Linear Collider Studies at Fermilab- Muon ID

C. Milstene- Fermilab –March-19- 2005 - Stanford ILCWS05

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1-Introduction

- The Stepper software simultaneously includes dE/dx and q*v x B effects, (Fermilab -TM-2274E). It improves the detection & reconstruction efficiency (14 hits in 14 layers for the SiD geometry), as well as the hadron particle reconstruction (by 40%) .

  The particle loss by ionization is calculated in term of a change in momenta components, this is not the standard way but provides flexibility and speed.
- In jets, however a combination of factors, e.g., the particle vicinity, the hadron punchthrough, the change in direction due to multiple-scattering, Bremstrahlung etc…, requires further implementations of the Algorithm
- The Kalman filter provides an answer by its step by step correction and has been implemented in the Muon package. We will describe the general principles and its implementation.

2a-Stepper Algorithm-The General Formula

•One starts with a particle at the interaction point (IP), at a given Position ~0,0,0, Momentum (p_x,p_y,p_z) and Mass.
•The Motion through matter in a magnetic field is given between step n and (n+1) by:

$$p_x(n+1) = p_x(n) + 0.3 * q * \frac{p_y(n)}{E(n)} * clight * B_z * \Delta T(n) + \gamma_x(n)$$

$$p_y(n+1) = p_y(n) + 0.3 * q * \frac{p_x(n)}{E(n)} * clight * B_z * \Delta T(n) + \gamma_y(n)$$

$$p_z(n+1) = p_z(n) + \gamma_z(n)$$

$$\gamma_i(n) = \Delta P_i^{Matter} = (\frac{dE}{di}) * \frac{E(n)}{P(n)} * \frac{p_i(n)}{P(n)} * \Delta s \text{ ; } i = x, y, z$$

The 2nd term in p_x and p_y is the usual qv×B term due to the field B_z and the 3rd term comes from energy loss in material.
Here p_x,p_y,p_z are in GeV/c, E(n) in GeV, clight =3E08m/s, Δt in seconds.

2aThe Stepper-The Particle Momentum

One can write for the term material dependant.

$$\gamma_x(n) = \Delta Px = \left(\frac{dE}{dx}\right) * \frac{E(n)}{P(n)} * \frac{p_x(n)}{P(n)} * \Delta s$$

$$\gamma_y(n) = \Delta Py = \left(\frac{dE}{dx}\right) * \frac{E(n)}{P(n)} * \frac{p_y(n)}{P(n)} * \Delta s$$

$$\gamma_z(n) = \Delta Pz = \left(\frac{dE}{dx}\right) * \frac{E(n)}{P(n)} * \frac{p_z(n)}{P(n)} * \Delta s$$

Remark: The effect of the energy loss by ionization by the way it affects the momentum components, although not standard, provides a greater flexibility.

2a-The Stepper- The Particle Position

The new position x(n+1),y(n+1),z(n+1), in cm, is re-calculated after each step as a function of the new values px,py,pz,E and the old Position x(n),y(n),z(n).

$$x(n+1) = x(n) + \frac{p_x(n+1)}{E(n+1)} * clight * \Delta t(n)$$

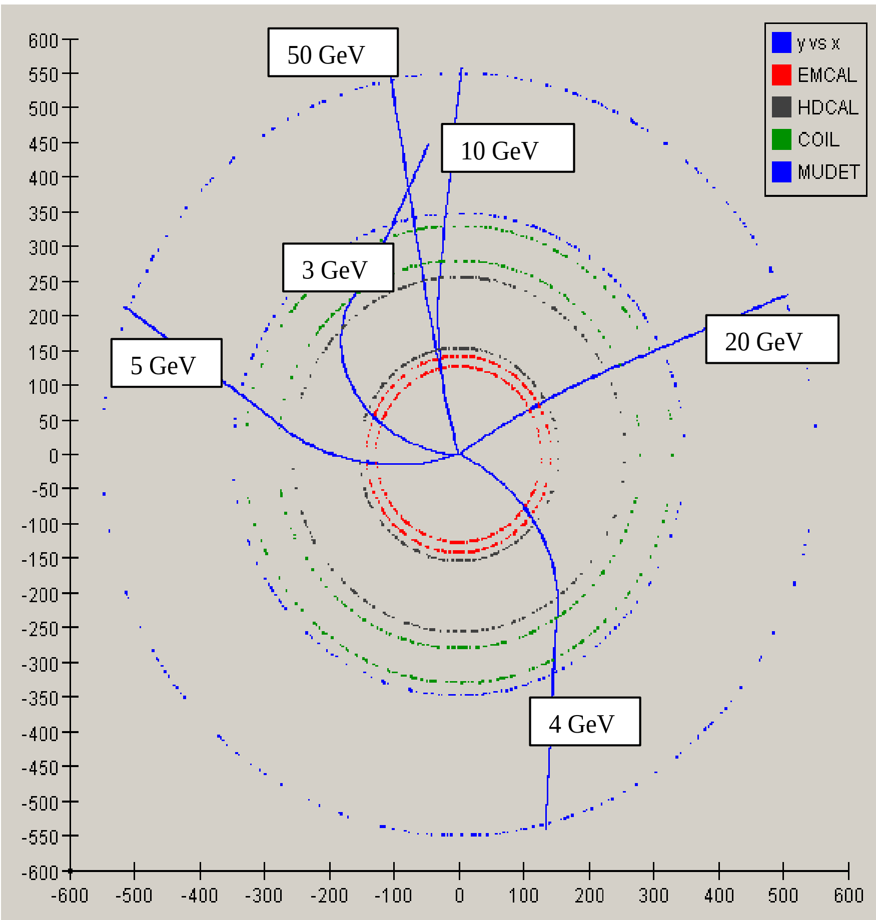
$$y(n+1) = y(n) + \frac{p_y(n+1)}{E(n+1)} * clight * \Delta t(n)$$

$$z(n+1) = z(n) + \frac{p_z(n+1)}{E(n+1)} * clight * \Delta t(n)$$

ΔT(n) is the time of flight in seconds of the particle at step n .

