Correlation Matrix Method for Pb/Scint Sampling Calorimeter

N.Nakajima, H.Miyata, A.Sanchez, H.Ono, S.Iba
Niigata University
For GLD Calorimeter Group

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1. Introduction

We study the electromagnetic and hadronic cascade shower fluctuations.

The correlation matrix was used for understanding the property of the fluctuation.

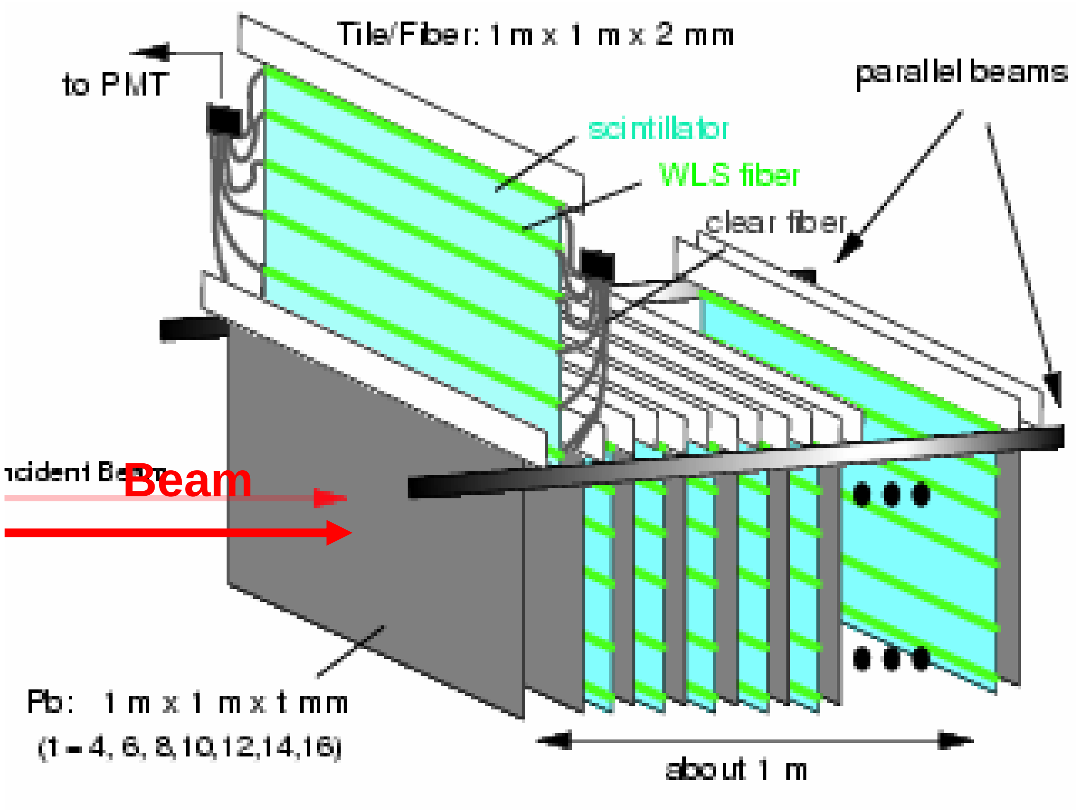
The objectives of this study are as follows;

1) To understand the behavior of EM and hadron shower fluctuations
2) To find a method to improve energy resolution by using the result of the fluctuation study

2. Experiment

We use the data of the beam test of GLD calorimeter prototype.

