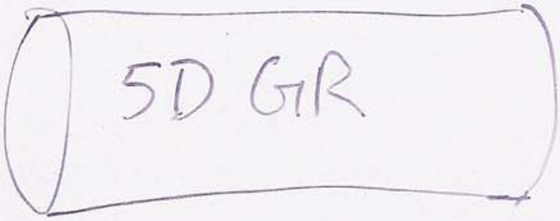
XD General Relativity (GR



$$\eta_{MN} \longrightarrow G_{MN}(x^\mu, \theta)$$

$$\Big\downarrow\, E \ll 1/R$$

$$G^{(0)}_{MN}(x)/\sqrt{2\pi R}$$

4D Massless fields on plugging into 5D Einstein action

$$\equiv \underbrace{\frac{G^{(0)}_{\mu\nu}(x)}{\sqrt{2\pi R}}}_{\text{effective 4D metric}}, \quad \underbrace{\frac{G^{(0)}_{\mu 5}(x)}{\sqrt{2\pi R}}}_{\text{4D vector}}, \quad \underbrace{\frac{G^{(0)}_{55}(x)}{\sqrt{2\pi R}}}_{\text{4D scalar}}$$

4D vector $\equiv$ 4D gauge field for KK charge n.

Can this K.K mechanism be origin of SM gauge fields?

Idea has not met with success in field theory, but mechanism can be seen to survive in construction of heterotic string theory (with some stringy enhancements).

4D Gravity: $G_{N\,4D} = G_5/2\pi R$

The RADION

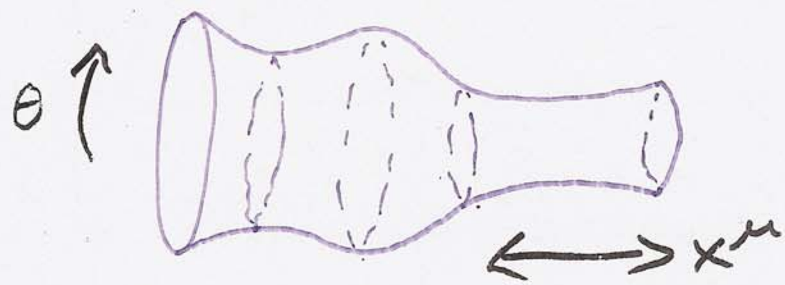
4D scalar $\equiv G_{55}^{(0)}(x)$ has no potential

(all terms in 5D Einstein action $\ni \partial_\mu$ or $\partial_5 G_{MN}^{(0)}(x) \to 0$)

Geometrically, these fluctuations $\equiv$

$$ds^2 \ni G_{55}(dx^5)^2 = G_{55} R^2 d\theta^2$$

$\Rightarrow$ $G_{55}^{(0)}(x) \equiv$ dynamical XD radius



$\therefore$ $\langle R_{physical}^{(x)} \rangle$ undetermined, & yet different VEVs are physically distinct

$\equiv \langle \sqrt{G_{55}} \rangle R$

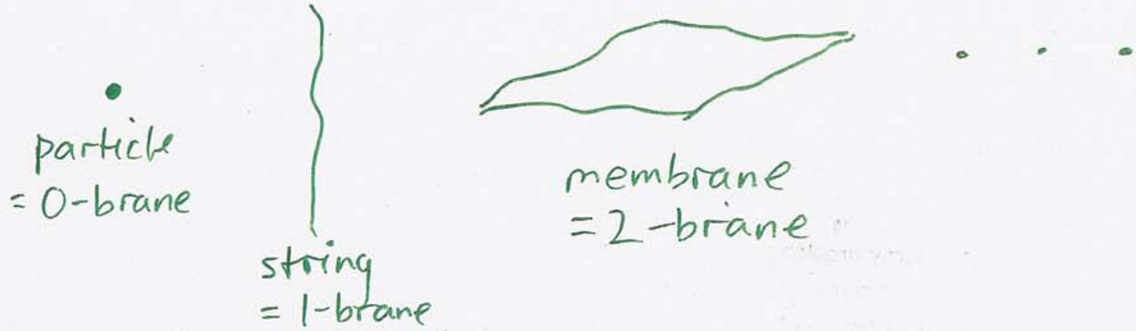
Eg. Adding 5D scalar,

$$\lambda_{4\,\text{eff}} = \frac{\lambda_5}{2\pi \langle R_{phys} \rangle} \quad \text{undetermi}$$

Such massless scalars are MODULI & greatly limit predictivity until VEVs determined.

