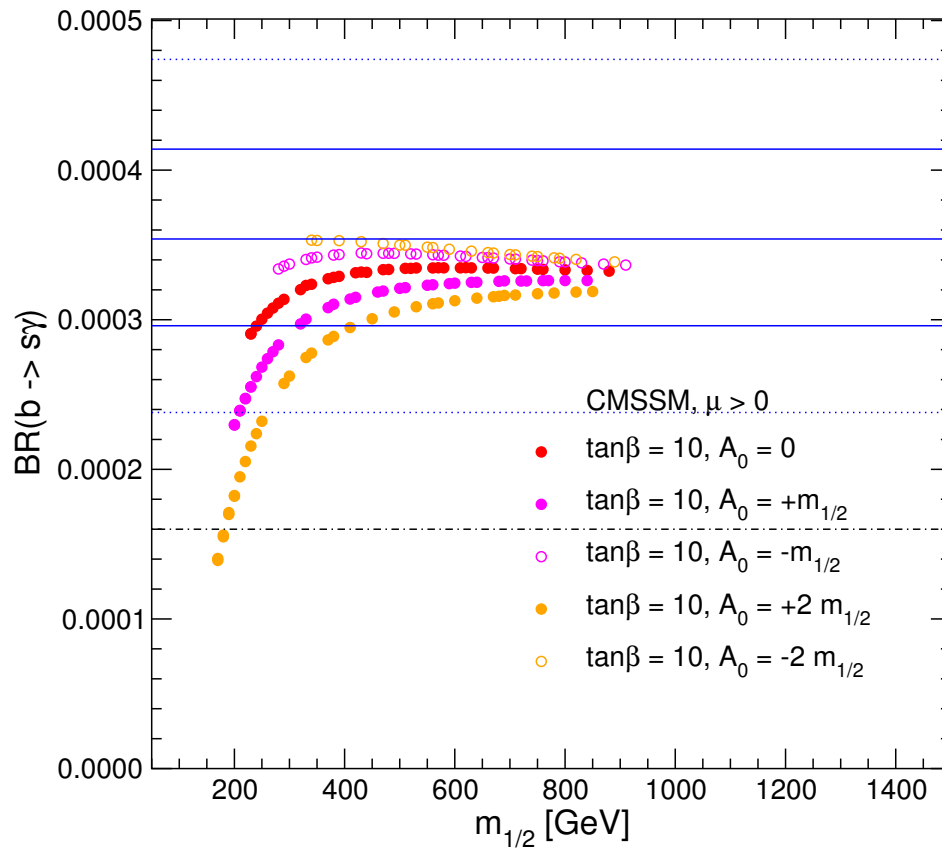


CMSSM prediction for Δa_μ vs. current precision (1σ and 2σ bands)



$\Rightarrow$ For $\tan\beta = 10$: relatively small values of $m_{1/2}$ favoured

CMSSM prediction for $\text{BR}(b \rightarrow s\gamma)$ vs. current precision (1σ and 2σ bands)



⇒ For $\tan\beta = 10$: relatively small values of $m_{1/2}$ allowed

Higgs mass prediction in the MSSM:

Prediction for $M_h, M_H, \dots$

Tree-level result for M_h, M_H :

$$m_{H,h}^2 = \frac{1}{2} \left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta} \right]$$