2b-MIP Tracks -Reconstructed By The Stepper In a B=5 Tesla Magnetic Field

y(cm)



x(cm)

EMCAL -2 RED Rings
HDCAL -2 BLACK Rings
COIL -2 GREEN Rings
MUDET -2 BLUE Rings

B= 5 Tesla

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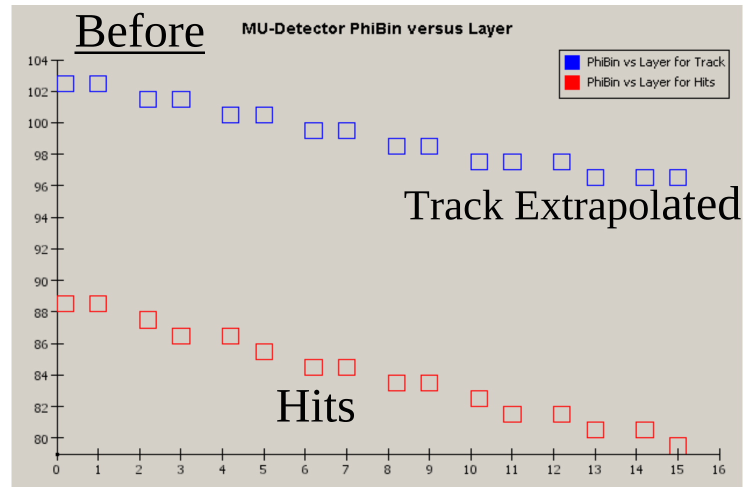
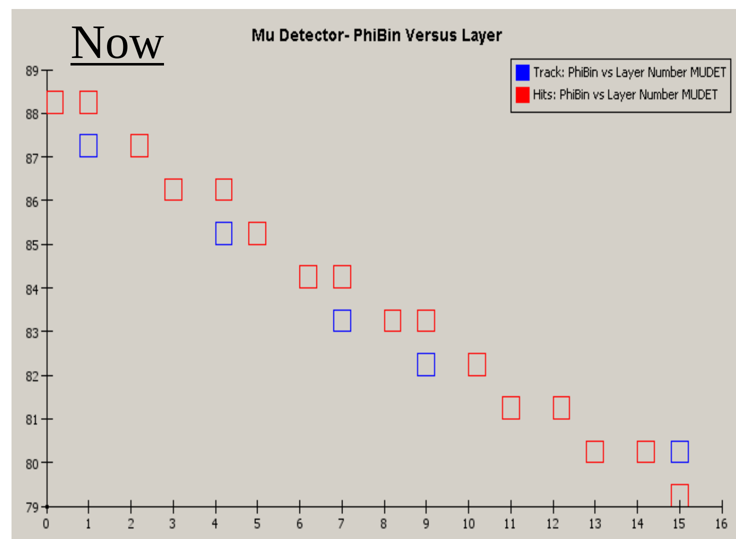
Mu-Detector 300 Phi Bins- 32 Layers