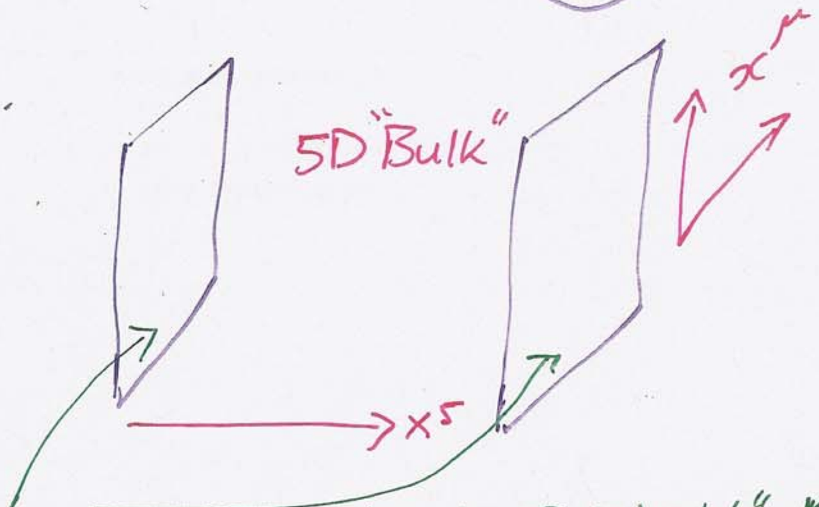
BRANES

≡ localized defects in higher dimensions



Eg. Suppose XD

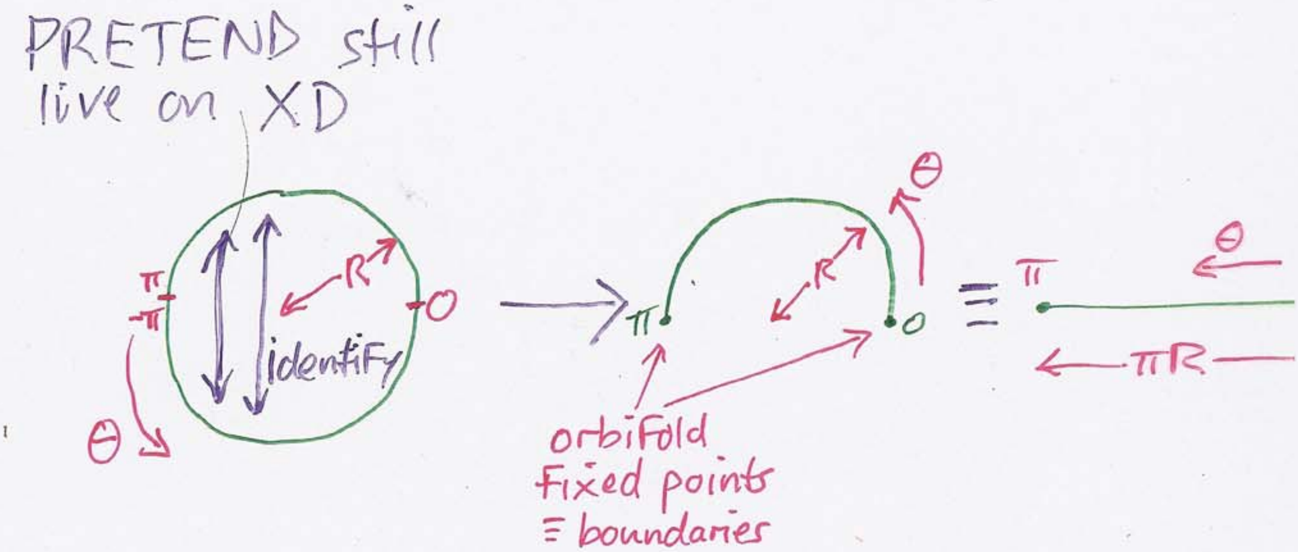
ie.