$\Rightarrow M_h \leq M_Z$ at tree level

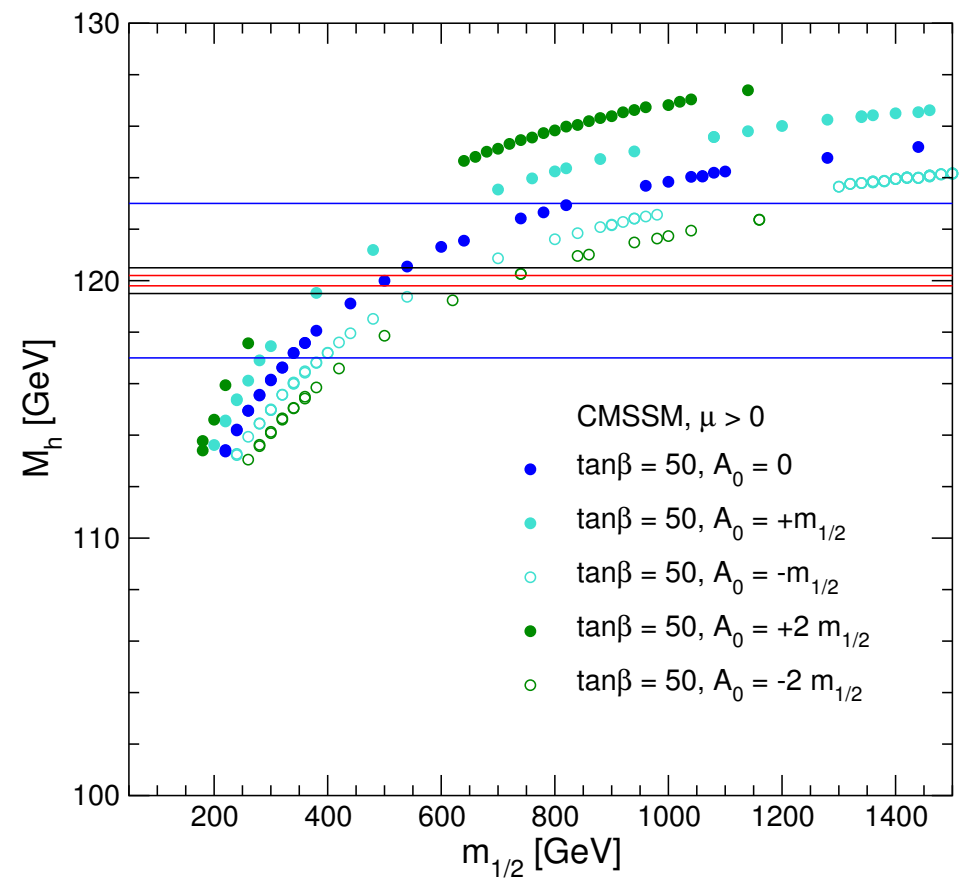
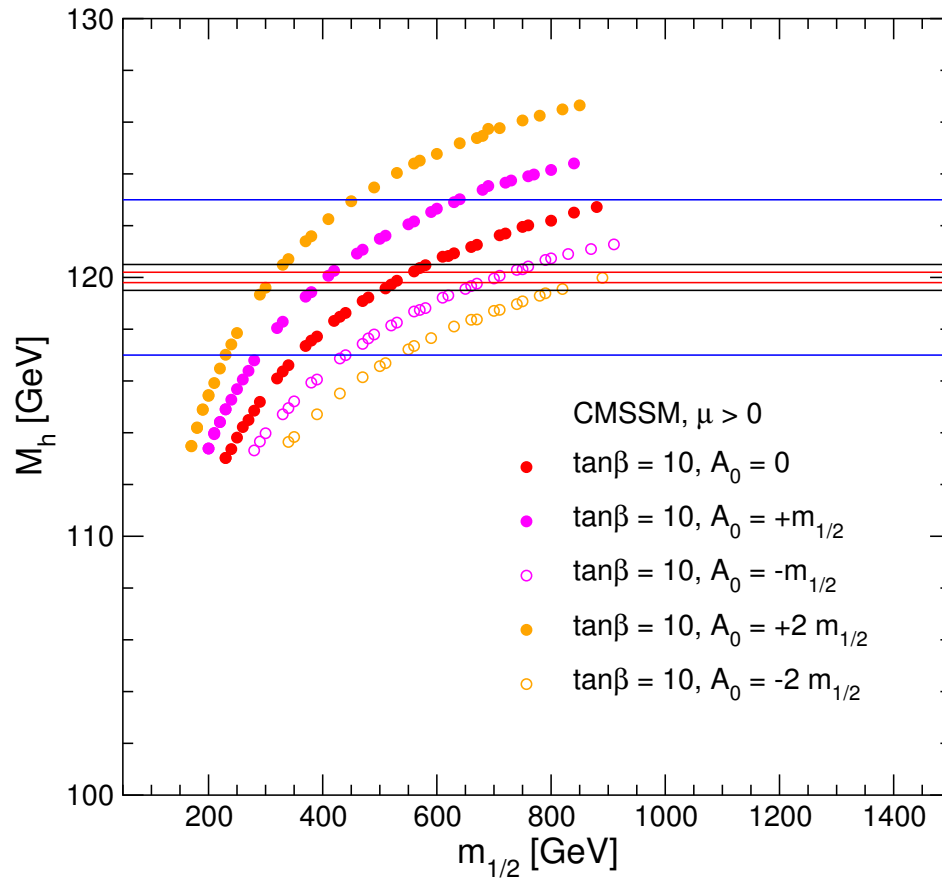
MSSM tree-level bound (gauge sector): **excluded by LEP!**

Large radiative corrections (Yukawa sector, ...):

Yukawa couplings: $\frac{e m_t}{2M_W s_W}, \frac{e m_t^2}{M_W s_W}, \dots$

$\Rightarrow$ Dominant one-loop corrections: $G_\mu m_t^4 \ln \left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right), \quad \mathcal{O}(100\%) !$

CMSSM prediction for M_h vs. assumed experimental value for current and different future theoretical uncertainties



⇒ High sensitivity to variations of $m_{1/2}$, A_0

⇒ constraints on SUSY parameter space

3. Combined sensitivity: present situation and ILC precision

Combined sensitivity investigated for
present situation + **ILC precision**

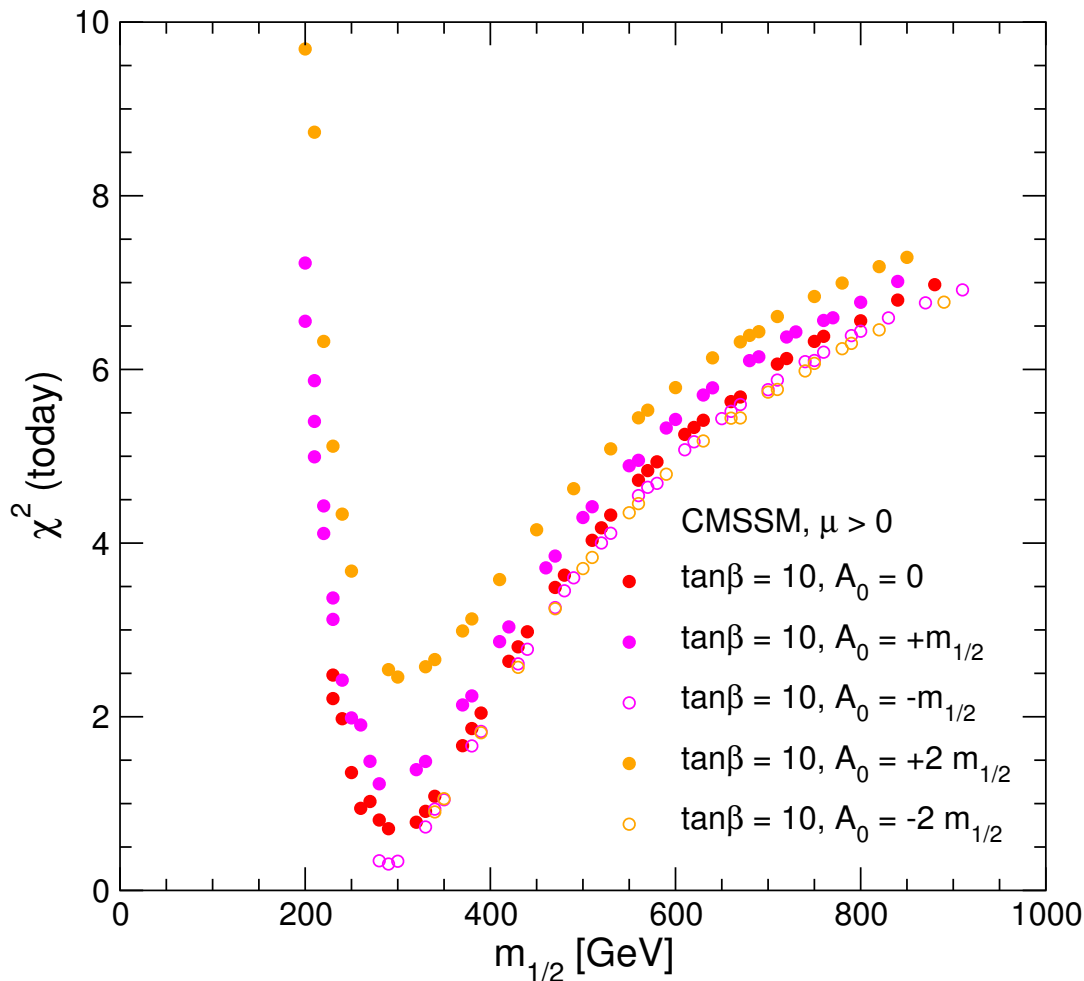
Two kinds of fits:

- χ^2 fit for fixed A_0
- χ^2 fit in $(m_{1/2}, A_0)$ plane
(scan of CMSSM parameter space)

χ^2 fit in CMSSM with dark matter constraints:

$M_W, \sin^2 \theta_{\text{eff}}, (g - 2)_\mu, \text{BR}(b \rightarrow s\gamma),$ **present situation**

$\tan \beta = 10:$



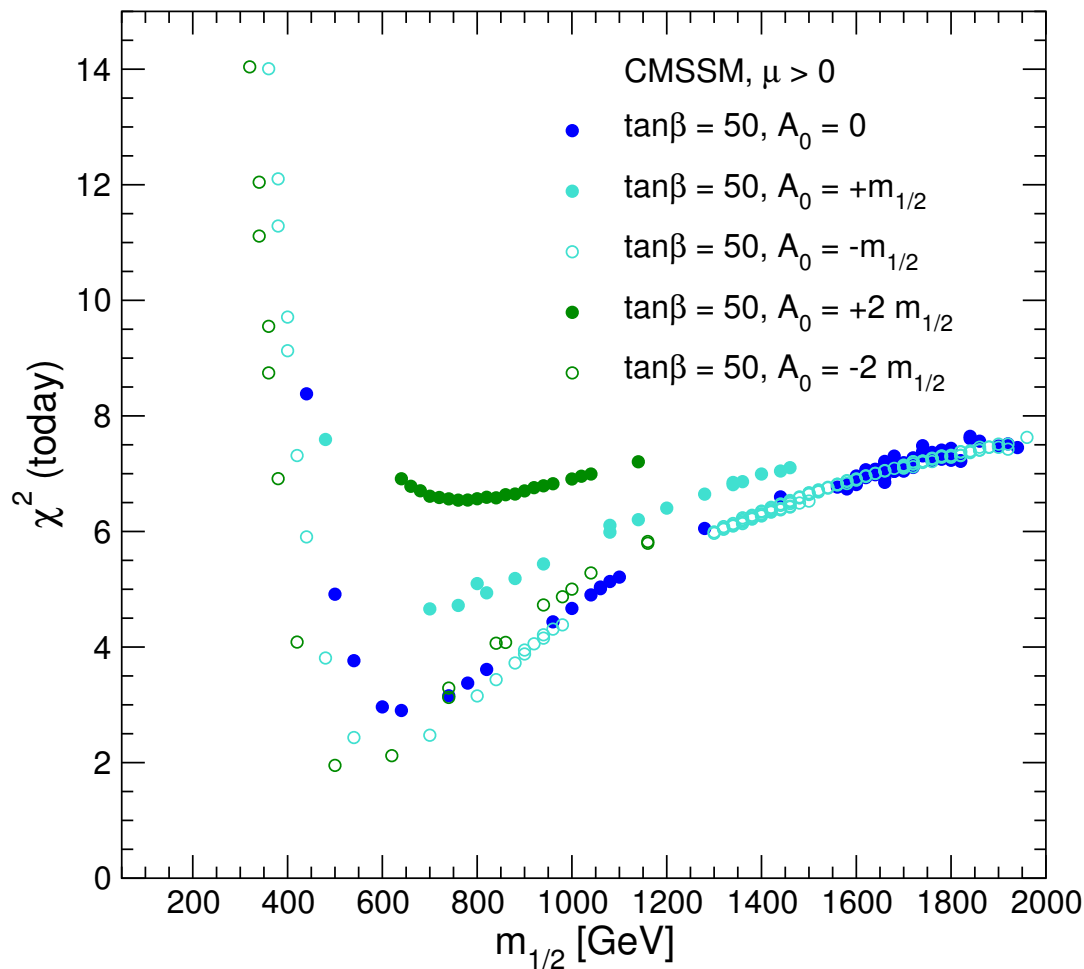
⇒ very good description
of the data

preference for relatively
small mass values

χ^2 fit in CMSSM with dark matter constraints:

$M_W, \sin^2 \theta_{\text{eff}}, (g - 2)_\mu, \text{BR}(b \rightarrow s\gamma),$ **present situation**

$\tan \beta = 50$:

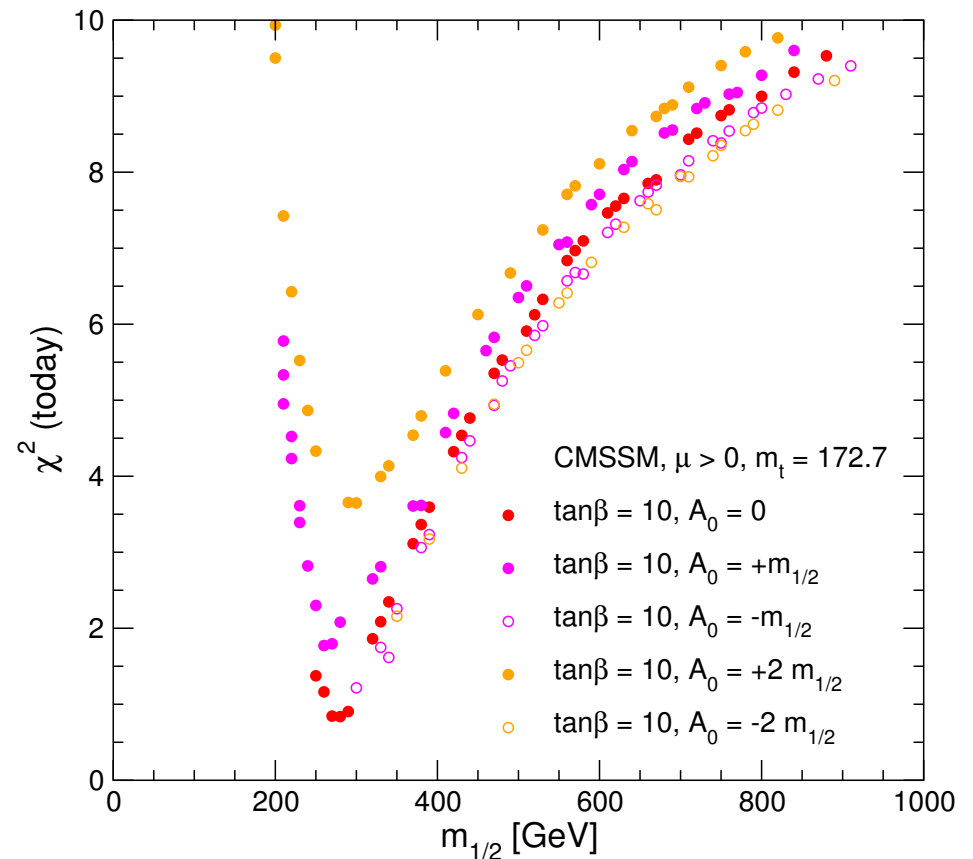
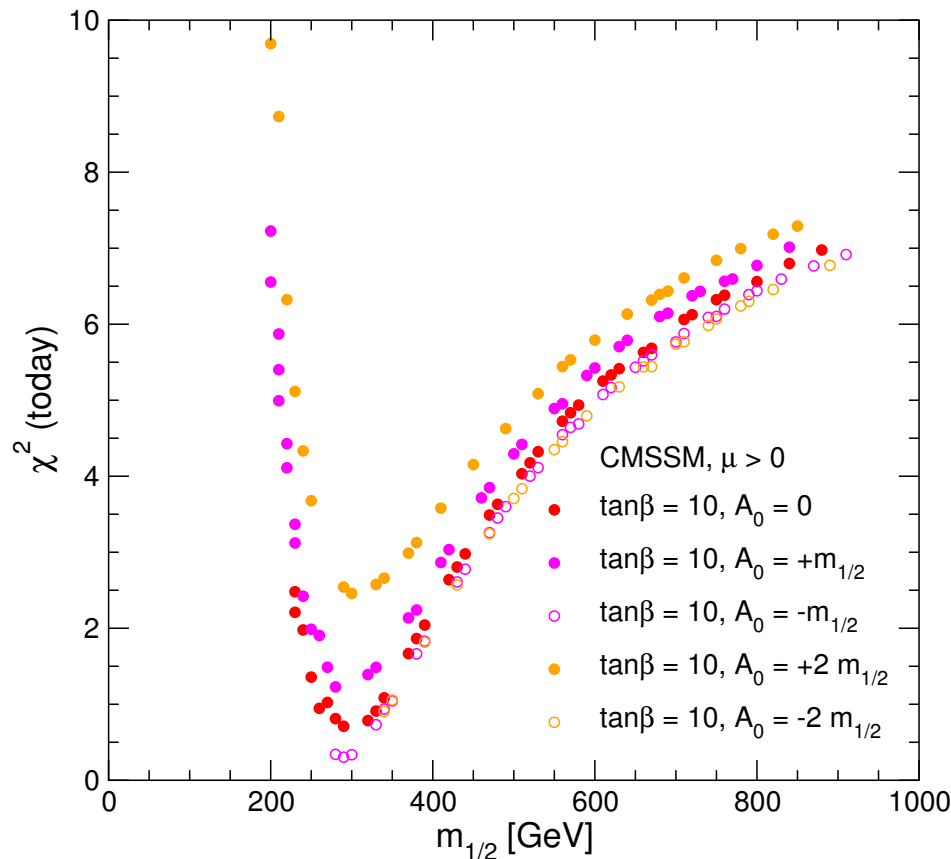


⇒ worse fit quality

preferred $m_{1/2}$ values
larger by 200–300 GeV
compared to $\tan \beta = 10$
case

Comparison: $m_t = 178.0 \pm 4.3$ GeV (*left*)

vs. $m_t = 172.7 \pm 2.9$ GeV (*right*)



