Signatures of Axinos and Gravitinos at the ILC

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based on

[hep-ph/0501287]

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Parallel Session: Cosmological Connections

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Dark Matter in the Universe: Evidence & Candidates

- ? Solar Neighborhood
  - * Planetary Motions
- □ Spiral Galaxies
  - * Rotation Curves
- □ (Super-) Clusters of Galaxies
  - * Weak Gravitational Lensing
  - * Strong Gravitational Lensing
- □ Large Scale Structure
  - * Structure Formation
- □ CMB Anisotropy: WMAP, ...
  - * $\Omega_{\mathrm{M}} = 0.27 \pm 0.016$
  - * $\Omega_{\mathrm{Tot}} = 1.02 \pm 0.02$

- □ Dark Baryons
  - * MACHO's
- □ Neutrinos
  - * Neutrino Oscillations
- □ Axion
  - * Strong CP Problem
- □ Lightest SUSY Particle (LSP)
  - ? Neutralino χ $\leftarrow$ MSSM
  - ? Gravitino $\tilde{G}$ $\leftarrow$ SUGRA
  - ? Axino $\tilde{a} \leftarrow$ SUSY + PQ-Symm.
- □ ... ?

Top 10 Puzzle: What is the Nature of Cold Dark Matter?

[Nilles, Raby, '82; Tamvakis, Wyler, '82; Kim, Nilles, '84]

Axino $\longleftarrow$ SM + Peccei-Quinn Symmetry + Supersymmetry

- strong CP problem $\rightarrow$ PQ Mechanism
- hierarchy problem $\rightarrow$ Supersymmetry

$\searrow$ $\swarrow$

$$\text{axion supermultiplet: } \Phi = \frac{1}{\sqrt{2}}(s + ia) + \sqrt{2}\tilde{a}\theta + F_\Phi \theta\theta$$

saxion axion axino

- □ Color Neutral and Electrically Neutral
- □ Interactions are suppressed by the PQ Scale

$$f_a \gtrsim 5 \times 10^9 \text{ GeV}$$

- □ Masses after Breaking of PQ Symmetry and SUSY

saxion: $m_s = \mathcal{O}(\text{TeV}) \leftarrow$ soft-mass term

axion: $m_a = \Lambda_{\text{QCD}}^2 / f_a \propto 10^{-2} - 10^{-5}\,\text{eV} \leftarrow$ chiral anomaly

axino: $\text{e}V \lesssim m_{\tilde{a}} \lesssim \text{GeV}$ $\leftarrow$ model dependent

Axino $\longrightarrow$ Lightest SUSY Particle $\longrightarrow$ Cold Dark Matter

Gravitino $\leftarrow$ Supergravity $\leftarrow$ Gauging Supersymmetry

- □ Gauge Field of Local SUSY Transformations
- □ Spin 3/2 Superpartner of the Graviton
- □ Color Neutral and Electrically Neutral
- □ Interactions are suppressed by the (reduced) Planck scale

$$\mathrm{M}_{\mathrm{Pl}} = 2.44 \times 10^{18}\,\mathrm{GeV}$$

! Enhancement for small gravitino masses $m_{\widetilde{G}}$ $\leftarrow$ Goldstino

- □ Mass $\leftarrow$ Breaking of Supersymmetry $m_{\widetilde{G}} \sim \sum_I \frac{\langle F_I \rangle}{\mathrm{M}_{\mathrm{Pl}}} + \sum_A \frac{\langle D_A \rangle}{\mathrm{M}_{\mathrm{Pl}}} \sim \frac{M^2_{\mathrm{SUSY}}}{\mathrm{M}_{\mathrm{Pl}}}$

gauge-mediated $m_{\widetilde{G}} \sim 10\ \mathrm{eV} - 1\ \mathrm{GeV}$

[Dine, Nelson, Shirman, '95; Dine, Nelson, Nir, Shirman, '96]

gravity-mediated $m_{\widetilde{G}} \sim 0.1 - 1\ \mathrm{TeV}$

cf. [Nilles, '84]

anomaly-mediated $m_{\widetilde{G}} \sim 10 - 100\ \mathrm{TeV}$

[Randall, Sundrum, '99; Giudice, Luty, Murayama, Rattazzi, '98]

Gravitino $\longrightarrow$ Lightest SUSY Particle $\longrightarrow$ Cold Dark Matter

Axino $\widetilde{a}$

- SM + PQ Mechanism + SUSY
  * hierarchy problem
  * strong CP problem
- Spin 1/2 Partner of the Axion
- Interactions: model-dependent, suppr. by the Peccei–Quinn scale
  $f_a \gtrsim 5 \times 10^9\,\mathrm{GeV}$
- Interactions are independent of $m_{\tilde{a}}$
- Mass: wide range, depends on the model & SUSY breaking scheme
  $\mathrm{e}V \lesssim m_{\tilde{a}} \lesssim \mathrm{GeV}$

Gravitino $\widetilde{G}$

- SM + local SUSY (SUGRA)
  * hierarchy problem
  * gravity
- Spin 3/2 Partner of the Graviton
- Interactions: model-independent, suppressed by the Planck scale
  $\mathrm{M_{Pl}} = 2.44 \times 10^{18}\,\mathrm{GeV}$
- Interactions depend on $m_{\widetilde{G}}$
- Mass: wide range, depends on the SUSY breaking scheme
  $\mathrm{eV} \lesssim m_{\widetilde{G}} \lesssim \mathrm{TeV}$

Axinos/Gravitinos $\stackrel{?}{=}$ Dominant Part of Cold Dark Matter

□ Assumption: Axino/Gravitino = LSP & stable ← R-parity conservation

? Relic LSP Abundance ← Cosmic Scenario & Production Mechanisms

$$\Omega_{\rm LSP} h^2 = \rho_{\rm LSP} h^2/\rho_c = m_{\rm LSP} n_{\rm LSP} h^2/\rho_c \quad \stackrel{?}{\longleftrightarrow} \quad \Omega_{\rm CDM}^{\rm WMAP} h^2 = 0.113^{+0.016}_{-0.018}$$

? Mass of the LSP (& NLSP) ← SUSY Breaking Scheme & Model

How COLD is Axino/Gravitino Dark Matter? → Structure Formation

? Cosmological Constraints ← NLSP Decays: photons & hadronic showers

– Abundances of Primordial D, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li
– Cosmic Microwave Background (CMB)
– Diffuse X-Ray and γ-Ray Backgrounds

[Kim, '79; Shifman, Vainshtein, Zakharov, '80]

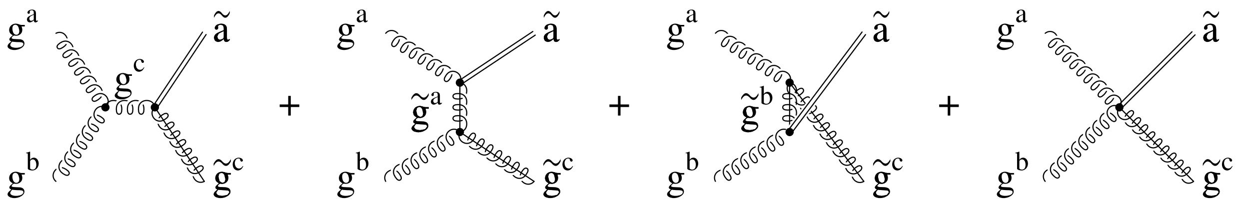
Axino Interactions $\longleftarrow$ Hadronic (KSVZ) Axion Models