Calorimeter module for the beam test



T411Beamtest@KEK in 1997

$$1 \sim 4GeV \quad e^-, \pi^-$$

Sampling Calorimeter

1layer =

Pb 4mm+ Scinti 2mm

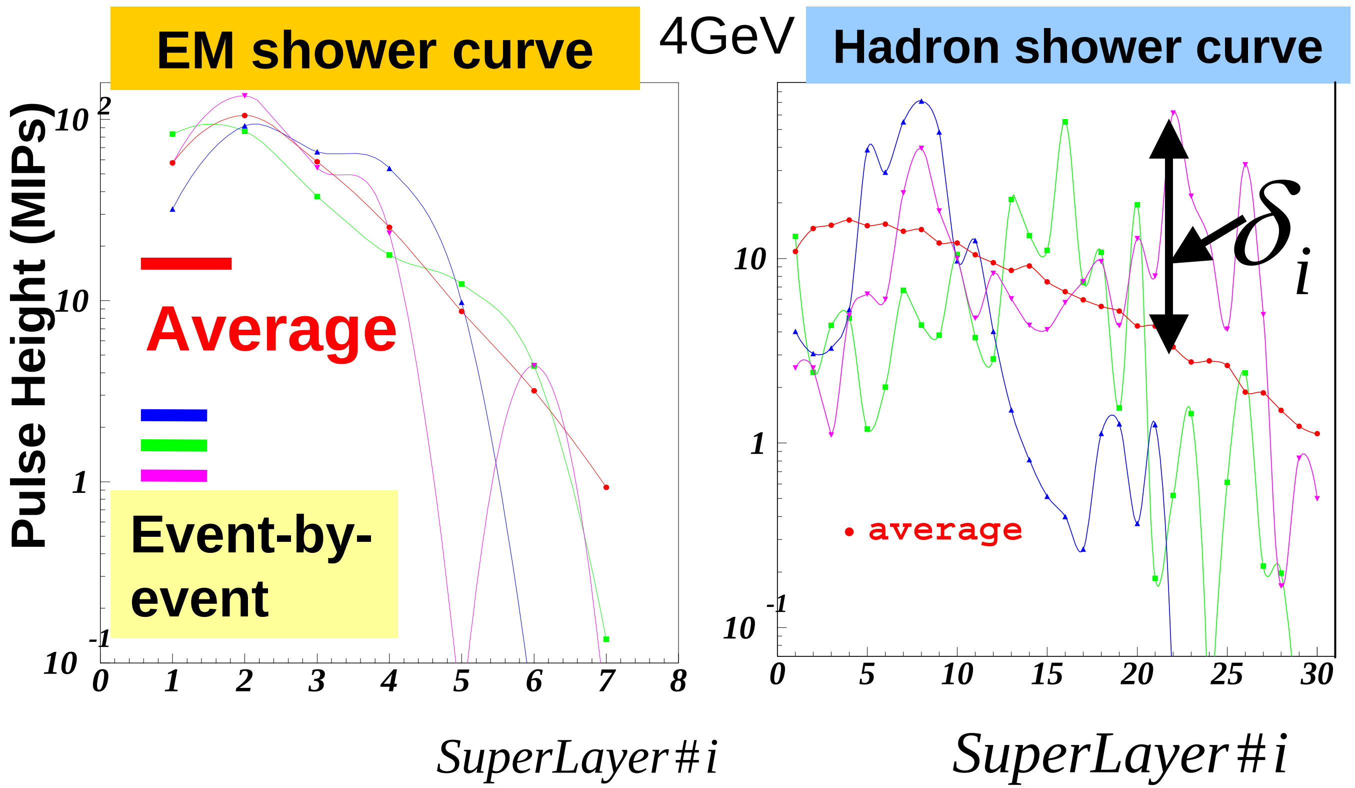
Read out :

1 Super layer = 5Layers

Number of Super Layers:

42 (9.6 λ_I)

3. Correlation Matrix and Fluctuation



Event-by-event shower fluctuation is much larger for hadron.

"Correlation matrix" represents the strength of the correlation between pulse height deviations at two depths. We use this matrix as a tool to investigate the shower fluctuation.

