Electroweak precision observables in the NMFV MSSM

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based on collaboration with
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1. Introduction

Precision Observables:

An alternative way, as compared to the direct search for SUSY or Higgs particles, is to probe SUSY via virtual effects of the additional non-standard particles to precision observables.

Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
$M_W, \sin^2\theta_{\text{eff}}, \ldots$	$\leftrightarrow$	SM, MSSM, ...

$\Downarrow$

Test of theory at quantum level: Sensitivity to loop corrections

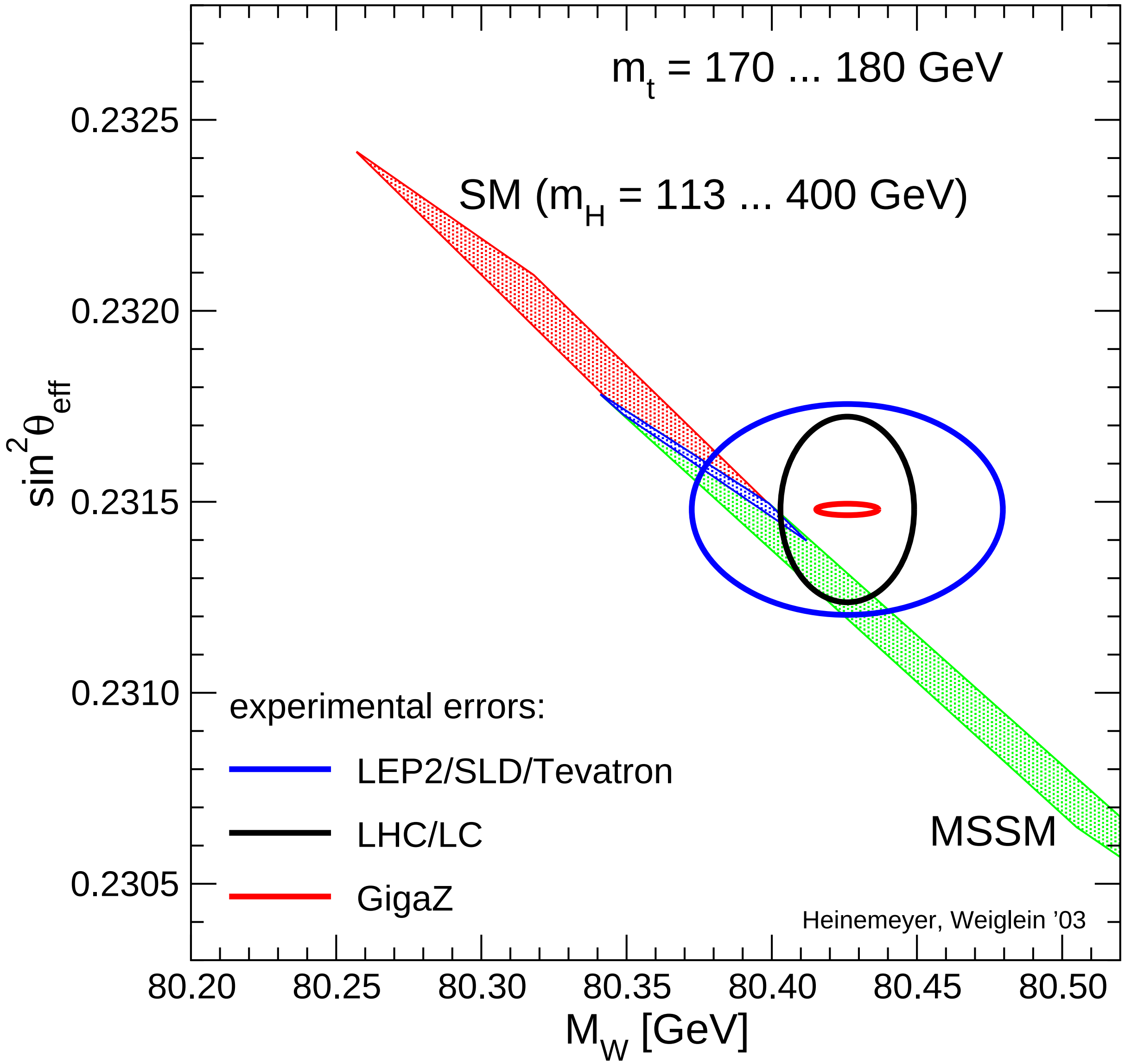
$\Downarrow$

Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

Example: Prediction for M_W and $\sin^2\theta_{\rm eff}$ in the SM and the MSSM :

hep-ph/0307177



MSSM uncertainty:
unknown masses
of SUSY particles

SM uncertainty:
unknown Higgs mass

NMFV in the MSSM

NMFV: Non Minimal Flavor Violation

$\rightarrow$ Mixing of scalar quark families (beyond CKM)

We consider the general case of mixing between the third and second generation of squarks ($\tilde{t}/\tilde{c}$ and $\tilde{b}/\tilde{s}$ sectors)