Boundaries ≡ "End-of-World" 3-branes

5D fields must ∴ have boundary conditions prescribed.

Orbifold boundary conditions



$$\phi(x,\theta) = (-1)^P \phi(x,-\theta)$$

orbifold parity respected by action

Brane localized interactions

allow different dynamics at branes than in bulk.

Eg. $S = \int d^4x \int_{-\pi}^{\pi} d\theta\, R \left\{ \frac{1}{2}(\partial_M \phi)^2 - \frac{m_5^2}{2}\phi^2 - K\phi\,\delta(x_5) - K\phi\,\delta(x_5 - \pi R) \right\}$

$\Rightarrow$ Equations $-\partial_\mu \partial^\mu \phi + \frac{\partial_\theta^2}{R^2}\phi - m_5^2 \phi = \frac{K\delta(\theta)}{R} + \frac{K\delta(\theta-\pi)}{R}$

$\Rightarrow$ Non-trivial Vacuum

$$\langle \phi(x,\theta) \rangle = \bar{\phi}(\theta),$$

$$\frac{1}{R^2}\partial_\theta^2\bar{\phi} - m_5^2\bar{\phi} = \frac{\kappa\,\delta(\theta)}{R} + \frac{\kappa\,\delta(\theta-\pi)}{R}$$

Ansatz: $\bar{\phi} = A\left[e^{m_5 R|\theta|} + e^{m_5 R(\pi-|\theta|)}\right]$

N.B. correct symmetries & 2π-periodicity.

Matching δ-functions,

$$2A\left(1-e^{m_5 R\pi}\right)\frac{m_5}{R} = \frac{\kappa}{R}$$

Plugging $\bar{\phi}$ back into action,

$$S = \int d^4x \int_{-\pi}^{\pi} d\theta\, R\left\{-\frac{\kappa}{2}\bar{\phi}\frac{\delta(\theta)}{R} - \frac{\kappa}{2}\bar{\phi}\frac{\delta(\theta-\pi)}{R}\right.$$

$$= -\int d^4x\, \frac{\kappa^2}{2m_5}\,\frac{1+e^{m_5\pi R}}{1-e^{m_5\pi R}}$$

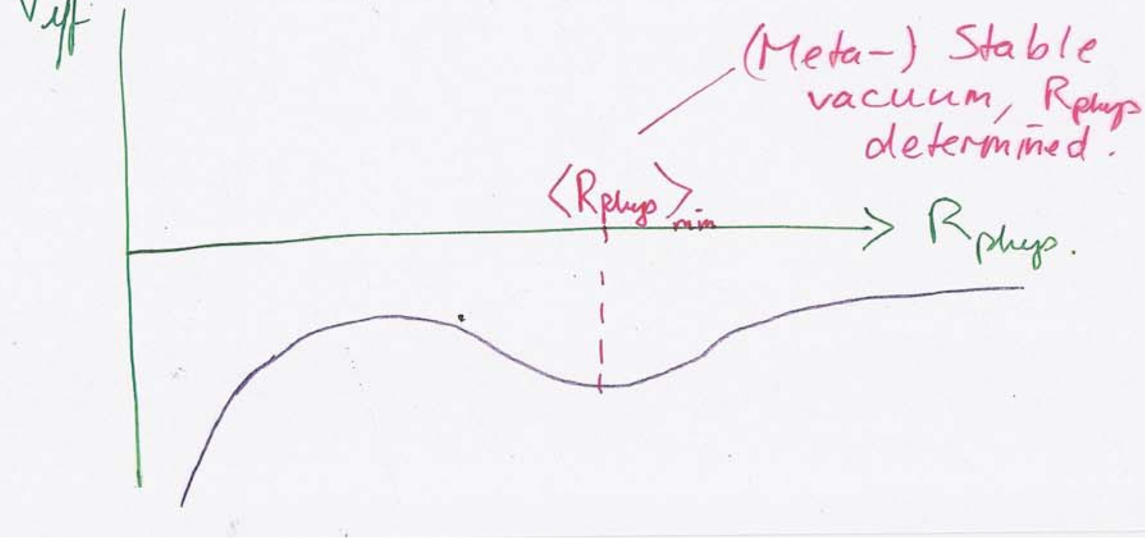
Looks like irrelevant constant, BUT with 5D GR added ...

