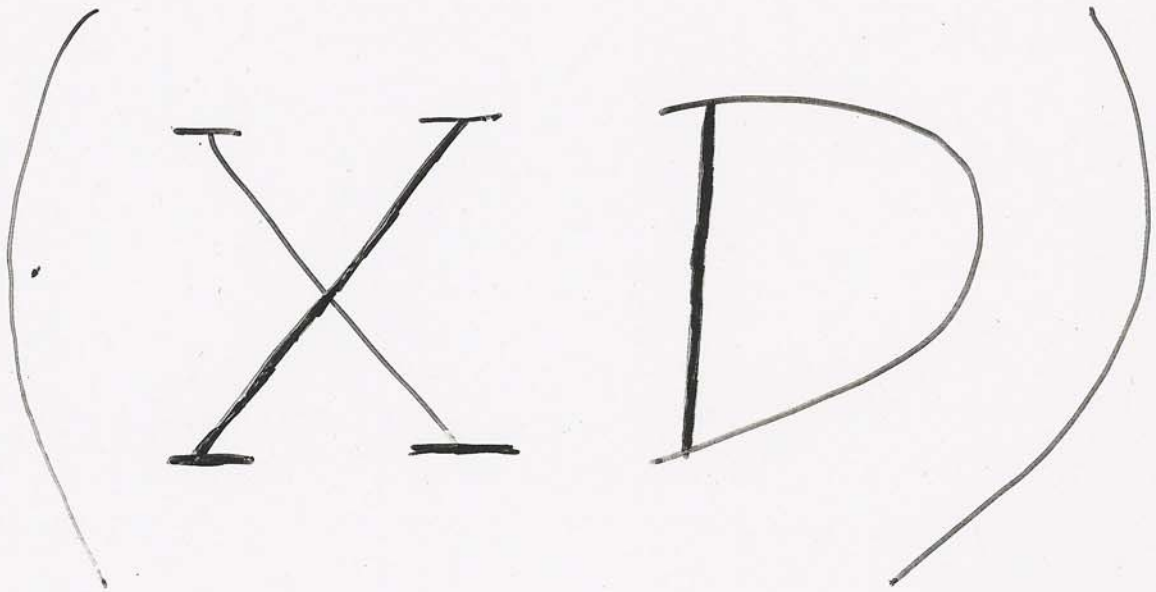
EXTRA DIMENSIONS



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OUTLINE

I FUNDAMENTALS

Why XD?

Hiding & Seeking XD

XD interactions & UV breakdown

Unification of spins
- Hierarchy Problem.

II AERIAL RECONNAISANCE

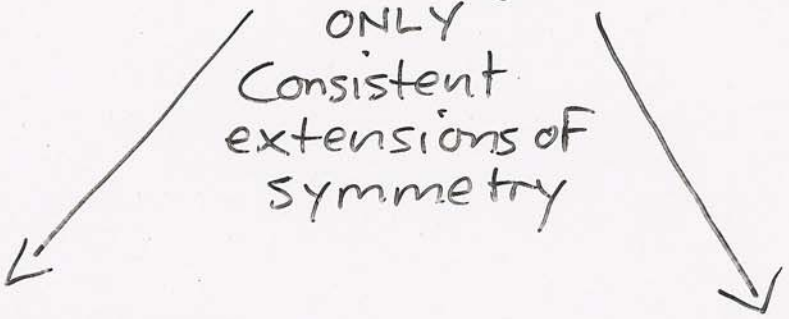
III FROM THE TRENCHES

WHY XD?

1) Spacetime Symmetry

Poincare Group $\ni$ Translations, Rotations, Boosts

+ new degrees of Freedom



Supersymmetry
relates particles
with Δspin $= \frac{1}{2}$

XD
relates particles
with Δspin $= 1$

(& extended symmetry)

New degrees of freedom must be hidden at present energies but revealed in UV.

Transition scale unknown.

2) These extensions naturally $\Rightarrow$ interesting new mechanisms,

perhaps essential to our understanding of the World.

3) String Theory (only well-developed framework reconciling gravity, relativity, QM) requires these extensions at highest energies.

4) Weakly-coupled phenomena in XD sometimes resemble strong-coupling phenomena (eg. QCD) without XD!

Sometimes,

Weakly-coupled XD theory
$\cong$ Strong-coupled theory without XD!