- axino–gluino–gluon interaction:

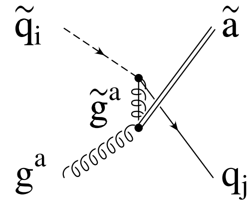
$$\mathcal{L}_{\tilde{a}\tilde{g}g} = i\,\frac{\alpha_{\rm s}}{16\pi(f_a/N)}\,\bar{\tilde{a}}\,\gamma_5\,[\gamma^\mu,\gamma^\nu]\,\tilde{g}^a\,G^a_{\mu\nu}$$

Thermal Axino Production in SUSY QCD

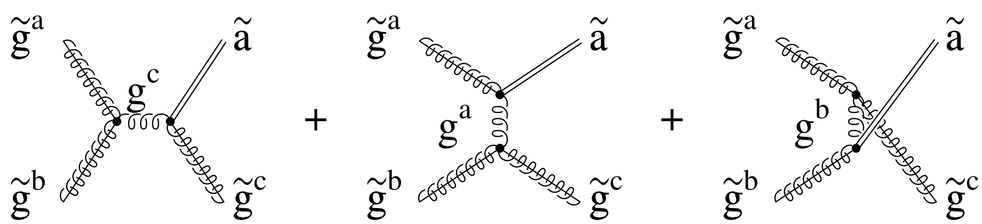
- A: $g^a + g^b \to \tilde{g}^c + \tilde{a}$



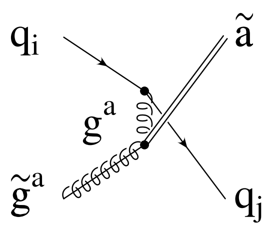
- B: $g^a + \tilde{g}^b \to g^c + \tilde{a}$ (crossing of A)
- C: $\tilde{q}_i + g^a \to \tilde{q}_j + \tilde{a}$



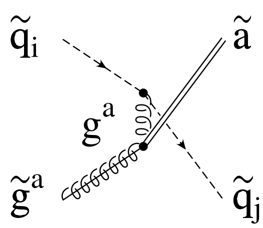
- D: $g^a + q_i \to \tilde{q}_j + \tilde{a}$ (crossing of C)
- E: $\bar{\tilde{q}}_i + q_j \to g^a + \tilde{a}$ (crossing of C)
- F: $\tilde{g}^a + \tilde{g}^b \to \tilde{g}^c + \tilde{a}$



- G: $q_i + \tilde{g}^a \to q_j + \tilde{a}$

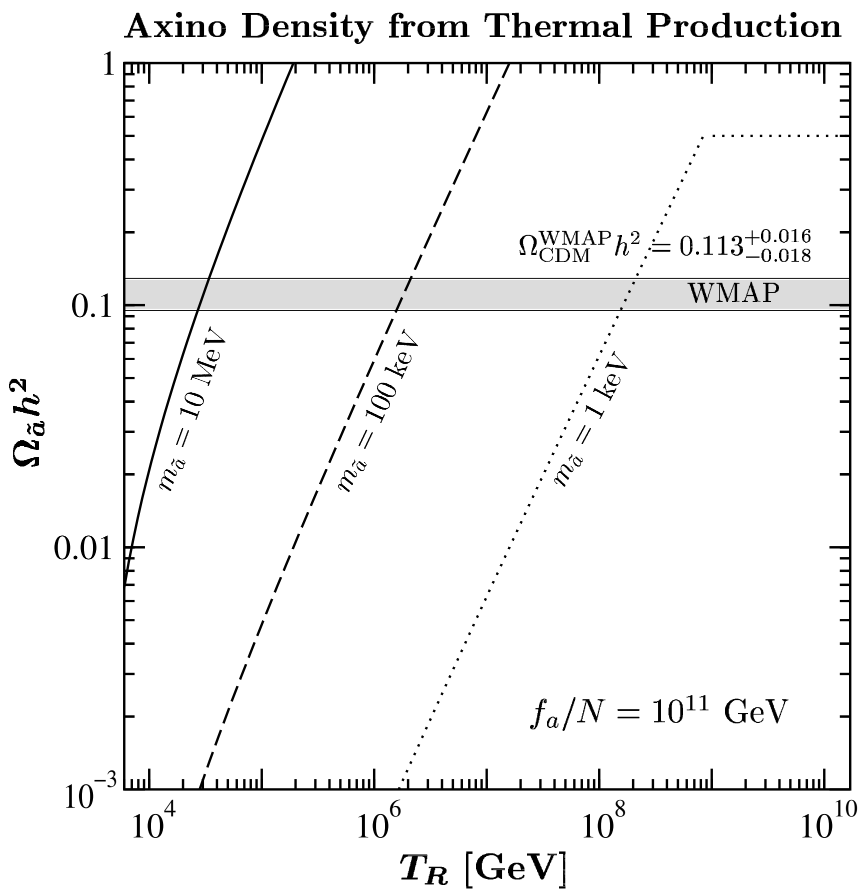


- H: $\tilde{q}_i + \tilde{g}^a \to \tilde{q}_j + \tilde{a}$



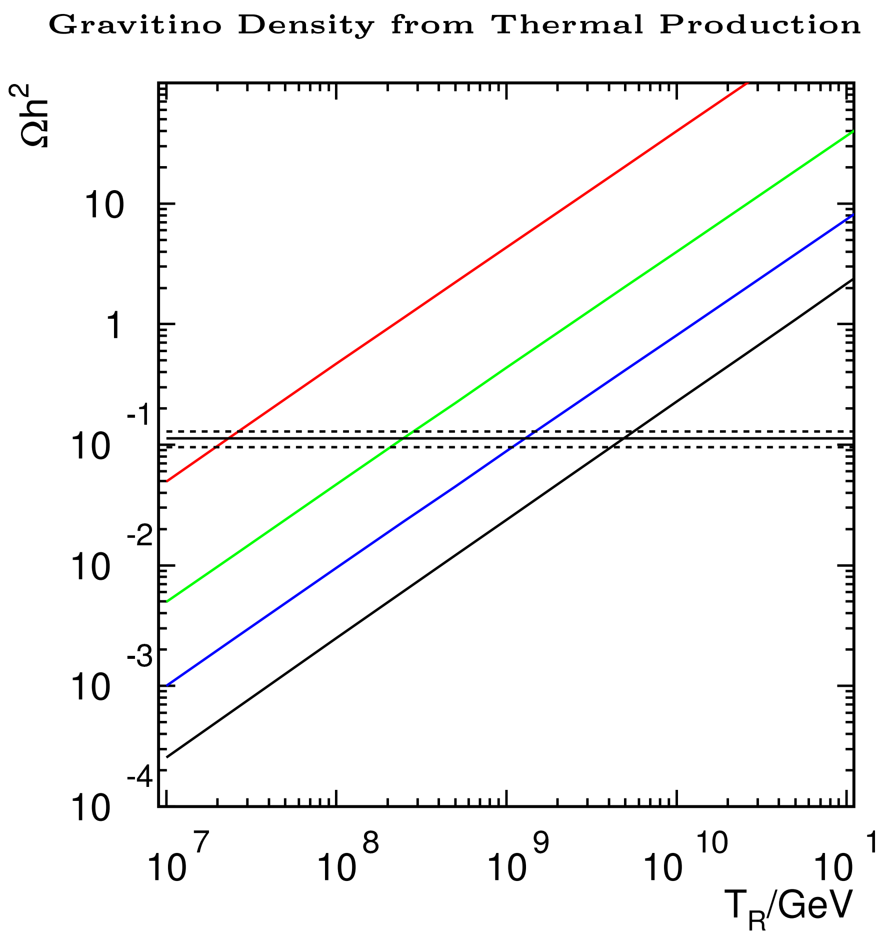
- I: $q_i + \bar{q}_j \to \tilde{g}^a + \tilde{a}$ (crossing of G)
- J: $\tilde{q}_i + \bar{\tilde{q}}_j \to \tilde{g}^a + \tilde{a}$ (crossing of H)

