Tests of Alternative Theories of Gravity

Gilles Esposito-Farèse
$\mathcal{G}\mathbb{R}\varepsilon\mathbb{C}\mathcal{O}$, Institut d'Astrophysique de Paris, UMR 7095-CNRS,
Université Pierre & Marie Curie, 98bis boulevard Arago, F-75014 Paris, France

These lecture notes complement the transparencies of the corresponding talk, available on the SLAC website at `<http://www-conf.slac.stanford.edu/ssi/2005/lec_notes/Esposito-Farese/>`. After recalling the various reasons why it is important to consider alternatives to general relativity (GR), we compare the probing power of the different classes of experimental tests. Einstein's theory is based on two independent assumptions, one of them being that matter is minimally coupled to a unique metric tensor $g_{\mu\nu}$, which implies the equivalence principle. Extra-dimensional theories generically predict deviations from this principle, and this is notably the case for superstring theory. We underline that free-fall experiments are the most constraining tests within such a string-inspired framework. The second assumption of GR, namely that gravity propagates as a spin-2 field, may be tested with different viewpoints. We first summarize the phenomenological "parametrized post-Newtonian" (PPN) formalism, then mention some tests of finite-range gravitational interactions, and finally describe a field-theoretical approach in which "scalar-tensor" theories of gravity play a privileged role. Our central conclusion is that there exists a *qualitative* difference between three classes of experimental tests: solar-system ones, binary-pulsar ones, and cosmological observations. In other words, they do not probe the same features of the theories. We finally discuss briefly the future detection of gravitational waves with laser interferometers, mention some puzzling observational issues, and comment on proposed theoretical modifications of gravity at large distances.

1. INTRODUCTION

As advertised in many publications, general relativity (GR) passes all present experimental tests with flying colors. There are however several reasons why it remains very important to consider alternative theories of gravity. The first one is that theoretical attempts at quantizing gravity or unifying it with other interactions generically predict deviations from Einstein's theory, because gravitation is no longer mediated by a pure spin-2 field but also by partners to the graviton (see Sec. 2.3 below). The second reason is that it is anyway extremely instructive to contrast GR's predictions with those of alternative models, even if there were no serious theoretical motivation for them. Indeed, such a comparison allows us to understand better which features of the theory are actually tested in a given experiment, and thereby to extract more definite information from experimental data. Moreover, this comparison can also suggest new tests able to discriminate between the various allowed models (see notably Secs. 3.1 and 3.3). The third reason is the existence of several puzzling experimental issues, which do not contradict GR in a direct way, but may nevertheless suggest that gravity does not behave at large distances exactly as Newton and Einstein predicted. Cosmological observations notably tell us that about 96% of the total energy density of the universe is composed of unknown, non-baryonic, fluids (72% of "dark energy" and 24% of "dark matter"), and the acceleration of the two Pioneer spacecrafts towards the Sun happens to be larger than what is expected from the $1/r^2$ law (see Sec. 4).

The present lecture notes will not attempt at cataloguing all alternative theories of gravity which have been proposed in the literature, not only because there are too many of them, but also because most of them are either mathematically ill-defined or physically unstable. In such a case, it is *a priori* not even worth testing them experimentally, since the behavior of their field equations suffices to rule them out. However, some of them may still be interesting from a phenomenological viewpoint, as contrasting alternatives to GR. We refer to Refs. [1–3], in which many classes of models have been carefully analyzed. We will focus below on the most natural and consistent alternatives to Einstein's theory.

General relativity is based on two independent assumptions, that textbooks sometimes present simultaneously, but which imply different kinds of physical phenomena. The simplest way to describe them is to write the action of the theory (instead of the field equations it implies):

$$S = \frac{c^2}{16\pi G} \int d^4x \sqrt{-g}\, R + S_{\substack{\text{standard} \\ \text{model}}} \left[{}^{\text{all}}\,{}^{\text{matter}}_{\text{fields}}, g_{\mu\nu}\right] . \tag{1}$$

The second term on the right-hand side (RHS) is a functional of all matter fields (including gauge bosons), assumed to be minimally coupled to one second-rank symmetric tensor, called $g_{\mu\nu}$. It defines how matter behaves in a given curved geometry, and notably how stars, planets or light rays propagate. We will analyze the observational consequences of this assumption in Sec. 2 below. On the other hand, the first term on the RHS of Eq. (1) is the famous Einstein-Hilbert action, which defines the dynamics of gravity itself, namely that of a spin-2 field. It tells us not only how gravitational waves propagate, but also the behavior of the gravitational field outside a material body generating it. Section 3 will be devoted to the tests of the dynamics of gravity.

2. MATTER-GRAVITY COUPLING

2.1. Metric Coupling

The assumption that all matter fields are universally coupled to one tensor $g_{\mu\nu}$, in action (1), is called "metric coupling". Einstein and Grossmann understood this was the appropriate mathematical way to implement the "equivalence principle" between acceleration and gravitation: An observer in a small closed and opaque room cannot determine from a *local* experiment if the room is accelerated by a rocket or static at the surface of a gravitating body. Another way to express this principle is to say that gravity is erased within a freely falling elevator (up to tidal forces, proportional to the ratio of the elevator's size and its distance to the center of gravitational attraction). The first conceptual cornerstone of GR can thus be summarized easily: Let us write the usual special relativistic laws of (non-gravitational) physics in a freely falling elevator, and a mere change of coordinates will then allow us to describe physical phenomena in an arbitrarily accelerated frame. The precise mathematical meaning of such a freely falling elevator is called a Fermi coordinate system: All along a given worldline (not necessarily a geodesic), there exist coordinates such that the metric is diagonal $g_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ and all its first derivatives vanish, $\Gamma^{\lambda}_{\mu\nu} = 0$.

2.2. Experimental Tests

This assumption of a metric coupling implies four classes of observable effects. The first two of them are obvious consequences of our writing of the special relativistic laws of physics within a freely falling elevator: (i) All constants entering the Standard Model of particle physics (gauge couplings, mass scales) are space and time independent. (ii) Local non-gravitational experiments are Lorentz invariant, i.e., do not exhibit preferred directions. We refer to Refs. [3–5] for detailed discussions of the corresponding experimental tests, but let us quote here the most precise ones: (i) The study of a natural fission reactor which took place two billion years ago in Oklo, Gabon, gives $|\dot\alpha/\alpha| < 7 \times 10^{-17}\,\text{yr}^{-1}$ for the time derivative of the fine structure constant [6], i.e., six orders of magnitude smaller than the inverse cosmological age $\sim 10^{-10}\,\text{yr}^{-1}$. (ii) The isotropy of space has notably been tested in Refs. [7] by looking for possible quadrupolar shifts of nuclear energy levels, confirming that matter is coupled to a unique metric tensor $g_{\mu\nu}$ at the 10^{-27} level.

The third observable effect is again an obvious consequence of the metric coupling, since is was actually used to *define* the equivalence principle: (iii) All (non self-gravitating) bodies fall with the same acceleration in an external gravitational field. This universality of free fall has been tested to a few parts in 10^{13} both with laboratory bodies [8, 9] and by analyzing the accelerations of the Moon and the Earth towards the Sun [10]. [Actually, the small ($\sim 10^{-9} mc^2$) gravitational binding energy of the Earth and the even smaller one of the Moon do enter this latter test, therefore it also probes the *strong* equivalence principle, as we will see in Sec. 3.1 below.]

The fourth observable consequence of a metric coupling $S_{\text{matter}}[\text{matter}, g_{\mu\nu}]$ seems less obvious: (iv) In a static Newtonian potential $g_{00} = -1 + 2U(\mathbf{x})/c^2 + \mathcal{O}(1/c^4)$, two clocks compared by means of electromagnetic signals exhibit a rate difference $\tau_1/\tau_2 = 1 + [U(\mathbf{x}_1) - U(\mathbf{x}_2)]/c^2 + \mathcal{O}(1/c^4)$. The fact that it is a consequence of the equivalence principle alone, but does not depend on the actual dynamics of gravity, can be understood easily. Let us consider a rocket accelerated upwards, with two clocks aft (i.e., at the bottom) and fore (at the top). Let also the first one send light signals towards the second at a given frequency. During the time needed for light to propagate from one clock to the other, the rocket accelerates, and therefore the second clock has a larger velocity when it receives the signal. Because of the classical Doppler effect (special relativistic corrections are not even needed for this reasoning), the second clock receives thus the signal with a lower frequency than it was emitted, exactly like a fire-truck siren sounds lower when it is going away. In conclusion, the second clock sees the first one redshifted, i.e., ticking at a lower rate. It now suffices to use the equivalence principle, which tells us that the same physical phenomenon should occur when one replaces the accelerated rocket by a gravitational field. If two clocks are located at the top and bottom of a tower, the lower one is therefore slower than the upper one. This gravitational redshift is called the "Einstein effect", and is independent of the composition of the clocks (in GR and theories satisfying the equivalence principle). It is amusing to mention one of its consequences: It is impossible to synchronize even *static* clocks in presence of gravity. The best experimental test of the Einstein effect dates back to the end of the 1970s [11]. By flying a hydrogen-maser clock on a rocket, the gravitational redshift was confirmed at the 2×10^{-4} level. The PHARAO/ACES project will probe the 5×10^{-6} level around 2008 [12].