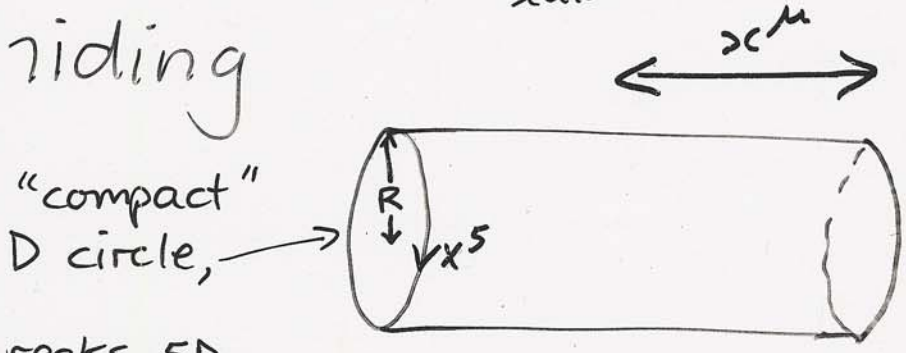
5D scalar field theory

$$X^M = x^{\mu=0,1,2,3}, x^5, \quad \text{metric } \eta_{MN} = \begin{pmatrix} 1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & -1 \end{pmatrix}$$

$$\mathcal{L}_{5D} = \frac{1}{2}\partial_M \phi \partial^M \phi - \frac{m_5^2}{2}\phi^2 - \lambda_5 \phi^4$$

(real scalar)

riding



reaks 5D
orentz invariance "softly"
For distances $> R$

$$x^5 \equiv R\theta, \quad -\pi \le \theta \le \pi$$

$$\phi(x^\mu, \theta) = \frac{1}{\sqrt{2\pi R}}\left\{\phi_0(x) + \sum_{n=1}^{\infty}\left(\phi_n(x) e^{in\theta} + \text{c.c.}\right)\right\}$$

(ϕ_0: real; ϕ_n: complex)

$$S = \int d^4x \int_{-\pi}^{\pi} d\theta\, R \left\{\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2R^2}(\partial_\theta \phi)^2 - \frac{m_5^2}{2}\phi^2 - \lambda_5 \phi^4\right\}$$

$$= \int d^4x \; \frac{1}{2}(\partial_\mu \phi_0)^2 - \frac{m_5^2}{2}\phi_0^2$$

$$+ \sum_{n=1}^{\infty}\left[|\partial_\mu \phi_n|^2 - \left(\frac{n^2}{R^2} + m_5^2\right)|\phi_n|^2\right]$$

$$- \frac{\lambda_5}{2\pi R}\sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty}\sum_{n_3=-\infty}^{\infty}\sum_{n_4=-\infty}^{\infty}\underbrace{\delta_{n_1+n_2+n_3+n_4,0}}_{\substack{\text{conservation} \\ \text{of XD angular momentum}}}\phi_{n_1}\phi_{n_2}\phi_{n_3}\phi_{n_4}$$

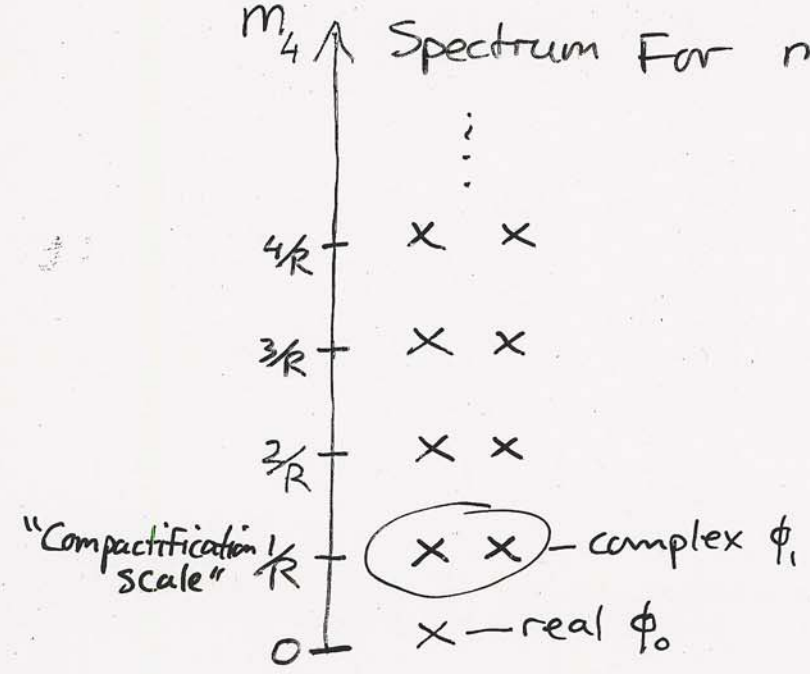
where $\phi_{n<0}(x) \equiv \phi^*_{-n}(x)$ for convenience.

