

Semi-numerical evaluation of one-loop corrections

G.Zanderighi

CERN, TH Division, CH-1211, Geneva 23, Switzerland

Fermilab, P.O. Box 500, 60510 Batavia, IL, US

We present a semi-numerical method to compute one-loop corrections to processes involving many particles. We treat in detail cases with up to five external legs and massless internal propagators, although the method is more general.

1. INTRODUCTION

The importance of next-to-leading order (NLO) corrections is today well-established. The benefits of NLO computations include the possibility of reliably estimating cross-section normalizations, of reducing renormalization scale dependences and of understanding the uncertainties due to the perturbative expansions. For all searches of new physics at upcoming collider experiments it is crucial to know the background in good detail, the more so, the smaller the signal to background ratio is. At the LHC and at the ILC most processes will involve many particles in the final state. It is therefore of increasing importance to predict cross sections for those processes at NLO. In QCD all $2 \rightarrow 2$ processes are today known, however a few $2 \rightarrow 3$, most $2 \rightarrow 4$ and all $2 \rightarrow N$ processes with $N > 5$ are not yet known at NLO.

A full N particle NLO calculation requires i) a tree level $(N+1)$ -particle amplitude; ii) the NLO N -particle amplitude; iii) the subtraction terms to regulate the divergences of both, i) and ii). While point i) has been extensively treated in the literature and automatized [1], and point iii) is also well-understood [2], the bottleneck in N -particle NLO calculations remains the complexity of the *analytical* evaluation of the virtual contribution. The aim of this project is therefore to seek a *semi-numerical* solution to this. Here we report on early steps along this direction. For other recent progress in developing algorithms to evaluate one-loop integrals see for instance refs. [3].

2. SEMI-NUMERICAL METHOD

We first introduce some notation. We define the D dimensional N -particle M-tensor integral as

$$I^{\mu_1 \dots \mu_M}(D; \nu_1, \dots, \nu_N) \equiv \int \frac{d^D l}{i\pi^{D/2}} \frac{l^{\mu_1} \dots l^{\mu_M}}{d_1^{\nu_1} \dots d_N^{\nu_N}}, \quad d_i \equiv (q_i + l)^2, \quad q_i \equiv \sum_{k=1}^i p_k, \quad \sigma \equiv \sum_{i=1}^N \nu_i. \quad (1)$$

Notice that we consider here only the case with massless internal propagators, although the method is more general.

2.1. The algorithm

The numerical procedure we used is simple and is based on the following few steps [4]

1. use a Feynman diagram generator (e. g. Qgraf [5]) to generate the amplitude A for a specific process at NLO;
2. use a symbolic manipulation program (e. g. Form [6]) to write the amplitude as

$$A(p_1, \dots, p_N; \epsilon_1, \dots, \epsilon_N; \dots) = \sum_n K_{\mu_1 \dots \mu_N}(p_1, \dots, p_N; \epsilon_1, \dots, \epsilon_N; \dots) \cdot I^{\mu_1 \dots \mu_N}(D; \nu_1, \dots, \nu_N), \quad (2)$$

where the kinematic tensor K depends on the particle properties (momenta p_i , polarization ϵ_i , ...) and is made up of four-dimensional objects only (i. e. dependence on the metric *must* be canceled analytically);

3. use Davydychev reduction formula [7]

$$I_{\mu_1 \dots \mu_M}(D; \{\nu_1 \dots, \nu_N\}) = \sum_{\substack{\lambda, \kappa_1, \kappa_2, \dots, \kappa_N \geq 0 \\ 2\lambda + \sum_i \kappa_i = M}} \left(-\frac{1}{2}\right)^\lambda \{[g]^\lambda [q_1]^{\kappa_1} \dots [q_N]^{\kappa_N}\}_{\mu_1 \dots \mu_M} (\nu_1)_{\kappa_1} \dots (\nu_N)_{\kappa_N} \quad (3)$$

$$\times I(D + 2(M - \lambda); \nu_1 + \kappa_1, \dots, \nu_N + \kappa_N),$$

to reduce *any* tensor integral to higher dimensional scalar integrals;