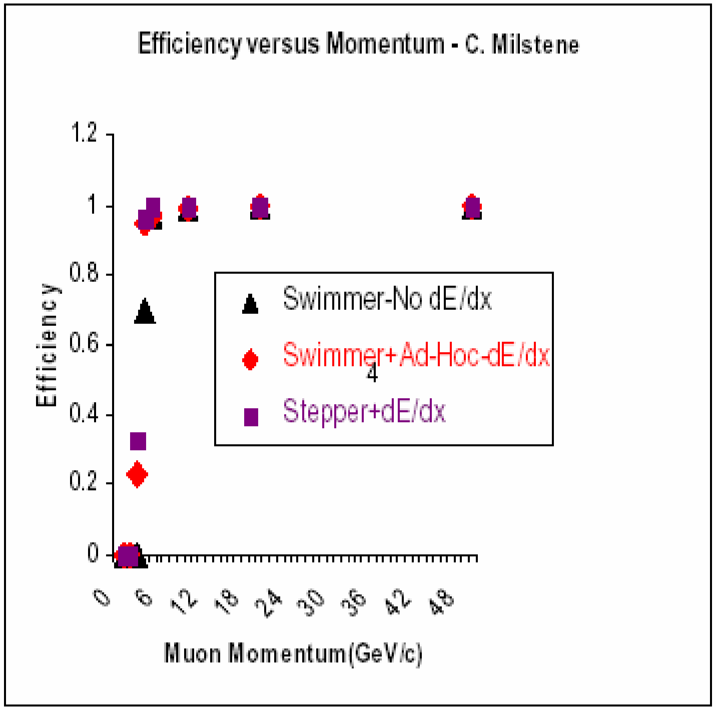
Phi





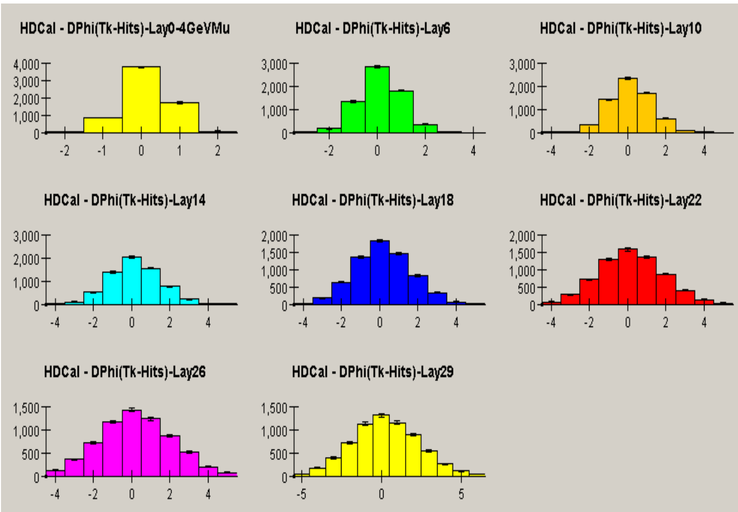
Layer_Number

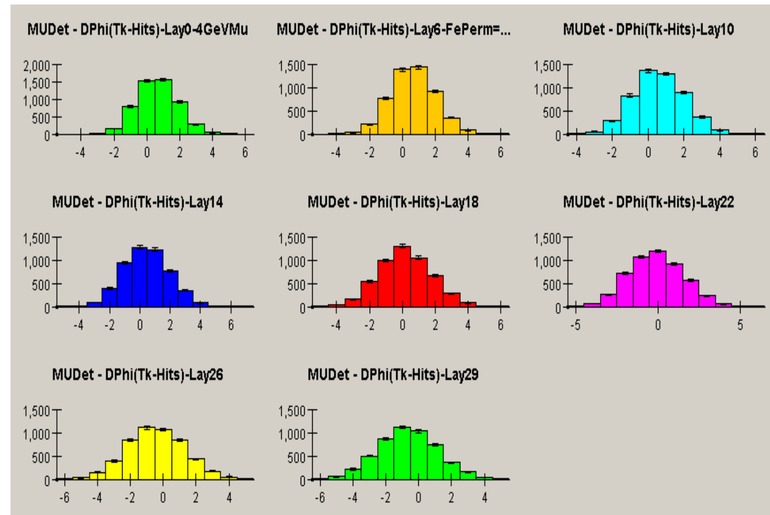
2b-Muon Reconstruction Efficiency

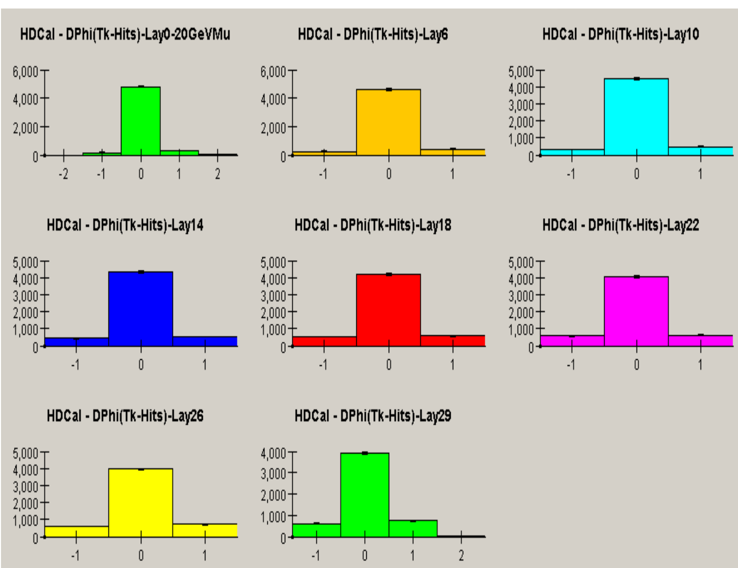


E(GeV) \ Techn.	3	4	5	10
No dE/dx	0.06 %	70%	97%	99.%
Ad-Hoc dE/dx	23%	95%	97%	99.%
V x B + dE/dx	33%	96%	99%	100%

2b-Angular resolution $\Delta\Phi$ =f(Layer Num) 4GeV/20 GeV Muon HCAL-left, MUDET-right