$\Rightarrow$ best fit value shifted to slightly smaller $m_{1/2}$

slightly larger minimum χ^2 value

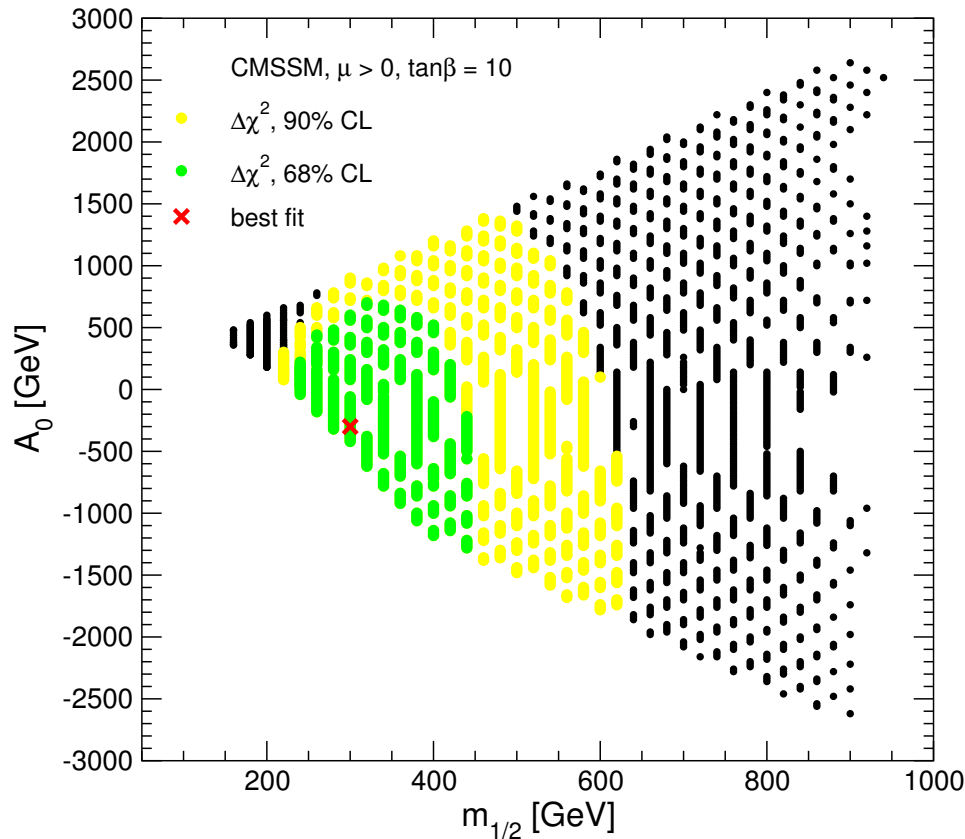
higher precision on m_t yields tighter bounds on $m_{1/2}$

χ^2 fit in CMSSM with dark matter constraints:

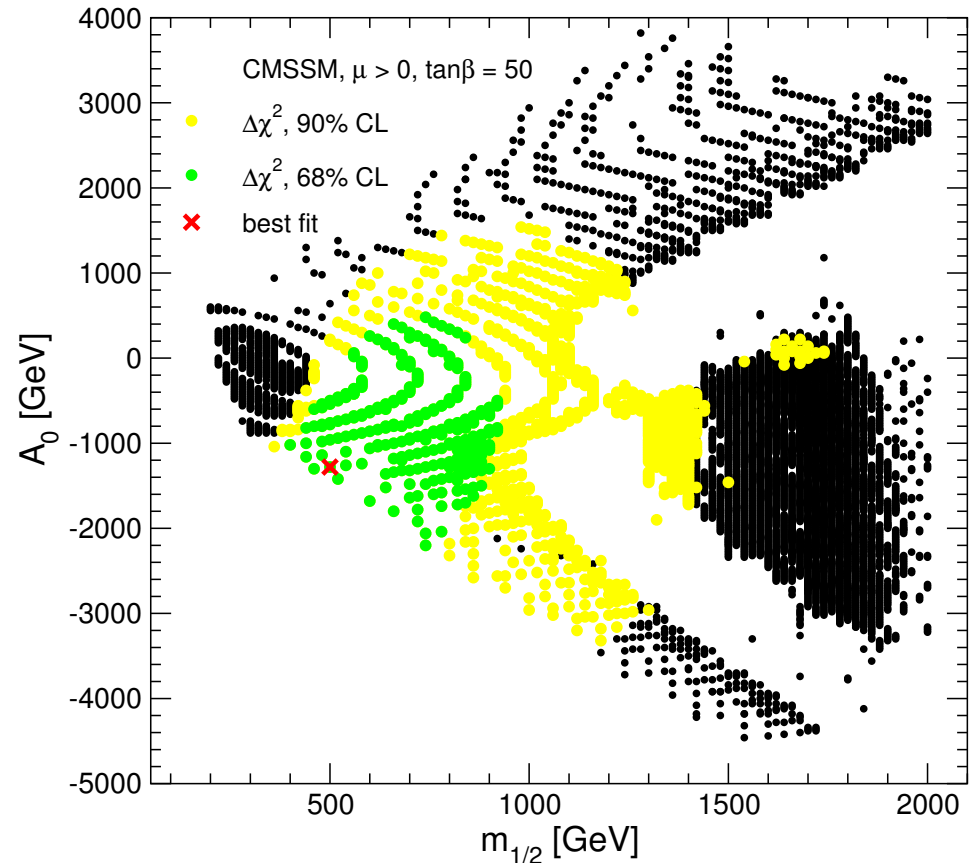
$M_W, \sin^2 \theta_{\text{eff}}, (g - 2)_\mu, \text{BR}(b \rightarrow s\gamma),$ **present situation**

68% and 90% C.L. regions in $m_{1/2}$ – A_0 plane:

$\tan \beta = 10$:

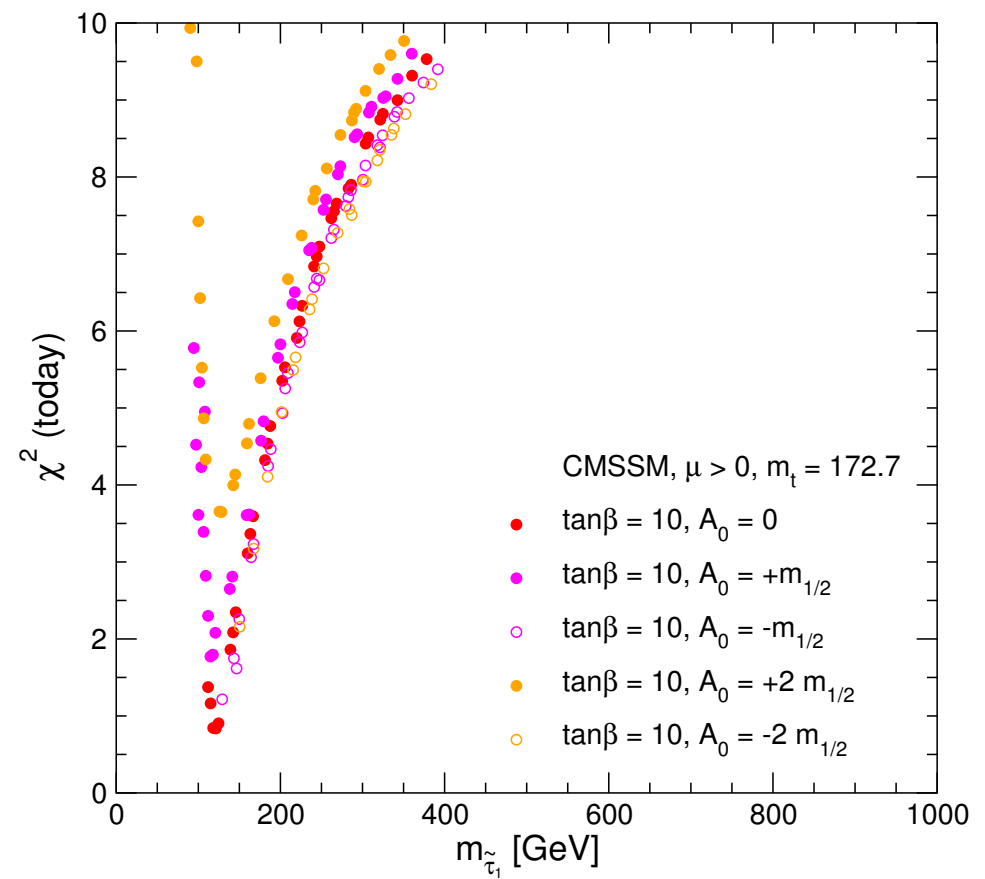
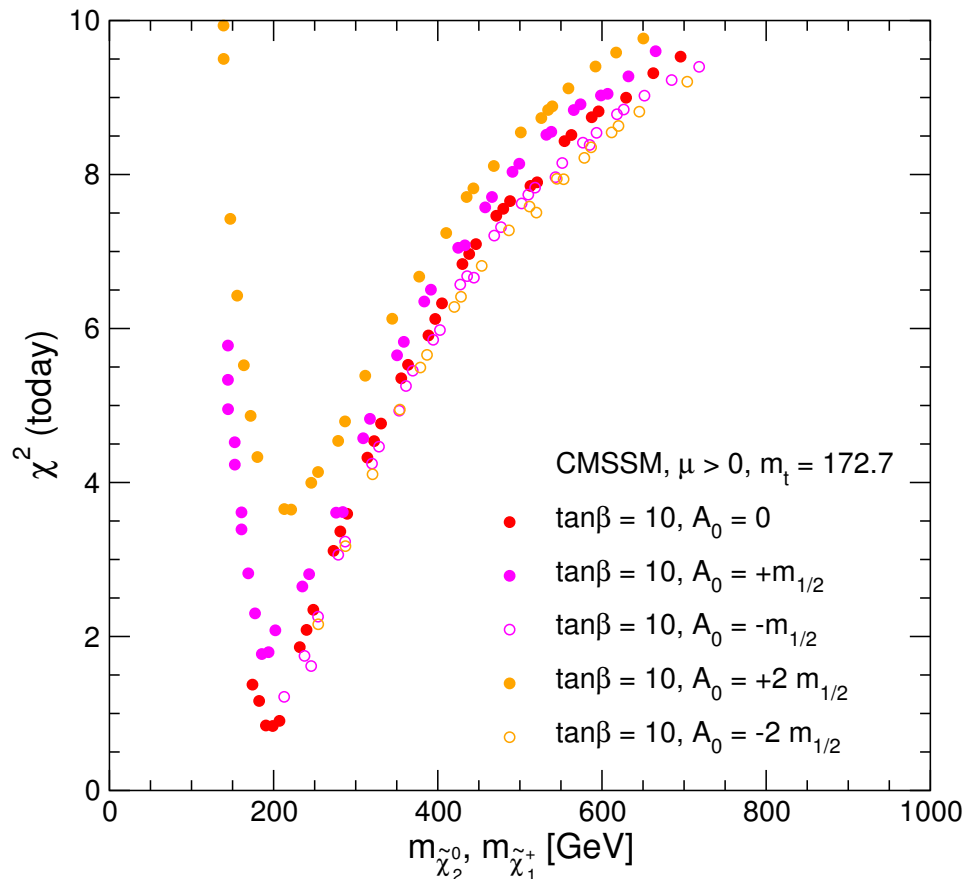


$\tan \beta = 50$:



Fit results (present) for particle masses,

$$\tan \beta = 10: m_{\tilde{\chi}_1^+} \approx m_{\tilde{\chi}_2^0}, \quad m_{\tilde{\tau}_1}, \quad m_t = 172.7 \pm 2.9 \text{ GeV}$$



⇒ Good prospects for the LHC and ILC

Comparison with other recent analysis

Fit involving all CMSSM parameters, but omitting electroweak precision observables M_W and $\sin^2 \theta_{\text{eff}}$