$\therefore \phi(x,\theta) \equiv \infty$ 4D fields, mostly very massive,

$$m_n^2 = \frac{n^2}{R^2} + m_5^2 \quad n>0 \equiv$$

"Kaluza-Klein (KK) excitations"

XD symmetry of circle $\xrightarrow[\text{"KK" decomposition}]{4D}$ internal symmetry, ϕ_n having charge n.



4D effective field theory (EFT) ?

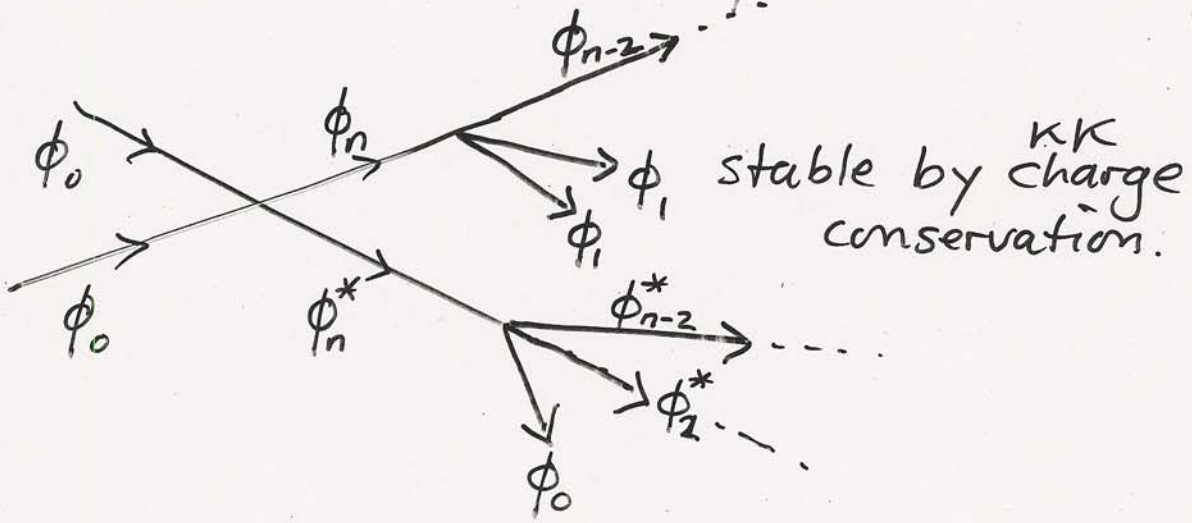
$$\mathcal{L}_{4D\,eff} = \tfrac{1}{2}(\partial_\mu \phi_0)^2 - \frac{m_{4\,eff}^2}{2}\phi_0^2 - \lambda_{4\,eff}\phi_0^4 + \mathcal{O}(E^2R^2)$$

$$m_{4\,eff}^2 = m_5^2$$

$$\lambda_{4\,eff} = \frac{\lambda_5}{2\pi R} + \text{loop-level corrections}$$

$[\text{length}] = [1/\text{Energy}]$

Collider Discovery, $E \gg 1/R$.

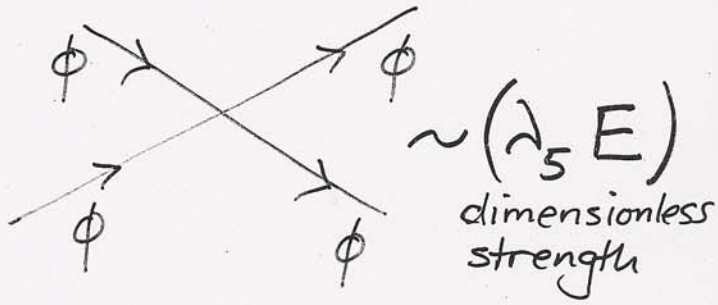


KK stable by charge conservation.