2.3. Non-Metric Coupling

Although the four observational consequences of the equivalence principle have already been confirmed with great accuracy, it remains important to test them at an even better level. Indeed, extra-dimensional models, and notably superstring theory, generically predict *non*-metric couplings, and thereby violations of the equivalence principle. These violations are caused by the existence of scalar partners to the graviton, participating in the gravitational interaction. In superstring theory, a first scalar field, called the dilaton, is already present in 10 dimensions in the supermultiplet of the graviton (because of supersymmetry, needed to incorporate fermions in the theory). But many extra scalar fields, called *moduli*, are also predicted when performing dimensional reduction down to our usual 4-dimensional spacetime. The idea may be illustrated with the simplest case of the original Kaluza-Klein theory, i.e., general relativity written in 5 dimensions. The 5-dimensional metric g_{mn} is decomposed schematically as

$$g_{mn} = \begin{pmatrix} g_{\mu\nu} & A_\mu \\ A_\nu & \varphi \end{pmatrix}, \tag{2}$$

where $g_{\mu\nu}$, A_μ and φ behave in 4 dimensions respectively as a metric tensor, an electromagnetic potential, and a scalar field. [Actually, it is more convenient to define the 4-dimensional metric as being orthogonal to the 5th dimension, i.e., as $g_{\mu\nu} - g_{\mu 5}g_{\nu 5}/g_{55}$, to write the g_{55} component as $e^{2\varphi}$ instead of φ to get a standard kinetic term for this scalar field, and to renormalize $g_{\mu\nu}$ and A_μ by powers of e^φ to simplify the expressions and their physical interpretation.] In more than 5 dimensions, the components of the metric tensor in the extra dimensions (say g_{ab} with $a, b \geq 5$) behave as a symmetric matrix of scalar fields. They generically do not couple in the same way to different matter fields. For instance, one gets a coupling to gauge bosons of the following exponential form

$$S_{\mathrm{EM}} = \int d^4x \sqrt{-g}\, \frac{e^\varphi}{4k_0^2}\, F_{\mu\nu}F_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}, \tag{3}$$

which is called dilatonic coupling. This only contribution to the matter action actually suffices to prove that it cannot be written in the purely metric form of the last term in (1). Indeed, let us define another tensor $\tilde{g}_{\mu\nu} \equiv A^2(\varphi)g_{\mu\nu}$ ($\Leftrightarrow \tilde{g}^{\mu\nu} \equiv A^{-2}(\varphi)g^{\mu\nu}$), depending on an arbitrary function A of φ. Then action (3) can be rewritten as

$$\begin{aligned} S_{\mathrm{EM}} &= \int d^4x \left[A^{-4}(\varphi)\sqrt{-\tilde{g}}\right] \frac{e^\varphi}{4k_0^2}\, F_{\mu\nu}F_{\rho\sigma} \left[A^2(\varphi)\tilde{g}^{\mu\rho}\right] \left[A^2(\varphi)\tilde{g}^{\nu\sigma}\right] \\ &= \int d^4x \sqrt{-\tilde{g}}\, \frac{e^\varphi}{4k_0^2}\, F_{\mu\nu}F_{\rho\sigma}\tilde{g}^{\mu\rho}\tilde{g}^{\nu\sigma}. \end{aligned} \tag{4}$$

In conclusion, the explicit coupling of gauge bosons to the scalar field φ cannot be eliminated by a redefinition of the metric, whatever the function $A(\varphi)$. [This is called the conformal invariance of the electromagnetic action in 4 dimensions.] Therefore, an observer within a freely falling elevator can have experimental access to the external value of φ from a *local* experiment involving photons, without looking out of a window. This already proves that gauge bosons do not behave like in special relativity within this freely falling elevator, in contradiction with the arguments of Sec. 2.1 defining the equivalence principle. Actually, since material bodies do contain a significant amount $\sim 1\%$ of electromagnetic binding (negative) energy, the dilatonic coupling (3) suffices to make different materials fall with different accelerations in a given gravitational field. Another way to understand this violation of the universality of free fall is to note that Eq. (3) is the standard electromagnetic action where the coupling constant k_0 has been replaced by an effective spacetime-dependent one $k_{\text{eff}} = k_0\, e^{-\varphi(\mathbf{x})/2}$. Because of the electromagnetic binding energy of material bodies, their masses $m(\varphi)$ will therefore also be spacetime dependent, and their accelerations will get an extra contribution $\delta\mathbf{a} = -\boldsymbol{\nabla} \ln m(\mathbf{x})$ which depends on their composition.

This spacetime dependence of masses and coupling constants is actually predicted in superstring theory even at the level of elementary particles. This theory indeed goes one step beyond GR. In Newtonian mechanics, both geometry and coupling constants were assumed to be rigid. General relativity promoted geometry to a dynamical field, but still assumed coupling constants to be fixed. In string theory, everything becomes dynamical, and notably masses and coupling constants, determined by the vacuum expectation value of some fields. However, tree-level predictions of strings happen to be experimentally ruled out by many orders of magnitude. Indeed, they generically predict differences in the accelerations of different materials of the order of $\Delta a/a \sim 10^{-5}$, whereas we saw in Sec. 2.2 that it is experimentally constrained at the 5×10^{-13} level. Similarly, tree-level strings predict time variations of coupling constants of the order of the inverse age of the Universe, say $\dot\alpha/\alpha \sim H_0 \sim 10^{-10}\,\text{yr}^{-1}$, i.e., six orders of magnitude larger than present experimental bounds. In Section 3.1 below, we will also see that the post-Newtonian parameters $\beta^{\text{PPN}} - 1$ and $\gamma^{\text{PPN}} - 1$ are observationally constrained respectively at the 10^{-4} and 10^{-5} level, whereas tree-level strings would predict ~ 40 for the first and ~ 1 for the second. In conclusion, superstring theory seems to be ruled out.

Fortunately, several mechanisms can reconcile this theory with experiment. A first argument is to claim that most scalar fields should acquire a mass (for instance because of supersymmetry breaking), so that their effect would be negligible at large enough distances. However, no natural mechanism is known to generate masses for *all* scalar fields in the theory, and a single massless one suffices to violate equivalence principle tests by several orders of magnitude. Moreover, difficult cosmological problems arise when considering such massive scalars, notably the fact that too much energy is stored in the cosmological oscillations of $\varphi(t)$. Another solution is to consider quantum loop corrections to the tree-level string action, as proposed in [13–15]. Then the exponential coupling e^{φ} in matter actions like (3) are replaced by more complicated functions $e^{a(\varphi)}$, where $a(\varphi)$ may depend on the matter species. If all these functions $a(\varphi)$ happen to have a minimum at the same value of φ (for instance as $\varphi \to \infty$ in the scenario of [15]), then one can prove that the cosmological evolution of the Universe drives φ very efficiently towards this minimum. Therefore, one explains naturally why matter is presently almost decoupled from any scalar field $[\partial_\varphi m(\varphi_{\text{min}}) = 0]$, and one can estimate that present deviations from GR should be of the order $[\partial_\varphi \ln m(\varphi_{\text{now}})]^2 \sim 10^{-19} \to 10^{-10}$, typically $\sim 10^{-14}$ in the scenario of [13] and $\sim 10^{-12}$ in that of [15].

One may then reanalyze the various experimental tests of the equivalence principle within such a string-inspired framework, and compare their probing power (see the table in the tenth transparency of my talk). For instance, it happens that the constraint imposed by the Oklo reactor on the time variation of the fine-structure constant, Sec. 2.2, translate as $[\partial_\varphi \ln m(\varphi_{\text{now}})]^2 < 10^{-3.5}$. In other words, although this experimental bound is six orders of magnitude tighter than what would be generically expected, it does not constrain at all the above string-inspired scenarii which predict much smaller values. On the other hand, free-fall experiments prove to be the most sensitive tests of such stringy deviations from the equivalence principle. While the (yet unfunded) NASA-ESA Satellite Test of the Equivalence Principle (STEP) [16] would probe values as small as $[\partial_\varphi \ln m(\varphi_{\text{now}})]^2 \sim 10^{-14}$, the approved CNES Microscope experiment [17] will reach deviations of order 10^{-11}, close to expected effects in the scenario of Ref. [15].

3. DYNAMICS OF GRAVITY

Since the equivalence principle is experimentally tested with great accuracy, let us now assume that it is exactly satisfied in order to focus on the second building block of a relativistic theory of gravity, namely the dynamics of gravity itself. We will thus consider an action of the form

$$S = S_{\text{gravity}} + S_{\text{matter}}\left[\text{matter}, g_{\mu\nu}\right], \tag{5}$$

in which all matter fields are assumed to be universally coupled to a single second-rank symmetric tensor $g_{\mu\nu}$, but where the kinetic term S_{gravity} of this tensor is not yet specified.

3.1. Parametrized Post-Newtonian formalism

A first very useful framework to analyze the experimental constraints on S_{gravity} is called the parametrized post-Newtonian (PPN) formalism. It actually does not assume anything about the dynamics of gravity, but the fact that it is described by a metric tensor $g_{\mu\nu}$ and that it does not involve any characteristic length scale. The idea is to write the most general form that $g_{\mu\nu}$ can take in presence of matter (described by perfect fluids), at the first post-Newtonian order (i.e., when considering corrections of order $1/c^2$ with respect to the Newtonian interaction). The first version dates back to 1923 [18], when Eddington wrote the usual Schwarzschild metric in isotropic coordinates

$$\begin{aligned} -g_{00} &= 1 - 2\frac{Gm}{rc^2} + 2\beta^{\text{PPN}}\left(\frac{Gm}{rc^2}\right)^2 + \mathcal{O}\left(\frac{1}{c^6}\right), \\ g_{ij} &= \delta_{ij}\left(1 + 2\gamma^{\text{PPN}}\frac{Gm}{rc^2}\right) + \mathcal{O}\left(\frac{1}{c^4}\right), \end{aligned} \tag{6}$$

but introduced inside some phenomenological parameters β^{PPN} and γ^{PPN} instead of the factors $\beta^{\text{PPN}} = \gamma^{\text{PPN}} = 1$ predicted by Einstein's theory. [The extra factor that one may introduce in front of the $2Gm/rc^2$ term in g_{00} can always be absorbed in the definition of the gravitational mass Gm.] General relativity is thus embedded in a two-dimensional space of theories, and one may now plot the various experimental constraints in the plane of the $(\beta^{\text{PPN}}, \gamma^{\text{PPN}})$ parameters, as shown in Fig. 1 and in the 12th transparency of my talk. Three different types of constraints can be distinguished. The measurement of Mercury perihelion shift [19] gives the bound

$$|2\,\gamma^{\text{PPN}} - \beta^{\text{PPN}} - 1| < 3 \times 10^{-3}. \tag{7}$$

Lunar laser ranging (LLR) consists in measuring the round-trip time of light travelling between Earth and corner reflectors placed on the Moon. It presently gives us the Earth-Moon distance with few millimeter accuracy. One can thus test very precisely if the Moon falls towards the Sun with the same acceleration as the Earth, and thereby if the equivalence principle is satisfied even for gravitational binding energy itself (this is called the *strong* equivalence principle, and the effect of its violation on the Moon orbit is called the Nordtvedt effect). The latest experimental data [10] give the following bounds on the Eddington PPN parameters:

$$4\,\beta^{\text{PPN}} - \gamma^{\text{PPN}} - 3 = (4.4 \pm 4.5) \times 10^{-4}. \tag{8}$$

The third class of post-Newtonian tests in the solar system involves light rays and the various deflection and retardation effects predicted by relativistic theories of gravity. All of them depend only on the γ^{PPN} parameter. Very long baseline interferometry [20] (VLBI) allows the analysis of light deflection all over the celestial sphere, and yields the bound $|\gamma^{\text{PPN}} - 1| < 4 \times 10^{-4}$, but the tightest experimental limits come from the measurement of the time delay variation to the Cassini spacecraft near solar conjunction [21]:

$$\gamma^{\text{PPN}} - 1 = (2.1 \pm 2.3) \times 10^{-5}. \tag{9}$$

(The 10^{-9} level should be reachable with the planned LATOR mission [22].) Equations (7), (8) and (9) define three thin strips with different slopes in the $(\beta^{\text{PPN}}, \gamma^{\text{PPN}})$ plane, and their intersection defines a tiny region around

