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Electroweak Corrections to Four-Fermion Production

Stefan Dittmaier

MPI Munich

(in collaboration with A. Denner, M. Roth and L.H. Wieders)

Contents

1 Introduction

W-pair production at LEP2

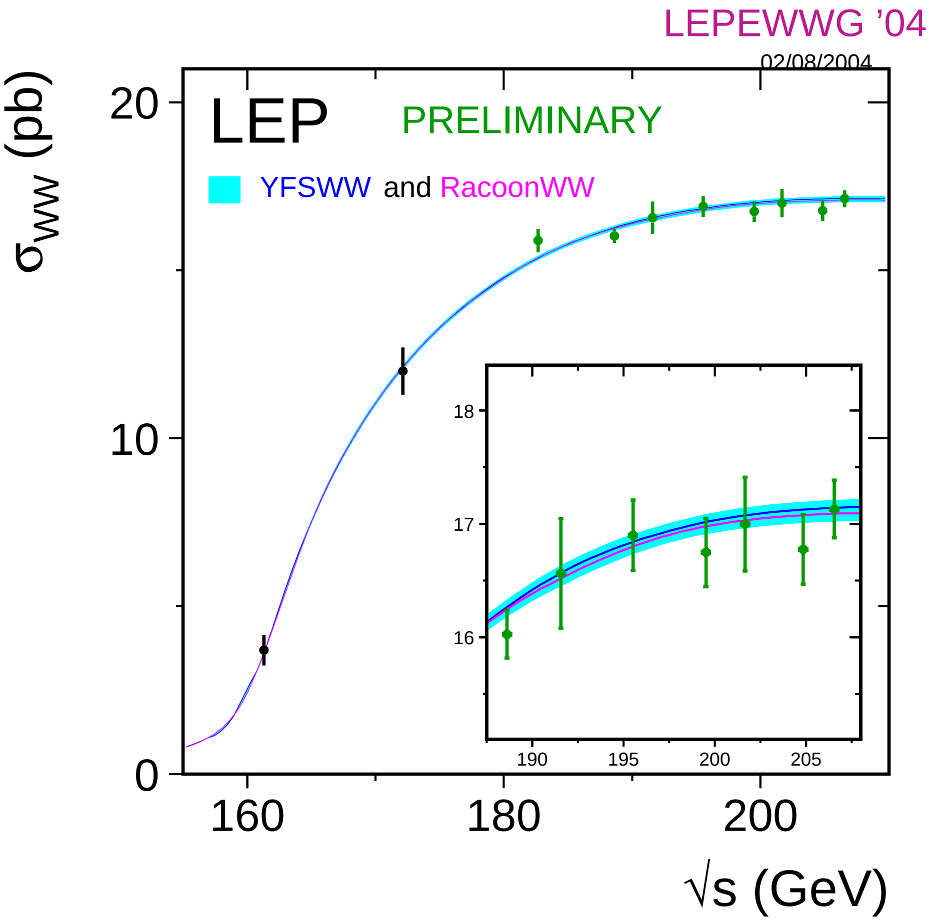
- cross-section measurement
  with $\Delta\sigma_{\mathrm{WW}}/\sigma_{\mathrm{WW}} \sim 1\%$

  $\hookrightarrow$ significance of non-universal electroweak corrections

- M_{W} from threshold cross section
  with $\Delta M_{\mathrm{W}} \sim 200\,\mathrm{MeV}$

- M_{W} from direct reconstruction
  with $\Delta M_{\mathrm{W}} \sim 40\,\mathrm{MeV}$

  $\hookrightarrow$ strengthening of M_{H} bounds



- constraints on anomalous
  triple gauge-boson couplings (TGC) at level of a few $\%$

  $\hookrightarrow$ verification of gauge structure

Predictions for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{WW} \to 4f(+\gamma)$ at LEP2

- lowest-order predictions based on full $\mathrm{e}^+\mathrm{e}^- \to 4f(+\gamma)$ matrix elements
- universal radiative corrections
  $\hookrightarrow$ "improved Born approximations" (IBA)
- non-universal radiative corrections from pole expansions
  $\hookrightarrow$ "double-pole approximations" (DPA)

