Precise predictions for the effective weak mixing angle

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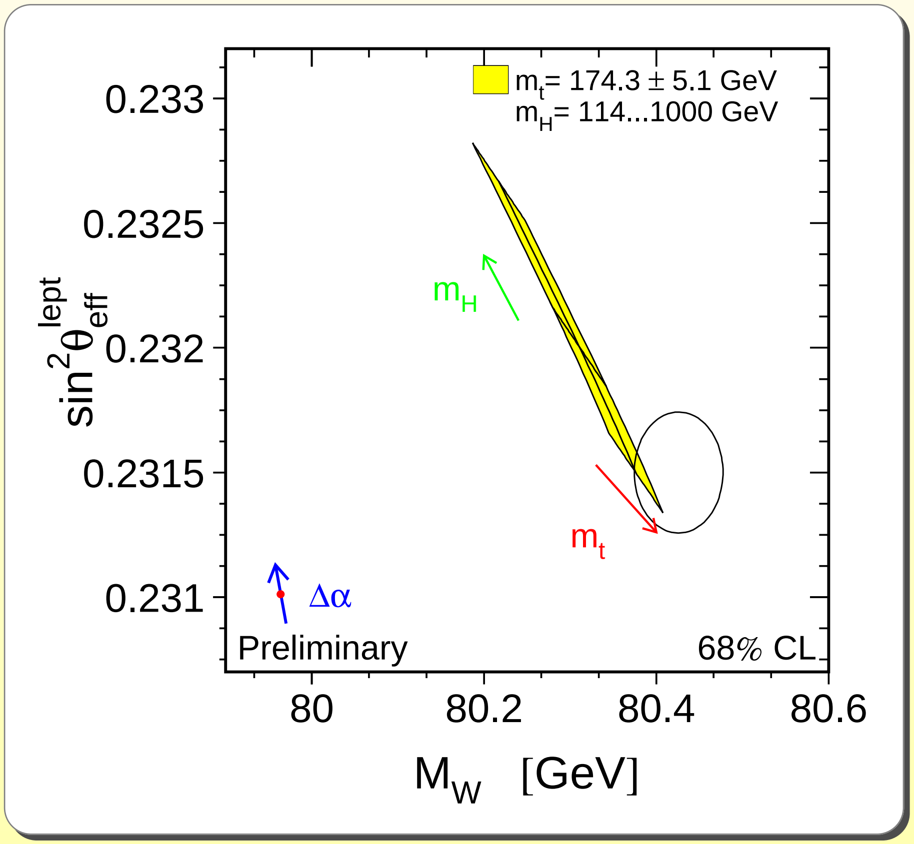
- *Introduction*
- *Electroweak 2-loop corrections*
- *Results*
- *Conclusions*

see Phys. Rev. Lett. **93**, 201805 (2004)

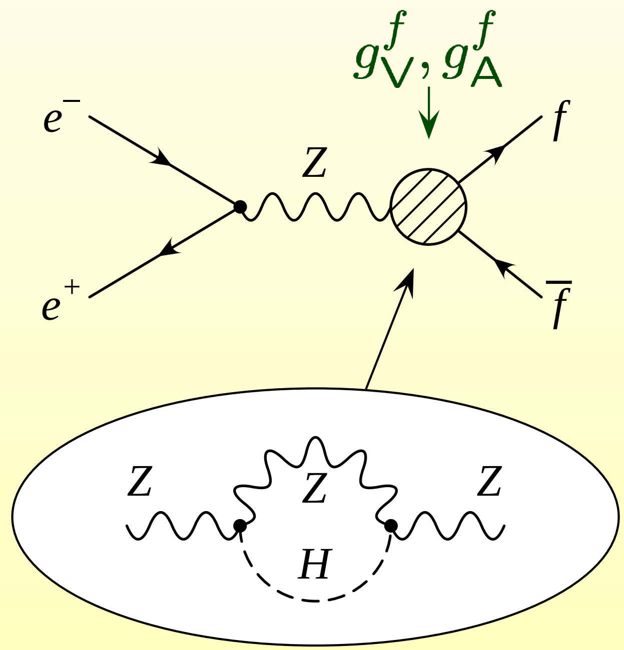
Introduction

$$\sin\theta^{f}_{\mathrm{W,eff}} = \frac{1}{4}\left[1 - \frac{g^{f}_{\mathrm{V}}}{g^{f}_{\mathrm{A}}}\right]$$

important quantity for testing the Standard Model and constraining M_{H}.