E.g. Mixing of stop/scharm

$$(\tilde{t}_L, \tilde{t}_R, \tilde{c}_L, \tilde{c}_R) \begin{pmatrix} \widetilde{T} & 0 \\ 0 & \widetilde{C} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \\ \tilde{c}_L \\ \tilde{c}_R \end{pmatrix} \quad \Rightarrow \quad (\tilde{t}_L, \tilde{t}_R, \tilde{c}_L, \tilde{c}_R) \begin{pmatrix} \widetilde{T} & \neq 0 \\ \neq 0 & \widetilde{C} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \\ \tilde{c}_L \\ \tilde{c}_R \end{pmatrix}$$

add NMFV

- mixing between the 3^{rd} and 2^{nd} generation squarks can be numerically significant due to the involved third-generation Yukawa couplings
- experimentally only partially restricted (most stringest constraints are given by $B(b \rightarrow s\gamma)$)
- strong experimental bounds involving the 1^{st} generation, coming from data on K^0–$\bar{K}^0$ and D^0–$\bar{D}^0$ mixing

F. Gabbiani et al, hep-ph/9604387; M. Misiak et al, hep-ph/9703442...

Squark Generation Mixing via Soft Breaking

Parametrization of non-diagonal squark mass matrices

$$M^2_{\tilde{u}} = \begin{pmatrix} M^2_{\tilde{L}_c} & \Delta^t_{LL} & m_c X_c & \Delta^t_{LR} \\ \Delta^t_{LL} & M^2_{\tilde{L}_t} & \Delta^t_{RL} & m_t X_t \\ m_c X_c & \Delta^t_{RL} & M^2_{\tilde{R}_c} & \Delta^t_{RR} \\ \Delta^t_{LR} & m_t X_t & \Delta^t_{RR} & M^2_{\tilde{R}_t} \end{pmatrix}$$

$$\begin{aligned} M^2_{\tilde{L}_q} &= M^2_{\tilde{Q}_q} + m_q^2 + \cos 2\beta\, M_Z^2 (T_3^q - Q_q s_W^2) \\ M^2_{\tilde{R}_q} &= M^2_{\tilde{U}_q} + m_q^2 + \cos 2\beta\, M_Z^2 Q_q s_W^2 \quad (q = t, c) \\ X_q &= A_q - \mu (\tan\beta)^{-2T_3^q} \end{aligned}$$

Similarly for the $\tilde{b}/\tilde{s}$ sector $(t \leftrightarrow b\,, c \leftrightarrow s)$

Mass eigenstates :

In order to diagonalize the two 4×4 squark mass matrices, two 4×4 rotation matrices, $R_{\tilde{u}}$ and $R_{\tilde{d}}$, are needed.

$\longrightarrow$ generate large splittings between the squark-mass eigenvalues

Analytical result:

evaluation with arbitrary NMFV couplings

Numerical result (Simplest scenario)

$\rightarrow$ RGE indicate that the largest entries are those connected to the SUSY partners of the left-handed quarks, Δ_{LL}

P. Brax, C. Savoy, *Nucl. Phys.* **B 447** (1995) 227, hep-ph/9503306

K. Hikasa and M. Kobayashi, *Phys. Rev.* **D 36** (1987) 724

$\rightarrow$ Δ_{LL} scale with the square of diagonal soft SUSY-breaking masses M_{SUSY}

Δ_{LR} and Δ_{RL} terms scale linearly and Δ_{RR} with zero power of M_{SUSY}

$\Rightarrow$ $\Delta_{LL} \gg \Delta_{LR,RL} \gg \Delta_{RR}$

Mixing only between the left-handed components of $\tilde{t}, \tilde{c}$ and $\tilde{b}, \tilde{s}$

$$\boxed{\begin{array}{ll} \Delta^t_{LL} = \lambda^t M_{\tilde{L}_t} M_{\tilde{L}_c}\,, & \Delta^t_{LR} = \Delta^t_{RL} = \Delta^t_{RR} = 0\,, \\ \Delta^b_{LL} = \lambda^b M_{\tilde{L}_b} M_{\tilde{L}_s}\,, & \Delta^b_{LR} = \Delta^b_{RL} = \Delta^b_{RR} = 0\,. \end{array}}$$

$\rightarrow$ $\lambda^t = \lambda^b = 0$ corresponds to the MSSM with MFV

$\rightarrow$ λ^t and λ^b correspond to $(\delta^u_{LL})_{23}$ and $(\delta^d_{LL})_{23}$

2. Results for M_W and $\sin^2\theta_{\rm eff}$

M_W Theoretical prediction for M_W in terms of M_Z, α, G_μ, Δr:

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}\left(\frac{1}{1-\Delta r}\right)$$

$$\Updownarrow$$

loop corrections

Δr depends on the entire set of input parameters $\Delta r = \Delta r(\alpha, M_W, M_Z, m_t, ..., M_{SUSY}...)$
contains photon vacuum polarization, γ, Z, W self-energies, box and vertex corrections

Effective mixing angle:

$$\sin^2\theta_{\rm eff}=\frac{1}{4\,|Q_f|}\left(1-\frac{\mathrm{Re}\,g_V^f}{\mathrm{Re}\,g_A^f}\right)$$

Higher order contributions:

$$g_V^f \to g_V^f + \underbrace{\Delta g_V^f}_{\Delta r}, \qquad g_A^f \to g_A^f + \underbrace{\Delta g_A^f}_{\Delta r}$$

All EW loop effects in Z-boson decays are concealed in the effective couplings g_V^f, g_A^f