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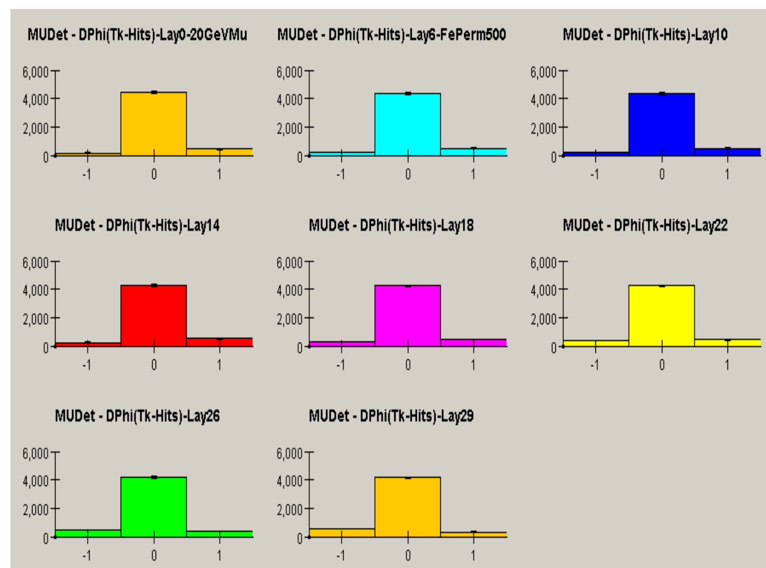
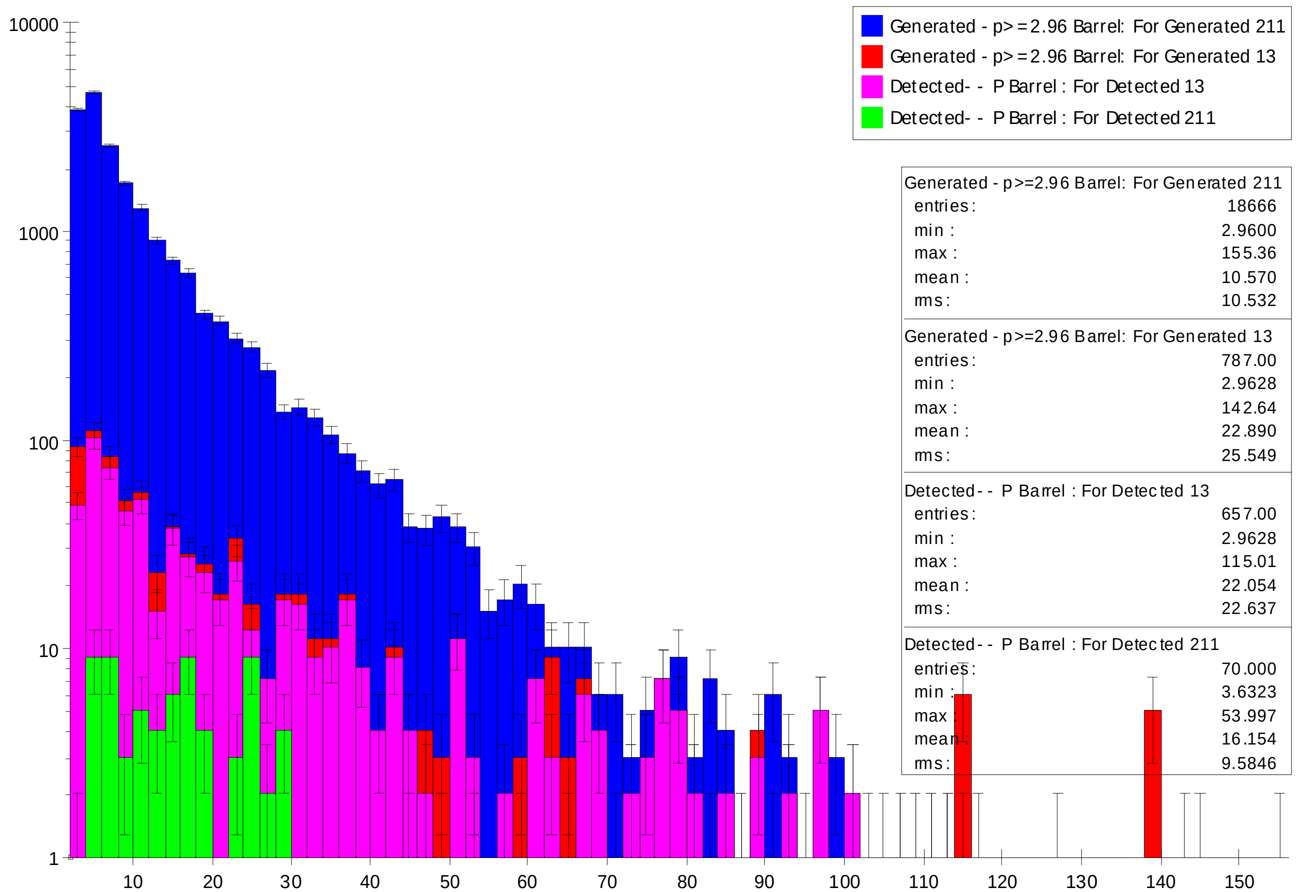


2c-The Stepper in Jets

- 10,000 b-quark pair produced events (20,000 b's) were studied.
- From the b-jets there were 18,666 (34% of the) produced pi's, 4,473(54%) K's and 1,622 (58%) protons that had momenta (> 3 GeV/c) sufficient to penetrate the first 4 plates of the muon detector, assuming they ranged out before interacting.
- From these hadrons, 70 pi's, 41 K's and 2 protons met the muon ID criteria. These numbers include12 pi's and 3 K's that decayed to muons. Thus, the fake rate probabilities for pi's, K's and protons, including punch-through, are 0.0037, 0.0092 and 0.0012, respectively, using the Stepper algorithm.

2c-Jets: Mu& Pions Background Generated/Detected by Mu Algorithm- Out Of 10000 Pairs of b-bbar



P(GeV/c)- 2GeV/bin

Signal:

•Generated Muons in Red

•Detected Muons in Magenta

Background:

Generated Pions in Blue

Detected Pions by Mu Algorithm In Green

Remark:

Below 2.96 GeV the Particles do not reach The Muon Detector

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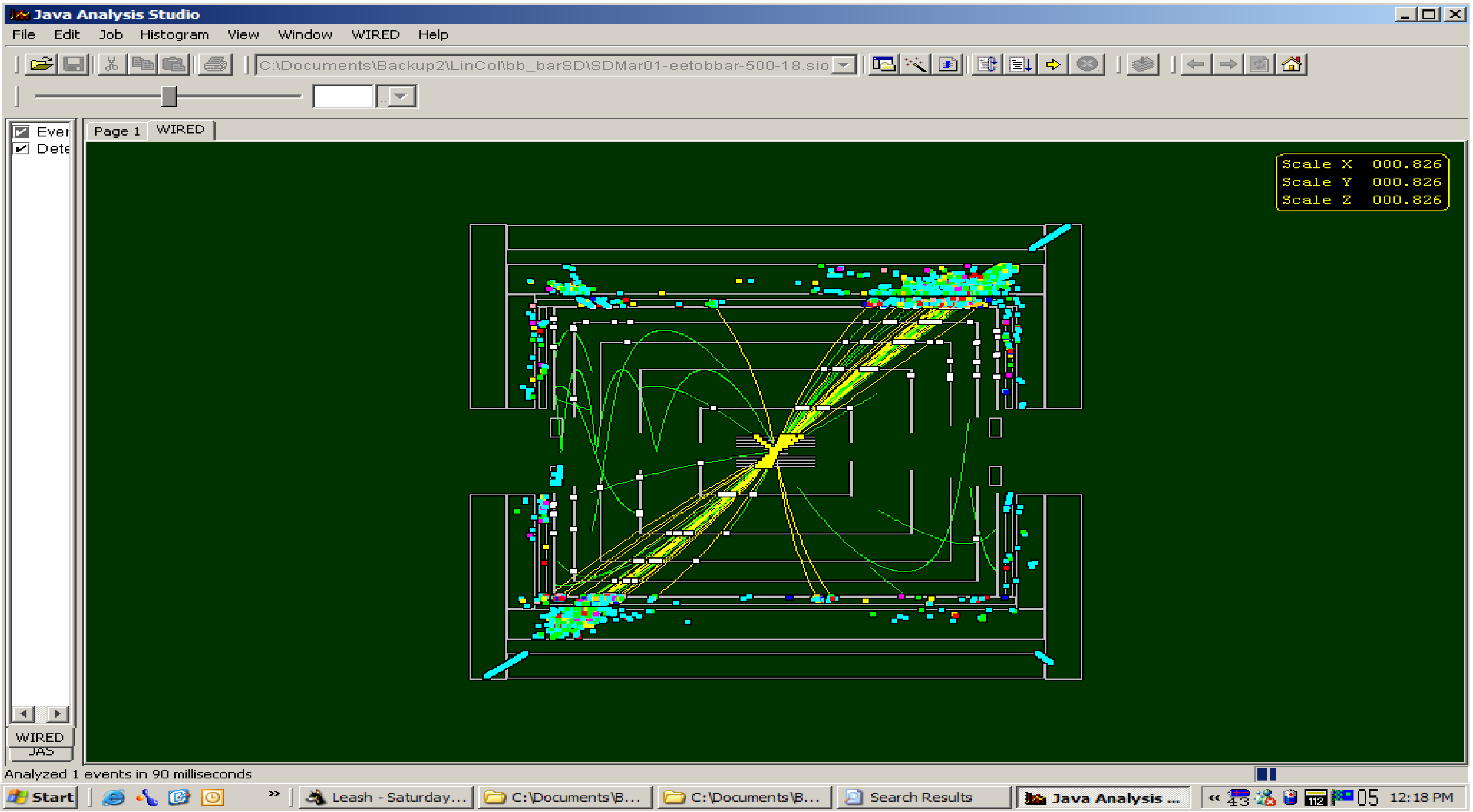
Two Muons