LEPEWWG '03



$$\sin^2\theta^{\mathrm{eff}}_{\mathrm{lept}} = \sin^2\theta^{\mathrm{eff}}_{\mathrm{lept}}(M_{\mathrm{H}}, M_{\mathrm{W}}, \ldots)$$

Existing calculations of $\sin^2\theta^{\mathrm{eff}}_{\mathrm{lept}}$

One-loop corrections $\mathcal{O}(\alpha)$ — W.J. Marciano, A. Sirlin '80

QCD corrections $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ and $\mathcal{O}(\alpha\alpha_{\mathrm{s}}^2)$ — A. Djouadi '88

K. Chetyrkin, J. Kühn, M. Steinhauser '95

$\mathcal{O}(\alpha^2)$ electroweak contributions:

Only results using large-mass expansions

Best available result:

Large-m_{t} expansion including $\mathcal{O}(\alpha^2 m_{\mathrm{t}}^4)$ and $\mathcal{O}(\alpha^2 m_{\mathrm{t}}^2)$

G. Degrassi, P. Gambino, A. Sirlin '97

$\rightarrow$ New result: Complete $\mathcal{O}(\alpha^2)$ corrections with closed fermion loops

M. Awramik, M. Czakon, A. Freitas, G. Weiglein '04

Electroweak 2-loop corrections

Define amplite as expansion around complex pole:

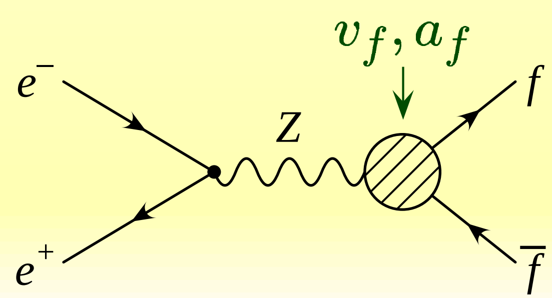
$$\mathcal{A}(e^+e^- \to f\bar{f}) = \frac{R}{s - \mathcal{M}_Z^2} + S + (s - \mathcal{M}_Z^2)\, S' + \cdots$$

$$\mathcal{M}_Z = M_Z^2 - iM_Z\Gamma_Z$$

Expanding up to $\mathcal{O}(\alpha^2)$ and using $\mathcal{O}(\Gamma_Z/M_Z) = \mathcal{O}(\alpha)$ one can identify the electroweak form factors $\rho_{ef}, \kappa_e, \kappa_f, \kappa_{ef}$

$$\kappa_f = \frac{1 - v_f/a_f}{1 - v_f^{(0)}/a_f^{(0)}}$$

where $\begin{matrix} v_f \\ a_f \end{matrix}$ are the $\begin{matrix}\text{vector} \\ \text{axial-vector}\end{matrix}$ $Zf\bar{f}$ couplings

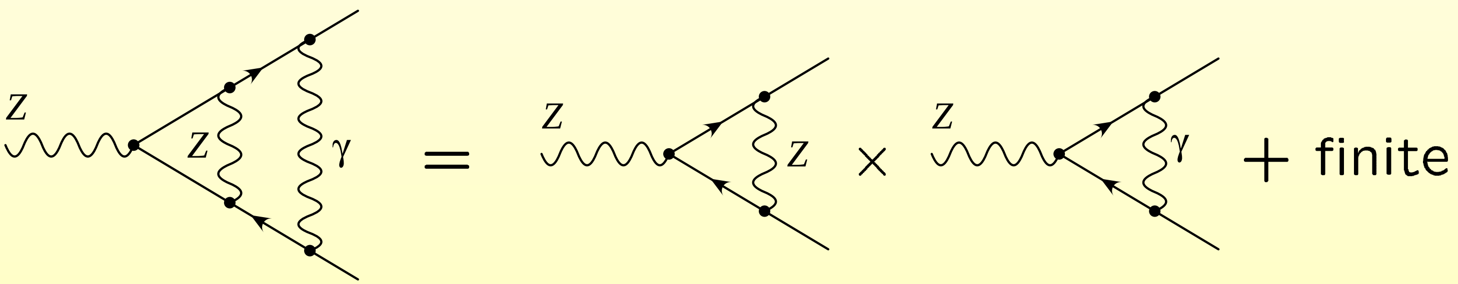


[(0) = tree-level]