PH deviation is defined as

$$\delta_i^{(k)} \equiv p_i^{(k)} - \langle p_i \rangle$$

k : Event #

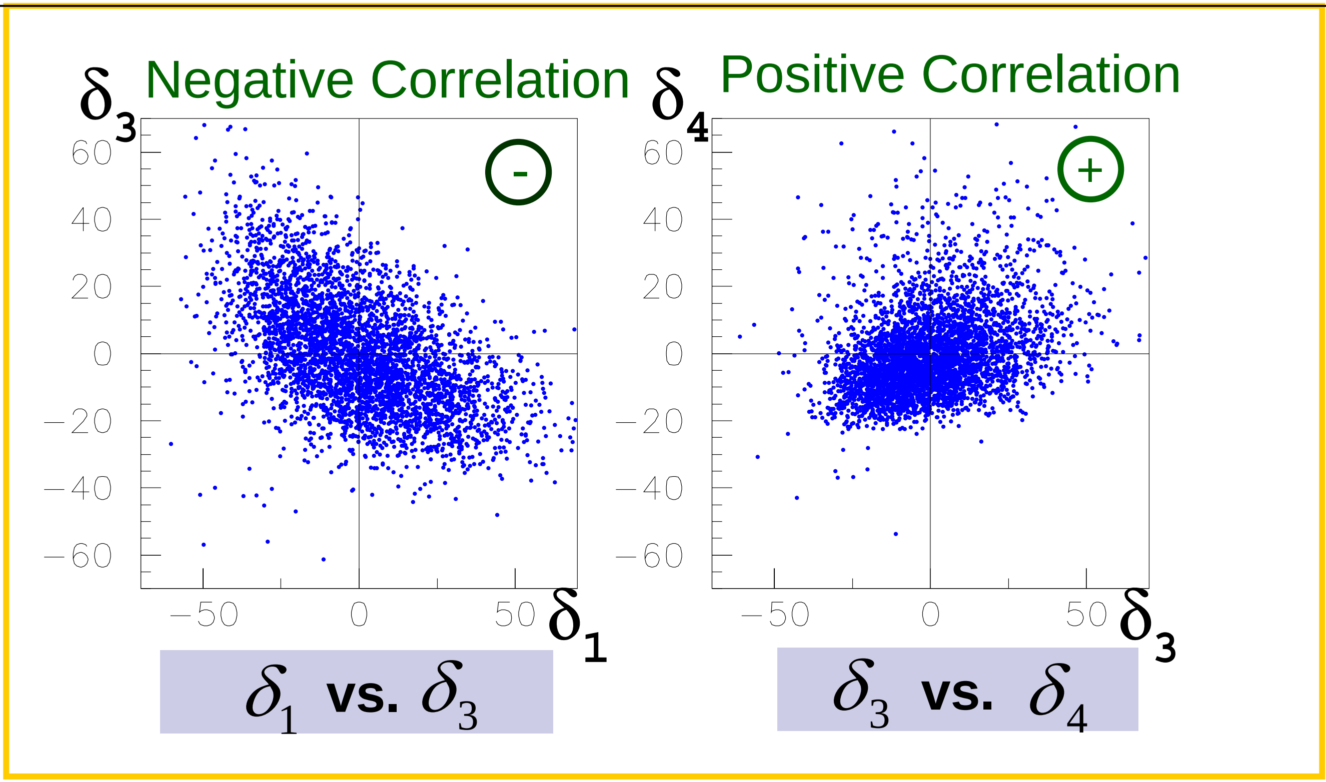
Correlation Matrix

$$C_{ij} \equiv \frac{1}{N} \sum_{k=1}^{N} \delta_i^{(k)} \delta_j^{(k)}$$

[H.Miyata et al.: J.Phys. Soc. Jpn. 69(2000)1645.]

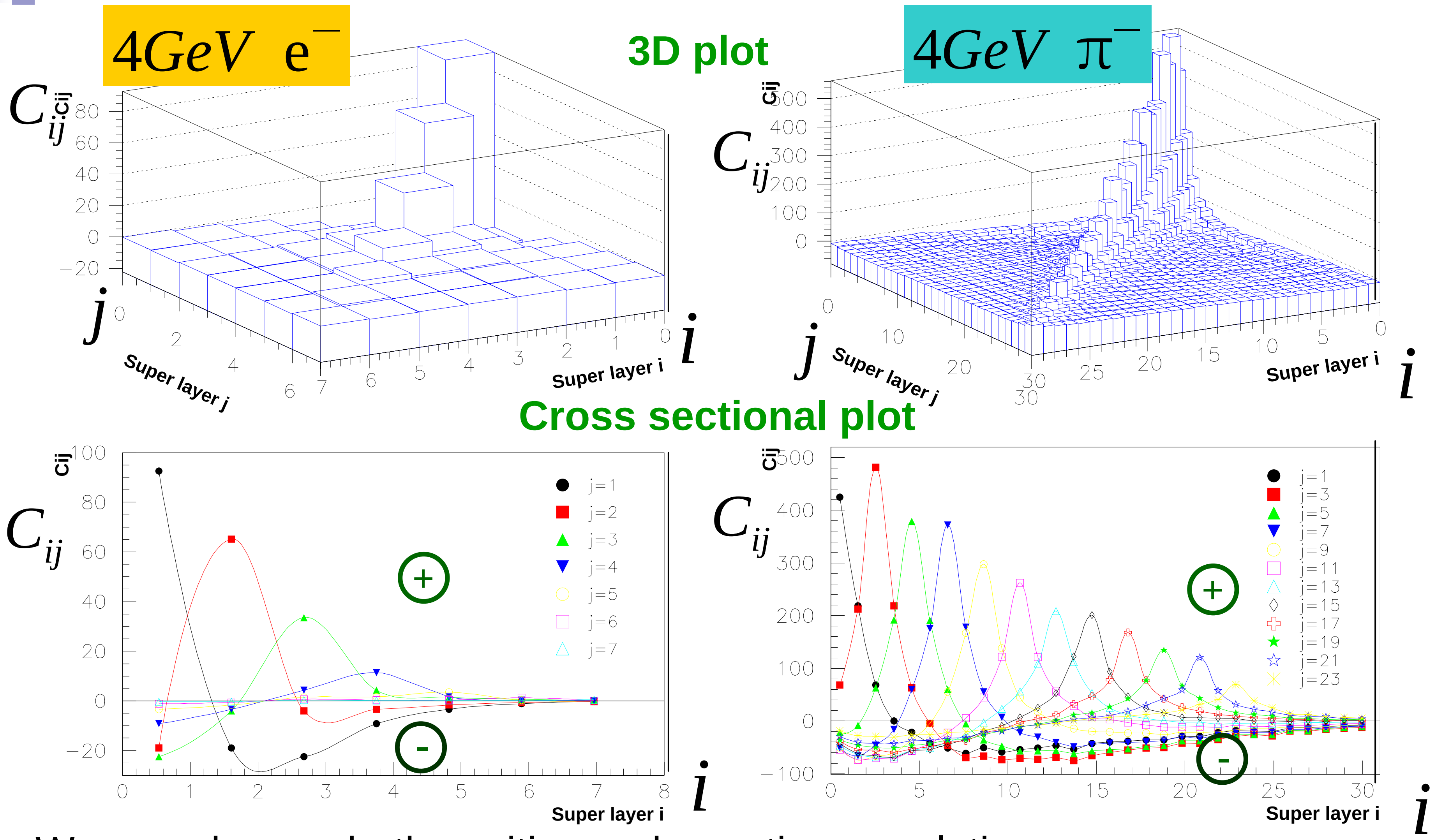
Correlation of PH Deviation at Two Different Depths