At The border Barrel-FB Detector



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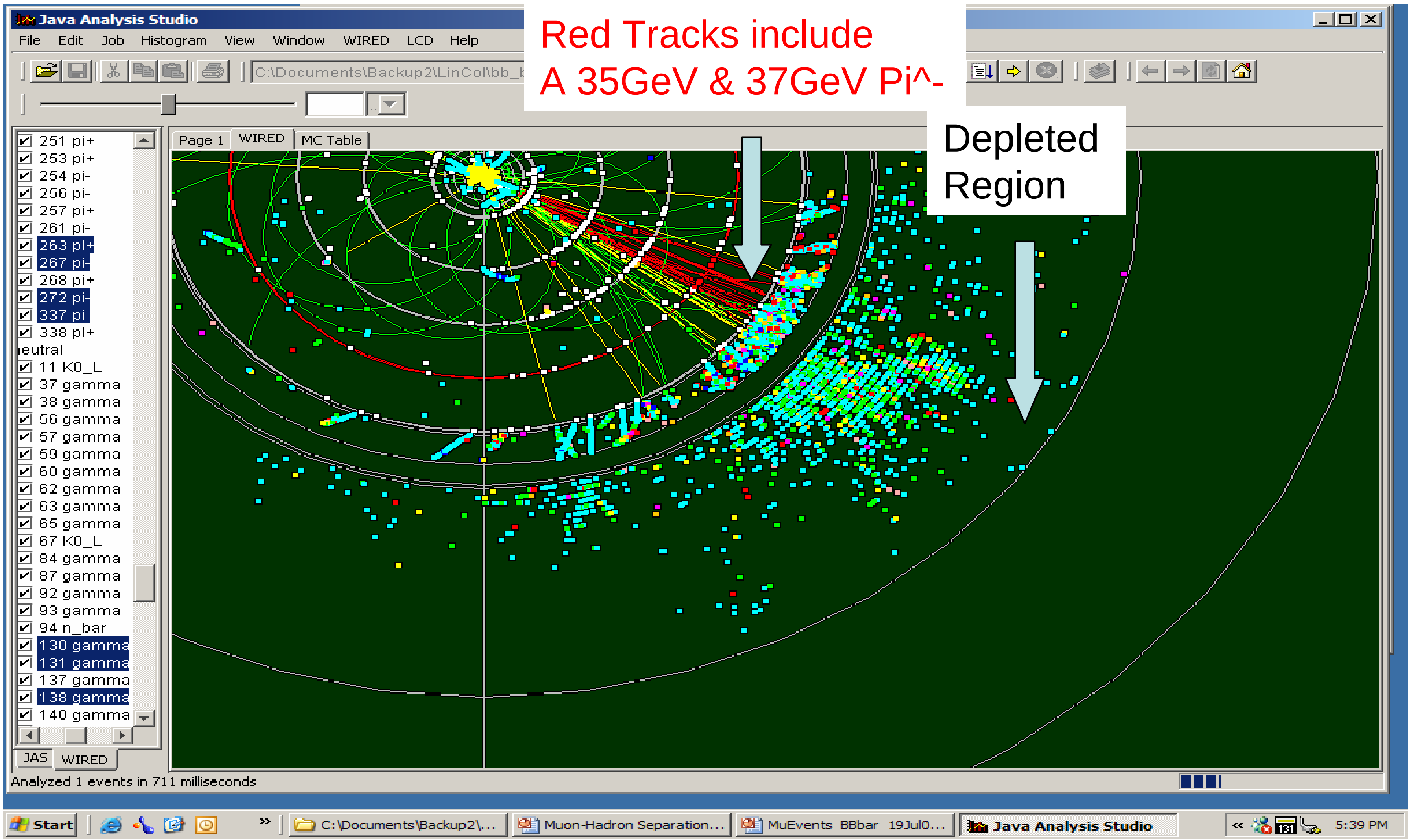
2c-Background Rejection in b-bbar jets

- Remark: With the stepper, more muons got reconstructed but more hadrons as well, for example, 40% more pions are now reconstructed and had to be discarded.