Beenakker, Berends, Chapovsky '98
Jadach et al. '99–'01
Denner, S.D., Roth, Wackeroth '99–'01
Kurihara, Kuroda, Schildknecht '01

$\Rightarrow$ state-of-the-art generators:
KoralW $\oplus$ *YFSWW* (Jadach,Płaczek,Skrzypek,Ward) and
RacoonWW (Denner, S.D.,Roth,Wackeroth)

Estimates of theoretical uncertainties (TU) for

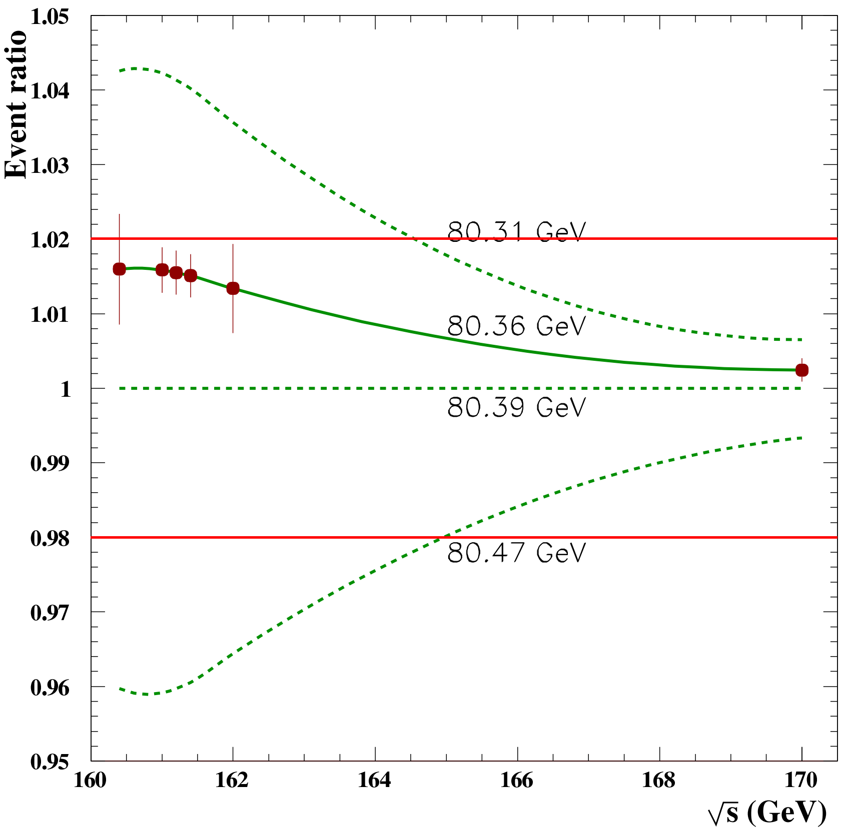
- total cross section (Denner et al., Jadach et al.)

$$\Delta\sigma_{\mathrm{WW}}/\sigma_{\mathrm{WW}} \lesssim \begin{cases} 2\% & \text{for} \quad \sqrt{s} < 170\,\mathrm{GeV} & \text{(IBA)} \\ 0.7\% & \text{for} \quad 170\,\mathrm{GeV} < \sqrt{s} < 180\,\mathrm{GeV} & \text{(DPA)} \\ 0.5\% & \text{for} \quad 180\,\mathrm{GeV} < \sqrt{s} < 500\,\mathrm{GeV} & \text{(DPA)} \end{cases}$$

- direct M_{W} reconstruction: $\Delta M_{\mathrm{W}} \lesssim 5\,\mathrm{MeV}$ (Jadach et al. '01) – $10\,\mathrm{MeV}$ (Cossutti '04)
- bounds on anomalous TGC λ: $\Delta\lambda \lesssim 0.005$ (Brunelière et al. '02)

W-pair production at future ILC (see e.g. TESLA-TDR '01)

- cross-section measurement with $\Delta\sigma_{\mathrm{WW}}/\sigma_{\mathrm{WW}} \lesssim 0.5\%$
- M_{W} from threshold cross section with $\Delta M_{\mathrm{W}} \sim 7\,\mathrm{MeV}$
  $\hookrightarrow$ IBA totally insufficient $\Longrightarrow$
- M_{W} from direct reconstruction with $\Delta M_{\mathrm{W}} \sim 10\,\mathrm{MeV}$
- constraints on anomalous TGC at level of 0.1%