RADION POTENTIAL

$$V_{eff}\left(R_{phys}^{(x)}\right) \equiv \frac{\kappa^2}{2m_5}\,\frac{1+e^{m_5 \pi R_{phys}}}{1-e^{m_5 \pi R_{phys}}}$$

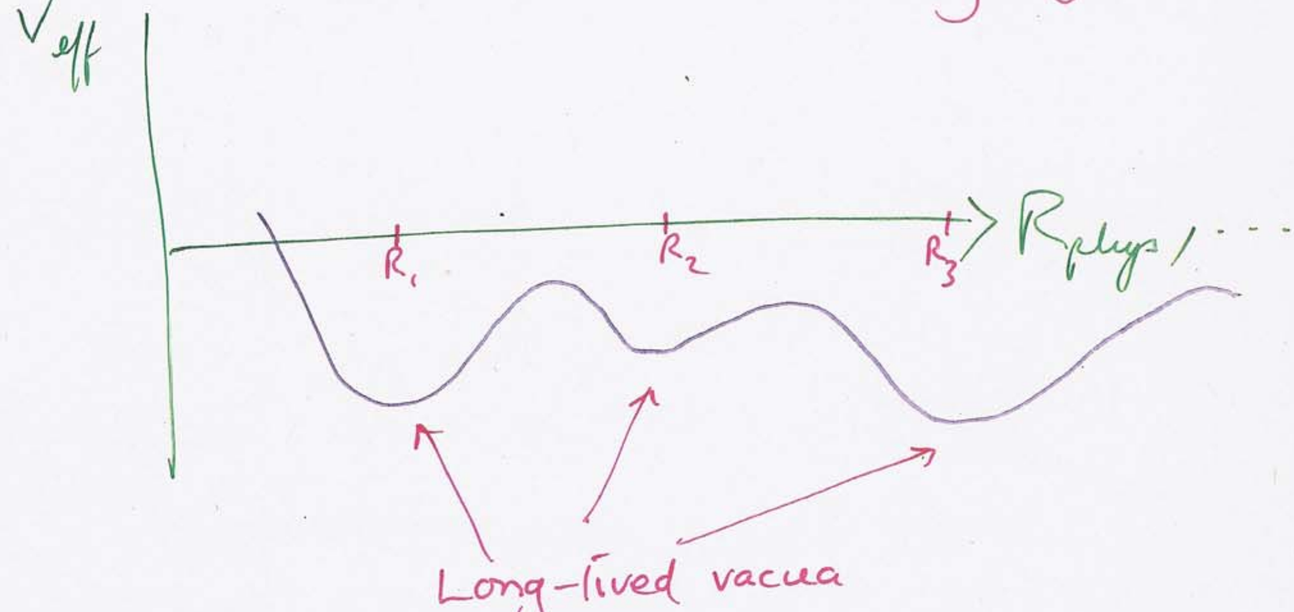
Adding contribution of a second scalar, with opposite sign but equal magnitude brane terms,

$$V_{eff}\left(R_{phys}^{(x)}\right) = \frac{\kappa^2}{2m_5}\,\frac{1+e^{m_5 \pi R_{phys}}}{1-e^{m_5 \pi R_{phys}}} + \frac{\tilde{\kappa}^2}{2\tilde{m}_5}\,\frac{1-e^{\tilde{m}_5 \pi R_{phys}}}{1+e^{\tilde{m}_5 \pi R_{phys}}}$$