  When a hadron interacts the signal get spread and a hit depletion appears in the path.

  Using the 4 last layers of the hadron calorimeter as an extension of the Muon detector which has a finer grain allowed to discard those hadrons, this suggests that 2 to 4 sensitive planes with a finer grain, located between the coil and the muon detector could be very useful.

2c-B-Bbar Event



2c-Background: Rejection Efficiency In b-bbar Jets

	Pion BG	K BG	Proton BG
Tot. Generated Gen.>3 Gev	(55805)* 18666	(8310)* 4473	(2816)* 1622
Detected	70	41	2
Rejection Efficiency	1 / 267	1 /109	1 / 811

•Generated Particles above >2.96 GeV used To calculate the Rejection Efficiencies.

Using he swimmer we were getting after cuts, out of 787 Generated Mu above 2.96GeV ~603 Muons

Now with the stepper & the HDCal filtering we get out of 787 Mu: 657 Muons

*The total Generated

69% of the Muons(787 out of 1147muons) have a Momentum above ~3 GeV. One notices that less than 34% of the Pions, 54% of the Kaons and 58% of the protons have a momentum above ~3 GeV.

A Case for the Kalman Filter

- It Allows to replace a static wider θ, Φ, Static road around the extrapolated tracks needed to reconstruct small momenta curling particles in jets, but as a consequence picks up hadrons in the vicinity.. A dynamic thin road driven by the particle itself corrected step by step is created in the Kalman filter instead.

- It allows to improve low momenta angular resolution further to match the resolution at and above 20 GeV, And at the same time is more realistic by including the multiple scattering in a realistic way.

3-The Kalman Filter

<u>The Kalman filter basic ideas</u>:

The Phase Space point has been chosen as the state vector point.

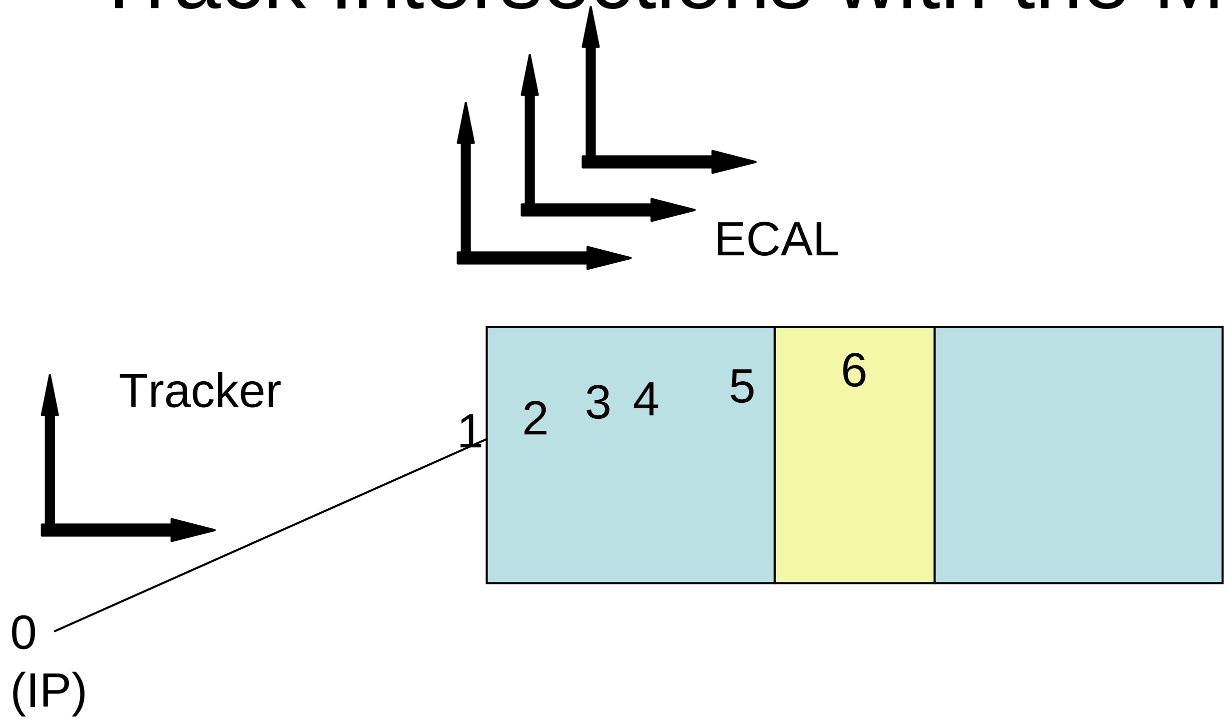
$$\begin{Bmatrix} x_{k-1}(-) \\ y_{k-1}(-) \\ z_{k-1}(-) \\ px_{k-1}(-) \\ py_{k-1}(-) \\ pz_{k-1}(-) \end{Bmatrix}$$

The state vector at location k-1, is propagated using a propagation matrix, to location k. The choice of the state vector as the phase point allows to use the stepper Algorithm written in a matriciel form as the propagation matrix.

This is done a few times in the dense material of the absorber whereas the multiple Scattering is transported with the covariant matrix

Then, In the active material the Kalman filter weighting procedure is applied using the estimate at k :$x_k(-)$& the actual measurement at k, z_k and at that point the changes in the state vector by the weighting procedure combining the extrapolation and measurement is applied and produces the vector $x_k(+)$.