2D scatter plots of PH deviations at two different depths for 4GeV electrons



Positive and negative correlations are observed.
Positive correlation comes from the layers near each other, but negative is from far layers.

Correlation Matrix



We can observe both positive and negative correlations.

For hadrons, there are 2 component correlations.

One is sharp peak like shape of short distance correlation. The other is gentle wave like shape of long distance correlation.

Diagonalization of Correlation Matrix C_{ij}

From the correlated PH deviation δ_i, we can derive uncorrelated independent fluctuations $\tilde{\delta}_i$ by diagonalizing the matrix C_{ij}

$$\vec{\tilde{\delta}}^{(k)} = T\,\vec{\delta}^{(k)}$$

$$TCT^T = \Lambda = \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & \lambda_n \end{pmatrix}$$

$$(\lambda_1 > \lambda_2 > \cdots > \lambda_n)$$

λ_i : eigenvalues of matrix Cij

$$T \equiv \begin{pmatrix} \vec{x}_1 \\ \vdots \\ \vec{x}_n \end{pmatrix}$$

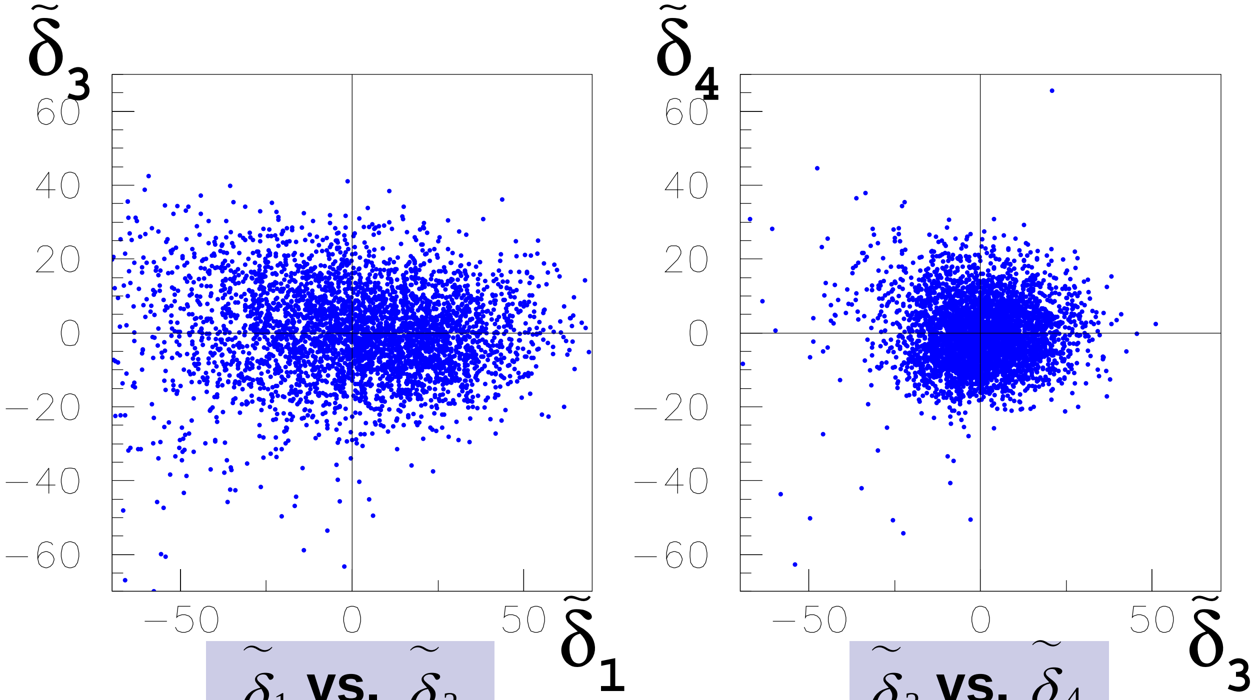
: orthogonal transformation matrix

$$\vec{x}_i = \left(x_{i1} \cdots x_{in}\right)$$

: the i – th eigenvector corresponding to eigenvalue λ_i

Independent PH fluctuation $\tilde{\delta}_i$

2D scatter plots of independent PH fluctuations for 4GeV electrons



$\tilde{\delta}_1$ **vs.** $\tilde{\delta}_3$

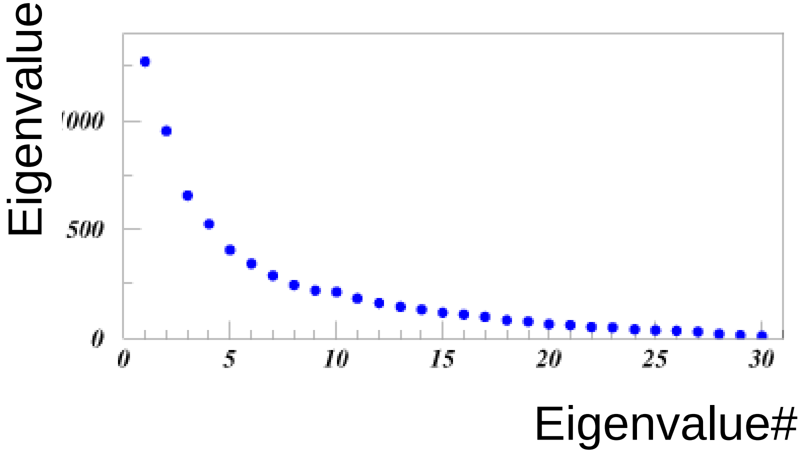
$\tilde{\delta}_3$ **vs.** $\tilde{\delta}_4$

These figures show no correlation between $\tilde{\delta}_i$ and $\tilde{\delta}_j$.

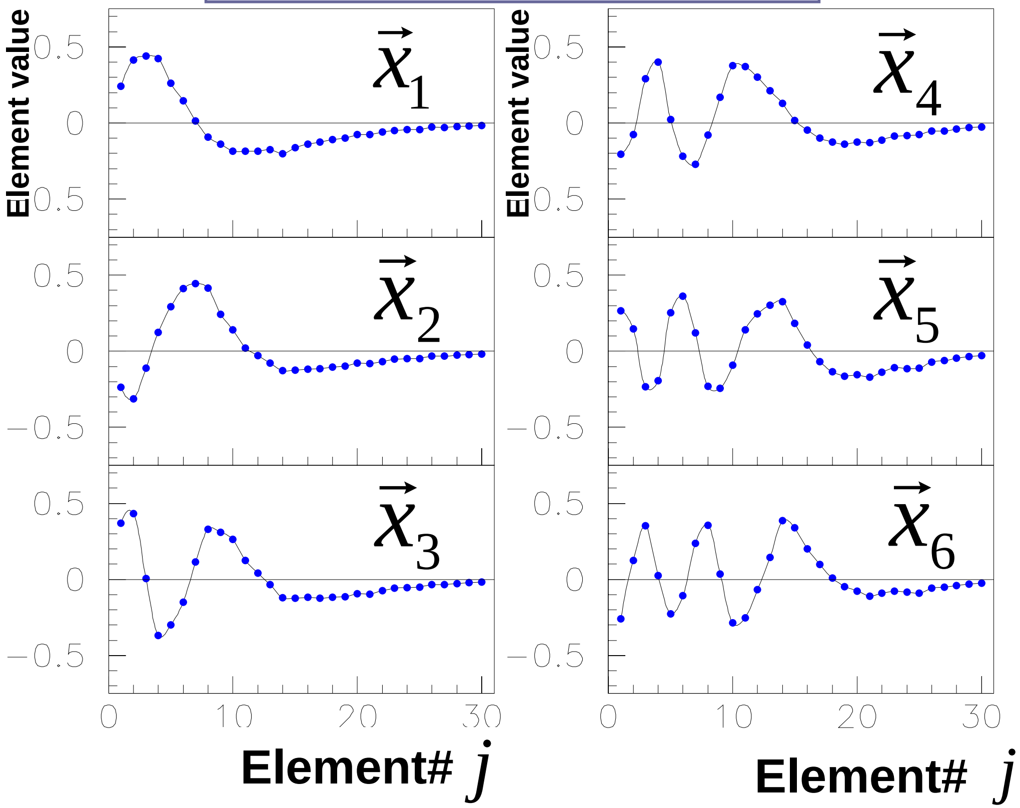
Eigenvalue and Eigenvector of Cij

$4GeV\ \pi^-$

Eigenvalue λ_i vs. i



Elements of eigenvector



1st eigenvector corresponds to largest eigenvalue.