Axino/Gravitino Dark Matter from Thermal Production



$$\Omega_{\tilde{a}} h^2 \propto \frac{1}{f_a^2}\, m_{\tilde{a}}\, T_R$$

[...; Brandenburg, FDS, '04]



$$\Omega_{\tilde{G}} h^2 \propto \frac{1}{\mathrm{M}_{\mathrm{Pl}}^2} \left(1 + \frac{m_{\tilde{g}}^2}{3 m_{\tilde{G}}^2}\right) m_{\tilde{G}}\, T_R$$

[...; Bolz, Brandenburg, Buchmüller, '01

Decays of the Next-to-Lightest Supersymmetric Particle

□ Next-to-Lightest Supersymmetric Particle (NLSP)

- Candidates $\in$ MSSM:
  - Neutralino $\widetilde{\chi}^0$
  - Slepton $\widetilde{l}$
  - Sneutrino $\widetilde{\nu}$
  - ...
- Decoupling Temperature $T_{\mathrm{D}}^{\mathrm{NLSP}} \ll T_{\mathrm{D}}^{\tilde{a}/\widetilde{G}}$ $\longleftarrow$ NLSP $\in$ MSSM
- Lifetime $\tau_{\mathrm{NLSP}} = \text{very long}$ $\longleftarrow$ $\tilde{a}/\widetilde{G} = \mathrm{LSP}$

□ Implications of NLSP Decays

- Axino/Gravitino Dark Matter from NLSP Decays
- Cosmological Constraints from Late NLSP Decays
- Cosmological Implications of Late NLSP Decays
- ...

Can we probe

Axino/Gravitino Dark Matter

in experiments?

Signatures of Axinos/Gravitinos in Experiments

– Direct Detection of $\tilde{a}/\widetilde{G}$

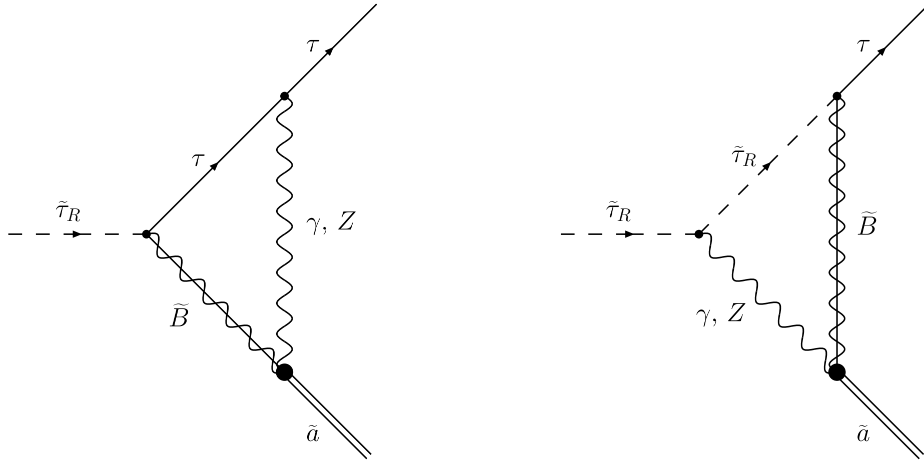
– Direct Production of $\tilde{a}/\widetilde{G}$

*** Decays of charged $\widetilde{l}$ NLSP's at the International Linear Collider**

[... ; Hamaguchi, Kuno, Nakaya, Nojiri, '04; Feng, Smith, '05]

$\tilde{a}$ LSP $\longrightarrow$ Probing the Peccei–Quinn Scale f_a & $m_{\tilde{a}}$ at Collider

□ Assumption: $\widetilde{\tau}_R$ NLSP & $\widetilde{\chi}^0 \simeq \widetilde{B}$



- 2-Body Decay $\widetilde{\tau}_R \to \tau + \tilde{a}$

$$\Gamma(\widetilde{\tau}_{\mathrm{R}} \to \tau \widetilde{a}) \simeq \xi^2 \, (25 \text{ sec})^{-1} C_{\mathrm{aYY}}^2 \left(1 - \frac{m_{\widetilde{a}}^2}{m_{\widetilde{\tau}}^2}\right) \left(\frac{m_{\widetilde{\tau}}}{100 \text{ GeV}}\right) \left(\frac{10^{11} \text{ GeV}}{f_a}\right)^2 \left(\frac{m_{\widetilde{B}}}{100 \text{ GeV}}\right)$$

- Peccei–Quinn Scale f_a $\longleftarrow$ NLSP Lifetime $\tau_{\widetilde{\tau}} \approx 1/\Gamma(\widetilde{\tau}_R \to \tau \, \tilde{a})$

$$f_a^2 \simeq \left(\frac{\tau_{\widetilde{\tau}}}{25 \text{ sec}}\right) \xi^2 \, C_{\mathrm{aYY}}^2 \left(1 - \frac{m_{\widetilde{a}}^2}{m_{\widetilde{\tau}}^2}\right) \left(\frac{m_{\widetilde{\tau}}}{100 \text{ GeV}}\right) \left(\frac{m_{\widetilde{B}}}{100 \text{ GeV}}\right)^2 (10^{11} \text{ GeV})^2$$

- Axino Mass $m_{\tilde{a}} = \sqrt{m_{\widetilde{\tau}}^2 + m_{\tau}^2 - 2 m_{\widetilde{\tau}} E_{\tau}}$ $\longleftarrow$ Kinematics

[Kim, '79; Shifman, Vainshtein, Zakharov, '80]

Axino Interactions $\longleftarrow$ Hadronic (KSVZ) Axion Models

- axino–gluino–gluon interaction:

$$\mathcal{L}_{\tilde{a}\tilde{g}g} = i\,\frac{\alpha_{\rm s}}{16\pi(f_a/N)}\,\bar{\tilde{a}}\,\gamma_5\,\left[\gamma^\mu,\gamma^\nu\right]\,\tilde{g}^a\,G^a_{\mu\nu}$$

- axino–bino–photon/Z-boson interaction:

$$\mathcal{L}_{\widetilde{a}\widetilde{B}\gamma/Z} = i\,\frac{\alpha_{\rm Y}C_{\rm aYY}}{16\pi f_a}\,\bar{\widetilde{a}}\,\gamma_5\,\left[\gamma_\mu,\gamma_\nu\right]\,\widetilde{B}\,\left(\cos\theta_W\,F_{\mu\nu}-\sin\theta_W\,Z_{\mu\nu}\right)$$

[Cremmer, Ferrara, Girardello, Van Proeyen, '83]