XD misbehave in UV

In UV, R irrelevant:

5D Language:



Perturbation theory, & any sensible interpretation, breaks down when

$$\frac{\lambda_5 E}{16\pi^2} \gg 1$$

← phase space considerations

unless theory drastically modified in this regime.

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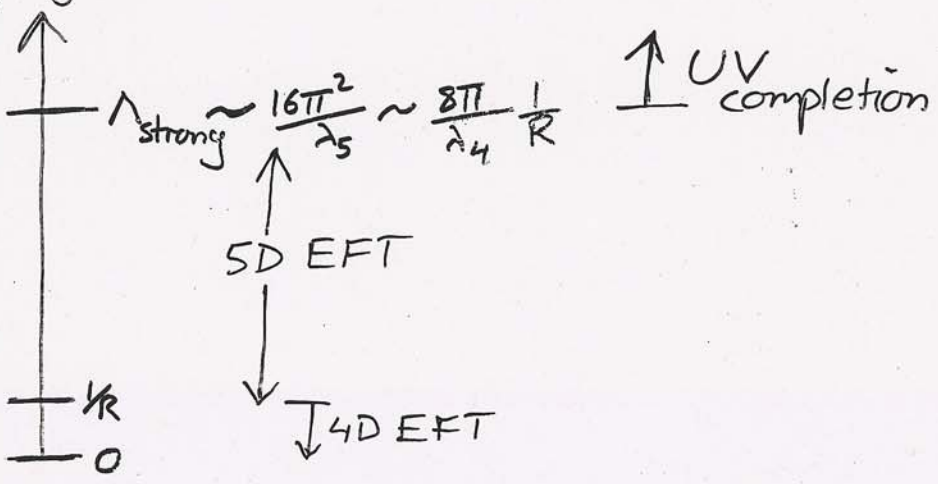


Unifying Spins

5D SU(2) Yang-Mills compactified

$$\mathcal{L}_5 = -\frac{1}{4} G^a_{MN} G^{MN}_{\substack{a=1,2,3 \\ \text{isospin}}}$$

$$= -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a - \frac{1}{2} G_{\mu 5} G^{\mu 5},$$

$$G^a_{MN} \equiv \partial_M A^a_N - \partial_N A^a_M - g_5 \varepsilon^{abc} A^b_M A^c_N$$

4D low-energy fluctuations:

$$A_M(x,\theta) = \frac{A^{(0)}_M(x)}{\sqrt{2\pi R}} + \cancel{\frac{\left(A^{(n)}_M(x) e^{in\theta} + c.c.\right)}{\sqrt{2\pi R}}}$$

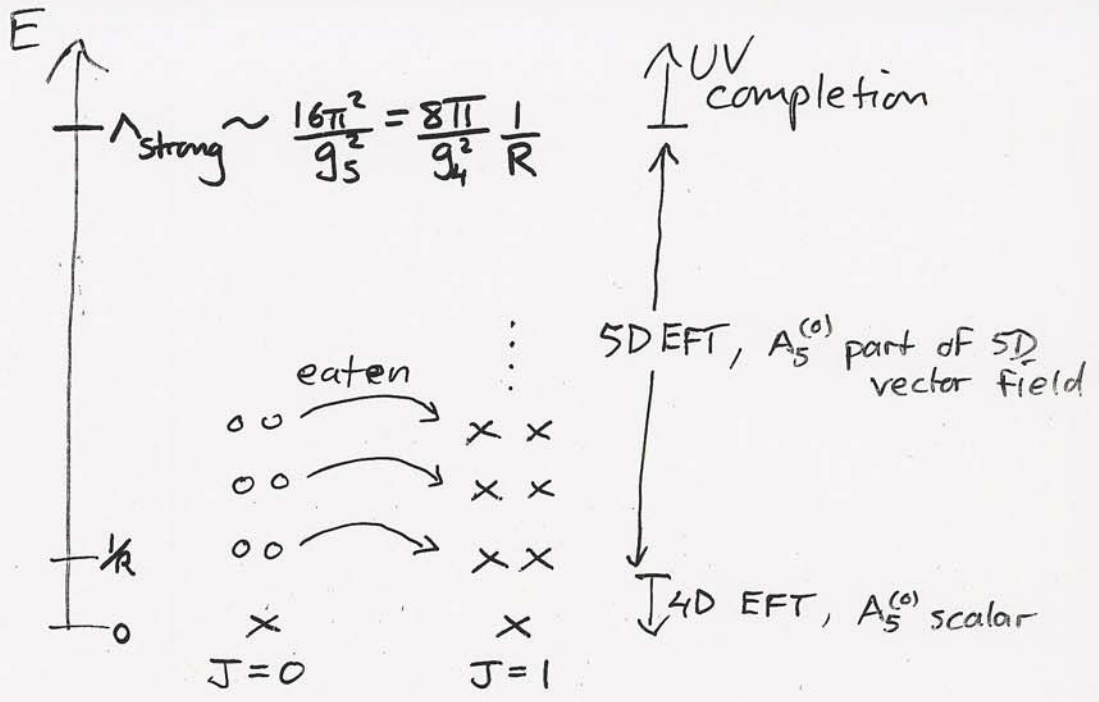
$$\Rightarrow G^a_{\mu\nu} = \partial_\mu A^{(0)}_\nu - \partial_\nu A^{(0)}_\mu - \frac{g_5}{\sqrt{2\pi R}} \varepsilon^{abc} A^b_\mu A^c_\nu$$