Corrections to M_W, $\sin^2\theta_{\rm eff}$

The shift in M_W and $\sin^2\theta_{\rm eff}$ caused by a variation of Δr reads

$$\begin{aligned}
\delta M_W &= -\frac{M_W}{2}\frac{s_W^2}{c_W^2 - s_W^2}\,\delta(\Delta r) \\
\delta \sin^2\theta_{\rm eff} &= \frac{c_W^2\, s_W^2}{c_W^2 - s_W^2}\,\delta(\Delta r) \\
&\quad - s_W c_W \left[\frac{\Sigma_{\gamma Z}(M_Z^2)}{M_Z^2} - \frac{c_W}{s_W}\left(\frac{\Sigma_Z(M_Z^2)}{M_Z^2} - \frac{\Sigma_W(M_W^2)}{M_W^2}\right)\right]
\end{aligned}$$

As fas as $\delta(\Delta r)$ originates from squarks -loop contributions to the self energies only:

$$\delta(\Delta r) = \Sigma'_\gamma(0) - \frac{c_W^2}{s_W^2}\left(\frac{\Sigma_Z(M_Z^2)}{M_Z^2} - \frac{\Sigma_W(M_W^2)}{M_W^2}\right) + \frac{\Sigma_W(0) - \Sigma_W(M_W^2)}{M_W^2}$$

Flavor mixing through the flavor non-diagonal entries in the squark-mass matrices

Numerically verified:

$\Delta\rho$ is an excellent approximation for the full calculation

$\Rightarrow$ concentrate on $\Delta\rho$ (but full calculation is available)

Corrections to M_W, $\sin^2\theta_{\text{eff}}$ can be approximated with the ρ-**parameter:**

ρ measures the relative strength between
neutral current interaction and charged current interaction

$$\rho = \frac{1}{1 - \Delta\rho} \qquad\qquad \Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

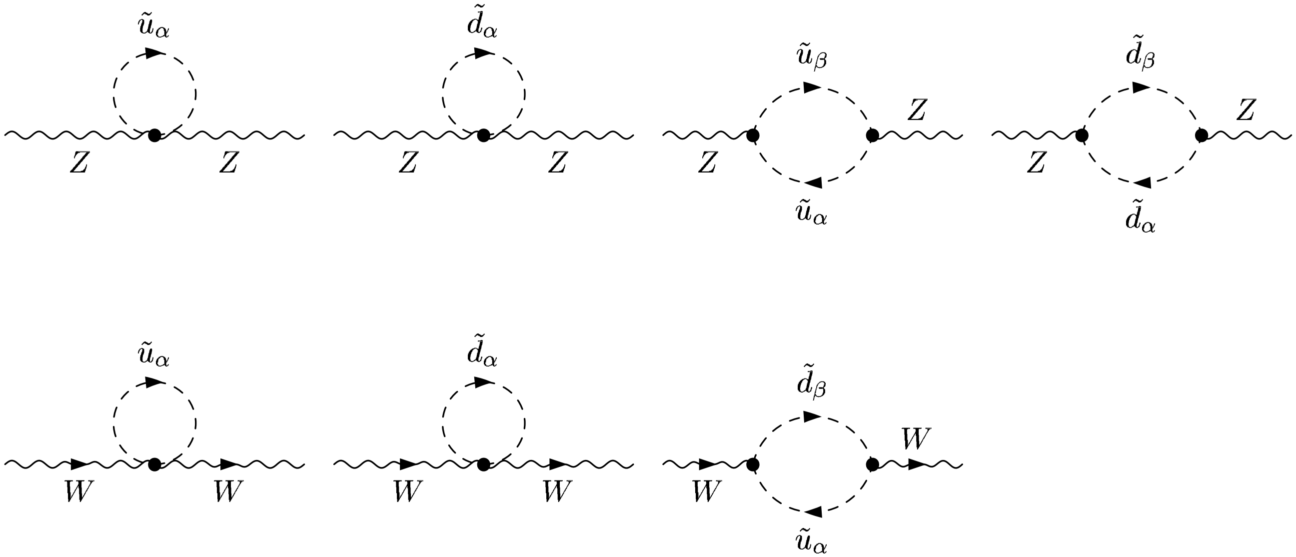
$\Delta\rho$ represents the leading universal corrections to the EW precision observables induced by mass splitting between partners in isospin doublets

$\Delta\rho$ gives the main contribution to:

$$\delta M_W \approx \frac{M_W}{2} \frac{c_W^2}{c_W^2 - s_W^2} \Delta\rho\,, \qquad\qquad \delta \sin^2\theta_W^{\text{eff}} \approx -\frac{c_W^2 s_W^2}{c_W^2 - s_W^2} \Delta\rho$$

$\Rightarrow$ Experimental bound: $\Delta\rho \lesssim 2 \times 10^{-3}$

Feynman diagrams for $\Delta\rho$:



Numerical analysis performed in 2 benchmark scenarios, but with a free scale M_{SUSY} :

M. Carena et. al, hep-ph/0202167

– $M_{h^0}^{\max}$ (A_t is not a free parameter, obeying $X_t = 2M_{\mathrm{SUSY}}$, with $X_t = A_t - \mu \cot\beta$)