Gravitino Interactions $\longleftarrow$ Supergravity Lagrangian

- gravitino–slepton–lepton interaction:

$$\mathcal{L}_{l\tilde{l}\widetilde{G}} = -\frac{1}{\sqrt{2}\,\mathrm{M_{Pl}}}\,\partial_\nu\,\tilde{l}\,\bar{l}\,\gamma^\mu\gamma^\nu\,\psi_\mu$$

- gravitino–slepton–lepton–gauge boson interaction:

$$\mathcal{L}_{l\tilde{l}V\widetilde{G}} = i\,\frac{g_V}{\sqrt{2}\,\mathrm{M_{Pl}}}\,A_\nu\,\tilde{l}\,\bar{l}\,\gamma^\mu\gamma^\nu\,\psi_\mu$$

- gravitino–gaugino–gauge boson interaction:

$$\mathcal{L}_{V\lambda\widetilde{G}} = -i\,\frac{1}{8\,\mathrm{M_{Pl}}}\,\overline{\psi}_\mu\,[\gamma_\nu,\gamma_\rho]\,\gamma^\mu\,\lambda\,F_{\mu\nu}$$

[Buchmüller, Hamaguchi, Ratz, Yanagida, '04]

$\widetilde{G}$ LSP $\longrightarrow$ Measuring the Planck Scale $\mathrm{M_{Pl}}$ & $m_{\widetilde{G}}$ at Colliders

□ Assumption: $\widetilde{\tau}_R$ NLSP

- 2-Body Decay $\quad \widetilde{\tau}_R \rightarrow \tau + \widetilde{G}$

$$\Gamma(\widetilde{\tau}_\mathrm{R} \rightarrow \tau\, \widetilde{G}) = \frac{m_{\widetilde{\tau}}^5}{48\pi\, m_{\widetilde{G}}^2\, \mathrm{M}_\mathrm{Pl}^2} \left(1 - \frac{m_{\widetilde{G}}^2}{m_{\widetilde{\tau}}^2}\right)^4 = \frac{1}{5.89\,\mathrm{sec}} \left(\frac{m_{\widetilde{\tau}}}{100\,\mathrm{GeV}}\right)^5 \left(\frac{10\,\mathrm{MeV}}{m_{\widetilde{G}}}\right)^2 \left(1 - \frac{m_{\widetilde{G}}^2}{m_{\widetilde{\tau}}^2}\right.$$

- Planck Scale $\mathrm{M_{Pl}}$ $\quad\longleftarrow\quad$ NLSP Lifetime $\quad \tau_{\widetilde{\tau}} \approx 1/\Gamma(\widetilde{\tau}_R \rightarrow \tau\, \widetilde{G})$

$$\mathrm{M}_\mathrm{Pl}^2 = \frac{\tau_{\widetilde{\tau}}}{48\pi} \frac{m_{\widetilde{\tau}}^5}{m_{\widetilde{G}}^2} \left(1 - \frac{m_{\widetilde{G}}^2}{m_{\widetilde{\tau}}^2}\right)^4 \quad \overset{???}{\longleftrightarrow} \quad \mathrm{M}_\mathrm{Pl}^2 = \frac{1}{8\pi\, G_N} = (2.44 \times 10^{18}\,\mathrm{GeV})^2$$

- Gravitino Mass $m_{\widetilde{G}} = \sqrt{m_{\widetilde{\tau}}^2 + m_\tau^2 - 2 m_{\widetilde{\tau}} E_\tau}$ $\quad\longleftarrow\quad$ Kinematics

Can one distinguish between the $\tilde{a}/\widetilde{G}$ LSP Scenarios?

- Lifetime of the NLSP $\longleftarrow$ Assumption: $\widetilde{\tau}_R = \text{NLSP}$ & $\widetilde{\chi}^0 \approx \widetilde{B}$

$\tilde{a} = \text{LSP}$

$\tau_{\widetilde{\tau}}^{\tilde{a}\,\text{LSP}} \longleftarrow m_{\widetilde{\tau}}, m_{\widetilde{B}}, m_{\tilde{a}}, f_a$

$\mathcal{O}(0.01\ \text{sec}) \lesssim \tau_{\widetilde{\tau}}^{\tilde{a}\,\text{LSP}} \lesssim \mathcal{O}(10\ \text{h})$

$\uparrow$ $f_a \sim 10^9\, GeV$ $\qquad$ $\uparrow$ $f_a \sim 10^{12}\,\text{GeV}$

$\widetilde{G} = \text{LSP}$

$\tau_{\widetilde{\tau}}^{\widetilde{G}\,\text{LSP}} \longleftarrow m_{\widetilde{\tau}}, m_{\widetilde{B}}, m_{\widetilde{G}}$

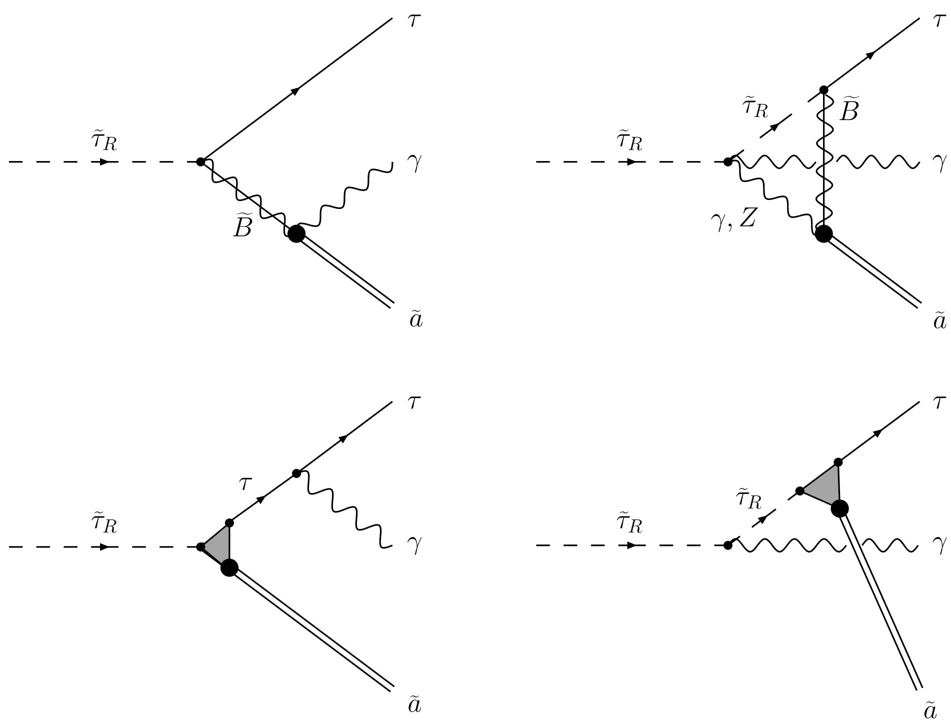
$\mathcal{O}(10^{-8}\ \text{sec}) \lesssim \tau_{\widetilde{\tau}}^{\widetilde{G}\,\text{LSP}} \lesssim \mathcal{O}(15\ \text{y})$

$\uparrow$ $m_{\widetilde{G}} \sim 1\ \text{keV}$ $\qquad$ $\uparrow$ $m_{\widetilde{G}} \sim 50\ \text{GeV}$