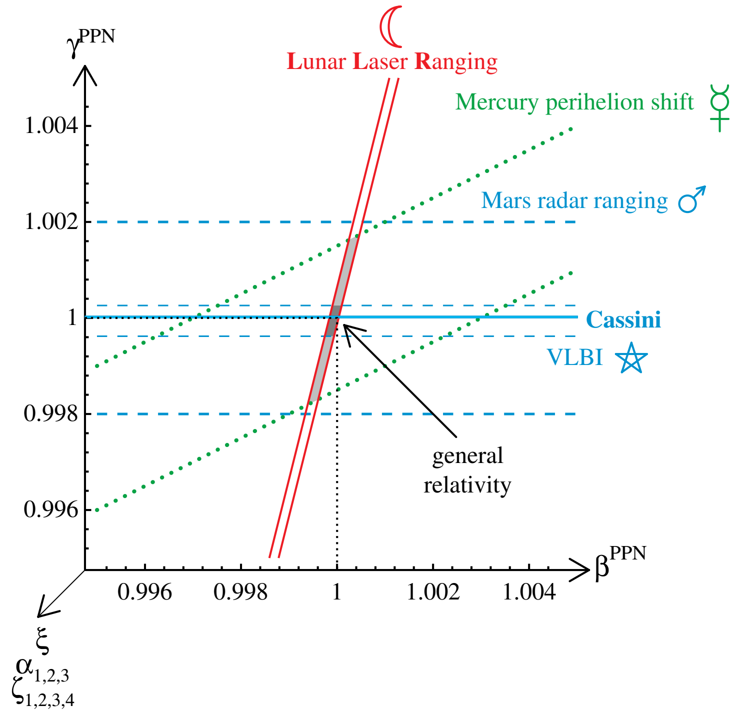

Figure 1: Solar-system constraints on the PPN parameters.

$\beta^{\rm PPN} = \gamma^{\rm PPN} = 1$. Instead of just confirming GR, we have thus reached now a much stronger conclusion: General relativity is basically the *only* theory consistent with solar-system experiments at the first post-Newtonian order.

This PPN formalism has been generalized by Schiff, Baierlin, and above all Nordtvedt and Will [1], who defined the most elaborate version. It introduces 10 phenomenological parameters (including $\beta^{\rm PPN}$ and $\gamma^{\rm PPN}$) to describe any possible deviation from GR at the first post-Newtonian order. All of them are presently constrained to be very close to their general relativistic values, at a few percent level for the least easily tested (denoted ζ_1, and parametrizing a particular violation of the conservation of total momentum), to the 10^{-20} level for the best constrained one (denoted α_3, and parametrizing a particular preferred-frame effect combined with a violation of the total momentum conservation). Their present experimental bounds are summarized in Table 4 of Ref. [3].

Several constraints on these PPN parameters come from the analysis of binary orbits, notably the Moon's orbit giving the above bounds (8). It is instructive to summarize the basic idea of such orbital tests. One needs to observe the time evolution of the eccentricity vector, which points from the center of mass of the binary system towards the periastron of the orbit, and whose norm is the eccentricity. If the two bodies do not have the same acceleration (towards a third body, say the Galaxy center, or because of some preferred-frame effect), then one can show that this eccentricity vector is the sum of a constant vector, proportional to the acceleration difference, and a rotating one, corresponding to the usual periastron advance predicted by GR. Therefore, its mean is given by the constant contribution, so that the orbit is polarized towards a particular direction, in a similar way as the Stark effect in electromagnetism. Very tight constraints on some PPN parameters come from the fact that we know several binary pulsars (composed of a neutron star and a white dwarf) whose eccentricity is vanishingly small. If there existed some significant deviations from GR, the only explanation would be that the unknown rotating contribution to the eccentricity vector precisely cancels the constant one at the time of our observation. But this is very improbable, and the fact that several systems have simultaneously a vanishing eccentricity is even more improbable. A statistical analysis then yields tight constraints on possible deviations from GR.

3.2. Finite-Range Effects

Since the PPN formalism assumes that there is no characteristic length scale entering the gravitational interaction, it does not allow us to test finite-range effects, caused for instance by massive partners to the graviton. As mentioned in Sec. 2.3 above, extra-dimensional theories do predict the existence of massive scalar fields participating in the gravitational interaction. However, their masses are expected to be very large in the standard Klein compactification mechanism, so that no deviation from Newton's $1/r^2$ law would be observable with present technology. On the contrary, recent brane models can predict *a priori* observable effects at macroscopic scales without being inconsistent with particle physics. The basic idea is that matter is described by the ends of open strings, constrained to live on a 4-dimensional surface embedded in the extra-dimensional one of string theory (called the bulk). Therefore, matter only feels 4 dimensions, and no deviation from the Standard Model of particle physics should occur. On the contrary, gravity is described by *closed* strings, so that it can propagate in the bulk. Therefore, the gravitational interaction may allow us to "feel" extra dimensions. A length scale is introduced in such brane models, either from the distance between two branes (describing two parallel universes), or because of a cosmological constant yielding a de Sitter metric for the bulk. Since this scale is only constrained by gravitational experiments, these models led to a renewal of "fifth-force" experimental searches. Assuming that the gravitational potential takes the generic form

$$V = \frac{Gm}{r}\left(1 + \alpha\, e^{-r/\lambda}\right), \tag{10}$$

where α and λ denote respectively the strength and range of an additional Yukawa potential (due to a massive partner to the graviton), one can then plot the present experimental limits in the plane of these two parameters. The latest results are displayed in Ref. [23]. The conclusion is that finite-range modifications of gravity at larger scales than a millimeter are known to be smaller than a percent of the Newtonian interaction. On the other hand, $\mathcal{O}(1)$ deviations from Newton's law are still experimentally allowed for scales $\lambda < 10^{-4}$ m, and they could even be 10^{10} larger than Newton's force for $\lambda \lesssim 10^{-6}$ m.

Several analyses have also been published to constrain the graviton mass itself, i.e., while assuming that the Newtonian potential takes the form

$$V_N = \frac{Gm}{r} e^{-r/\lambda_g}, \tag{11}$$

without any infinite range contribution contrary to Eq. (10) above. However, serious theoretical difficulties occur when defining the notion of massive graviton. Around a flat background, Pauli and Fierz indeed showed that its mass term must take a very specific form, otherwise negative-kinetic energy degrees of freedom ("ghosts") are excited and the vacuum becomes violently unstable. However, it was later realized that the Pauli-Fierz mass term is inconsistent with solar-system tests even if the mass is vanishingly small [24]: There is a discontinuity between the case of a strictly massless graviton (general relativity) and a extremely light one. In other words, the Mercury perihelion shift suffices to rule out the existence of a massive graviton. Recent theoretical works have underlined that this van Dam-Veltman-Zakharov discontinuity can been avoided when considering other types of mass terms, but they sometimes did not realize that the presence of ghosts was deadly for the models. On the other hand, some brane models have been shown to describe massive gravity without exhibiting the above discontinuity (see e.g. [25] and references therein), so that they can pass all solar-system tests if the graviton mass scale is small enough. However, in some particular background geometries, these models do contain ghost excitations, and their stability around other backgrounds is not fully proven yet.

Since the notion of massive graviton is far for being cleanly defined, let us just quote briefly the *phenomenological* constraints one may put on its mass m_g, or equivalently, on its range $\lambda_g \equiv h/(m_g c)$. Assuming a Yukawa potential of the form (11), Ref. [26] obtained the bound $\lambda_g > 3 \times 10^{15}$ m from solar-system planetary observations. Binary pulsars (see Sec. 3.3.5 below) put a less stringent bound, $\lambda_g > 10^{13}$ m, but depend on the radiative structure of the theory [27]. Tighter limits on the graviton mass will be obtained when the Laser Interferometer Space Antenna (LISA) will detect gravitational waves, by correlating their arrival time with that of photons coming from the same source. The analysis of Refs. [28] shows that a bound $\lambda_g > 2 \times 10^{17}$ m would be reachable. But even better constraints will be obtained by analyzing the time evolution of the gravitational waves emitted by an inspiralling binary. The basic idea is that a massive graviton should have a dispersion relation $E = \gamma m_g c^2 \Leftrightarrow v_g^2/c^2 = 1 - m_g^2 c^4/E^2$, so that high-energy gravitational waves propagate with a velocity $v_g \approx c$ whereas low-energy ones have a lower velocity. Since the gravitational waveform of an inspiralling binary is a "chirp", i.e., a modulated sinusoid varying from low frequencies to high ones, its propagation on a large distance from the source to the Earth will deform its shape. By analyzing the correlation of the detected signals with the templates predicted by GR, one can thus put constraints on the graviton mass. With the LIGO/VIRGO ground detectors, a bound $\lambda_g > 6 \times 10^{15}\,\mathrm{m}$ would be reachable, i.e., of the same order of magnitude as present solar-system limits. On the other hand, as shown in Refs. [29–31], the LISA space antenna would allow us to probe values $\lambda_g > 6 \times 10^{19}\,\mathrm{m}$, i.e., to obtain the best constraint on the graviton mass, $m_g < 2 \times 10^{-26}\,\mathrm{eV}/c^2$.