Track Intersections with the Material



• The Change in the vector state accounts for the dE/dx and Bz and is taken into account in the propagation Matrix (step 1,2,3,4,5) in the passive material.
with a Change in the covariant matrix for the Multiple Scattering (1,2,3,4,5)

• The change in the vector state which accounts for the Kalman Weighting as a result of all the above, takes place when the hit is recorded in the active material (step 6)

Muon Code Implementation of the Filter

$$\left.\begin{cases} \vec{x}_{k+1}(-) = \Phi_k \bullet \vec{x}_k(-) \\ P_{k+1}(-) = \Phi_k \bullet P_k(-) \bullet \Phi_k^T + Q_k \end{cases}\right\} \quad (1)$$

Propagation In Passive material

$$\left.\begin{cases} \vec{x}_k(+) = \vec{x}_k(-) + K_k \bullet [\vec{z}_k - H_k \bullet \vec{x}_k(-)] \\ P_k(+) = [1 - K_k \bullet H_k] \bullet P_k(-) \end{cases}\right\} \quad (2)$$

Kalman Filter in Scintillator after Measurement of the signal

$$K_k = P_k(-) \bullet H_k^T \bullet [H_k \bullet P_k(-) \bullet H_k^T + R_k]^{-1} \quad (3)$$

$$Q_k = |\vec{p}| \bullet \Theta_0 \bullet \mathrm{I}$$

Q_k is the noise from Multiple scattering
R_k = Measurement Error, dx, dy, dz-
H_k = Measurement Matrix
Φ_k = Propagation Matrix, applied in passive material
$X_k(-)$ is the extrapolated vector state (x,y,z,px,py,pz)
Z_k is the measured quantities (Φ,θ,r) translated to the cartesian system(x,y,z)
Xk(+)= the state vector after applying the Kalman filter, applied at measurement (6)

Muon Code Implementation of the Filter

$$\begin{Bmatrix} x_k(-) \\ y_k(-) \\ z_k(-) \\ px_k(-) \\ py_k(-) \\ pz_k(-) \end{Bmatrix} = dT * \left(\begin{pmatrix} aa & ab \\ ba & bb \end{pmatrix} + I \right) \begin{Bmatrix} x_{k-1}(+/-) \\ y_{k-1}(+/-) \\ z_{k-1}(+/-) \\ px_{k-1}(+/-) \\ py_{k-1}(+/-) \\ pz_{k-1}(+/-) \end{Bmatrix}$$

$$ab = \begin{pmatrix} (cdedx) & Bz & 0 \\ -Bz & (cdedx) & 0 \\ 0 & 0 & (cdedx) \end{pmatrix}; \mathrm{bb} = \begin{pmatrix} 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{pmatrix} \quad ; \mathrm{cdedx} = \mathrm{de/dx} \bullet \mathrm{v/En}$$

$$\mathrm{R}_0 = \begin{pmatrix} \mathrm{dx} & 0 & 0 \\ 0 & \mathrm{dy} & 0 \\ 0 & 0 & \mathrm{dz} \end{pmatrix} \; ; \Theta_0 = (13.6 MeV / P \bullet \beta c)\sqrt{x / X0} \bullet (1 + 0.038 \bullet lan(x / X0)$$

$$x = r \bullet \sin\Phi; y = r \bullet \cos\Phi; z = r \bullet ctg\Theta$$

Conclusion

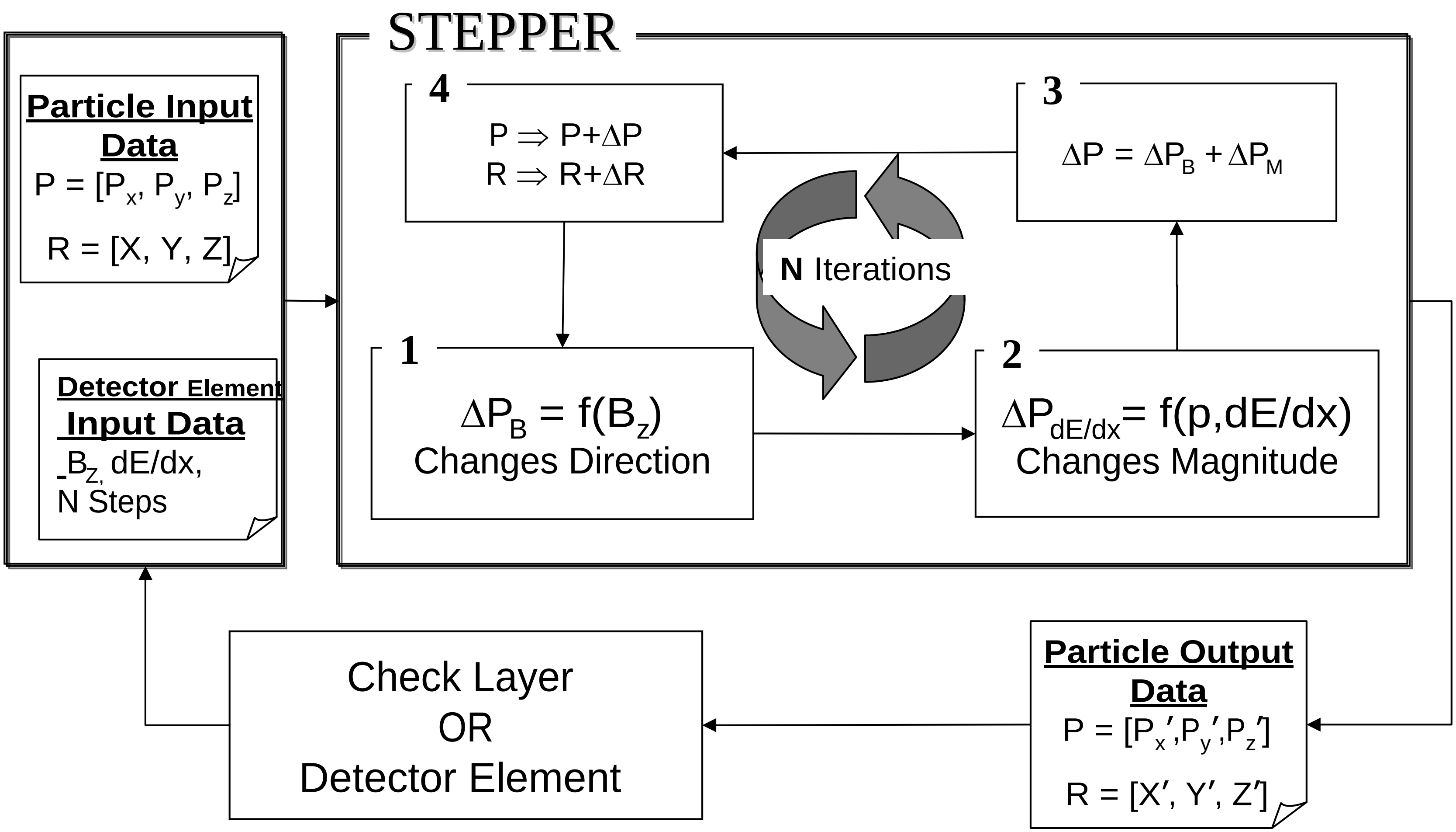
- The Kalman filter has been implemented in the Muon code. It has to be fully tested and optimized and will be available for release in a few weeks.
- It includes the multiple scattering and the dE/dx, the error at starting point is the angle bin size and ½ the layer size of the calorimeter.
- The Kalman filter applies a realistic propagation at each step and allows hits to be collected in a narrow kinematic band dynamically implemented using information from the data