Very Short/Very Long Lived NLSP $\longrightarrow$ $\widetilde{G}$ LSP Scenario

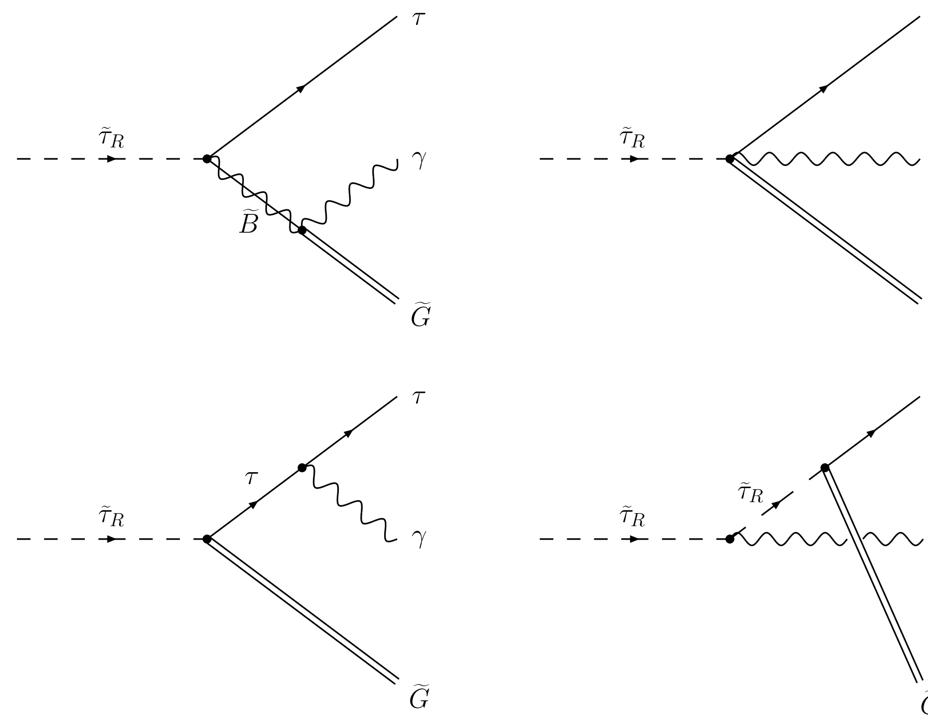
The 3-Body Decays

$$\tilde{a} = \text{LSP:} \quad \widetilde{\tau}_R \rightarrow \tau + \gamma + \tilde{a}$$



$$\frac{d^2\Gamma(\widetilde{\tau}_\text{R} \rightarrow \tau\,\gamma\,\widetilde{a})}{dx_\gamma\,d\cos\theta} = \dots$$

$$\widetilde{G} = \text{LSP:} \quad \widetilde{\tau}_R \rightarrow \tau + \gamma + \widetilde{G}$$



$$\frac{d^2\Gamma(\widetilde{\tau}_\text{R} \rightarrow \tau\,\gamma\,\widetilde{G})}{dx_\gamma\,d\cos\theta} = \dots$$

$\tilde{a}$ LSP $\longrightarrow$ The Differential Decay Rate for $\widetilde{\tau}_R \longrightarrow \tau + \gamma + \tilde{a}$

$$\frac{d^2\Gamma(\widetilde{\tau}_{\rm R} \to \tau\,\gamma\,\widetilde{a})}{dx_\gamma\, d\cos\theta} = \frac{m_{\widetilde{\tau}}}{512\pi^3}\,\frac{x_\gamma(1-A_{\widetilde{a}}-x_\gamma)}{[1-(x_\gamma/2)(1-\cos\theta)]^2}\,\sum_{\rm spins}|\mathcal{M}(\widetilde{\tau}_{\rm R}\to\tau\,\gamma\,\widetilde{a})|^2\ ,$$

where

$$\sum_{\rm spins}|\mathcal{M}(\widetilde{\tau}_{\rm R}\to\tau\,\gamma\,\widetilde{a})|^2 \quad = \quad \frac{\alpha^3\,C^2_{\rm aYY}}{\pi\cos^4\theta_W}\,\frac{m^2_{\widetilde{\tau}}}{f_a^2}\,F^{(\widetilde{a})}_{\rm diff}(x_\gamma,\cos\theta,A_{\widetilde{a}},A_{\widetilde{B}})\ ,$$

with

$$x_\gamma \equiv \frac{2E_\gamma}{m_{\widetilde{\tau}}}\ , \qquad A_{\widetilde{a}} \equiv \frac{m^2_{\widetilde{a}}}{m^2_{\widetilde{\tau}}}\ , \qquad A_{\widetilde{B}} \equiv \frac{m^2_{\widetilde{B}}}{m^2_{\widetilde{\tau}}}\ ,$$

and

$$F^{(\widetilde{a})}_{\rm diff}(x_\gamma,\cos\theta,A_{\widetilde{a}},A_{\widetilde{B}}) = \frac{x_\gamma^2(1-A_{\widetilde{a}}-x_\gamma)[1+\cos\theta+A_{\widetilde{a}}(1-\cos\theta)][1+\cos\theta+A_{\widetilde{B}}(1-\cos\theta)}{\{x_\gamma(1+\cos\theta)+2A_{\widetilde{a}}-A_{\widetilde{B}}[2-x_\gamma(1-\cos\theta)]\}^2}$$

$$+\ \frac{3\alpha}{\pi\cos^2\theta_W}\,\xi\log\left(\frac{f_a}{m}\right)\left\{\frac{\sqrt{A_{\widetilde{a}}A_{\widetilde{B}}}(1+\cos\theta)(1-A_{\widetilde{a}}-x_\gamma)}{x_\gamma(1+\cos\theta)+2A_{\widetilde{a}}-A_{\widetilde{B}}[2-x_\gamma(1-\cos\theta)]}\right.$$

$$\left. +\ \frac{A_{\widetilde{B}}\,[(1+\cos\theta)(1-A_{\widetilde{a}})+A_{\widetilde{a}}x_\gamma(1-\cos\theta)]}{x_\gamma(1+\cos\theta)+2A_{\widetilde{a}}-A_{\widetilde{B}}[2-x_\gamma(1-\cos\theta)]}\right\}$$

$$+\ \frac{9\alpha^2}{4\pi^2\cos^4\theta_W}\,\xi^2\log^2\left(\frac{f_a}{m}\right)A_{\widetilde{B}}\left\{\frac{1+\cos\theta+A_{\widetilde{a}}(1-\cos\theta)}{(1-\cos\theta)(1-A_{\widetilde{a}}-x_\gamma)}+\frac{2(1+\cos\theta)(1-A_{\widetilde{a}})}{x_\gamma^2(1-\cos\theta)}\right.$$

$\widetilde{G}$ LSP $\to$ Diff. Decay Rate $\widetilde{\tau}_R \to \tau + \gamma + \widetilde{G}$