Two-loop contribution:

$$\kappa_l^{(2)} = \left. \frac{a_l^{(2)} v_l^{(0)} a_l^{(0)} - v_l^{(2)} \left(a_l^{(0)}\right)^2 - \left(a_l^{(1)}\right)^2 v_l^{(0)} + a_l^{(1)} v_l^{(1)} a_l^{(0)}}{\left(a_l^{(0)}\right)^2 \left(a_l^{(0)} - v_l^{(0)}\right)} \right|_{s=M_Z^2}$$

- Interplay between 2-loop terms and products of 1-loop terms to cancel IR-divergencies



$\rightarrow$ $\sin^2\theta^{\text{eff}}_{\text{lept}}$ is IR-safe !

- Genuine 2-loop contributions contain products of imaginary parts of 1-loop terms

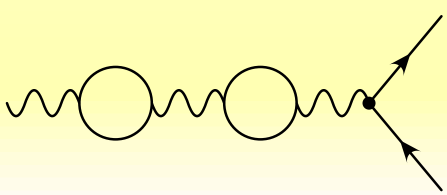


Diagram classes

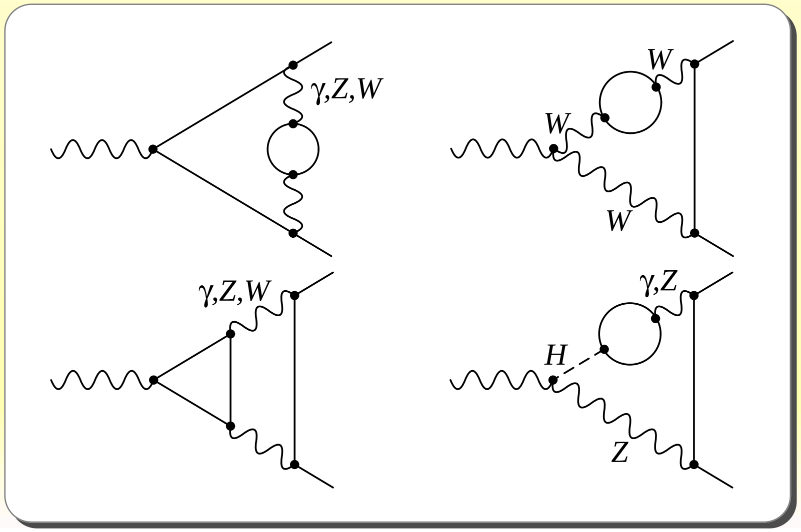
- On-shell renormalization requires on-shell two-loop propagators, e.g. weak-mixing angle counterterm

$$\delta s_{\mathrm{W}(2)} = \frac{M_\mathrm{W}^2}{2 s_\mathrm{W} M_\mathrm{Z}^2} \left[\frac{\Sigma^\mathrm{Z}_{\mathrm{T}(2)}(M_\mathrm{Z}^2)}{M_\mathrm{Z}^2} - \frac{\Sigma^\mathrm{W}_{\mathrm{T}(2)}(M_\mathrm{W}^2)}{M_\mathrm{W}^2} \right] + \text{(1-loop terms)}$$

$\rightarrow$ Well known and tested for two-loop corrections to M_W, Δr

A. Freitas, W. Hollik, W. Walter, G. Weiglein '00
M. Awramik, M. Czakon '02
A. Onishchenko, O. Veretin '02

- New complication:
  Two-loop vertex diagrams



- Divide into two classes
  - With closed fermion loops
  - No closed fermion loops