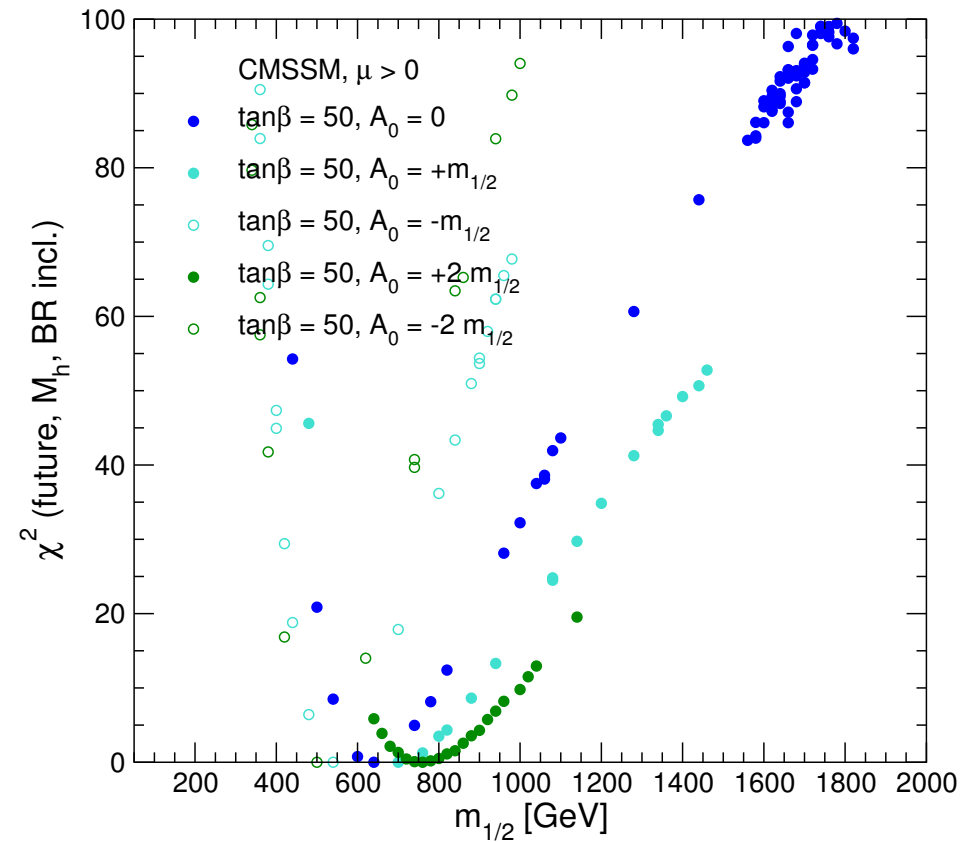
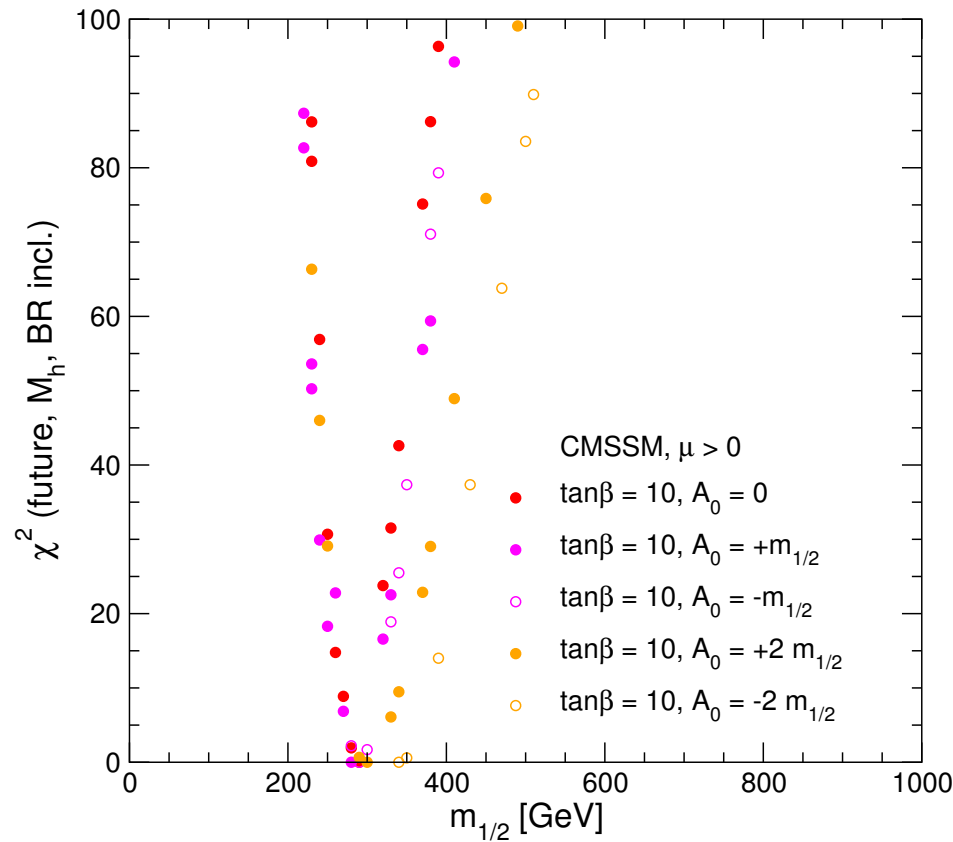
[*B. Allanach, C. Lester '05*]

⇒ preference for larger values of $\tan \beta$ and somewhat larger $m_{1/2}$

⇒ M_W and $\sin^2 \theta_{\text{eff}}$ are crucial for the fit,
give rise to preference for relatively small $m_{1/2}$

χ^2 fit in **CMSSM with CDM constraints**: $M_W, \sin^2 \theta_{\text{eff}}, (g-2)_\mu,$

$\text{BR}(b \rightarrow s\gamma), M_h, \text{BR}(h \rightarrow b\bar{b})/\text{BR}(h \rightarrow WW^*),$ **ILC precision**