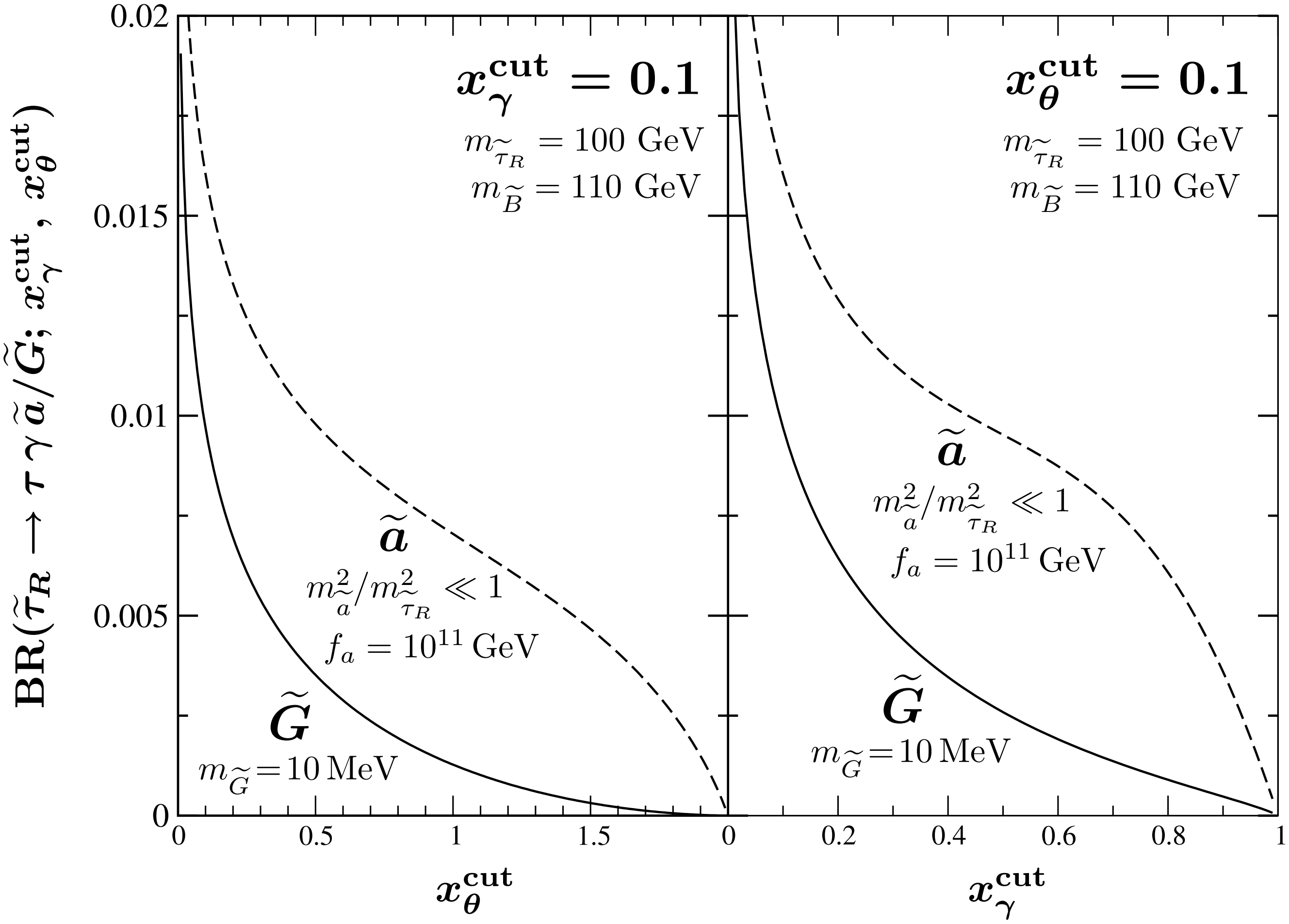
$$\frac{d^2\Gamma(\widetilde{\tau}_R \to \tau\,\gamma\,\widetilde{G})}{dx_\gamma\, d\cos\theta} = \frac{m_{\widetilde{\tau}}}{512\pi^3}\,\frac{x_\gamma(1 - A_{\widetilde{G}} - x_\gamma)}{[1 - (x_\gamma/2)(1-\cos\theta)]^2}\sum_{\text{spins}}|\mathcal{M}(\widetilde{\tau}_R \to \tau\,\gamma\,\widetilde{G})|^2,$$

where $\displaystyle\sum_{\text{spins}}|\mathcal{M}(\widetilde{\tau}_R \to \tau\,\gamma\,\widetilde{G})|^2 = \frac{8\pi\alpha}{3}\,\frac{m^2_{\widetilde{\tau}}}{\mathrm{M}^2_{\mathrm{Pl}}\,A_{\widetilde{G}}}\,F^{(\widetilde{G})}_{\text{diff}}(x_\gamma, \cos\theta, A_{\widetilde{G}}, A_{\widetilde{B}})$

with $\displaystyle x_\gamma \equiv \frac{2E_\gamma}{m_{\widetilde{\tau}}}\,,\qquad A_{\widetilde{a}} \equiv \frac{m^2_{\widetilde{a}}}{m^2_{\widetilde{\tau}}}\,,\qquad A_{\widetilde{B}} \equiv \frac{m^2_{\widetilde{B}}}{m^2_{\widetilde{\tau}}}\,,\qquad A_{\widetilde{G}} \equiv \frac{m^2_{\widetilde{G}}}{m^2_{\widetilde{\tau}}},$