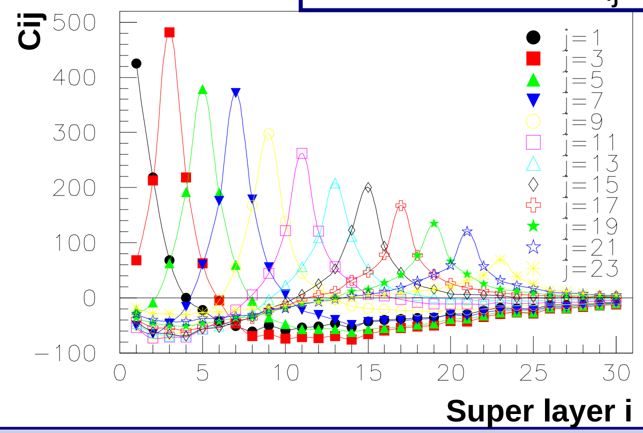
The number of wave nodes increases as the corresponding eigenvalue number becomes bigger, as if the normal vibration, the 2nd vibration, the 3rd vibration etc.

Each eigenvector represents a mode of shower fluctuations.

4.Application of shower fluctuation study

4GeV π^- C_{ij}

For the hadron shower correlation matrix, we assume short distance correlation comes from π^0 production.
So, at first, we identify π^0 in a shower.



π^0 ID

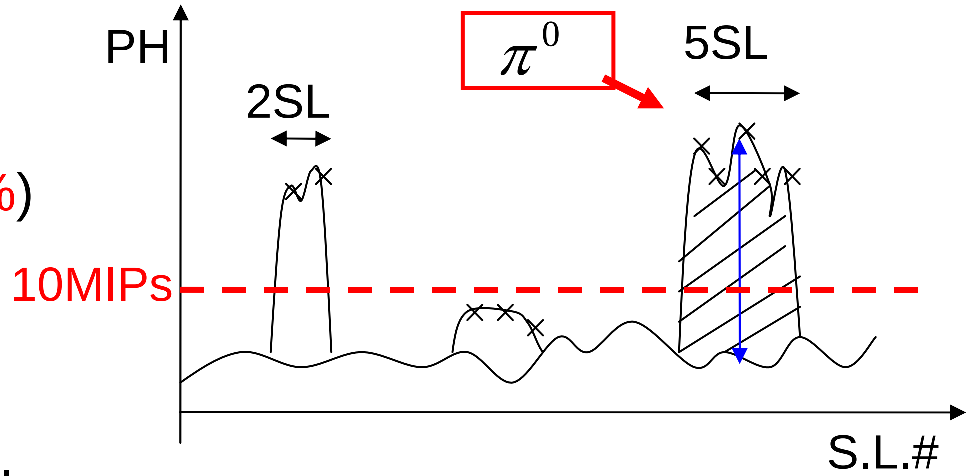
The response having SL's PH more than 10MIPs for at least 3 consecutive SLs($11X_0$) in event-by-event longitudinal shower development.

The EM shower made by π^0's has smaller longitudinal spread than the hadron shower.

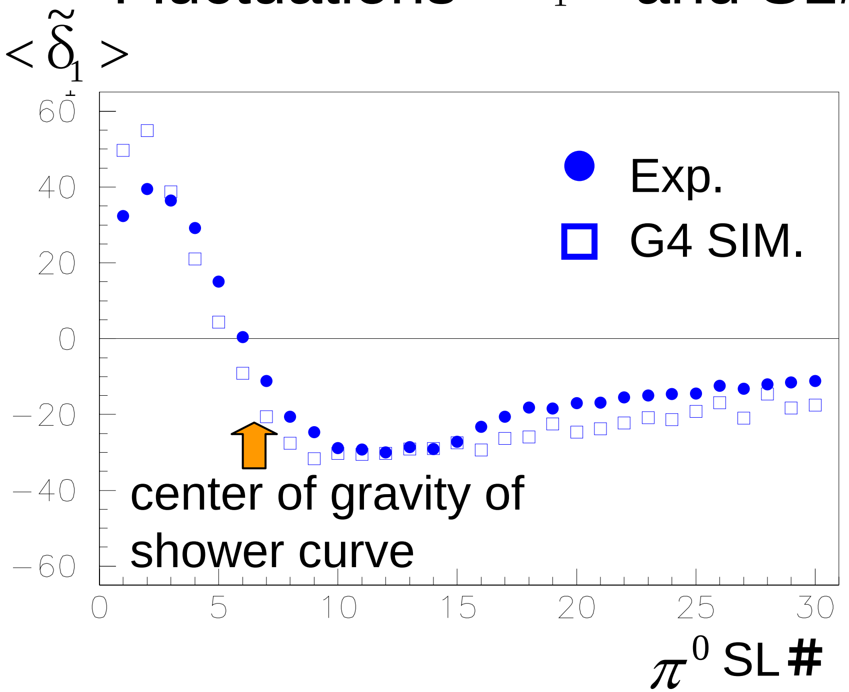
The total number of events : 4246
The number of identified π^0 events : 3840 (90%)

Geant4 simulation gives the real π^0 quantity of 87% for the same selection criteria.