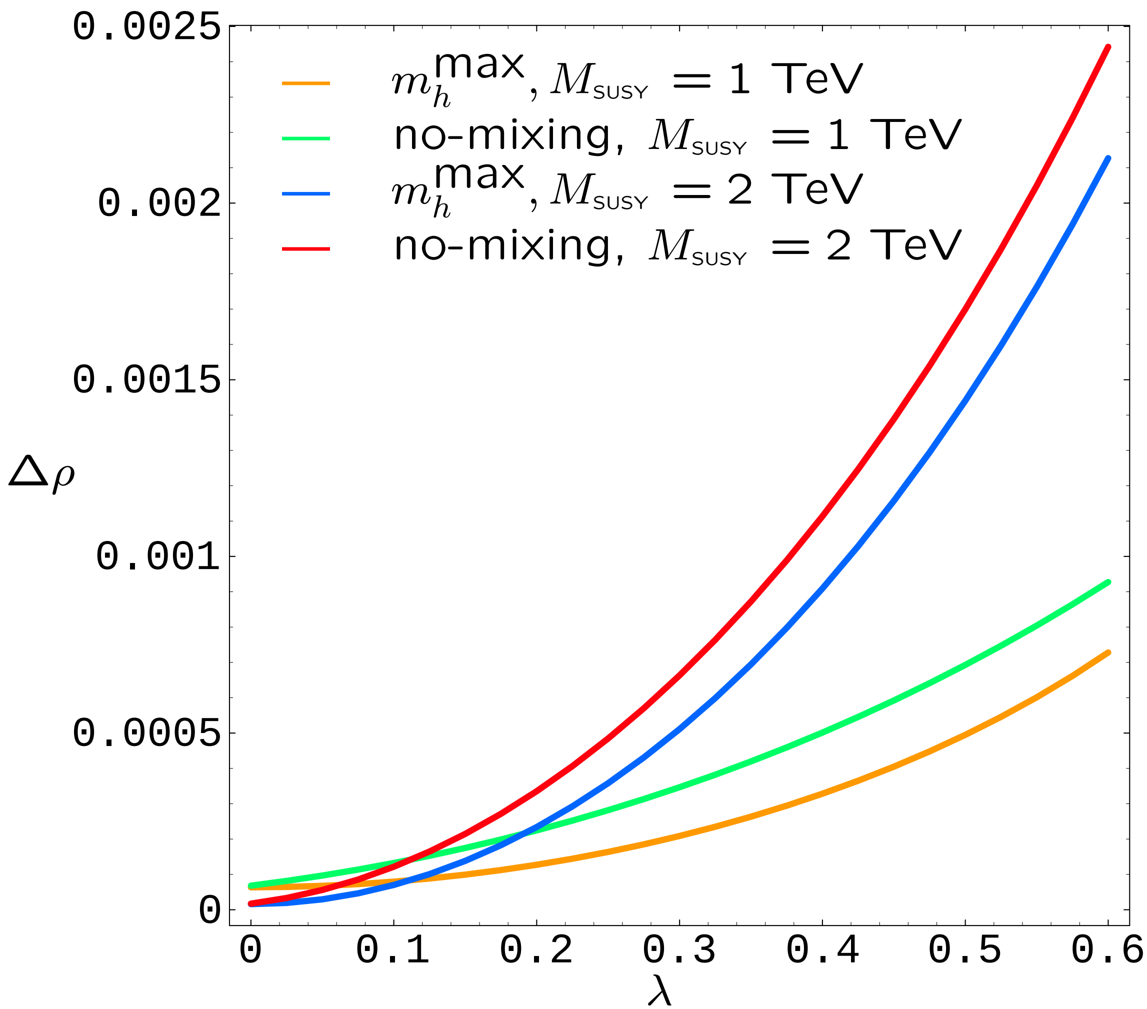
– no-mixing (no mixing in the MFV $\tilde{t}$ sector ($X_t = 0$))

The same flavor mixing parameter in the $\tilde{t}/\tilde{c}$ and $\tilde{b}/\tilde{s}$ sectors is assumed: $\lambda = \lambda^t = \lambda^b$

A large difference between λ^t and λ^b is not allowed: LL blocks of the up- and down-squark mass matrices are not independent because of the $SU(2)$ gauge invariance.

M. Misiak, S. Pokorski and J. Rosiek,hep-ph/9703442.

$\Delta\rho$ as a function of λ:



increasing λ

$\Rightarrow$ increasing mixing, splitting in the squarks sector

$\Rightarrow$ increasing $\Delta\rho$

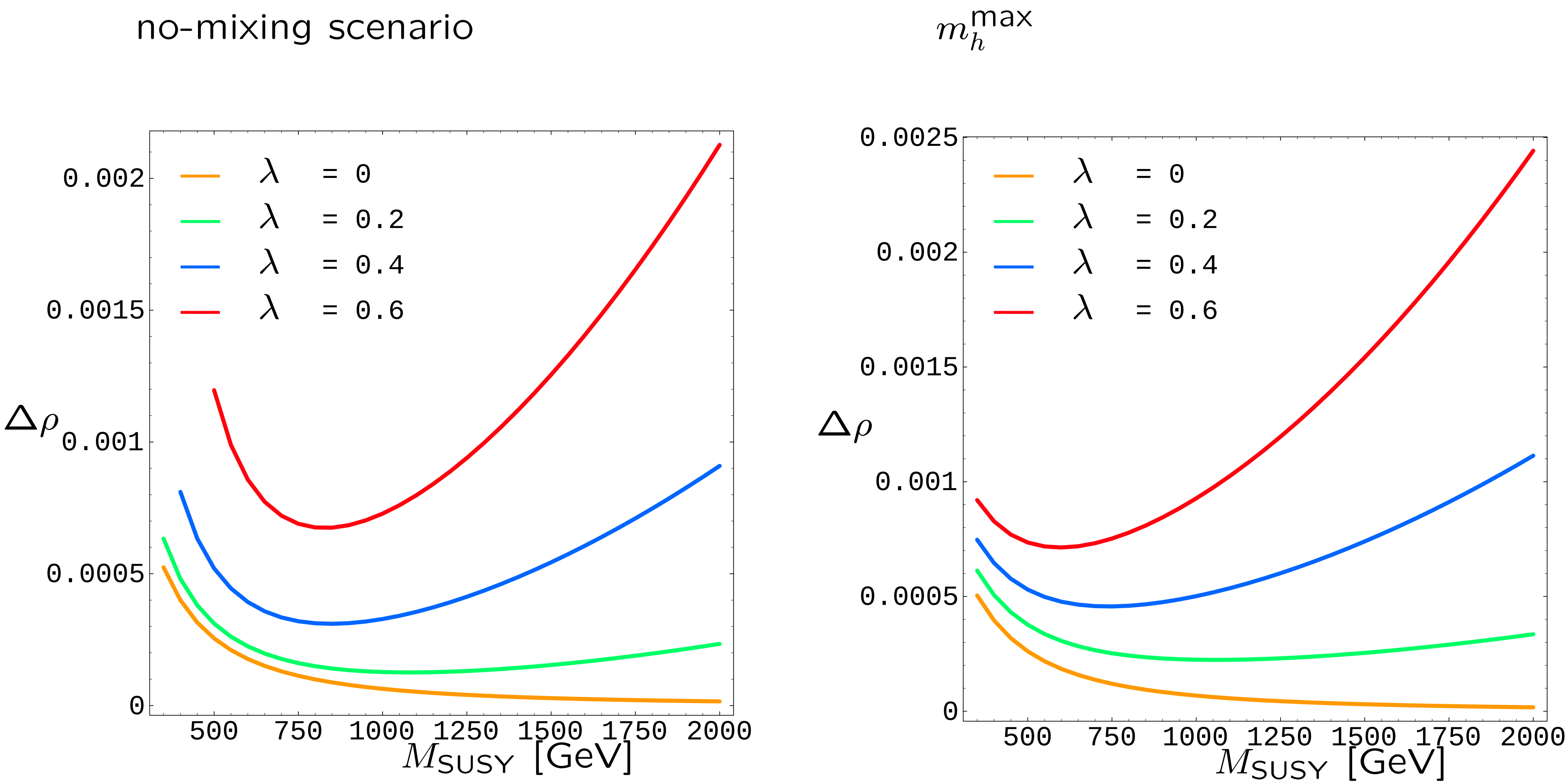
idem when increasing M_{SUSY}

Decoupling for $\lambda = 0$ as expected

$\Delta\rho^{\tilde{q}}$ grows with the λ parameter, being close to zero for $\lambda = 0$ and $M_{\text{SUSY}} = 2$ TeV.

$\Delta\rho \lesssim 2 \times 10^{-3}$ can be saturated

$\Delta\rho$ as a function of M_{SUSY}:

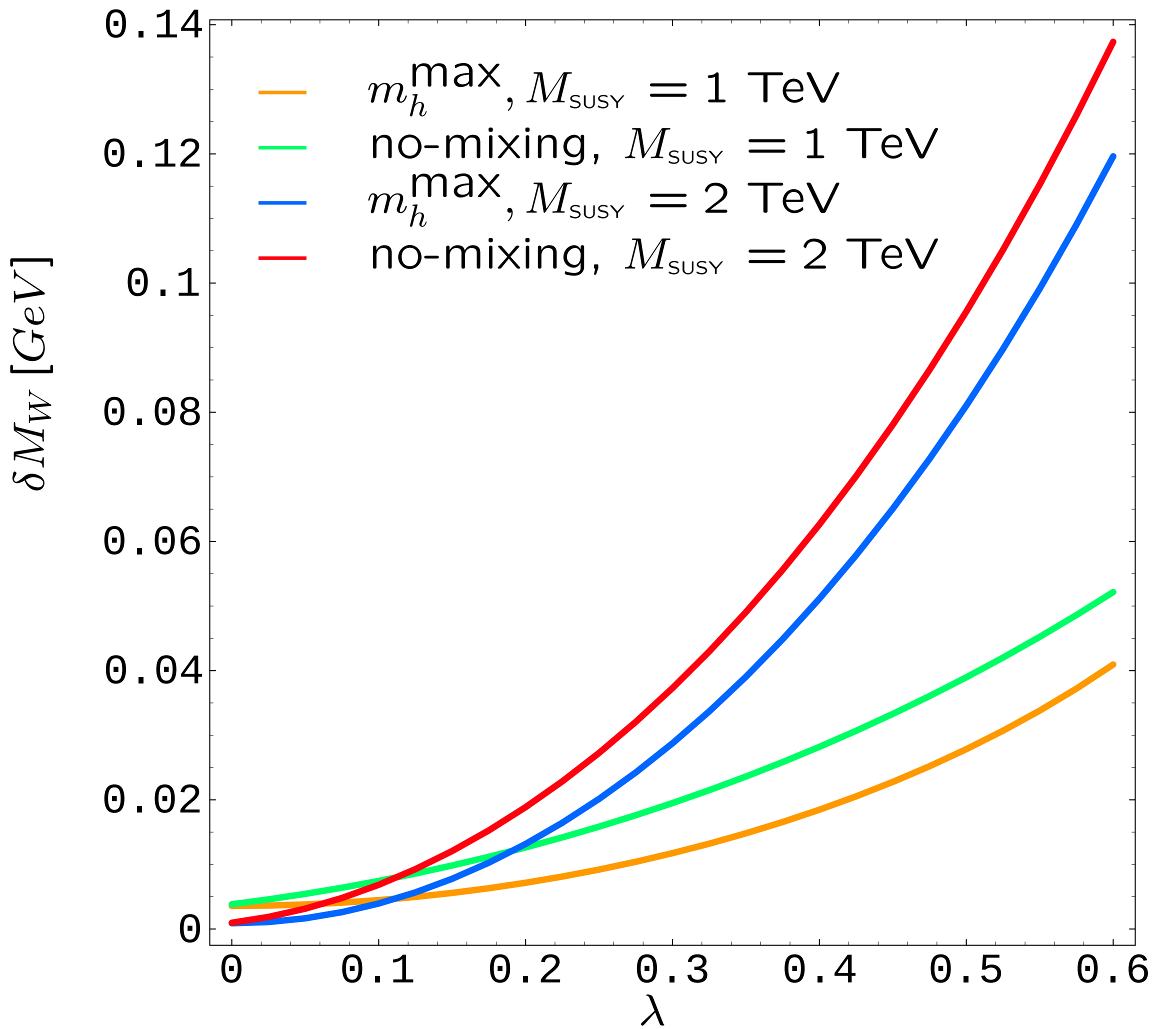


$\rightarrow$ decoupling for $\lambda = 0$ as expected

$\rightarrow$ $\lambda \neq 0$: minimum at moderate M_{SUSY}

increase for large M_{SUSY} (due to enlarged mixing)

δM_W as a function of λ:



follows the behavior of $\Delta\rho$

$\rightarrow$ The induced shifts in M_W can become as large as 0.14 GeV for no-mixing, $M_{\text{SUSY}} = 2$ TeV, $\lambda = 0.6$.

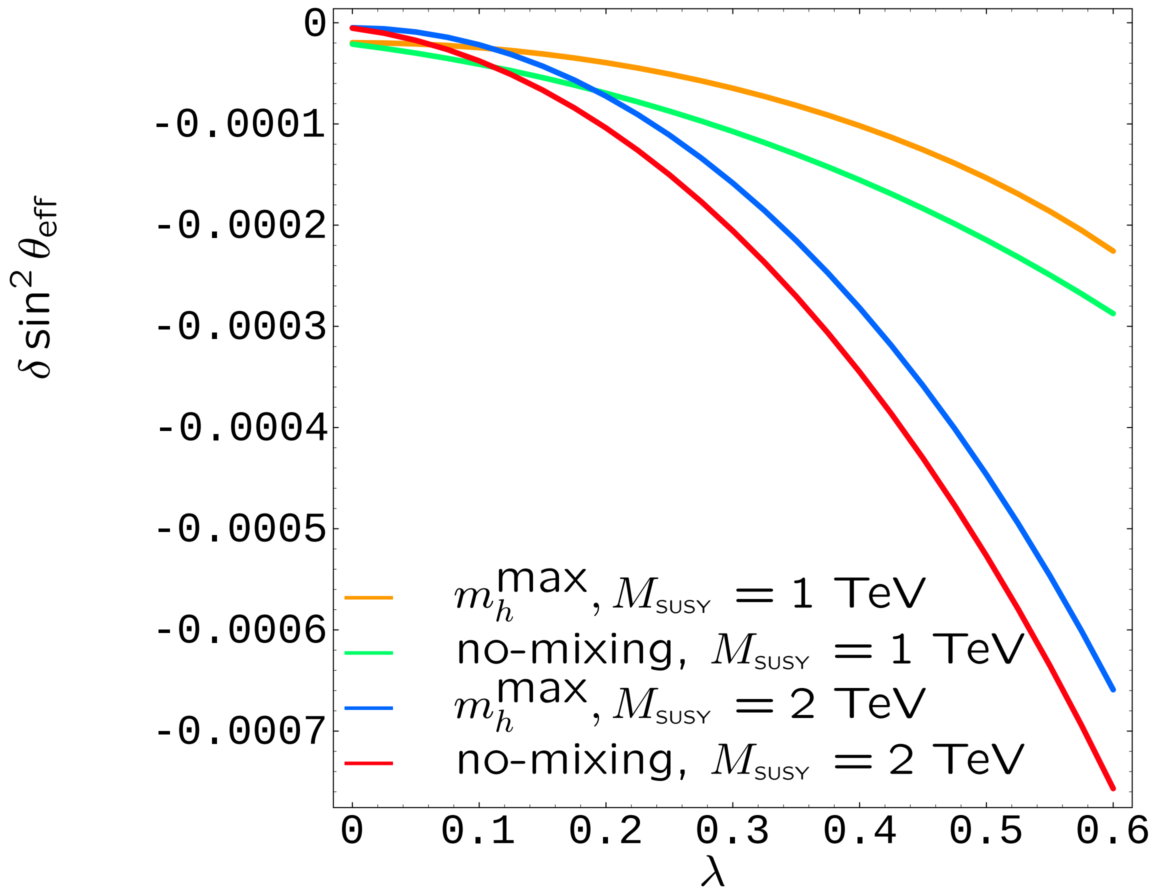
$\rightarrow$ $\delta M_W \lesssim 0.05$ GeV in the less favorable scenario, but still sizeable.