3.3. Field-Theoretical Approach

3.3.1. Scalar-tensor theories as a privileged class of models

Instead of adopting a purely phenomenological viewpoint, one may also test the dynamics of gravity within a well defined field-theoretical framework. We still assume an action of the form (5), where matter is universally coupled to a unique metric tensor, $S_{\mathrm{matter}}[\mathrm{matter}, g_{\mu\nu}]$, but we now wish to specify how $g_{\mu\nu}$ propagates, i.e., to define its action S_{gravity}. This physical metric tensor (defining the lengths and times measured by material rods and clocks) may be *a priori* a combination of many different fields, for instance

$$g_{\mu\nu} = A^2(\varphi)\left[g^*_{\mu\nu} + a_1 B_\mu B_\nu + a_2 g^*_{\mu\nu} B_\rho B^{*\rho} + a_3 B_{\mu\rho} B^{*\rho}_{\nu} + \cdots\right], \tag{12}$$

where $A(\varphi)$ is an arbitrary function of a (spin-0) scalar field φ, $g^*_{\mu\nu}$ describes the usual (spin-2) graviton whose kinetic term is the Einstein-Hilbert one $\int\sqrt{-g^*}R^*$, B_μ is a (spin-1) vector field, $B_{\mu\nu}$ is an antisymmetric tensor field, etc. However, such vector of tensor partners to the graviton enter as squares in Eq. (12), and their field equations take thus the form $\Box B \propto BT$, where T denotes schematically the matter sources. In other words, they are not linearly coupled to matter, and if their background value vanishes, matter will not generate them. On the other hand, if their vacuum expectation value happens not to vanish, then this violates Lorentz invariance because there exists a preferred frame in which they take simple forms (say, $B_\mu = (1, 0, 0, 0)$, defining an "ether"). Since there exist tight experimental constraints on possible violations of Lorentz invariance of gravity (see Sec. 3.1 and Refs. [1–3]), vector or extra tensor fields participating in the gravitational interaction seem thus almost excluded. Moreover, such fields generically lead to serious theoretical difficulties when they are massless, notably the existence of ghosts (negative-kinetic energy modes), causality violations, discontinuities in the field degrees of freedom, or ill-posedness of the Cauchy problem. Actually, a massless vector field needs to be coupled to a conserved current (see [32]), say to the baryonic number of nucleons if one wishes to mimic a gravitational force. However, the baryonic and gravitational masses of a macroscopic body do differ significantly (because of the various binding energies), so that such a force would violate equivalence principle tests by several orders of magnitude. Most of these problems can be avoided if the vector or antisymmetric tensor fields are massive, but no theoretical motivation is known to couple them to matter in the "metric" way $S_{\mathrm{matter}}[\mathrm{matter}, g_{\mu\nu}]$. Moreover, the effect of massive fields at large enough distances is exponentially small with respect to the Newtonian interaction. It should be underlined that a consistent class of gravity theories involving a constant-norm dynamical vector field, not directly coupled to matter, has been studied in depth (see [33] and references therein). As expected, Lorentz invariance is violated, but post-Newtonian tests in the solar system happen to be passed for a particular subclass of these models. As expected too, ghost degrees of freedom and causality violations also generically occur, but there remains a nonzero domain of viable models. In conclusion, vector partners to the graviton are *not* ruled out, but their analysis must be undertaken very carefully in order to avoid the numerous theoretical problems they generically cause.

On the contrary, scalar partners to the graviton are both consistent and easier to study. Not only their existence is predicted in all extra-dimensional theories, but they also play a crucial role in modern cosmology (notably during the accelerated expansion phases of the Universe, i.e., during inflation or as "quintessence" fields). They also respect most of GR's symmetries, namely conservation laws, constancy of non-gravitational constants, and local Lorentz invariance even if a subsystem is influenced by external masses. They can also satisfy exactly the weak equivalence principle (universality of free fall of laboratory-size objects) even for a massless scalar field. We will thus focus below on this specific class of alternative theories. To simplify the discussion, we will also restrict it to models involving a single scalar field, although the study of tensor–multi-scalar theories can also be done in great detail [34]. It suffices to note that their phenomenology is richer but similar to the single scalar case, at least when all scalar degrees of freedom carry positive energy, as required for the vacuum to be stable. The class of gravity theories we shall thus consider below is defined by the action [35]

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g^*}\,(R^* - 2g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi) + S_{\rm matter}\left[{\rm matter}; g_{\mu\nu} \equiv A^2(\varphi)g^*_{\mu\nu}\right], \tag{13}$$

which depends on one function $A(\varphi)$ characterizing how matter is coupled to the scalar field. A potential $V(\varphi)$ may also be considered in this action, yielding finite-range effects as those discussed in Sec. 3.2 above. We will focus below on "infinite-range" modifications of GR, i.e., actually on a scalar field whose mass (and other self-interactions described by $V(\varphi)$) is small enough to be negligible at the solar-system or binary-pulsar scale.

3.3.2. Nordström's theory

Before analyzing the predictions of general scalar-tensor theories in the weak-field (Sec. 3.3.3) and strong-field (Sec. 3.3.4) regimes, it is instructive to have a quick look at the simplest example: the limiting case of a purely scalar theory of gravity (without any spin-2 graviton), which was actually proposed by Nordström in 1913, two years before Einstein's general relativity. Its action reads

$$S = -\frac{c^3}{8\pi G}\int d^4x\,\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + S_{\rm matter}\left[{\rm matter}, g_{\mu\nu} \equiv \varphi^2\eta_{\mu\nu}\right], \tag{14}$$

where $\eta_{\mu\nu} = {\rm diag}(-1,1,1,1)$ denotes the flat metric, so that the physical metric $g_{\mu\nu}$ (to which matter is universally coupled) is conformally flat. This theory reproduces Newton's law at the lowest order, it satisfies the weak equivalence principle because of the metric coupling $S_{\rm matter}[{\rm matter}, g_{\mu\nu}]$ assumed for the matter action, and it can even be proven that it satisfies exactly the *strong* equivalence principle, i.e., that self-gravitating bodies fall with the same acceleration as test particles in an externally imposed gravitational field. However, it does not predict any light deflection, since null geodesics are such that $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = 0 \Rightarrow \eta_{\mu\nu}dx^\mu dx^\nu = 0$. Therefore, light propagates in the flat metric $\eta_{\mu\nu}$ without feeling the conformal factor φ^2 entering $g_{\mu\nu}$. Another way to prove this result is to recall the conformal invariance of the electromagnetic action, that we derived in Eq. (4) above [just remove the e^φ factor in this derivation, and write $g_{\mu\nu} = \varphi^2\eta_{\mu\nu}$ instead of $\tilde g_{\mu\nu} = A^2(\varphi)g_{\mu\nu}$]: There is strictly no coupling between photons and the scalar field φ. This already suffices to rule out this theory experimentally [see notably Eq. (9) above], but this also points out an instructive paradox. Since Nordström's theory satisfies by construction the weak equivalence principle, the physical effects of a gravitational field should be equivalent to those of an acceleration. But in an accelerated rocket, light rays are necessarily curved, just because of the coordinate transformation which passes from a local inertial frame (where light rays are straight lines) to the accelerated one. In conclusion, the equivalence principle suffices to know that light *must* be attracted by a massive body. Therefore, like GR, Nordström's theory should predict light deflection! Actually, both theories do predict the same *local* effect on photons, consistently with the equivalence principle. But when observing distant stars, a *global* effect also enters the observable deflection angle, because of the curvature of *space* (i.e., the fact that the spatial metric g_{ij} is not flat). In Nordström's theory, it happens that this spatial contribution to the deflection angle exactly cancels the local one predicted by the equivalence principle. On the contrary, both contributions are equal in GR, so that the global observable effect is exactly twice the naive result given by the equivalence principle alone. This explains why Einstein had predicted half the correct value from 1911 to 1914, and derived the exact result only in 1915, when he wrote and solved the complete field equations for $g_{\mu\nu}$.

This discussion underlines that simple reasonings, using the local equivalence between an accelerated rocket and a gravitational field, can easily give correct results, as in Sec. 2.2 above, but may also be misleading when global effects make the observable quantities more subtle to define. The absence of light deflection in the limiting case of

Nordström's theory, in which gravity is mediated by a (spin-0) scalar field but no (spin-2) graviton, also gives us a hint of a generic effect in all scalar-tensor theories: Light deflection is always predicted to be *smaller* than the general relativistic result.

3.3.3. Solar-system tests

Let us now come back to the general scalar-tensor action (13), characterized by a matter-scalar coupling function $A(\varphi)$. It is convenient to expand it around the (cosmologically-imposed) background value φ_0 of the scalar field as

$$\ln A(\varphi) = \alpha_0(\varphi - \varphi_0) + \frac{1}{2}\beta_0(\varphi - \varphi_0)^2 + \mathcal{O}(\varphi - \varphi_0)^3, \tag{15}$$

where α_0 defines the linear coupling constant of matter to scalar excitations, β_0 its quadratic coupling to two scalar lines, etc. Newtonian and post-Newtonian predictions depend only on these first two coupling constants, α_0 and β_0. For instance, the effective gravitational constant between two bodies is not given by the bare constant G entering action (13), but by $G_{\text{eff}} = G(1 + \alpha_0^2)$, in which a contribution G comes from the exchange of a (spin-2) graviton whereas $G\alpha_0^2$ is due to the exchange of a (spin-0) scalar field, each matter-scalar vertex bringing a factor α_0. [Actually, the value of a gravitational constant depends on the chosen units, and the expression $G_{\text{eff}} = G(1 + \alpha_0^2)$ corresponds to the "Einstein-frame" representation used to write action (13). An extra factor $A_0^2 = A(\varphi_0)^2$ enters when using the physical metric $g_{\mu\nu} = A^2(\varphi)g^*_{\mu\nu}$ to define observable quantities, and the actual gravitational constant which is measured reads $GA_0^2(1+\alpha_0^2)$. No such extra factors A_0 enter the computation of dimensionless observable quantities, such as the post-Newtonian parameters β^{PPN} and γ^{PPN} below.]

Let us first study light deflection within such scalar-tensor models. We saw in Eq. (4) above that the electromagnetic action is conformal invariant in 4 dimensions. In our present metrically-coupled framework, this means that $S_{\text{EM}} = \frac{1}{4}\int d^4x\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} = \frac{1}{4}\int d^4x\sqrt{-g^*}g_*^{\mu\rho}g_*^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}$, so that photons are coupled as in GR to the gravitons $g^*_{\mu\nu} - \eta_{\mu\nu}$, but they do not feel at all the global factor $A^2(\varphi)$ of the physical metric. In other words, they are totally decoupled from the scalar field $\varphi - \varphi_0$. Therefore, we can immediately deduce that the light deflection angle should take strictly the same expression as in GR, namely $\Delta\theta = 4Gm/bc^2$, where b denotes the impact parameter of the light ray. But contrary to GR, we do not have direct experimental access to Gm in scalar-tensor theories, since planets feel a gravitational potential $G_{\text{eff}}m/r = G(1 + \alpha_0^2)m/r$. Therefore, although the deflection angle $\Delta\theta$ is actually the same as in GR, we *interpret* it in terms of observable quantities as $\Delta\theta = 4G_{\text{eff}}m/(1 + \alpha_0^2)bc^2 < 4G_{\text{eff}}m/bc^2$. In conclusion, when comparing the light-deflection angle to the gravitational force felt by test masses, we get a *smaller* result than in GR. This is consistent with the limiting case of Nordström's theory which does not predict any light deflection (see Sec. 3.3.2 above), but the physical reason of this result is noteworthy: The light deflection itself is not smaller, but the gravitational force felt by test masses is larger than in GR because of the extra attractive force caused by the scalar partner to the graviton.