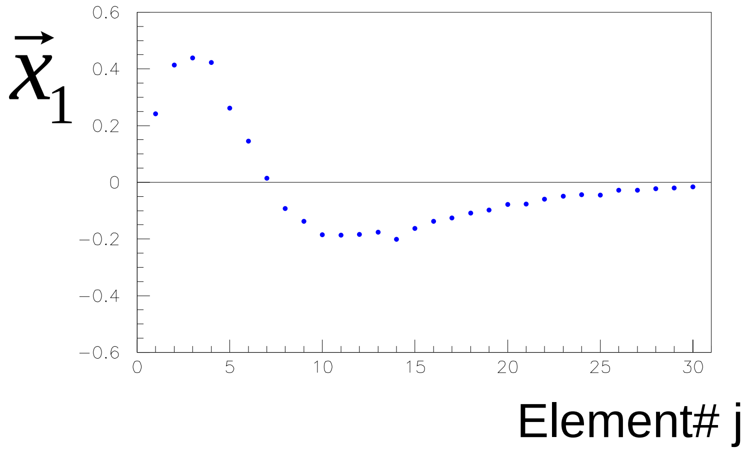
This selection criteria is reasonable.



Relation between the Average of Independent First Fluctuations $<\tilde{\delta}_1>$ and SL# of π^0 Production



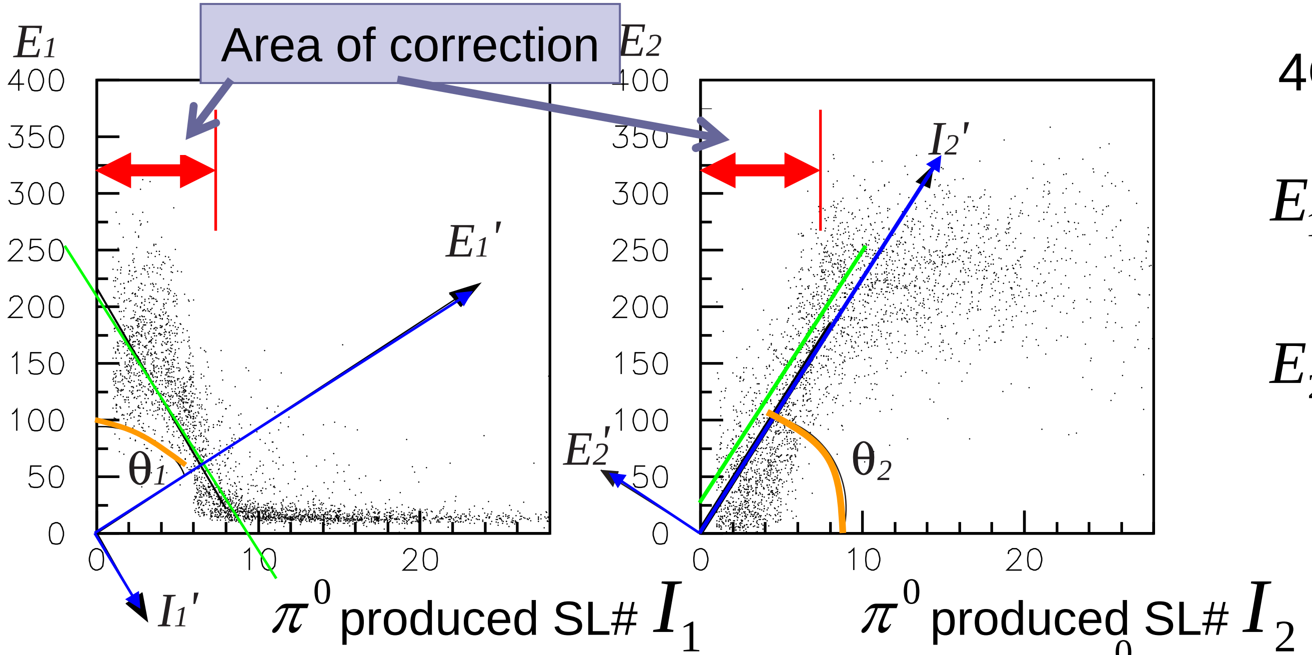
This figure shows similar pattern of $\vec{x}_1$



- We believe the first fluctuation mode($\vec{x}_1$) comes mainly from π^0 production.
- The position that $<\tilde{\delta}_1>$ becomes zero at the center of gravity (6th SL) of the hadron shower curve.
- The first fluctuation $\tilde{\delta}_1$ vibrates in before and in after the center of gravity.

For these feature, we treated the fluctuations in the front part and in the rear part of 6th SL separately.