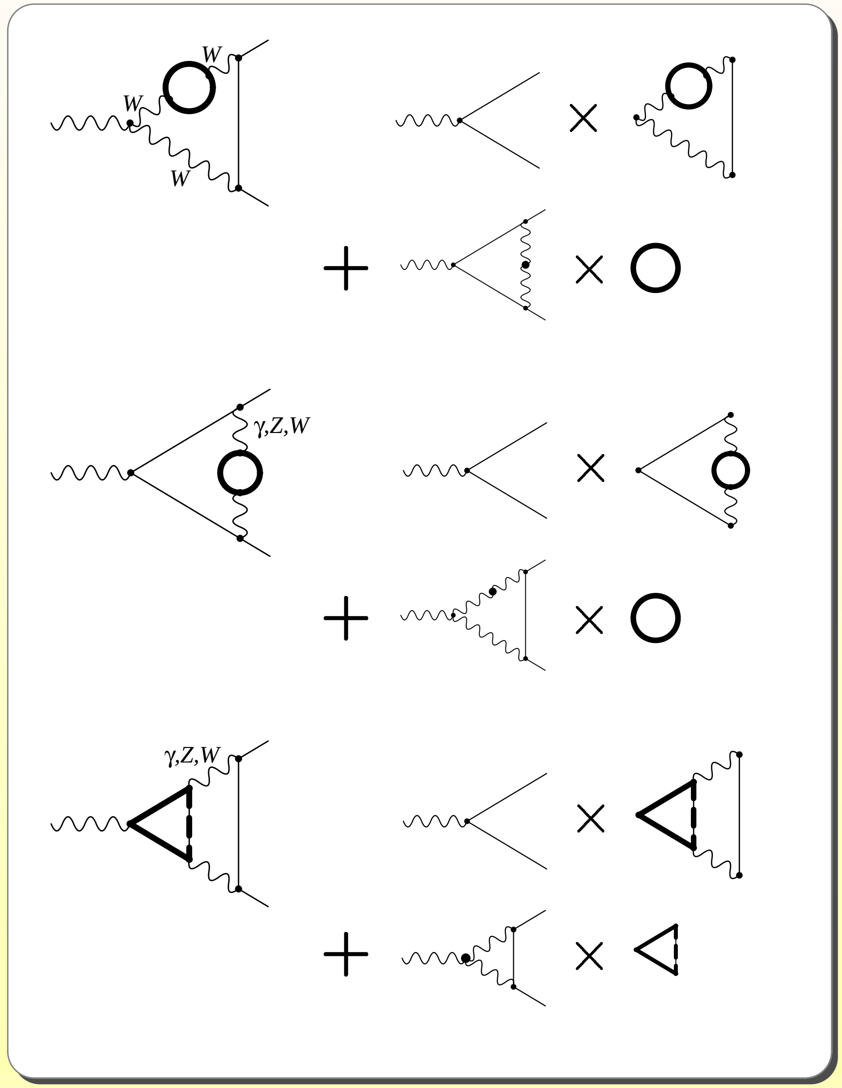
Method 1: Asymptotic expansions and analytical results

Top quark contributions

- Exploit large scale difference between top mass and other masses:

$$M_Z^2/m_t^2 \approx 1/4$$

- Simplifies diagrams to 2-loop tadpoles and 1-loop vertices
- Fast numerical evaluation
- Previously considered: m_t^4 and m_t^2 contribution only
  G. Degrassi, P. Gambino, A. Sirlin '97



- 10th order expansion has relative error estimate: $\pm 1.3 \times 10^{-5}$

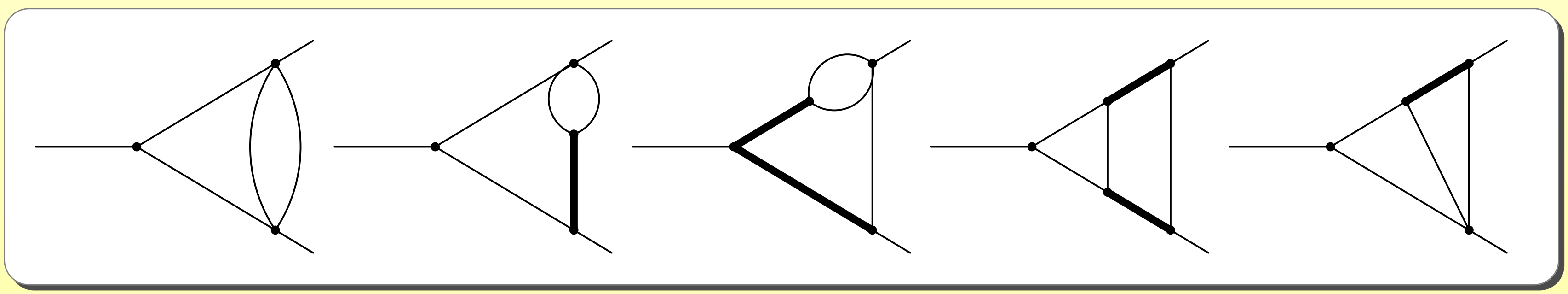
Light fermion contributions

- Take light fermions (all except top quark) massless in 2-loop diagrams

- Integration-by-parts identities and Lorentz-invariance identities to reduce diagrams to master integrals