$$G^a_{\mu 5} = \partial_\mu A^{(0)a}_5 - \frac{g_5}{\sqrt{2\pi R}} \varepsilon^{abc} A^{(0)b}_\mu A^{(0)c}_5 \quad \leftarrow \text{4D scalar!}$$

$$\equiv D_\mu A^{(0)}_5$$

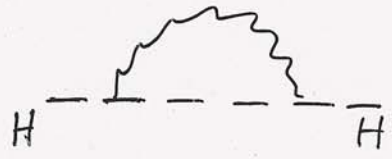
$$\Rightarrow \mathcal{L}_{4\text{eff}} = -\frac{1}{4} G^{(0)a}_{\mu\nu} G^{\mu\nu(0)}_a + \frac{1}{2}\left(D_\mu A^{(0)}_5\right)^2$$

$$g_{4\text{eff}} \equiv \frac{g_5}{\sqrt{2\pi R}}$$



Usual Hierarchy Problem of 4D scalars (Higgs.

J=0:



$\Rightarrow \delta_{QM} m_H^2 \sim \frac{g_4^2}{16\pi^2} \Lambda_{UV}^2$

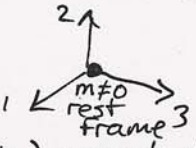
UV cutoff on calculation

$\Rightarrow$ theory cannot naturally be extrapolated far above m_H^2.

J=1: Why γ travels at speed of light ($m_\gamma = 0$)

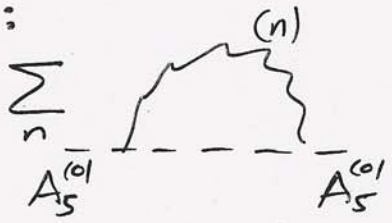
J=1, m=0 has 2 polarizations

J=1, m≠0 has 3 polarizations



Discrete qualitative effect (Higgs mechanism = superconductivity) need to give mass & add degree of freedom.

$A_5^{(0)}$ From XD:



$$\Rightarrow \delta_{QM} m^2_{A_5^{(0)}} \sim \frac{g_4^2}{16\pi^2}\left(\frac{1}{R}\right)^2$$

because for $p_{loop} \gg 1/R$,
5D Lorentz invariance recovered which protects A_M-mass.

XD naturally allow greater separation of scales

$$\Lambda_{UV}\left(\begin{matrix}\text{unknown,}\\ \text{dangerous}\\ \text{physics}\end{matrix}\right) : m_H\left(\sim \begin{matrix}\text{weak}\\ \text{scale}\end{matrix}\right)$$

Usual 4D scalar: $\frac{\Lambda_{UV}}{m_H} \underset{\text{naturally}}{\sim} \frac{4\pi}{g_4}$

$A_5^{(0)}$ scalar: $\frac{\Lambda_{strong}}{m^{(0)}_{A_5}} \underset{\text{naturally}}{\sim} \underbrace{\left(\frac{8\pi}{g_4^2}\right)}_{\text{Winnings}}\left(\frac{4\pi}{g_4}\right)$

[Winnings are lower when all realistic properties of Higgs realized, but still useful.]