THE "LANDSCAPE" OF XD SHAPES & SIZES

Consider several stabilizing agents



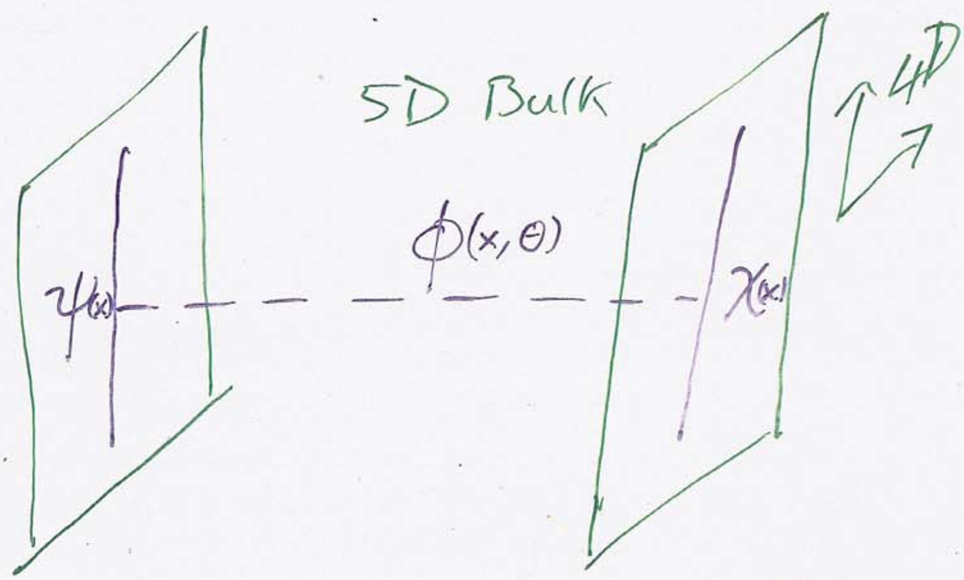
$\Rightarrow$ v. different possible low-energy worlds

$$\lambda_{4\,eff}(R_i)$$
??

String theory has run into this loss of predictivity.

BRANEWORLD

Some degrees of freedom may reside (localized) entirely on the defects,



Eg. ψ = Standard Model

$\phi \equiv$ gravity

$\chi \equiv$ "Hidden/Dark" sector.

Large Extra Dimensions

Arkani-Hamed, Dimopoulos, Dvali '98

Gravity appears v. weak because of XD "dilution"

$$G_{N_{4d}} \sim \frac{G_{\text{higher dimensions}}}{(2\pi R)^n}$$

rough generalization to arbitrary number of XD

Perhaps $G_{\text{Higher}} \sim G_{\text{Fermi}}$ & gravity strongly coupled above weak scale!

PARALLEL UNIVERSE FORM-FACTORS

Arkani-Hamed, Schmaltz ~ '99 -'00

Consider $\underbrace{\psi + \chi \xrightarrow[\phi\text{-exchange}]{} \psi + \chi}_{\text{all massless}}$

$\propto$ ϕ-propagator (spacelike 4-momentum, Δx_5 = brane separation)

$$\propto \sum_{p_5 = \frac{n}{R}} \frac{e^{i p_5 \pi R}}{\underbrace{p_\mu p^\mu}_{<0} - p_5^2 + i\varepsilon}$$

$e^{i p_5 \pi R}$ ← ie ± 1

← neglecting orbifold boundary conditions for simplicity

assuming no brane-localized terms for ϕ alone.