All other post-Newtonian corrections due to the scalar field can easily be derived [1, 34]. Since we are considering metric theories, they fit within the PPN framework described in Sec. 3.1 above. Only two PPN parameters, out of the 10 possible, take different values than in GR. This is due to the large number of symmetries respected by scalar-tensor theories. They precisely correspond to the original Eddington parameters β^{PPN} and γ^{PPN}, which can now be written in terms of the coupling constants entering Eq. (15):

$$\gamma^{\text{PPN}} - 1 = -\frac{2\alpha_0^2}{1 + \alpha_0^2}\,, \qquad \beta^{\text{PPN}} - 1 = \frac{1}{2}\,\frac{\alpha_0\beta_0\alpha_0}{(1 + \alpha_0^2)^2}\,. \tag{16}$$

As in the case of the effective gravitational constant G_{eff}, the factor α_0^2 comes from the exchange of a scalar particle between two bodies, whereas $\alpha_0\beta_0\alpha_0$ comes from a scalar exchange between three bodies. We notably recover the above result for light deflection, since it reads in the PPN formalism $\Delta\theta = 2(1+\gamma^{\text{PPN}})G_{\text{eff}}m/bc^2 = 4G_{\text{eff}}m/(1+\alpha_0^2)bc^2$.

All solar-system constraints quoted in Sec. 3.1 can thus be translated as bounds on the matter-scalar coupling constants α_0 and β_0, as shown in Fig. 2 and on the 25th transparency of my talk. The tightest constraint comes from the Cassini limit (9), which implies that the linear coupling constant $|\alpha_0|$ must be smaller than 3×10^{-3}, i.e., that matter must be very weakly coupled to a possible scalar field in the solar system. On the other hand, the quadratic

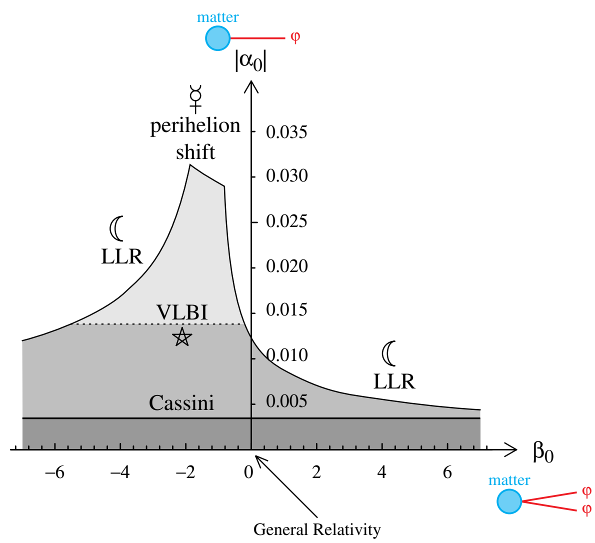

Figure 2: Solar-system constraints on the matter-scalar coupling function $\ln A(\varphi) = \alpha_0(\varphi - \varphi_0) + \frac{1}{2}\beta_0(\varphi - \varphi_0)^2 + \mathcal{O}(\varphi - \varphi_0)^3$. The allowed region is shaded. The vertical axis ($\beta_0 = 0$) corresponds to Brans-Dicke theory with a parameter $2\omega_{\rm BD} + 3 = 1/\alpha_0^2$. The horizontal axis ($\alpha_0 = 0$) corresponds to theories which are perturbatively equivalent to GR, i.e., which predict strictly no deviation from it (at any order $1/c^n$) in the weak-field conditions of the solar system.

coupling constant β_0 is almost not constrained by solar-system tests, because it appears multiplied by the already small factor $\alpha_0^2 < 10^{-5}$ in the observable PPN parameter $\beta^{\rm PPN}$, Eq. (16) above. In particular, it is important to note that these weak-field experiments allow any sign for β_0.

3.3.4. Strong-field predictions

In spite of their precision, present solar-system tests allow us to probe only the first post-Newtonian corrections to GR, i.e., the terms explicitly displayed in Eq. (6) above, but not the higher-order corrections in powers of $1/c$. This is because the gravitational field is very weak in the solar system: The largest deviation from flat space is at the surface of the Sun, and is of order $Gm_\odot/R_\odot c^2 \approx 2 \times 10^{-6}$, where $m_\odot$ and $R_\odot$ denote the mass and radius of the Sun. For the Earth, this dimensionless ratio $Gm_\oplus/R_\oplus c^2$ is even of order 7×10^{-10}. It may happen that the correct theory of gravity coincides with GR at the first post-Newtonian order but differs significantly from it at higher orders. Let us therefore analyze scalar-tensor theories in the strong gravitational field regime, when the mass-to-radius ratio of a body, Gm/Rc^2, is not negligibly small. For instance, it is of order 0.2 for a neutron star, not far from the theoretical maximum of 0.5 corresponding to black holes.

If one analyzes scalar-tensor theories in a perturbative expansion in powers of $1/c$, it is easy to show that any deviation from GR involves at least two factors α_0, and has the schematic form

$$\text{deviation from GR} = \alpha_0^2 \times \left[\lambda_0 + \lambda_1 \frac{Gm}{Rc^2} + \lambda_2 \left(\frac{Gm}{Rc^2}\right)^2 + \cdots\right], \tag{17}$$

where λ_0, λ_1, ... are constants built from the coefficients α_0, β_0, ... of expansion (15). Indeed, the classical gravitational interactions we are studying may be described by tree-level Feynman diagrams, in which matter sources are connected by graviton of scalar lines. All possible deviations from GR correspond to diagrams involving at least one scalar exchange. But any scalar line must have at least two ends, and each of them involves the linear coupling constant α_0. We can thus conclude that any deviation from GR, at any post-Newtonian order, always involves a global factor $< \alpha_0^2$, that we know from solar-system tests to be smaller than 10^{-5} (see Sec. 3.3.3 above). We thus expect scalar-tensor theories to be close to GR at any order.

However, nonperturbative effects may occur in strong-field conditions: If the compactness Gm/Rc^2 of a body is greater than a critical value, the square brackets of Eq. (17) can become large enough to compensate even a vanishingly

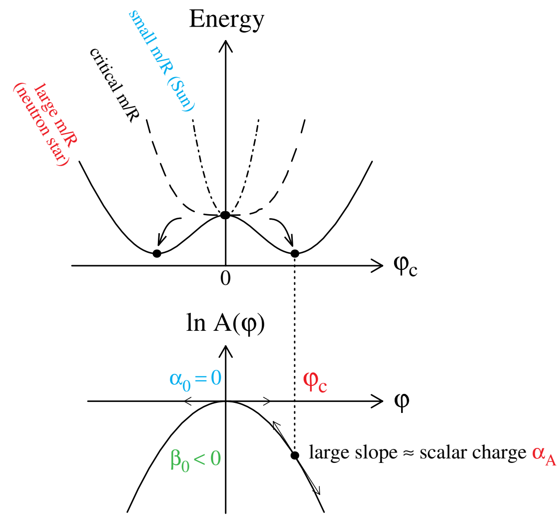

Figure 3: Heuristic argument to explain the phenomenon of "spontaneous scalarization". When $\beta_0 < 0$ and the compactness Gm/Rc^2 of a body is large enough, it is energetically favorable to create a local scalar field different from the background value. The body becomes thus strongly coupled to the scalar field.

small α_0^2. To illustrate this, let us consider a model for which α_0 vanishes strictly, i.e., which is perturbatively equivalent to GR: There is strictly no deviation from GR at any order in a perturbative expansion in powers of $1/c$. A parabolic coupling function $\ln A(\varphi) = \frac{1}{2}\beta_0\varphi^2$ suffices for our purpose (we set here $\varphi_0 = 0$ to simplify). At the center of a static body, the scalar field takes a particular value φ_c, and it decreases as $1/r$ outside. The energy of such a scalar field configuration involves two contributions, coming respectively from the kinetic term and from the matter-scalar coupling function in action (13). As a rough estimate of its value, one can write

$$\text{Energy} \approx \int \left[\frac{1}{2}(\partial_i\varphi)^2 + \rho\, e^{\beta_0\varphi^2/2}\right] \approx mc^2\left(\frac{\varphi_c^2/2}{Gm/Rc^2} + e^{\beta_0\varphi_c^2/2}\right). \tag{18}$$

When $\beta_0 < 0$, this is the sum of a parabola and a Gaussian, and if the compactness Gm/Rc^2 is large enough, the function Energy(φ_c) takes the shape of a Mexican hat (see Fig. 3 and the 27th transparency of my talk). The value $\varphi_c = 0$ now corresponds to a local *maximum* of the energy. It is therefore energetically favorable for the star to create a nonvanishing scalar field φ_c, and thereby a nonvanishing "scalar charge" $d\ln A(\varphi_c)/d\varphi_c = \beta_0\varphi_c$. This phenomenon is analogous to the spontaneous magnetization of ferromagnets.

This heuristic argument has been verified by explicit numerical calculations, taking into account the coupled differential equations of the metric and the scalar field, and using various realistic equations of state to describe nuclear matter inside a neutron star [36]. The correct definition of the linear coupling strength between a compact body A and the scalar field reads $\alpha_{\rm A} \equiv \partial \ln m_{\rm A}/\partial\varphi_0$. It is plotted in Fig. 4 for the particular model $\beta_0 = -6$. One finds that there exists indeed a "spontaneous scalarization" above a critical mass (whose value decreases as $-\beta_0$ grows). On the other hand, if $\beta_0 > 0$, both the above heuristic argument and the actual numerical calculations show that $|\alpha_{\rm A}| < |\alpha_0|$. In that case, one finds that neutron stars are even less coupled to the scalar field than solar-system bodies.

The scalar charge $\alpha_{\rm A}$ enters the predictions of the theory in the same way as α_0 in weak-field conditions. For instance, in the orbital motion of two bodies A and B, the Eddington parameter $\gamma^{\rm PPN}$ keeps the form of Eq. (16), but it now involves the product $\alpha_{\rm A}\alpha_{\rm B}$ of the two scalar charges instead of α_0^2. Similarly, the strong-field analogue of $\beta^{\rm PPN}$ involves products of scalar charges and their derivatives $\beta_{\rm A} \equiv \partial\alpha_{\rm A}/\partial\varphi_0$. Since numerical calculations show that $|\alpha_A|$ can be of order 1 when spontaneous scalarization develops (see Fig. 4), one thus expects $\mathcal{O}(1)$ deviations from GR in strong-field conditions, even if the theory is perturbatively equivalent to GR in the weak-field conditions of the solar system. Moreover, the quadratic coupling strength β_A can take very large numerical values near the

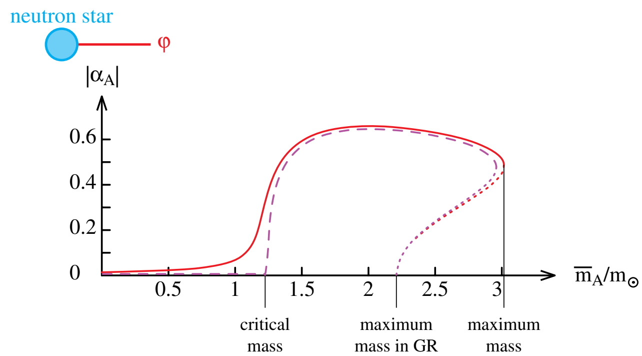

Figure 4: Scalar charge $\alpha_{\rm A}$ versus baryonic mass $\overline{m}_{\rm A}$, for the model $A(\varphi) = \exp(-3\varphi^2)$ (i.e., $\beta_0 = -6$). The solid line corresponds to the maximum value of α_0 allowed by Eqs. (7)-(8), and the dashed line to $\alpha_0 = 0$. The dotted lines correspond to unstable configurations of the star.

critical mass, like the magnetic susceptibility of ferromagnets. Therefore, even larger deviations from GR are found when the mass of a neutron star happens to be close to the critical one.