4. use the basic equation of the integration-by-parts method [8],

$$\int \frac{d^D l}{i\pi^{D/2}} \frac{\partial}{\partial l^\mu} \left(\frac{\left(\sum_{i=1}^N y_i \right) l^\mu + \left(\sum_{i=1}^N y_i q_i^\mu \right)}{d_1^{\nu_1} d_2^{\nu_2} \dots d_N^{\nu_N}} \right) = 0, \quad (4)$$

to derive a complete set of reduction relations. Here complete means that any integral is reduced to a linear combination of *analytically* known scalar integrals. A sample reduction relation one obtains is for instance

$$I(D; \{\nu_k\}_{k=1}^N) = \frac{1}{(D-1-\sigma)B} \left(I(D-2; \{\nu_k\}_{k=1}^N) - \sum_{i=1}^N b_i I(D-2; \{\nu_k - \delta_{ik}\}_{k=1}^N) \right), \quad (5)$$

where $S_{ij} \equiv (q_i - q_j)^2$, $b_i \equiv \sum_{j=1}^N S_{ij}^{-1}$ and $B \equiv \sum_{i=1}^N b_i$.

The method is semi-numerical in the sense that point 1) and 2) are done analytically, once and for all for a given process, while step 3) and 4) are repeated numerically for each phase space point. A key point which makes this method efficient is that a record is kept of all previously computed scalar integrals, so that each one is computed only once.

2.2. Treatment of exceptional phase space points

Implicit in the numerical use of recursive relations such as eq. (5) is the assumption that the kinematic matrix S_{ij} is not singular and that B does not vanish. If B or $\det(S)$ are exactly zero one obtains a simpler set of relations [9]. This is needed for instance in the treatment of cases with high N , where the particle momenta are not all linearly independent. More problematic is the treatment of so called *exceptional momentum configurations*, where an accidental degeneracy causes B or $\det(S)$ to be very small. In this case, relations such as eq. (5) are in principle still valid but become numerically unstable. A solution to this problem was first suggested in [10]. The idea is to exploit the existence of a small quantity, parameterizing the closeness to the exceptional phase space configuration, to define expanded reduction relations. For instance if $B \ll 1$ one rewrites eq. (5) as

$$I(D; \{\nu_k\}_{k=1}^N) = \sum_{i=1}^N b_i I(D; \{\nu_k - \delta_{ik}\}_{k=1}^N) + (D+1-\sigma) B I(D+2; \{\nu_k\}_{k=1}^N), \quad (6)$$

where the first term consists of simpler integrals (lower D and/or σ) and the second term consists of more difficult integrals, which are however suppressed by the small parameter B in front. In a similar way, if $\det(S)$ is small one determines the eigenvalue(s) corresponding to a small eigenvector and defines modified, expanded reduction relations.

2.3. Higgs plus four partons

As a first application of the method we considered the yet uncalculated gluon-gluon fusion amplitude $p_1 + p_2 \rightarrow p_3 + p_4 + H$ at NLO in the large m_t limit, where the Higgs couples directly to the gluons via an effective coupling [11]. The main mechanisms for Higgs production at hadron colliders are vector boson fusion (VBF) and gluon gluon fusion (GGF). The former allows a precise determination of the Higgs properties, in particular of the Higgs couplings. Due

to the gluons in the t-channel, GGF is characterized by more QCD activity in the central region, therefore VBF and GGF can be partly discriminated with suitable kinematical cuts. However, GGF remains the dominant background to VBF. A precise determination of it is therefore mandatory. We performed the numerical, partonic calculation [12]. In the four quark case we performed also an analytical calculation [12], which confirmed the validity of the numerical results. In the two quark-two gluon and in the four gluon case we verified the Ward identities and known identities between gluonic amplitudes (cyclicity, reflection and dual Ward identity and the decoupling identity for number of flavours $n_f = 0$) [13]. Since the method is based on an analytical evaluation of basis integrals no loss of accuracy is expected. Using double precision the accuracy of the results is of the order of 10^{-12} (10^{-6}) for normal (exceptional) phase space points (a even higher accuracy can be achieved at the price of including more terms in the expansions).

3. WORK IN PROGRESS

The combination of real and virtual results and phase space integration to obtain full predictions for Higgs + dijet production at the LHC is currently begin performed. We are also exploring the possibility of refining the method and of finding a more efficient numerical algorithm and phase space integration procedure.

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