Energy Deposits E_1,E_2 as a Function of the SL# of π^0 Production



4GeV pion

$$E_1 = \sum_{i=1}^{5} p_i$$

$$E_2 = \sum_{i=6}^{30} p_i$$

The energy deposit is correlated to the produced π^0 SL# in the front part. The correction to reduce these correlations is applied for 1~7-th SL, by making coordinate transformation for 2~4 GeV pion injection.

$$\begin{pmatrix} I'_m \\ E'_m \end{pmatrix} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & con\theta_m \end{pmatrix} \begin{pmatrix} I_m \\ E_m \end{pmatrix} + \begin{pmatrix} a_m \\ b_m \end{pmatrix}$$

Corrected total energy for π^0 inclusive events

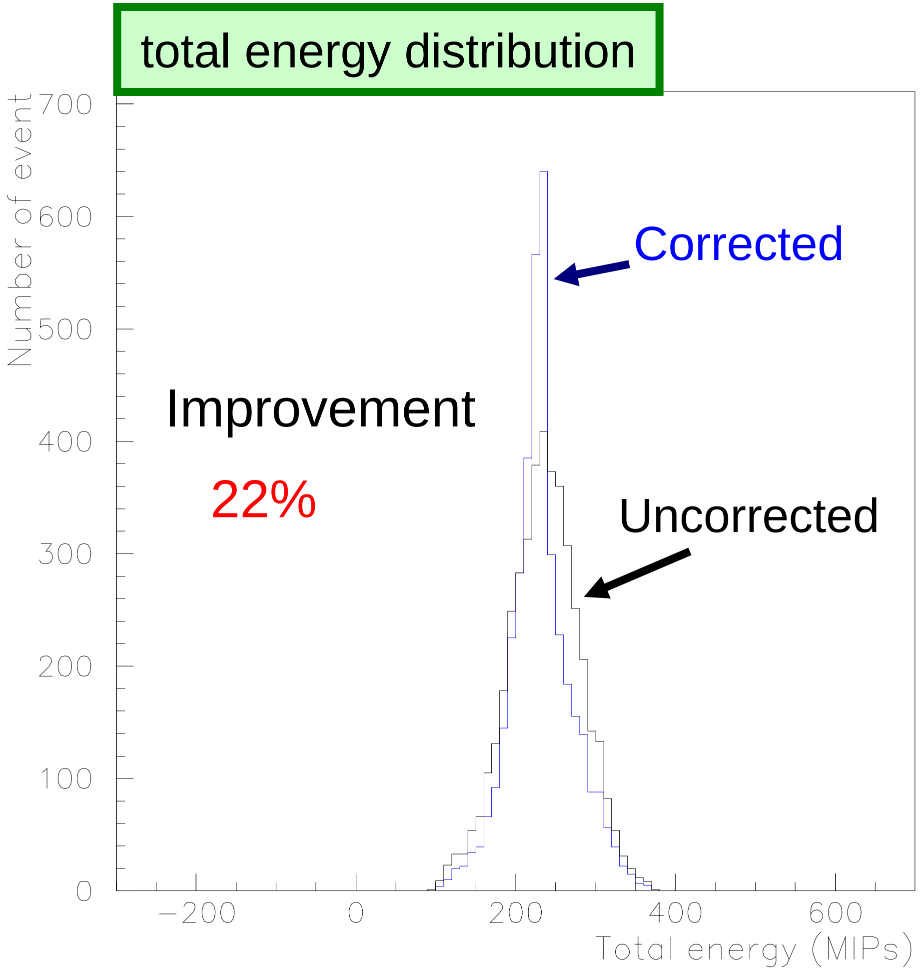
$$E'_{\pi^0} = E'_1 + E'_2$$

$$E'_m = -\sin\theta_m \cdot I_m + \cos\theta_m \cdot E_m + b_m \quad (m = 1,2)$$

I_m: π^0 SL numbers, θ_m : the rotation angles, b_m : the parallel transformations

Corrected total energy E'

The reconstructed total energy for 4GeV incident pions with the π^0 correction



without π^0 correction :

σ = 44.7
<E> = 236.4
s/<E> = 18.7(%)

with π^0 correction:

σ = 38.8
<E> = 233.9
s/<E> = 14.6(%)

The corrected energy resolution improves by 22% at 4GeV compared to the uncorrected one.

This is a software method to reduce the effects of fluctuations

5. Summary

We studied the longitudinal cascade shower fluctuations

1. The general properties of the fluctuations were observed for both EM and hadron showers
   - Independent fluctuations and basic vibrations of the shower development were obtained by diagonalizing the correlation matrix.

2. Application of shower fluctuation study
   - The software method for improvement of hadronic energy resolution was proposed for a energy range of 2~4GeV. We could get 22% better resolution than usual method for 4GeV incident pions.