$$\propto \int_{\mathcal{C}} dp_5 \, \frac{e^{i p_5 \pi R}}{(p_5 + i\sqrt{-p^2})(p_5 - i\sqrt{-p^2})} \, \frac{1}{e^{2 i p_5 \pi R} - 1}$$

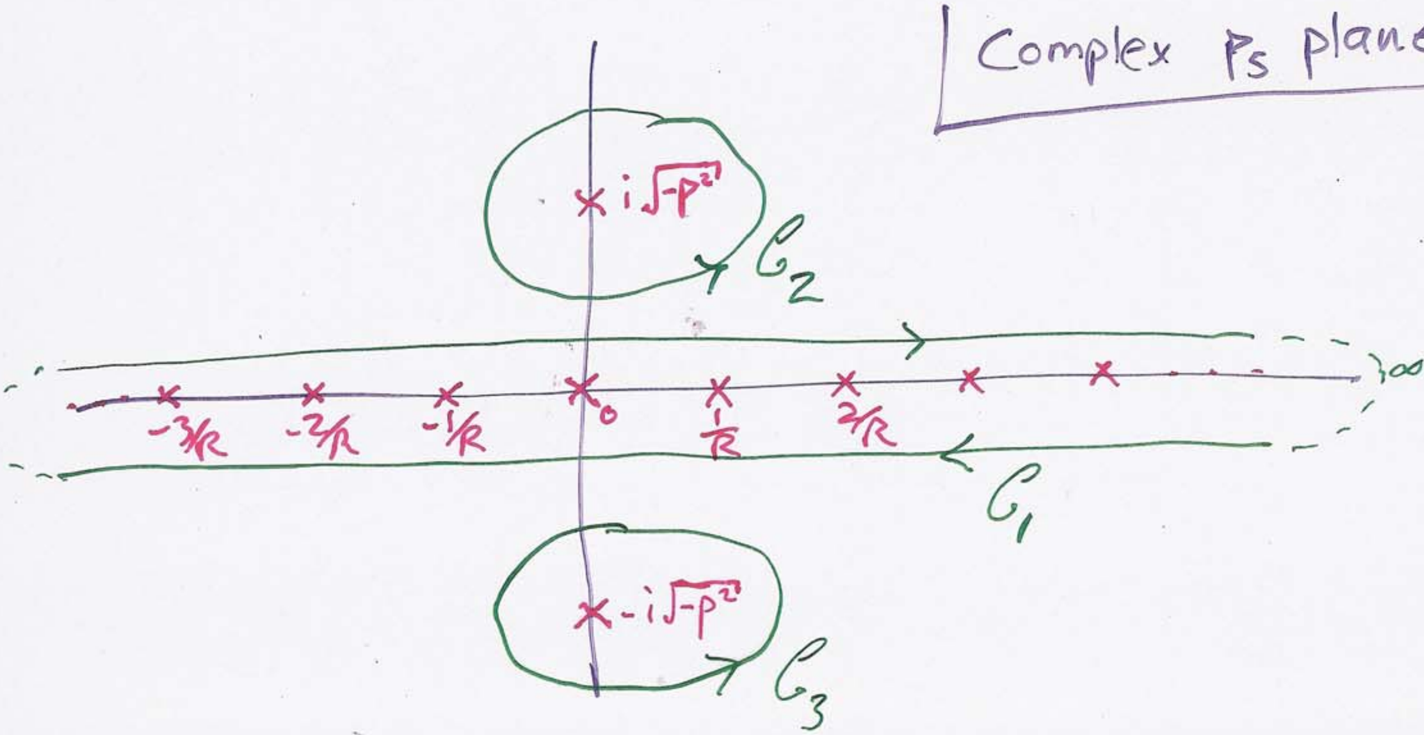
$$\propto \int_{\mathcal{C}_2+\mathcal{C}_3} dp_5 \; \frac{e^{ip_5\pi R}}{\left(p_5+i\sqrt{-p^2}\right)\left(p_5-i\sqrt{-p^2}\right)} \; \frac{1}{e^{2ip_5\pi R}-1}$$

$$\propto \frac{e^{+\sqrt{-p^2}\,\pi R}}{\sqrt{-p^2}\left[e^{2\sqrt{-p^2}\,\pi R}-1\right]}$$

$\overbrace{-p^2 \gg 1/R}$ $\qquad \dfrac{e^{-\sqrt{-p^2}\,\pi R}}{\sqrt{-p^2}}$ Weird form-factor.