3.3.5. Binary-pulsar tests

In order to test gravity in the strong-field regime, one needs to observe very compact objects. As mentioned above, black holes seem to be good candidates because their compactness Gm/Rc^2 takes the maximum theoretical value of $\frac{1}{2}$. However, they can only be observed indirectly (*via* the radiation emitted by accreted matter), and they also have a serious theoretical drawback: Even if a scalar partner to the graviton does exist, black holes are anyway totally decoupled from it, because of the "no hair theorem". Therefore, they behave exactly as in GR in a given background metric, and they cannot be used to discriminate scalar-tensor models from GR.

On the other hand, neutron stars are still quite compact bodies ($Gm/Rc^2 \simeq 0.2$), but they can be strongly coupled to a putative scalar field (because of the "spontaneous scalarization" phenomenon described above), and above all they can be directly observed as pulsars. A pulsar is indeed a rapidly rotating and highly magnetized neutron star, emitting a beam of radio waves, like a lighthouse. Experiment tells us that isolated pulsars are very stable clocks when they are old enough. A pulsar A orbiting a companion B is thus a moving clock, the best tool that one could dream of to test a relativistic theory. Indeed, by precisely timing its pulse arrivals, one gets a stroboscopic information on its orbit, and one can measure several relativistic effects. Such effects do depend on the two masses $m_{\rm A}$, $m_{\rm B}$, which are not directly measurable. However, two different effects suffice to determine them, and a third relativistic observable then gives a test of the theory.

For instance, in the case of the famous Hulse-Taylor binary pulsar PSR B1913+16, three relativistic parameters have been determined with great accuracy [37]: (i) the Einstein time delay parameter γ_T, which combines the second-order Doppler effect ($\propto v_{\rm A}^2/2c^2$, where $v_{\rm A}$ is the pulsar's velocity) together with the redshift due to the companion ($\propto Gm_{\rm B}/r_{\rm AB}c^2$, where $r_{\rm AB}$ the pulsar-companion distance); (ii) the periastron advance $\dot{\omega}$ (relativistic effect of order v^2/c^2); and (iii) the rate of change of the orbital period, $\dot{P}$, caused by gravitational radiation damping (an effect of order v^5/c^5 in GR, but of order v^3/c^3 in scalar-tensor theories; see below). The same parameters have also been measured for the neutron star-white dwarf binary PSR J1141−6545, but with much less accuracy [38]. In addition to these three parameters, (iv) the "range" (global factor $Gm_{\rm B}/c^3$) and (v) "shape" (time dependence) of the Shapiro time delay have also been determined for two other binary pulsars, PSR B1534+12 [39] and PSR J0737−3039 [40]. The latter system is particularly interesting because both bodies have been detected as pulsars. Since their independent timing gives us the (projected) size of their respective orbits, the ratio of these sizes provides a direct measure of (vi) the mass ratio $m_{\rm A}/m_{\rm B} \approx 1.07$. In other words, 6 relativistic parameters have been measured for the double pulsar PSR J0737−3039. After using two of them to determine the masses $m_{\rm A}$ and $m_{\rm B}$, this system thereby provides $6 - 2 = 4$ tests of relativistic gravity in strong-field conditions.

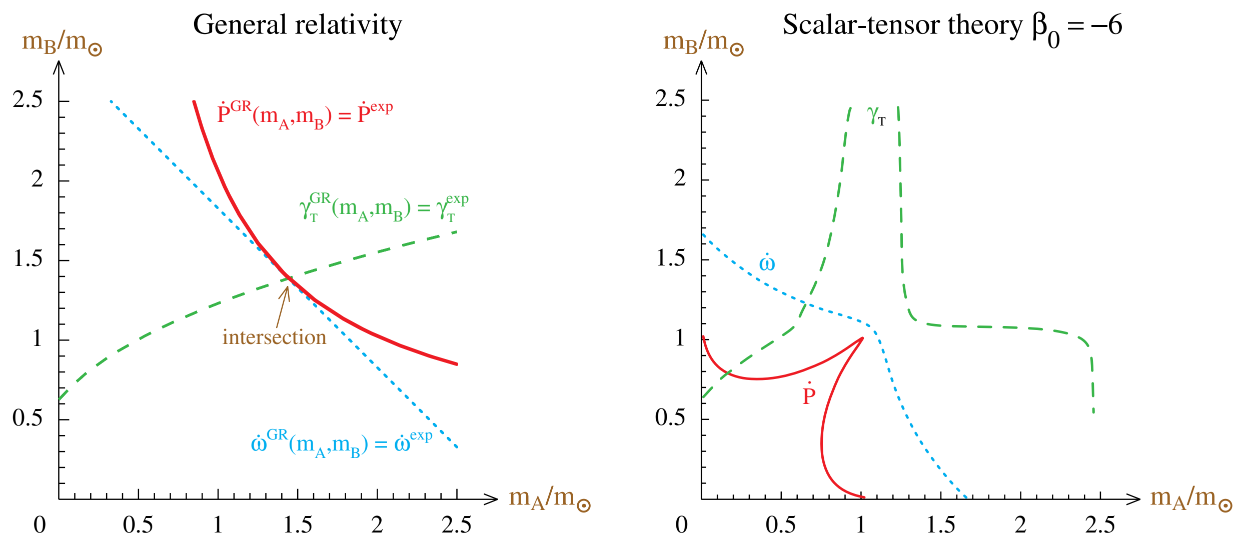

Figure 5: Mass plane ($m_{\rm A}$ = pulsar, $m_{\rm B}$ = companion) of the Hulse-Taylor binary pulsar PSR B1913+16 in general relativity (left panel) and for a scalar-tensor theory with $\beta_0 = -6$ (right panel). The widths of the lines are larger than 1σ error bars. While GR passes the test with flying colors, the value $\beta_0 = -6$ is ruled out.

The clearest way to illustrate these tests is to plot the various experimental constraints in the mass plane $(m_{\rm A}, m_{\rm B})$, for a given theory of gravity. Any theory indeed predicts the expressions of the various timing parameters in terms of these unknown masses and other (Keplerian) observables, such as the orbital period and the eccentricity. The equations *predictions*$(m_{\rm A}, m_{\rm B})$ = *observed values* thereby define different curves in the mass plane, or rather different *strips* if one takes into account experimental errors. If these strips have a common intersection, there exists a pair of masses which is consistent with all observables, and the theory is confirmed. On the other hand, if the strips do not meet simultaneously, the theory is ruled out.

Since Ingrid Stairs' contribution to the present SLAC Summer Institute is precisely devoted to binary pulsars, I refer to her lecture notes for up-to-date plots of the mass planes within GR (see also Fig. 5 and the 30th to 33rd transparencies of my talk). Einstein's theory passes with flying colors the 9 tests provided by the above 4 binary pulsars. In scalar-tensor theories, the same mass planes may be plotted, but since the theoretical predictions for the various relativistic parameters differ from those of GR, the various strips are deformed. For some theories, these strips still have a common intersection (not necessarily for the same values of the masses $m_{\rm A}$ and $m_{\rm B}$ that were consistent with GR). When they do not, we can conclude that the corresponding theory is ruled out. These deformations of strips in the mass plane are illustrated in Fig. 5 and the 35th transparency of my talk.

The allowed region of the theory space $(|\alpha_0|, \beta_0)$ is displayed in Fig. 6, as well as in transparencies 36 and 38–40 of my talk. This illustrates vividly the *qualitative* difference between solar-system and binary-pulsar observations. Indeed, while weak-field tests only constrain the linear coupling constant $|\alpha_0|$ to be small without giving much information about the quadratic coupling constant β_0 (see Sec. 3.3.3 above), binary pulsars impose

$$\beta_0 > -4.5\,, \tag{19}$$

even for a vanishingly small α_0. This constraint is due to the spontaneous scalarization of neutron stars, which occurs when $-\beta_0$ is large enough. Equations (16) allow us to rewrite this inequality in terms of the Eddington parameters $\beta^{\rm PPN}$ and $\gamma^{\rm PPN}$, which are both consistent with 1 in the solar system. One finds

$$\frac{\beta^{\rm PPN} - 1}{\gamma^{\rm PPN} - 1} < 1.1\,. \tag{20}$$

The singular (0/0) nature of this ratio underlines why such a conclusion could not be obtained in weak-field experiments.

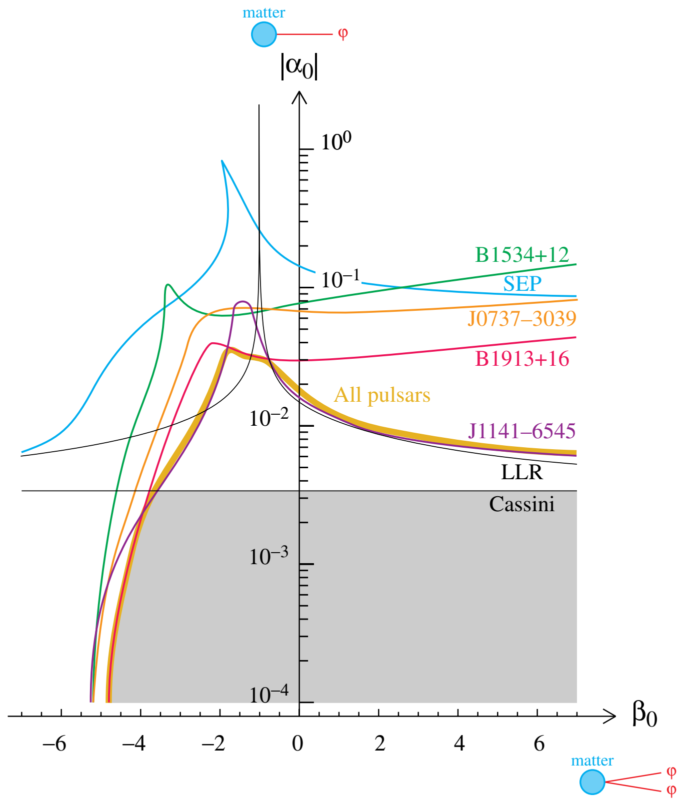

Figure 6: Same theory plane as in Fig. 2, but now with a logarithmic scale for the linear matter-scalar coupling constant $|\alpha_0|$. General relativity corresponds to $\alpha_0 = \beta_0 = 0$, i.e., to the point at $-\infty$ down the vertical axis. This plot displays both solar-system and binary-pulsar constraints, and the allowed region is shaded. LLR stands as before for "lunar laser ranging", and the curve labeled SEP corresponds to tests of the "strong equivalence principle" using a set of several neutron star-white dwarf low-eccentricity binaries. The most constraining binary-pulsar tests are those of PSR B1913+16 and PSR J1141−6545. While solar system tests impose a small value of $|\alpha_0|$, binary pulsars impose the orthogonal constraint $\beta_0 > -4.5$.