Theoretical requirements for ILC:

- full $\mathcal{O}(\alpha)$ correction for $\mathrm{e}^+\mathrm{e}^- \to 4f$
  $\hookrightarrow$ subject of this talk !
- leading corrections beyond $\mathcal{O}(\alpha)$

2 Features of the calculation

Complete $\mathcal{O}(\alpha)$ corrections to $\mathrm{e}^+\mathrm{e}^- \to \nu_\tau \tau^+ \mu^- \bar{\nu}_\mu$ leptonic

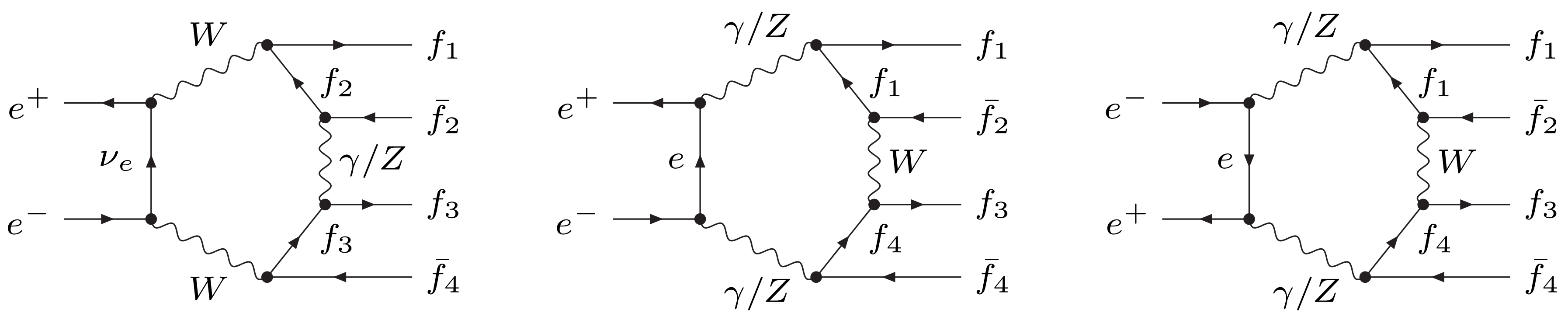
$\mathrm{u}\bar{\mathrm{d}}\mu^-\bar{\nu}_\mu$ semileptonic

$\mathrm{u}\bar{\mathrm{d}}\mathrm{s}\bar{\mathrm{c}}$ hadronic final state

$\hookrightarrow$ $\mathcal{O}(10^3)$ one-loop diagrams: generation with FeynArts versions 1 and 3

Küblbeck, Böhm, Denner '90 Hahn '00

- 40 hexagons



+ graphs with reversed fermion-number flow in final state

- 112 pentagons
- 227 boxes ('t Hooft–Feynman gauge)
- many vertex corrections and self-energy diagrams

Calculational details:

- algebraic reduction of $\mathcal{O}(10^3)$ spinor chains

  such as $\bar{v}_{e^+}\gamma^\mu\gamma^\nu \not{p}_i u_{e^-} \times \bar{u}_{f_1}\gamma_\nu\gamma^\rho \not{p}_j v_{\bar{f}_2} \times \bar{u}_{f_3}\gamma_\mu\gamma_\rho \not{p}_k v_{\bar{f}_4}$

  $\hookrightarrow$ $\mathcal{O}(10)$ standard structures with well-behaved coefficients

- complex gauge-boson masses everywhere, i.e., also in couplings

  $$c_W^2 = 1 - s_W^2 = \frac{M_W^2 - iM_W\Gamma_W}{M_Z^2 - iM_Z\Gamma_Z}$$

  $\hookrightarrow$ gauge-invariant result !