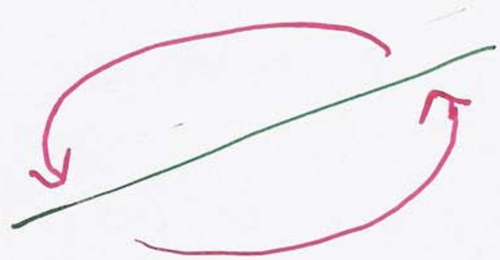
$\overbrace{-p^2 \ll 1/R}$ $\qquad \dfrac{1}{p^2}$ Usual 4D scattering

Complex p_5 plane



When is it XD?

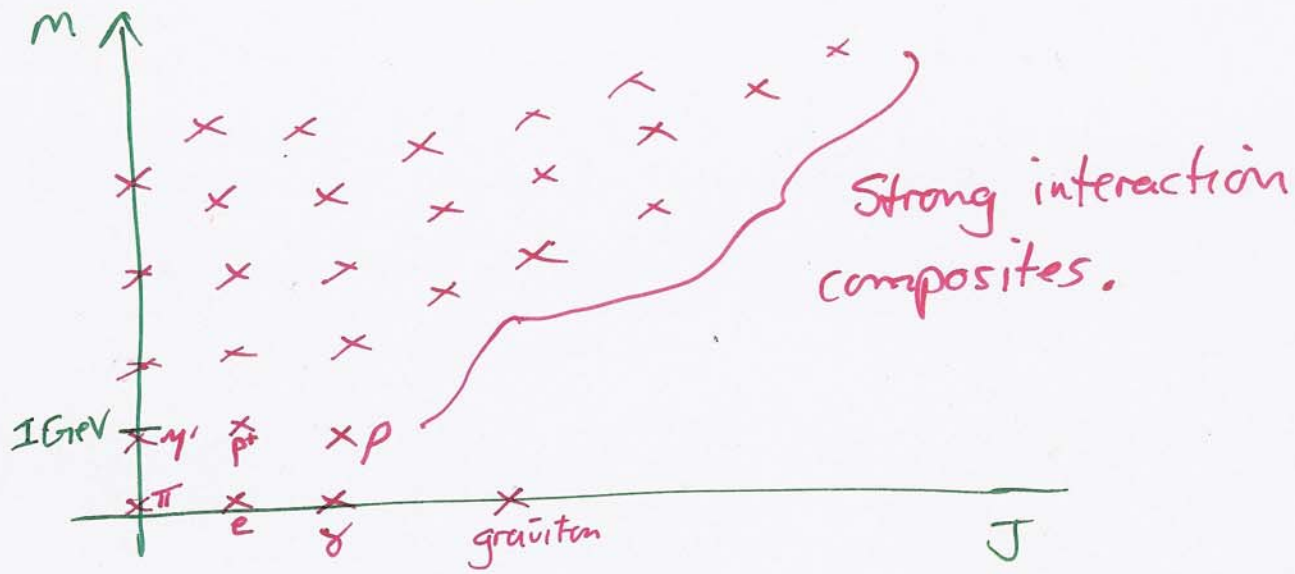
The only known UV completion of XD and/or GR (preserving extended spacetime symmetry in UV) is string theory. When stringiness relevant, there are Regge trajectories (proliferation of high spin excitation.



open string stretched by its orbital motion.