The two most constraining binary pulsars, for scalar-tensor theories, are the Hulse-Taylor one (PSR B1913+16), which has been observed for more than 30 years and has thus very small experimental uncertainties, and paradoxically the neutron star-white dwarf system PSR J1141−6545, in spite of its still large experimental errors. The reason why the latter system is so constraining is its asymmetry. Indeed, while the pulsar is a neutron star which can spontaneously scalarize and acquire a large scalar charge α_A, the white dwarf is much less compact and keeps a negligible scalar charge $\approx \alpha_0$. This causes a huge deviation from GR in the orbital period variation $\dot{P}$. Indeed, the energy flux carried out by gravitational waves is of the form

$$\begin{aligned}\text{Energy flux} \;=\; & \left\{\frac{\text{Quadrupole}}{c^5} + \mathcal{O}\left(\frac{1}{c^7}\right)\right\}_{\text{helicity 2}} \\ & + \left\{\frac{\text{Monopole}}{c}\left(0 + \frac{1}{c^2}\right)^2 + \frac{\text{Dipole}}{c^3}(\alpha_A - \alpha_B)^2 + \frac{\text{Quadrupole}}{c^5} + \mathcal{O}\left(\frac{1}{c^7}\right)\right\}_{\text{helicity 0}} . \qquad (21)\end{aligned}$$

The first curly brackets contain the prediction of general relativity, of order v^5/c^5, whereas the second ones contain the extra contributions predicted in tensor-scalar theories. In particular, the dipolar contribution is of order v^3/c^3, much larger than the usual quadrupole of GR if the two scalar charge are significantly different. This dipolar emission of (helicity-0) gravitational waves has been derived long ago [41], and already used to constrain scalar-tensor theories with different neutron star-white dwarf binaries (notably PSR B0655+64) [34, 36, 42], but the better timing of PSR J1141−6545 makes it much more constraining. It is not only *infinitely* more constraining than solar-system tests in the region $\beta_0 < -4.5$ of the theory space (like all binary pulsars), but it is even almost as constraining as them in the region $\beta_0 > 0$.

3.3.6. Gravitational waves antennas

The LIGO and VIRGO interferometers will detect gravitational waves emitted by inspiralling binaries. Since matter is known to be weakly coupled to the scalar field in the solar system (small value of $|\alpha_0|$), these detectors will not be sensitive of the helicity-0 waves. On the other hand, the time evolution of the helicity-2 chirp does depend on the energy flux (21), which differs significantly from the GR prediction when the scalar charges α_A, α_B are nonzero. Therefore, the GR wave templates used for matched filtering in LIGO and VIRGO may not be accurate if there exists a scalar partner to the graviton, and the signal-to-noise ratio may then drop. Fortunately, it was shown in [36] that binary-pulsar data are so precise that they already exclude the models which would have predicted significant effects in the gravitational waveforms (see transparencies 47 and 48 of my talk). Therefore, although these interferometers are *a priori* more sensitive to the scalar field than classic solar-system tests, one may securely use the GR wave templates for their data analysis. On the other hand, it was shown in [29–31] that the LISA space interferometer can be sensitive to scalar effects which are still allowed by all present tests. However, future binary-pulsar data should probe them before the LISA mission is launched (see transparencies 49–52 of my talk and Ref. [43]).

4. PUZZLING ISSUES

The various tests of GR described in the previous sections, both in weak-field and strong-field conditions, give us a strong confidence in this theory. Nevertheless, there exist some puzzling experimental issues, which may either be a hint that the behavior of gravity needs to be changed at large distances, or may be interpreted as the effect of rather strange matter components filling our Universe, without modifying gravity itself.

Within the solar system, at distances from 20 to 70 AU, the two Pioneer 10 and 11 spacecrafts exhibit an anomalous constant acceleration towards the Sun, of about $8.5 \times 10^{-10}\,\mathrm{m.s}^{-2}$. This extra acceleration has the same direction and the same sign as the standard Newtonian force, and could not be explained by any known source of noise in the system. There are actually four different directions which are almost parallel for these spacecrafts: their spin axis, their velocity, the Sun, and the Earth. To understand better the physical origin of this anomalous force, a careful reanalysis of all recorded data (over ~ 10 years) has been undertaken, and preliminary results seem to confirm that it is directed towards the Sun, rather than the three other possible directions [44]. It should thus be caused by a *gravitational* phenomenon.

The combination of all cosmological observations is consistent with a Universe filled with about 72% of "dark energy", 24% of "dark matter", and only 4% of ordinary baryonic matter. Although this is *a priori* not a problem for the theory of gravity itself, these adjectives "dark" nevertheless underline that we do not know the nature of 96% of the energy content of the Universe.

The existence of dark energy is suggested mainly by the observation of type Ia supernovae, which indicate that the expansion of the Universe is presently accelerating. This dark energy is a fluid with a negative pressure approximately opposite to its energy density. It may be easily described by a cosmological constant, but its observed value $\Lambda \approx 3\times 10^{-122} c^3/\hbar G$ is so small that its interpretation as the vacuum energy is very problematic. Such small numbers can however be obtained naturally within "quintessence" models, in which the cosmological constant is actually replaced by the potential $V(\varphi)$ of a scalar field slowly rolling down towards its minimum. However, there remains to explain why the dark energy density ($\Omega_\Lambda \approx 0.7$ in units of the critical density) and the matter density ($\Omega_m \approx 0.3$) are numerically close at present, whereas they differ by many orders of magnitude during most of the Universe evolution.

Aside from these fine-tuning problems, one may adopt a phenomenological point of view and try to describe the recent evolution of our Universe with a simple scalar-tensor theory of gravity. Then one finds that cosmological data *a priori* allow us to reconstruct both the matter-scalar coupling function $A(\varphi)$ entering action (13) and the possible potential $V(\varphi)$ of the scalar field. Of course, such a reconstruction is very noisy as compared to solar-system and binary-pulsar tests of scalar-tensor theories, but it is important to underline the qualitative difference of cosmological observations. Instead of just constraining the first derivative α_0 of $A(\varphi)$, like solar-system tests, or its second derivative β_0, like binary-pulsar tests, cosmology allows us to reconstruct the full shape of both $A(\varphi)$ and $V(\varphi)$ for the values of φ probed by the data [45, 46].

Dark matter is a fluid with vanishingly small pressure, like ordinary baryonic matter, but it is seen only *via* its gravitational field. It has notably no interaction with light, and is therefore dark and transparent. It has also vanishingly small interaction with itself, aside from its gravitational influence. Its existence is suggested by several data. For instance, when combining the value of Ω_Λ consistent with type Ia supernovae with the fact that $\Omega_\Lambda + \Omega_m \approx 1$ as given by the analysis of the first acoustic peak of the Cosmic Microwave Background (CMB), one gets a matter density $\Omega_m \approx 0.3$ which is an order of magnitude larger than our estimates of baryonic matter in the Universe. Moreover, galaxies (and clusters of galaxies) rotate as is they were rigid bodies [47], whereas Newton's law predicts that outer stars should rotate around the galaxy center with a much slower velocity than inner stars. This can be explained without modifying gravity by invoking the existence of a halo of dark matter surrounding the galaxy (or cluster), rounder than the galaxy itself because of the absence of self-interaction of dark matter. The amount of such dark matter needed to account for galaxy rotation curves is consistent with the above cosmological estimate $\Omega_m \approx 0.3$.

Nevertheless, such a dark component of matter remains puzzling, and various theorists have tried to account for experimental data *via* a modification of gravity at large distances. For instance, Milgrom proposed in 1983 a phenomenological modification of Newton's law [48] which still superbly fits galaxy rotation curves [49] (although clusters anyway require some amount of dark matter), and automatically recovers the Tully-Fisher law [50] $v_\infty^4 \propto M$ (where M denotes the baryonic mass of a galaxy, and v_∞ the asymptotic circular velocity of visible matter in its outer region). The norm a of a particle's acceleration is assumed to be given by its Newtonian value a_N when it is greater than a universal constant denoted $a_0 \approx 1.2 \times 10^{-10}\,\mathrm{m.s}^{-2}$, but to read $a = \sqrt{a_N a_0}$ in the small-acceleration regime $a < a_0$. In particular, the gravitational acceleration should now read $a = \sqrt{GMa_0}/r$ at large distance, instead of the usual GM/r^2 law. Various attempts have been made to derive such a modified Newtonian dynamics (MOND) from a consistent relativistic field theory. However, many models contain tachyons (negative mass squared) or ghosts (negative kinetic energy) and are thus unstable. Moreover, many models write actions which depend on the mass M of the considered galaxy, i.e., are actually using a different theory for each galaxy. At present, the best candidate is certainly the Bekenstein-Sanders model [51, 52], although its stability is not fully proven yet. It takes into account all the subtleties understood in the previous attempts, and therefore looks quite complicated. It incorporates a vector and two scalar fields in addition to the usual graviton of general relativity, and depends on two coupling constants and two functions of one scalar field. These functions can been tuned to give the right phenomenology (including for the Pioneer anomaly, if needed), but there is no fundamental principle to fix them. The vector field is introduced in order to predict the right light deflection. Indeed, we saw in Secs. 3.3.2 and 3.3.3 above that scalar-tensor theories always predict a smaller light deflection than GR. Therefore, a trick is needed to make it larger, as if there were more matter curving spacetime. The idea is to introduce a preferred frame (i.e., an "ether"), and to couple the scalar field differently to g_{00} and the spatial metric g_{ij}. An inverse coupling $e^{2\varphi} g_{00}$ and $e^{-2\varphi} g_{ij}$ mimics for instance the Schwarzschild metric of GR, so that the contribution of the scalar field now *increases* light deflection. A vector field u_μ is just a convenient way to define such a preferred frame in a covariant way: If $u_\mu = (1, 0, 0, 0)$ in the preferred frame, then $-u_\mu u_\nu$ behaves as g_{00}, whereas $g_{\mu\nu} + u_\mu u_\nu$ defines the corresponding g_{ij}. We thus understand why Bekenstein and Sanders assume that matter is coupled to a physical metric $\tilde{g}_{\mu\nu} = e^{-2\varphi}(g_{\mu\nu} + u_\mu u_\nu) - e^{2\varphi} u_\mu u_\nu$. It should be noted that the precise exponentials of φ introduced in this expression are chosen in order to reproduce the same light deflection as the corresponding "dark matter". In other words, this is not a prediction of the model, and one would just need to change these factors to increase or decrease the amount of light deflection caused by dark matter if experiment told us so. In spite of this lack of predictiveness, this model does work well, and the preferred frame defined by the vector field does not seem to cause observational problems in the solar system. However, modified gravity models seem to have serious difficulties in reproducing the right CMB power spectrum. For instance, it has been computed in [53] for the above Bekenstein model, and the positions and heights of the acoustic peaks was found to be inconsistent with observed data. By assuming a significant amount of energy density due to massive neutrinos

($\Omega_\nu = 0.17$), it was however possible to reproduce perfectly the first two peaks. Since such a value 0.17 is not so far from the needed amount of dark matter ($\Omega_m = 0.24$), this looks like a failure of modified-gravity models. However, massive neutrinos are too light to cluster on the size of galaxies, whereas they could form the amount of dark matter which is still needed for galaxy clusters in the MOND scenario. Therefore, it is still important to study in depth such modifications of gravity at large distances, although the dark matter paradigm seems today more and more robust.