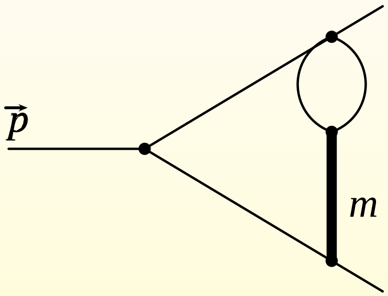
  Chetyrkin, Tkachov '81
  Gehrmann, Remiddi '00
  Laporta '00

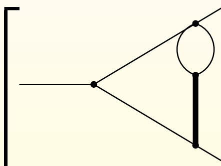
- Differential equations to get analytical results for master integrals

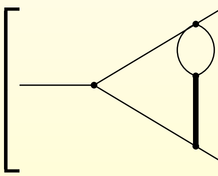


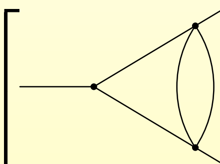
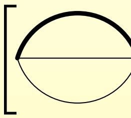
Method 1, light fermion contributions

Evaluate prototype master integrals by differential equations



$p^2 \dfrac{\mathrm{d}}{\mathrm{d}p^2}$ [] $=$

$\dfrac{1}{2}\dfrac{p^2}{p^2+m^2}\Big((4-D)(4+5\dfrac{m^2}{p^2})$ []

$+\,(10-3D)$ [] $-\,(2-D)$ [] $\Big)$

Result: $\quad (x = p^2/m^2)$

$$-\,\mathrm{Li}_2(-\mathrm{x})\Big(-2+2\log(\mathrm{m}^2)+3\log(-\mathrm{x})+\log(1+\mathrm{x})\Big)+4\mathrm{Li}_3(-\mathrm{x})-\mathrm{S}_{1,2}(-\mathrm{x})$$

$$+\,\frac{1}{2}\log(1+x)\Big[2\zeta_2-\log(-x)\Big((-4+4\log(m^2)+2\log(-x)+\log(1+x)\Big)\Big]$$

Method 1, light fermion contributions

- Other prototypes can be more complicated
- In total:
  - 8 Integration-by-parts identities
  - 1 Lorentz-invariance identity

  $\rightarrow$ Tensor reduction and differential equation solving is non-trivial
- Use automated computer tools, capable to solve systems of $\mathcal{O}(10^4)$ integrals

Internal tests:

- Some master integrals tested by means of Padé resummed Mellin-Barnes representations
- Other master integrals tested by with low momentum expansion
- Complete set tested by means of low momentum expansion

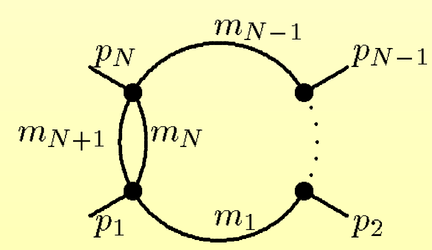
Method 2: Semi-numerical integral evaluation

Topologies with **self-energy sub-loop** can easily be integrated by using dispsersion relation for B_0 function: S. Bauberger et al. '95

$$B_0(p^2, m_1^2, m_2^2) = -\int_{(m_1+m_2)^2}^{\infty} \mathrm{d}s \, \frac{\Delta B_0(s, m_1^2, m_2^2)}{s - p^2}$$

with $$\Delta B_0(s, m_1^2, m_2^2) = (4\pi\mu^2)^{4-D} \frac{\Gamma(D/2-1)}{\Gamma(D-2)} \frac{\lambda^{(D-3)/2}(s, m_1^2, m_2^2)}{s^{D/2-1}},$$

$\lambda(a,b,c) = (a-b-c)^2 - 4bc$



$$T_{N+1}(p_i; m_i^2) = -\int_{s_0}^{\infty} \mathrm{d}s \, \Delta B_0(s, m_N^2, m_{N+1}^2)$$
$$\times \int \mathrm{d}^4 q \, \frac{1}{q^2 - s} \, \frac{1}{(q+p_1)^2 - m_1^2} \cdots \frac{1}{(q+p_1+\cdots+p_{N-1})^2 - m_{N-1}^2}$$

Method 2, numerical integration

Dispersion relations for diagrams with **triangle subloop** possible, but difficult