$$\begin{aligned}
F^{(\widetilde{G})}_{\text{diff}}(x_\gamma, \cos\theta, A_{\widetilde{G}}, A_{\widetilde{B}}) &= -3A^2_{\widetilde{G}} - 7x_\gamma A_{\widetilde{G}} + \frac{2(2-5\cos\theta)A_{\widetilde{G}}}{1-\cos\theta} - \frac{x_\gamma(1+\cos\theta)}{(1-\cos\theta)} \\
&- \frac{(1+\cos\theta)(3+\cos\theta)}{(1-\cos\theta)^2} + \frac{2(1-A_{\widetilde{G}})^3(1+\cos\theta)}{x^2_\gamma(1-\cos\theta)} + \frac{A_{\widetilde{G}}(1-A_{\widetilde{G}})^2}{1-A_{\widetilde{G}}-x_\gamma} \\
&+ \frac{(1-A_{\widetilde{G}})^2(1+\cos\theta)}{(1-A_{\widetilde{G}}-x_\gamma)(1-\cos\theta)} - \frac{4\left[1+\cos\theta + A_{\widetilde{G}}(1-\cos\theta)\right]^2}{[2-x_\gamma(1-\cos\theta)]^2\,(1-\cos\theta)^2} \\
&+ \frac{2\left\{3+\cos\theta\left[4-\cos\theta+2A_{\widetilde{G}}(1-\cos\theta)\right]\right\}\left[1+\cos\theta+A_{\widetilde{G}}(1-\cos\theta)\right]}{[2-x_\gamma(1-\cos\theta)]\,(1-\cos\theta)^2} \\
&+2(1-A_{\widetilde{G}}-x_\gamma)\left\{\frac{1+x_\gamma - x^2_\gamma - 2A_{\widetilde{G}}(1+3x_\gamma-2x^2_\gamma) + A^2_{\widetilde{G}}(1+5x_\gamma)}{x_\gamma(1-A_{\widetilde{B}})(1-A_{\widetilde{G}}-x_\gamma)}\right. \\
&- \frac{2\left[1+x_\gamma(2+A_{\widetilde{B}}) - x^2_\gamma + 2A_{\widetilde{G}}(1-x_\gamma)\right]}{x_\gamma\,[2-x_\gamma(1-\cos\theta)]} + \frac{4(1-A_{\widetilde{G}}-x_\gamma)}{[2-x_\gamma(1-\cos\theta)]^2} \\
&- \frac{\sqrt{A_{\widetilde{B}}A_{\widetilde{G}}}\left[2(1+\cos\theta)(1-A_{\widetilde{G}}) + 3x_\gamma A_{\widetilde{G}}(1-\cos\theta)\right]}{x_\gamma(1+\cos\theta) + 2(A_{\widetilde{G}}-A_{\widetilde{B}}) + A_{\widetilde{B}}x_\gamma(1-\cos\theta)} \\
&- \frac{2\left\{A^2_{\widetilde{G}}\left[-3-6x_\gamma + A_{\widetilde{B}}(2+x_\gamma)\right] + 4A_{\widetilde{B}}A_{\widetilde{G}}(1+x_\gamma-x^2_\gamma)\right\}}{x_\gamma(1-A_{\widetilde{B}})\left[x_\gamma(1+\cos\theta)+2(A_{\widetilde{G}}-A_{\widetilde{B}})+A_{\widetilde{B}}x_\gamma(1-\cos\theta)\right]} \\
&\left.+ \frac{2A^2_{\widetilde{B}}\left[(1-x_\gamma)(1+2A_{\widetilde{G}}+x_\gamma) + x_\gamma A_{\widetilde{B}}\right]}{x_\gamma(1-A_{\widetilde{B}})\left[x_\gamma(1+\cos\theta)+2(A_{\widetilde{G}}-A_{\widetilde{B}})+A_{\widetilde{B}}x_\gamma(1-\cos\theta)\right]}\right\} \\
&+(1-A_{\widetilde{G}}-x_\gamma)\left\{\frac{(-1+3A_{\widetilde{G}})(1-A_{\widetilde{G}})}{(1-A_{\widetilde{B}})} + \frac{2\left[2-x_\gamma-2(A_{\widetilde{G}}-A_{\widetilde{B}})\right]}{2-x_\gamma(1-\cos\theta)}\right. \\
&- \frac{4(1-A_{\widetilde{G}}-x_\gamma)}{[2-x_\gamma(1-\cos\theta)]^2} - \frac{2(A_{\widetilde{G}}-A_{\widetilde{B}})\left[3A_{\widetilde{G}}(2-2A_{\widetilde{G}}-x_\gamma) + A_{\widetilde{B}}(2-2A_{\widetilde{B}}+x_\gamma)\right]}{(1-A_{\widetilde{B}})\left[x_\gamma(1+\cos\theta)+2(A_{\widetilde{G}}-A_{\widetilde{B}})+A_{\widetilde{B}}x_\gamma(1-\cos\theta)\right]} \\
&\left.+ \frac{4(1-A_{\widetilde{G}}-x_\gamma)(3A_{\widetilde{G}}+A_{\widetilde{B}})(A_{\widetilde{G}}-A_{\widetilde{B}})^2}{(1-A_{\widetilde{B}})\left[x_\gamma(1+\cos\theta)+2(A_{\widetilde{G}}-A_{\widetilde{B}})+A_{\widetilde{B}}x_\gamma(1-\cos\theta)\right]^2}\right\}
\end{aligned}$$

Branching Ratios of $\widetilde{\tau}_R \to \tau\,\gamma\,\widetilde{a}/\widetilde{G}$ with Cuts



- Branching Ratio of the Integrated Rate with Cuts

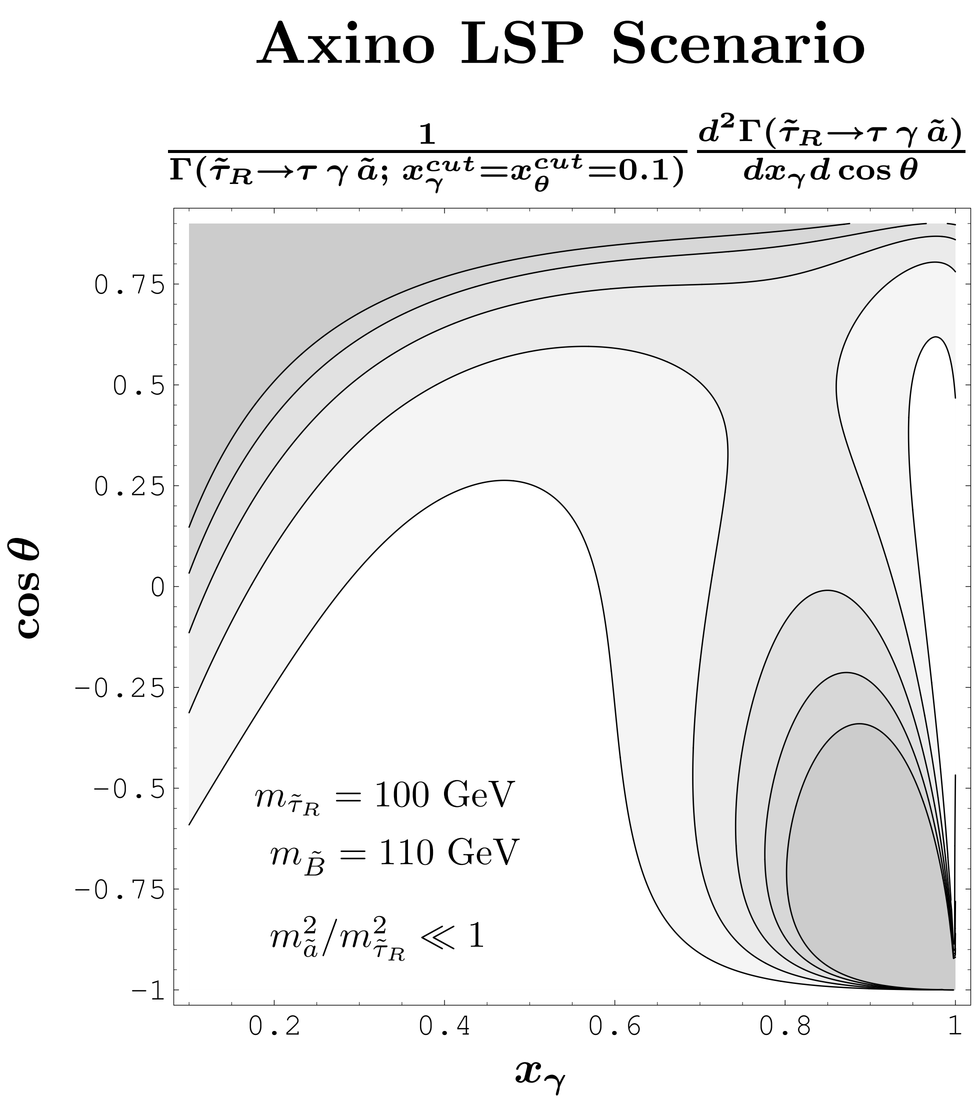
$$\mathrm{BR}(\widetilde{\tau}_\mathrm{R} \to \tau\,\gamma\,i\,;x_\gamma^{\rm cut}, x_\theta^{\rm cut}) \equiv \frac{\Gamma(\widetilde{\tau}_\mathrm{R} \to \tau\,\gamma\,i\,;x_\gamma^{\rm cut}, x_\theta^{\rm cut})}{\Gamma^{\rm total}_{\tilde{\tau}_R \to i\,X}}$$