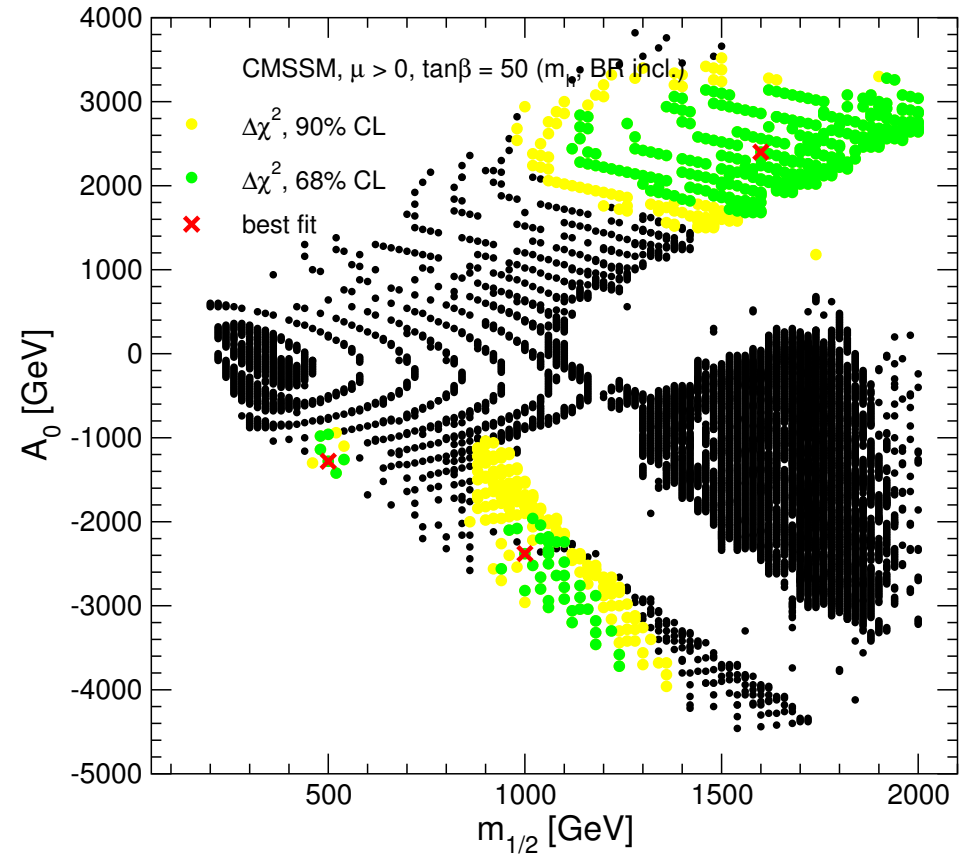
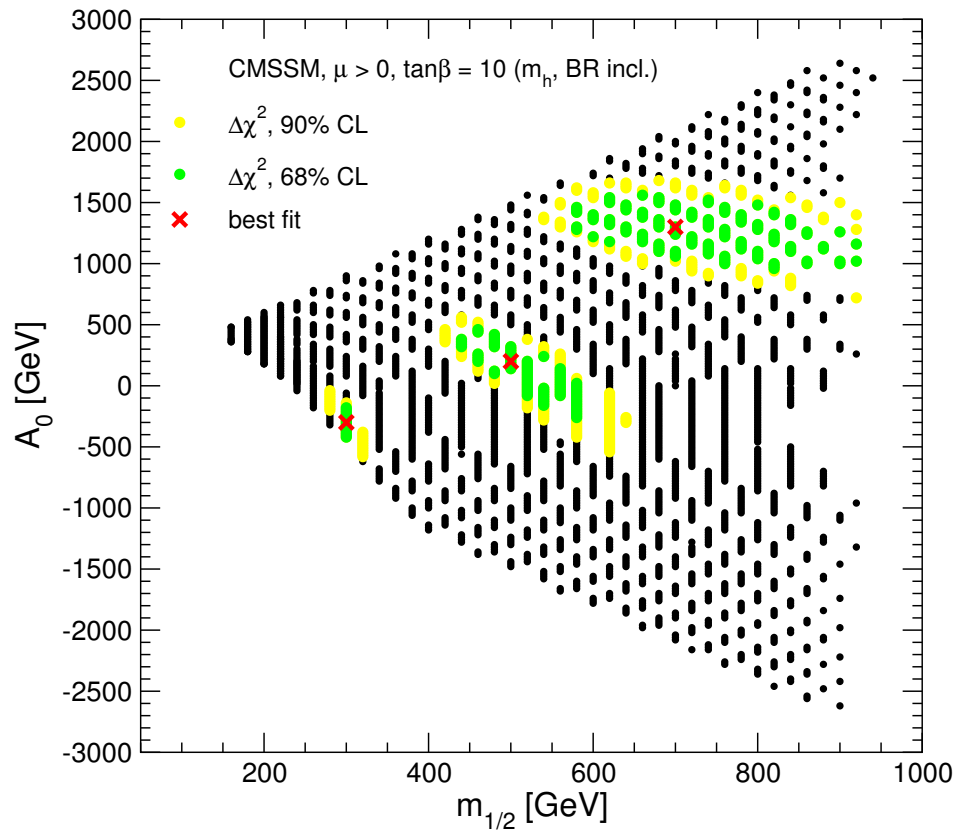


⇒ Great increase in sensitivity

⇒ tight constraints on particle masses

χ^2 fit in CMSSM with CDM constraints: $M_W, \sin^2 \theta_{\text{eff}}, (g-2)_\mu,$

$\text{BR}(b \rightarrow s\gamma), M_h, \text{BR}(h \rightarrow b\bar{b})/\text{BR}(h \rightarrow WW^*),$ **ILC precision**



$\Rightarrow$ ILC precision greatly improves sensitivity to $m_{1/2}, A_0$

high indirect sensitivity up to $m_{1/2} \lesssim 1$ TeV

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Comparison with direct experimental information $\Rightarrow$ test of CMSSM at the loop level