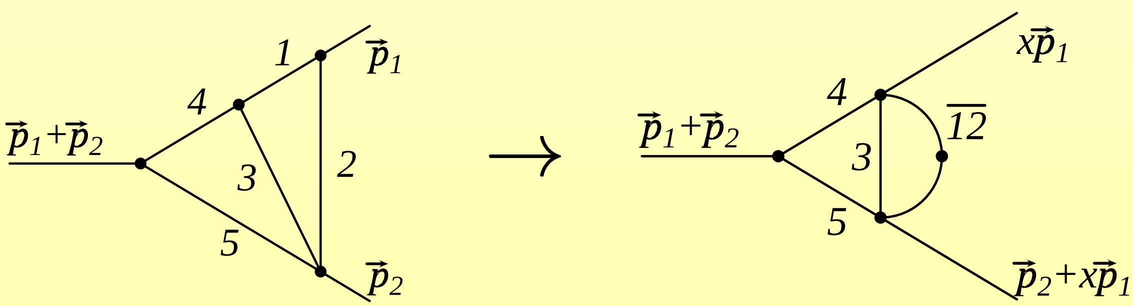
→ Alternative approach: Introduce Feynman parameters

J. v.d.Bij, A. Ghinculov '94

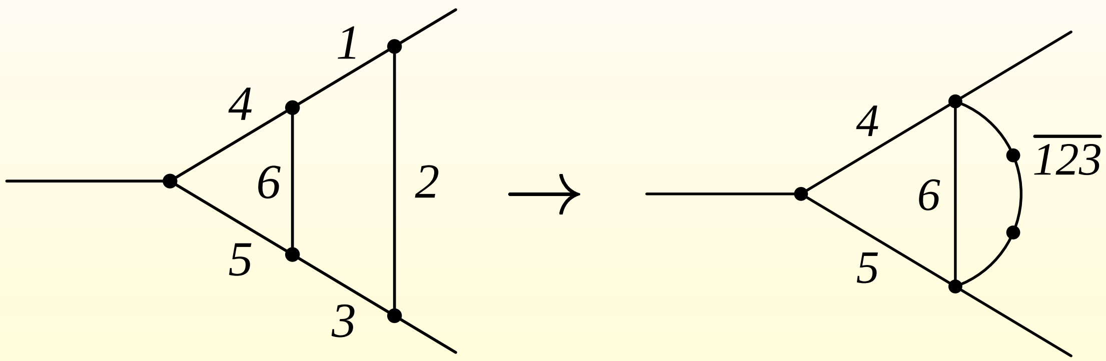
$$\frac{1}{(q+p_1)^2 - m_1^2}\,\frac{1}{(q+p_2)^2 - m_2^2} = \int_0^1 \mathrm{d}x\, \frac{1}{[(q+\bar{p})^2 - \overline{m}^2]^2}$$

$$\bar{p} = x\,p_1 + (1-x)p_2, \qquad \overline{m} = x\,m_1 + (1-x)m_2 - x(1-x)(p_1-p_2)^2$$

Reduces triangle to self-energy sub-loops:



Correspondingly:



(two Feynman parameters)

- Integration over Feynman parameters performed numerically
- At most 3-dim. numerical integrations for 2-loop vertex diagrams
- Divergent parts need to subtracted in integrand
  e.g. terms that evaluate to products of one-loop functions

+ Applicable for arbitrary mass patterns

− Slow, large expressions

Fermion loop triangle and treatment of γ_5

- Probem in dimensional regularization:

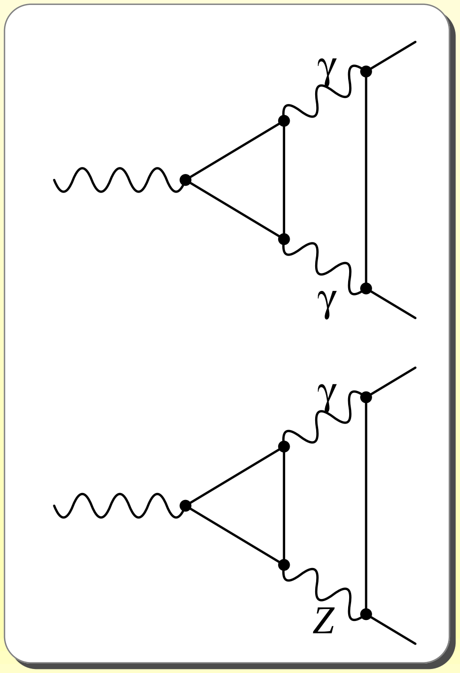
$$\{\gamma_\mu, \gamma_5\}, \qquad \mathrm{Tr}(\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta\gamma_5) = 4i\,\epsilon^{\alpha\beta\gamma\delta}$$

cannot be simultaneously fulfilled in $D \neq 4$ dimensions

- Solution:

Use 4-dim. Dirac algebra for calculation
of terms arising from $\mathrm{Tr}(\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta\gamma_5) = 4i\,\epsilon^{\alpha\beta\gamma\delta}$

$\rightarrow$ These terms are UV-finite
$\rightarrow$ Regulate photonic divergencies
with photon mass



Results

Results for form factor κ_f using M_W as input:

M_H [GeV]	$\mathcal{O}(\alpha)$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2)_{\text{ferm}}$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2)_{\text{tb}}$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2)_{\text{lf}}$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2)_{\text{tr}\gamma_5}$ $\times 10^{-4}$
100	438.94	-0.63	-16.96	-2.84	0.27
200	419.60	-2.16	-17.10	-3.08	0.27
600	379.56	-5.01	-16.89	-3.77	0.27
1000	358.62	-4.73	-14.90	-4.25	0.27

M_H [GeV]	$\mathcal{O}(\alpha\alpha_s)$ $\times 10^{-4}$	$\mathcal{O}(\alpha\alpha_s^2)$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2\alpha_s m_t^4)$ $\times 10^{-4}$	$\mathcal{O}(\alpha^3 m_t^6)$ $\times 10^{-4}$	reducible 3-loop $\times 10^{-4}$
100	-36.83	-7.32	1.25	0.17	0.92
200	-36.83	-7.32	2.08	0.09	0.94
600	-36.83	-7.32	4.07	0.07	0.97
1000	-36.83	-7.32	5.01	0.99	0.98

Final result for $\sin^2\theta^{\rm eff}_{\rm lept}$ uses G_μ as input
$\rightarrow$ include corrections to $M_{\rm W}$

Present result in terms of fitting formula:

$$
\begin{aligned}
\sin^2\theta^{\rm eff}_{\rm lept} = \sin^2\theta^{\rm eff}_{{\rm lept}(0)} &+ d_1 \log\left(\frac{M_{\rm H}}{100}\right) + d_2 \log^2\left(\frac{M_{\rm H}}{100}\right) + d_3 \log^4\left(\frac{M_{\rm H}}{100}\right) \\
&+ d_4\left[\left(\frac{M_{\rm H}}{100}\right)^2 - 1\right] + d_5\left[\frac{\Delta\alpha}{0.05907} - 1\right] \\
&+ d_6\left[\left(\frac{m_{\rm t}}{178}\right)^2 - 1\right] + d_7\left[\left(\frac{m_{\rm t}}{178}\right)^2 - 1\right]^2 \\
&+ d_8\left[\left(\frac{m_{\rm t}}{178}\right)^2 - 1\right] \times \left[\frac{M_{\rm H}}{100} - 1\right] \\
&+ d_9\left[\frac{\alpha_{\rm s}}{0.117} - 1\right] + d_{10}\left[\frac{M_{\rm Z}}{91.1876} - 1\right]
\end{aligned}
$$