Multiple Scattering Added to the Stepper

For completion a term accounting for the multiple scattering has been added to the stepper.

A vector n of length +/-1 perpendicular to p has been created and its component multiplied by a random Gaussian term of 0 mean and a σ of θ_0 .

2a- Stepper Processing Flow



2-A case for the stepper in the SiD

Amount of Material in front of MuCal

EMCAL 22X0 – 0.87 Λ –190MeV lost by dE/dx

HCAL 39.5X0 – 4.08 Λ – 800MeV lost by dE/dx

The Coil 5.6 X0 – 1.27 Λ – 218MeV lost by dE/dx

Total = *67* X0 – *6.22* Λ – *1200*MeV *lost by dE/dx*

A Magnetic Field of 5 Tesla

MuCal 9 X0 – 9.6 Λ – 1600MeV lost by dE/dx

MuCal:

Outer_Radius 660.5cm(up to 550cm Instrumented)

Inner_Radius 348.5cm

A Total 312 cm(202cm instrumented)

The Unit: Fe 5cm + Gap 1.5cm scintillator

48 Layers /32 Layers Instrumented

80cm Fe = 16 planes

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The Time Of Flight

Below one expresses the components of the velocity as a function Of p,E and the light velocity. If d is the step size one gets for the Radii between steps n and n+1 the following relations

$$V_i(n) = \frac{p_i(n)}{E(n)} * clight\,;\; i = x, y, z$$

$$r(n+1)^2 - r(n)^2 = [x(n+1)^2 + y(n+1)^2] - [x(n)^2 + y(n)^2]$$

$$= [\{x(n) + \mathrm{v}_x(n) * \Delta T(n)\}^2 + \{y(n) + \mathrm{v}_y(n) * \Delta T(n)\}^2] - [x(n)^2 + y(n)^2]$$

$$r(n+1)^2 = r(n)^2 + 2 * d * r(n) + d^2$$

ΔT(n) is the solution of an equation of the second order.

Time of Flight (cont.)

$$\Delta T(n) = \frac{-b + \sqrt{b^2 - 4*a*c}}{2*a}; \quad c = -[2*d*r(n) + d^2]$$

$$a = v_x^2(n) + v_y^2(n); \quad b = 2*[x(n)*v_x(n) + y(n)*v_y(n)]$$

3a-The Muon Candidate

•The original code of the μ package of R. Markeloff has been modified to use a stepper in order to extrapolate the tracks and collect the hits. This allows to include the effects of the Magnetic field and account for the dE/dx

• A set of **hits in HDCal &&EMCal** within ($3\Delta\phi,3\Delta\theta$) bins from the track (HDCal bin=$\pi/600$); and ($2\Delta\phi,2\Delta\theta$) (EMCal bin= $\pi/840$) is collected.

• At least **14hits in MuCal** within ($2\Delta\phi,2\Delta\theta$) bins from the track (MuCal bin= $\pi/150$), *in14 layers or more & a mean*♌ *< 2hits/layer*.

<u>Remark:</u> We are looking only in the Barrel Detector, accounted by a cut in Θ, 0.95 rd<Θ<=2.2 rd

3c-The Hadron Calorimeter End - Extension of the Muon Detector

The information of the very end of the Hadron calorimeter, the 4 last layers, is used as an extension of the Muon Detector to improve the hadronic rejection.

- In the next figures, one can see that in jets events as well as in single events, the muon has a continuous pattern of hits in each layer up to the end of HCal, on the other hand, hadrons tend to interact well before the end of the Hadron Calorimeter. The products of the interactions are scattered from the region spanned by the extrapolated track and leave a void of hits at the back of the Hadron Calorimeter.

- By requiring a hit in each one of the4 last layers of the Hadron calorimeter, we will take advantage of:

  1_The Muon distinctive repetitive pattern in the detector

  2_ The refined granularity of the hadron Calorimeter , 5mrd bins

  3_ This fine granularity is specially useful since the hadron calorimeter is located before the Coil and less material on which to scatter is on the way.

Using those 3 properties HCal will be able to filter out part of the hadrons before they reach the Muon Detector and improve the rejection efficiency.

3b)The Candidate-Extension of the Algorithm

The actual Algorithm requires:

- A Charged track with a good fit in the tracker
- In the Muon detector we require at least 14 Layers/14 hits.
- In order to allow for random scattering which are bigger for lower particle momenta, we use the same Momentum dependant angle cut than with the swimmer for the track below ~ 10 GeV, namely, ($\Delta\phi$, $\Delta\theta \sim 1/p$) .
- We use the very last end of the Hadron Calorimeter as an extension of the muon detector in order to improve the hadron rejection and the hadron-muon separation.