∴ look for this:

THE PARTICLE DATA BOOK



How to distinguish XD from discovery of new strong interaction ~~scale (in sense~~ composites of new high scale variant of QCD ("Technicolor" in its loosest sense)

(i) XD can be weakly coupled (if $m_{string} < \Lambda_{strong}$), but new strong dynamics will be "strong".

(ii) In XD spectrum, high spins appear after many low spin ~~[illegible]~~ recurrences.

(iii) At v. high energies strong dynamics becomes asymptotically free, revealing constituent partons.

BUT

(i) In gauge theories with $N_{color} \gg 1$ the strong dynamics can just confine quarks into hadrons, the residual interactions between hadrons can be $\sim 1/N_{color}$.

(ii) There are strongly interacting 4D dynamics where high spins are delayed to much higher mass than low spin recurrences.

(iii) There are strongly interacting 4D theories where coupling does not run & remains strong at high energy (conformal field theories).

In fact there are examples where

Weakly-coupled XD theory $\equiv$ Strongly-coupled 4D gauge theory

in every physical consequence

Eg. see Strassler TASI lectures

Analogy: Wave-Particle duality in quantum mechanics

- Central module of such "duality" is the AdS_{5D}/CFT_{4D} Correspondence.

"Anti-de Sitter"

review Aharony, Gubser, Maldacena, Ooguri, Oz '99

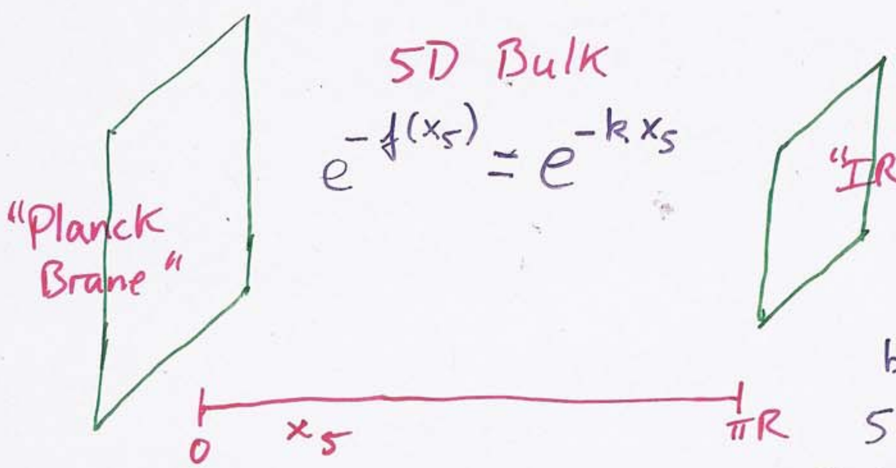
- The catch in this duality is that

$$\eta_{MN} \longrightarrow \begin{pmatrix} \eta_{\mu\nu}\, e^{-2f(x_5)} & 0 \\ 0 & -1 \end{pmatrix}$$

"warp factor" preserves 4D Poincaré symmetry

which makes calculations harder.

- Simplest warped compactification, used in phenomenological effective field theorie



$$e^{-f(x_5)} = e^{-k x_5}$$

Randall-Sundrum I (
background solves 5D Einstein Equations supplemented with suitable bulk + brane cosmological const

String realization:

Analogy — Inflating universe (dS spacetime)

$$\eta_{\mu\nu} \longrightarrow g_{\mu\nu} = \begin{pmatrix} +1 & 0 \\ 0 & -e^{Ht}\delta_{ij} \end{pmatrix}$$

"scale factor"

solve 4D Einstein Equations with positive cosmological constant. Space inflates in time.

ie. $AdS_5 \equiv$ "inflation" of 4D spacetime in XD

KK decomposition of bulk fields is more subtle (not sinusoidal).

General features:

- Low-lying KK excitations localize near IR brane
- Lowest mode shape is v. sensitive to boundary conditions & 5D spin, mass