Results

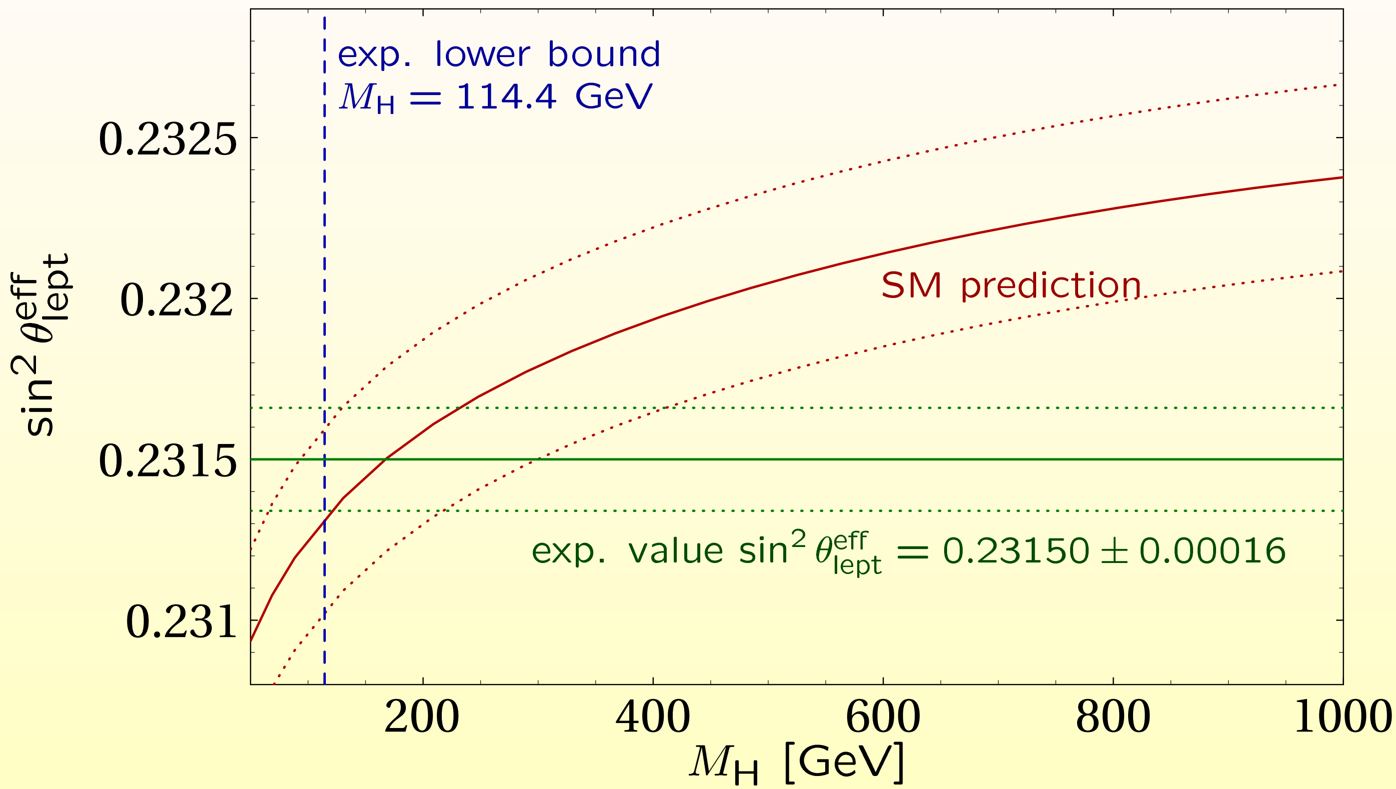
Comparison to previous result with large-m_{t} expansion up to $O(\alpha^2 m_{\mathrm{t}}^2)$

G. Degrassi, P. Gambino, A. Sirlin ’97
G. Degrassi, P. Gambino, M. Passera, A. Sirlin ’98

M_H GeV	$\left(\Delta \sin^2\theta^{\mathrm{eff}}_{\mathrm{lept}}\right)_{\mathrm{DGPS}}$ $\times 10^{-4}$	$\left(\Delta \sin^2\theta^{\mathrm{eff}}_{\mathrm{lept}}\right)_{\mathrm{ZFITTER}}$ $\times 10^{-4}$
100	-0.45	-0.40
200	-0.69	-0.72
300	-0.85	-0.83
600	-1.17	-0.94
1000	-1.60	-1.28

Current experimental precision: $\sin^2\theta^{\mathrm{eff}}_{\mathrm{lept}} = 0.23150 \pm 0.00016$

Results



Extraction of M_H from $\sin^2\theta^{eff}_{lept}$:

central value for $m_t = 174.3$ GeV:

previously: 119 GeV

new result: 132 GeV

central value for $m_t = 178.0$ GeV:

previously: 148 GeV

new result: 168 GeV

Conclusions and outlook

- Complete fermionic $\mathcal{O}(\alpha^2)$ corrections to $\sin^2\theta^{\mathrm{eff}}_{\mathrm{lept}}$ calculated with two independent methods
- New corrections to $\sin^2\theta^{\mathrm{eff}}_{\mathrm{lept}}$ have sizeable impact on indirect determination of M_{H}
- New result presented in terms of fitting formula and incorporated into ZFITTER 6.41
- Error estimate from missing higher-order corrections $\delta_{\mathrm{th}}\sin^2\theta^{\mathrm{eff}}_{\mathrm{lept}} = 4.9\times 10^{-5}$
- Bosonic corrections will be approached with same tools