5. CONCLUSIONS

General relativity passes all present tests with flying colors, but alternative theories are anyway very important to study. Deviations from GR are indeed predicted in all extra-dimensional theories, notably in superstrings. Moreover, contrasting alternatives to GR are useful in order to understand precisely which features of the theory have been tested in a particular experiment, and to suggest new experiments probing the remaining features. The main conclusion is illustrated by our study of the privileged class of scalar-tensor theories: There exists a qualitative difference between solar-system tests (probing weak-field gravity at the 10^{-5} level), binary-pulsar tests (probing strong-field gravity), and cosmological observations (probing the time-evolution of the Universe). While solar-system tests impose that the slope of the matter-scalar coupling function $A(\varphi)$ must be small, binary-pulsar data impose that its second derivative cannot take large and negative values, and cosmological observations *a priori* allow us to reconstruct the full shape of both $A(\varphi)$ and the scalar field potential $V(\varphi)$, but with much more noisy results. We finally mentioned some puzzling issues, which still need to be understood either theoretically or experimentally. They do not contradict GR in a direct way, but may suggest modifications of Newton's law at large distances.

Acknowledgments

I wish to thank the SLAC team for its kind invitation to this Summer Institute.

References

[1] C. M. Will, *Theory and experiment in gravitational physics* (Cambridge U. Press, Cambridge, 1993).
[2] C. M. Will, Living Rev. Rel. **4**, 4 (2001), gr-qc/0103036.
[3] C. M. Will, gr-qc/0510072.
[4] J.-P. Uzan, Rev. Mod. Phys. **75**, 403 (2003), hep-ph/0205340.
[5] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. **B592**, 1 (2004), contribution by T. Damour entitled *Experimental tests of gravitational theories*, p. 186.
[6] A. I. Shlyakhter, Nature **264**, 340 (1976); T. Damour and F. Dyson, Nucl. Phys. **B480**, 37 (1996), hep-ph/9606486; Y. Fujii *et al.*, Nucl. Phys. **B573**, 377 (2000), hep-ph/9809549.
[7] J. D. Prestage *et al.*, Phys. Rev. Lett. **54**, 2387 (1985); S. K. Lamoreaux *et al.*, Phys. Rev. Lett. **57**, 3125 (1986); T. E. Chupp *et al.*, Phys. Rev. Lett. **63**, 1541 (1989).
[8] S. Baessler *et al.*, Phys. Rev. Lett. **83**, 3585 (1999).
[9] E. G. Adelberger, Class. Quantum Grav. **18**, 2397 (2001).
[10] J. G. Williams, S. G. Turyshev, and D. H. Boggs, Phys. Rev. Lett. **93**, 261101 (2004), gr-qc/0411113.
[11] R. F. C. Vessot and M. W. Levine, Gen. Rel. Grav. **10**, 181 (1978); R. F. C. Vessot *et al.*, Phys. Rev. Lett. **45**, 2081 (1980).
[12] C. Salomon and V. Veillet, in Proc. Symp. on Space Station Utilisation, ESA-SP **385**, 295 (1996).
[13] T. Damour and A. M. Polyakov, Nucl. Phys. **B423**, 532 (1994), hep-th/9401069; Gen. Relativ. Gravit. **26**, 1171 (1994), gr-qc/9411069.
[14] T. Damour and A. Vilenkin, Phys. Rev. **D53**, 2981 (1996), hep-th/9503149.

[15] T. Damour, F. Piazza, and G. Veneziano, Phys. Rev. **D66**, 046007 (2002), hep-th/0205111.

[16] P. W. Worden, in *Proc. 7th Marcel Grossmann Meeting on General Relativity* (1995), ed. R. J. Jantzen and G. MacKeise (World Scientific, Singapore), p. 1569.

[17] P. Touboul *et al.*, C. R. Acad. Sci. Paris (série IV) **2**, 1271 (2001).

[18] A. S. Eddington, *The Mathematical Theory of Relativity*, Cambridge University Press (1923).

[19] I. I. Shapiro, in *General Relativity and Gravitation 12* (1990), ed. N. Ashby, D. F. Bartlett, and W. Wyss (Cambridge University Press), p. 313.

[20] T. M. Eubanks *et al.*, Bull. Am. Phys. Soc., Abstract No. K 11.05 (1997), unpublished; draft available at `ftp://casa.usno.navy.mil/navnet/postscript/prd_15.ps` (1999); S. S. Shapiro *et al.*, Phys. Rev. Lett. **92**, 121101 (2004).

[21] B. Bertotti, L. Iess, and P. Tortora, Nature **425**, 374 (2003).

[22] S. G. Turyshev *et al.*, gr-qc/0506104.

[23] C. D. Hoyle *et al.*, Phys. Rev. **D70**, 042004 (2004), hep-ph/0405262.

[24] H. van Dam and M. J. Veltman, Nucl. Phys. **B22**, 397 (1970); V. I. Zakharov, Sov. Phys. JETP Lett. **12**, 312 (1970).

[25] C. Deffayet, Phys. Rev. **D71**, 103501 (2005), gr-qc/0412114.

[26] C. Talmadge *et al.*, Phys. Rev. Lett. **61**, 1159 (1988).

[27] L. S. Finn and P. J. Sutton, Phys. Rev. **D65**, 044022 (2002), gr-qc/0109049; P. J. Sutton and L. S. Finn, Class. Quant. Grav. **19**, 1355 (2002), gr-qc/0112018.

[28] S. L. Larson and W. A. Hiscock, Phys. Rev. **D61**, 104008 (2000), gr-qc/9912102; C. Cutler, W. A. Hiscock, and S. L. Larson, Phys. Rev. **D67**, 024015 (2003), gr-qc/0209101; A. Cooray and N. Seto, Phys. Rev. **D69**, 103502 (2004), astro-ph/0311054.

[29] C. M. Will, Phys. Rev. **D57**, 2061 (1998), gr-qc/9709011.

[30] P. D. Scharre and C. M. Will. Phys. Rev. D **65**, 042002 (2002); C. M. Will and N. Yunes, Class. Quantum Grav. **21**, 4367 (2004).

[31] E. Berti, A. Buonanno, and C. M. Will, Phys. Rev. **D71**, 084025 (2005), gr-qc/0411129.

[32] T. Damour, in *Gravitation and Quantizations*, ed. B. Julia and J. Zinn-Justin, Les Houches, Session LVII (Elsevier, Amsterdam, 1995), pp. 1–61.

[33] C. Eling, T. Jacobson, D. Mattingly, gr-qc/0410001; B. Z. Foster and T. Jacobson, gr-qc/0509083.

[34] T. Damour and G. Esposito-Farèse, Class. Quant. Grav. **9**, 2093 (1992).

[35] P. G. Bergmann, Int. J. Theor. Phys. **1**, 25 (1968); K. Nordtvedt, Astrophys. J. **161**, 1059 (1970); R. Wagoner, Phys. Rev. D **1**, 3209 (1970).

[36] T. Damour and G. Esposito-Farèse, *Phys. Rev. Lett.* **70**, 2220 (1993); *Phys. Rev. D* **54**, 1474 (1996), gr-qc/9602056; *Phys. Rev. D* **58**, 042001 (1998), gr-qc/9803031.

[37] J. M. Weisberg and J. H. Taylor, *ASP Conference Proceedings* **302**, 93 (2003), astro-ph/0211217.

[38] M. Bailes *et al.*, *Astrophys. J.* **595**, L49 (2003), astro-ph/0307468.

[39] I. Stairs *et al.*, *Astrophys. J.* **581**, 501 (2002), astro-ph/0208357.

[40] M. Burgay *et al.*, *Nature* **426**, 531 (2003), astro-ph/0312071; A. G. Lyne *et al.*, *Science* **303**, 1153 (2004), astro-ph/0401086; M. Kramer *et al.*, eConf **C041312**, 0038 (2004), astro-ph/0503386.

[41] D. M. Eardley, Astrophys. J. **212**, L91 (1977).

[42] C. M. Will and H. W. Zaglauer, Astrophys. J. **346**, 366 (1989).

[43] G. Esposito-Farèse, gr-qc/0402007, to be published in the proceedings of the tenth Marcel Grossmann Meeting, July 2003; T. Damour and G. Esposito-Farèse, in preparation.

[44] S. G. Turyshev, M. M. Nieto, and J. D. Anderson, Amer. J. Phys. **73**, 1033 (2005), physics/0502123; H. Dittus *et al.*, gr-qc/0506139; S. G. Turyshev, M. M. Nieto, and J. D. Anderson, gr-qc/0510081.

[45] B. Boisseau, G. Esposito-Farèse, D. Polarski, and A. A. Starobinsky, Phys. Rev. Lett. **85**, 2236 (2000).

[46] G. Esposito-Farèse and D. Polarski, Phys. Rev. D **63**, 063504 (2001).

[47] V. C. Rubin, N. Thonnard, and W. K. Ford, Astrophys. J. **225**, L107 (1978).

[48] M. Milgrom, Astrophys. J. **270**, 365 (1983).
[49] R. H. Sanders and S. S. McGaugh, Ann. Rev. Astron. Astrophys. **40**, 263 (2002), astro-ph/0204521.
[50] R. B. Tully and J. R. Fisher, Astron. Astrophys. **54**, 661 (1977).
[51] J. D. Bekenstein (2004), astro-ph/0412652; J. D. Bekenstein, Phys. Rev. **D70**, 083509 (2004), astro-ph/0403694.
[52] R. H. Sanders, astro-ph/0502222.
[53] C. Skordis *et al.*, astro-ph/0505519.