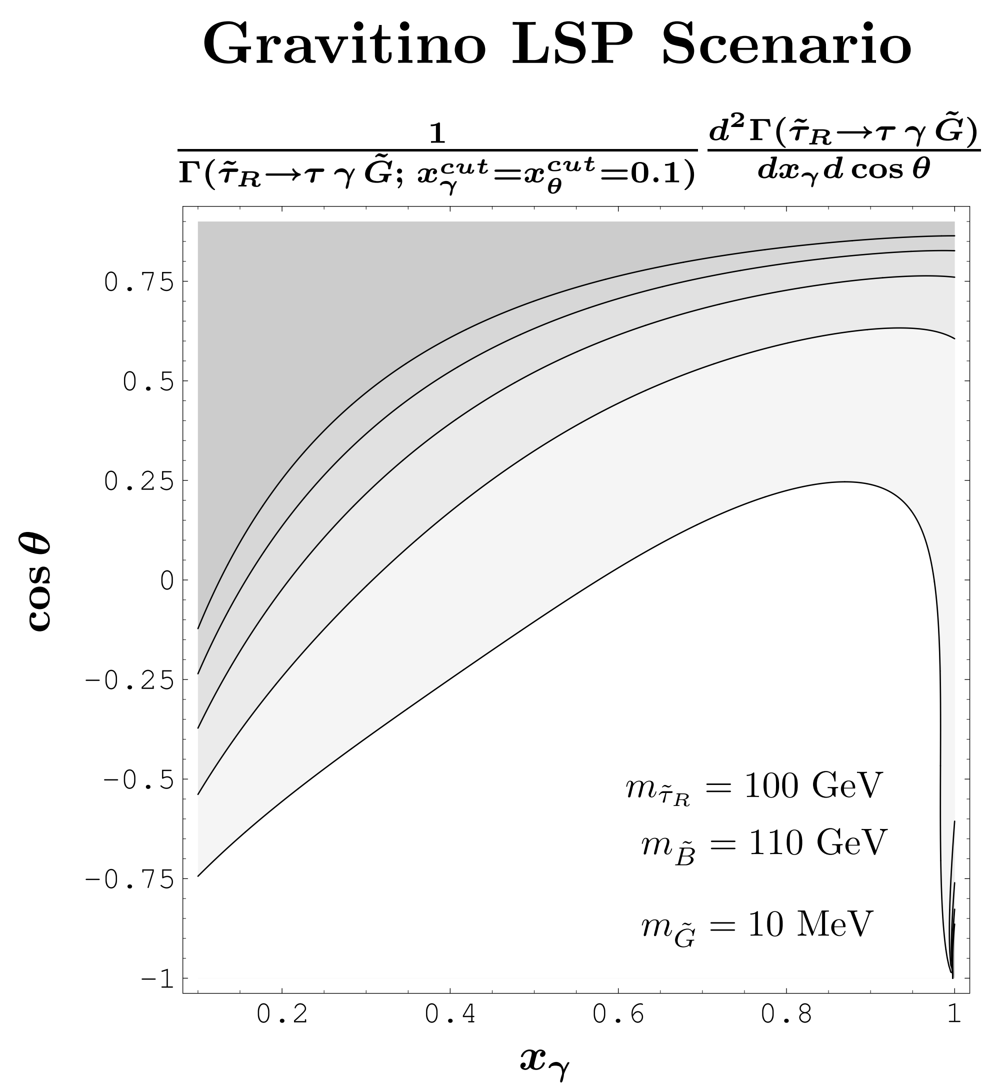
- Integrated Rate $\quad \leftarrow$ Cuts on $x_\gamma = 2\,E_\gamma/m_{\widetilde{\tau}}$ & $\cos\theta$

$$\Gamma(\widetilde{\tau}_\mathrm{R} \to \tau\,\gamma\,i\,;x_\gamma^{\rm cut}, x_\theta^{\rm cut}) = \int_{x_\gamma^{\rm cut}}^{1-A_i} dx_\gamma \int_{-1}^{1-x_\theta^{\rm cut}} d\cos\theta \,\frac{d^2\Gamma(\widetilde{\tau}_\mathrm{R} \to \tau\,\gamma\,i)}{dx_\gamma\, d\cos\theta}$$

- Total Decay Rate of the Stau NLSP $\leftarrow$ 2-Body Decay

$$\Gamma^{\rm total}_{\tilde{\tau}_R \to i\,X} \simeq \Gamma(\widetilde{\tau}_\mathrm{R} \to \tau\,i)\ , \quad i = \widetilde{a}, \widetilde{G}$$





Differential Distribution of the Visible Decay Products

[Buchmüller, Hamaguchi, Ratz, Yanagida, '04]

Polarizations of τ and γ $\longrightarrow$ Spin of the LSP

Conclusion

? What is the Nature of Cold Dark Matter ? $\longleftarrow$ Thermal Production of $\tilde{a}/\widetilde{G}$'s

Axino LSP $\leftarrow$ **SM + PQ + SUSY**

$\tilde{a}\,\mathrm{CDM} \leftarrow (m_{\tilde{a}}, T_R) \approx (100\,\mathrm{keV}, 10^6\ \mathrm{GeV})$

$\tilde{a}\,\mathrm{WDM} \leftarrow (m_{\tilde{a}}, T_R) \approx (100\ \mathrm{eV}, 10^{10}\,\mathrm{GeV})$

Gravitino LSP $\leftarrow$ **SM + local SUSY**

$\widetilde{G}\,\mathrm{CDM} \leftarrow (m_{\widetilde{G}}, T_R) \approx (10\ \mathrm{MeV}, 10^6\ \mathrm{GeV}$

$\widetilde{G}\,\mathrm{CDM} \leftarrow (m_{\widetilde{G}}, T_R) \approx (100\,\mathrm{GeV}, 10^{10}\,\mathrm{GeV}$

! Probing the $\tilde{a}/\widetilde{G}$ LSP Scenarios at the ILC $\longleftarrow$ Assumption: $\boldsymbol{\tilde{\tau}}$ **= NLSP** !

Axino LSP: $\boldsymbol{\tilde{\tau} \rightarrow \tau + \tilde{a}}$

* Estimate of the Peccei-Quinn Scale f_a
* Measurement of the Axino Mass $m_{\tilde{a}}$

Gravitino LSP: $\boldsymbol{\tilde{\tau} \rightarrow \tau + \widetilde{G}}$

* Measurement of the Planck Scale $\mathrm{M_{Pl}}$
* Measurement of the Gravitino Mass m

! Identifying the LSP Scenario at the ILC $\longleftarrow$ $\boldsymbol{\tilde{\tau} \rightarrow \tau + \gamma + \tilde{a}/\widetilde{G}}$!

* Branching Ratio
* Differential Distribution
* τ & γ Polarizations: $\tilde{a}/\widetilde{G}$ Spin

Outlook

International Linear Collider

- $\tilde{a}/\widetilde{G}$ LSP
- $\widetilde{\tau}$ NLSP
- Analysis of $\widetilde{\tau}$ Decays

Nature of Cold Dark Matter

Axino $\tilde{a}$

Gravitino $\widetilde{G}$

Proving the Existence of

Peccei–Quinn Mechanism

$\downarrow$

No Strong CP Problem

Supergravity

$\downarrow$

String Theory