$$\delta M_W^{\text{exp,today}} = 34 \text{ MeV}$$

$$\delta M_W^{\text{exp,future}} = 7 \text{ MeV}$$

$\Rightarrow$ extreme parameter regions already ruled out

$\delta \sin^2\theta_{\rm eff}$ as a function of λ:



follows the behavior of $\Delta\rho$

$\to$ The shifts $\delta \sin^2\theta_{\rm eff}$ can reach values up 7×10^{-4} for no-mixing scenario, $M_{\rm SUSY} = 2$ TeV, $\lambda = 0.6$,

$\to$ smaller, but still sizeable, for the other scenarios.

$\delta \sin^2\theta_{\rm eff}^{\rm exp,today} = 17 \times 10^{-5}$

$\delta \sin^2\theta_{\rm eff}^{\rm exp,future} = 1.3 \times 10^{-5}$

$\Rightarrow$ extreme parameter regions already ruled out

$\Rightarrow$ highly sensitive test in the future

3. Results for M_{h^0}

- Contrary to the SM: M_{h^0} is not a free parameter

- Large radiative corrections:

  Dominant one-loop corrections: $\sim G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

  The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

- Measurement of M_{h^0}, Higgs couplings $\Rightarrow$ test of the theory

- LHC: $\Delta M_{h^0} \approx 0.2$ GeV

  LC: $\Delta M_{h^0} \approx 0.05$ GeV

$\Rightarrow M_{h^0}$ will be electroweak precision observable

MSSM with MFV

Dominant one-loop contributions are described by loop diagrams involving third-generation quarks and squarks.

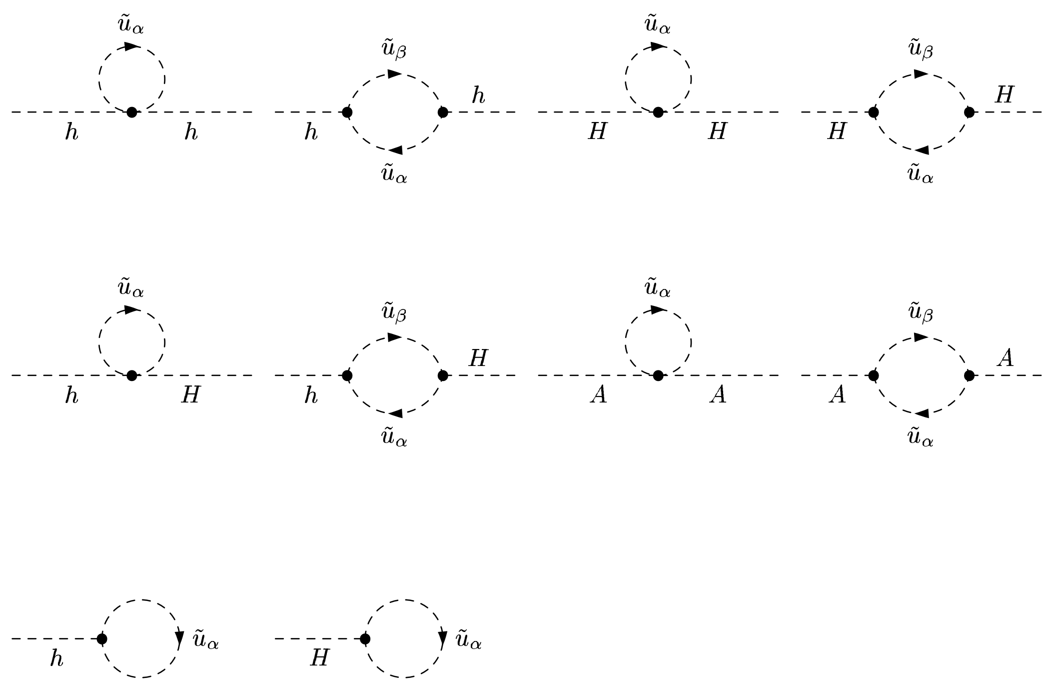
MSSM with NMFV

The squark loops have to be modified by introducing the generation-mixed squarks.

Feynman diagrams for M_{h^0}:

$\Rightarrow$ For not too large $\tan\beta$: only $\tilde{t}/\tilde{c}$ sector relevant

$\Rightarrow$ Evaluation of Σ_h, Σ_H, Σ_{hH}, Σ_A, T_h, T_H (contributions from $t/\tilde{t}$ and $c/\tilde{c}$ only)



Higgs boson sector analysis performed in 5 benchmark scenarios:

M. Carena et. al, hep-ph/0202167

– $M_{h^0}^{\text{max}}$: $X_t = 2M_{\text{SUSY}}$, with $X_t = A_t - \mu \cot\beta$
to maximize the lightest Higgs boson mass

– constrained $M_{h^0}^{\text{max}}$: with $X_t/M_{\text{SUSY}} = -2$ for $b \to s\gamma$

– no-mixing : with no mixing in the MFV $\tilde{t}$ sector

– gluophobic Higgs : with reduced ggh coupling

– small α_{eff} : with reduced $h\bar{b}b$ and $h\tau^+\tau^-$ coupling

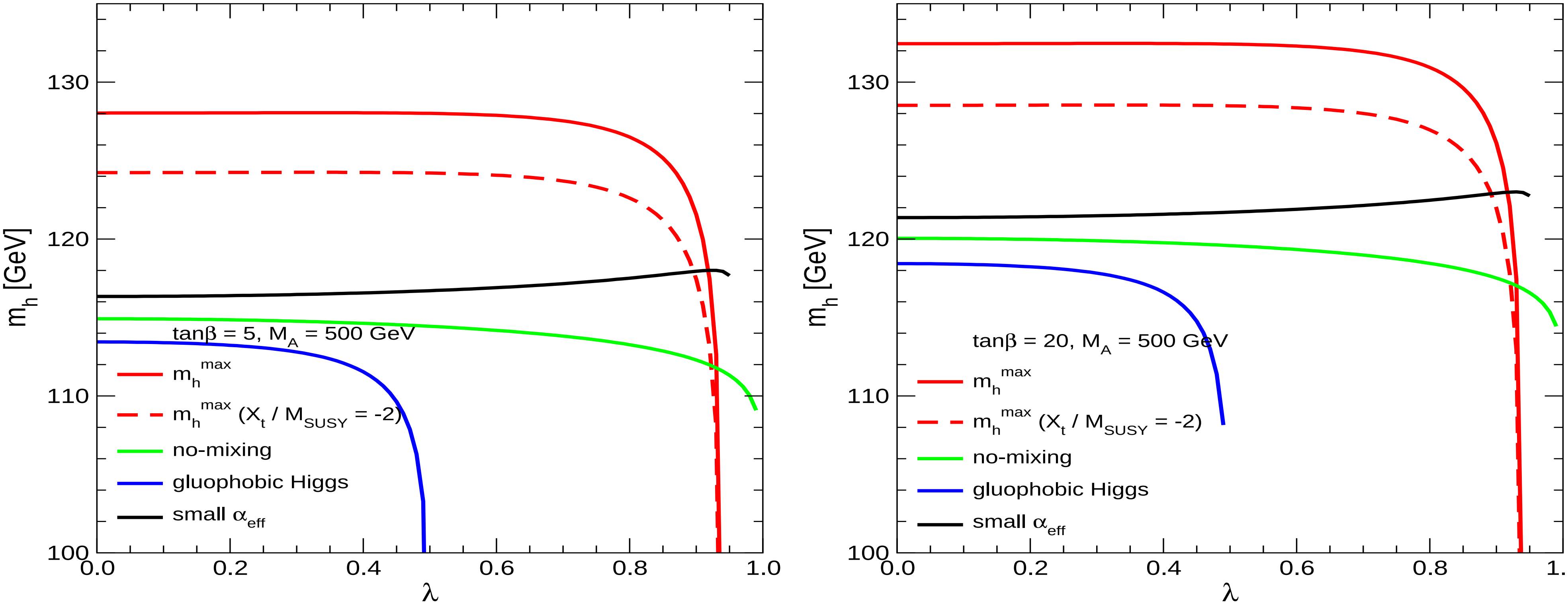
For all these benchmark scenarios the soft SUSY-breaking parameters in the three generations of scalar quarks are equal,

$$M_{\text{SUSY}} = M_{\tilde{Q}_q} = M_{\tilde{U}_q} = M_{\tilde{D}_q} \text{ and } A_s = A_b = A_c = A_t$$

$\Rightarrow$ Results implemented in *FeynHiggs*2.1 (www.feynhiggs.de)

M_{h^0}, mixing angle α and $\Delta\rho$ included

M_{h^0} as a function of λ:



All scenarios show a similar behavior

$\Rightarrow$ small effects for small/moderate λ

$\Rightarrow$ $\delta M_{h^0} = \mathcal{O}\,(5\text{ GeV})$ only for very large λ

(around 0.5 in the gluophobic Higgs scenario, and around 0.9 in the other four scenarios)

$\Rightarrow$ mostly decreasing M_{h^0}, but also increase possible

(in small α_{eff}-scenario it can be enhanced by up to 2 GeV)

4. Conclusions

- Precision observables can

  constrain MSSM parameter space already today, and even more for the increasing precision at future colliders

- MSSM with NMFV:

  general 4×4 mixing in $\tilde{t}/\tilde{c}$ and $\tilde{b}/\tilde{s}$ sectors

  $\Rightarrow$ Evaluation of M_W, $\sin^2\theta_{\text{eff}}$, M_{h^0}

- Analytical results: for arbitrary mixing

  Numerical results: only for LL mixing, parametrized with λ ($(\delta_{LL})_{23}$)

- Large effects possible for M_W, $\sin^2\theta_{\text{eff}}$:

  $\lambda \lesssim 0.2 \Rightarrow \delta M_W \lesssim 20$ MeV $\qquad$ $\lambda \lesssim 0.2 \Rightarrow \delta\sin^2\theta_{\text{eff}} \lesssim 10^{-4}$

  $\longrightarrow$ We have shown that the effects of scalar quark generation mixing enters essentially through $\Delta\rho$

- Moderate effects possible for M_{h^0} only for large λ

- FeynArts, FormCalc, LoopTools include:

  NMFV MSSM : 6×6 generalized squarks mixing matrices