  (Slavnov–Taylor identities and gauge-parameter independence)

  $\hookrightarrow$ unitarity cancellations respected !

  but: complex gauge-boson masses in all loop integrals

- algebraic reduction of 5- and 6-point integrals (4-dim. of space-time) without inverse Gram determinants:

  5-point integrals $\rightarrow$ five 4-point integrals — Melrose '65; Denner, S.D. '02

  6-point integrals $\rightarrow$ six 5-point integrals — Melrose '65; Denner '93

Numerical problems:

Passarino–Veltman reduction of 2-point to 4-point tensor integrals

$\hookrightarrow$ inverse Gram determinants of up to three momenta

$\hookrightarrow$ serious numerical instabilities where $\det G \to 0$
(at phase-space boundary but not only !)

Solutions:

- 2-point functions: numerically stable direct calculation possible
- 3-/4-point functions: two alternative "rescue systems"

Variant 1: appropriate expansions of tensor coefficients
in small Gram determinants

Variant 2: numerical evaluation of one appropriate tensor coefficient
(logarithmic Feynman-parameter integral)
and algebraic reduction to this basis integral

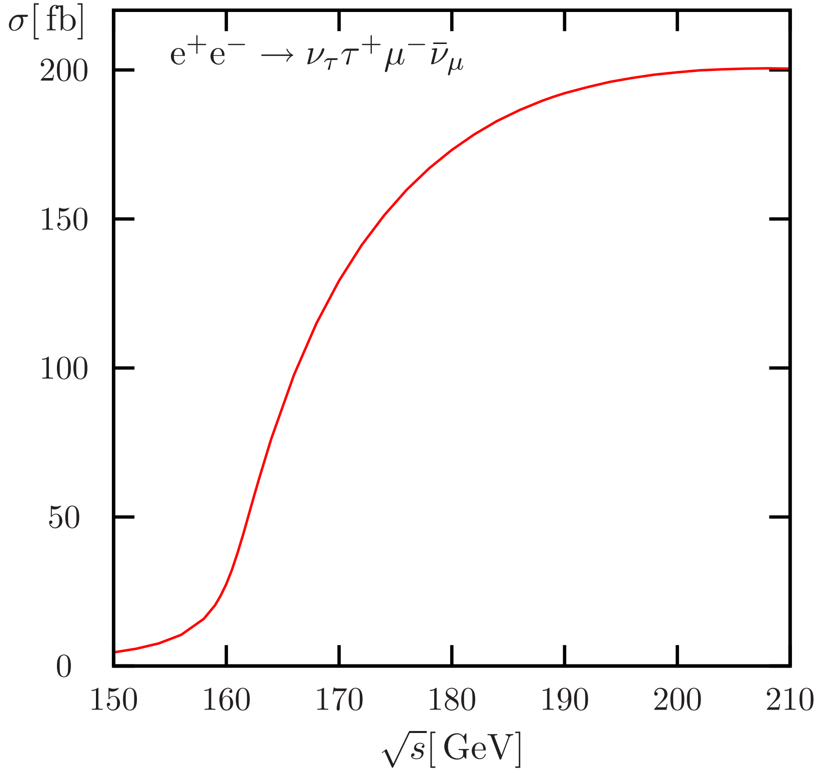
Checks:

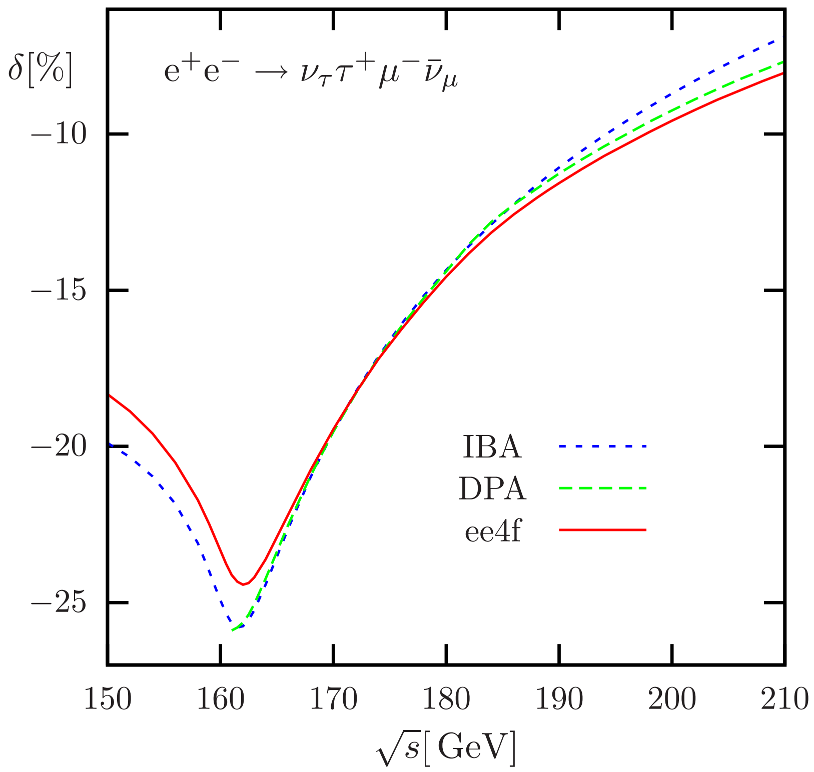
- UV structure of virtual corrections
  $\hookrightarrow$ independence of reference mass μ of dimensional regularization
- IR structure of virtual + soft-photonic corrections
  $\hookrightarrow$ independence of $\ln m_\gamma$ (m_γ = infinitesimal photon mass)
- mass singularities of virtual + related collinear photonic corrections
  $\hookrightarrow$ independence of $\ln m_{f_i}$ (m_{f_i} = small masses of external fermions)
- gauge invariance of amplitudes with $\Gamma_{\mathrm{W}}, \Gamma_{\mathrm{Z}} \neq 0$
  $\hookrightarrow$ identical results in 't Hooft–Feynman and background-field gauge
  Denner, S.D., Weiglein '94
- real corrections
  $\hookrightarrow$ taken from RACOONWW
- combination of virtual and real corrections
  $\hookrightarrow$ identical results with two-cutoff slicing and dipole subtraction
  S.D. '99; Roth '00
- two completely independent calculations of all ingredients !

3 Numerical results

Complete $\mathcal{O}(\alpha)$ corrections to the total cross section – LEP2 energies

Denner, S.D., Roth, Wieders '05



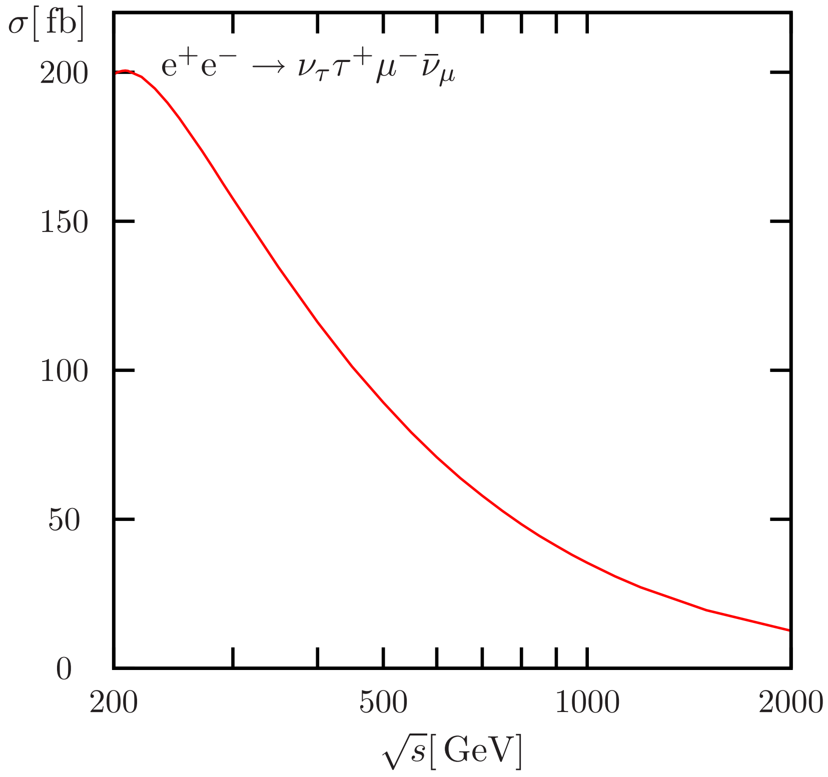


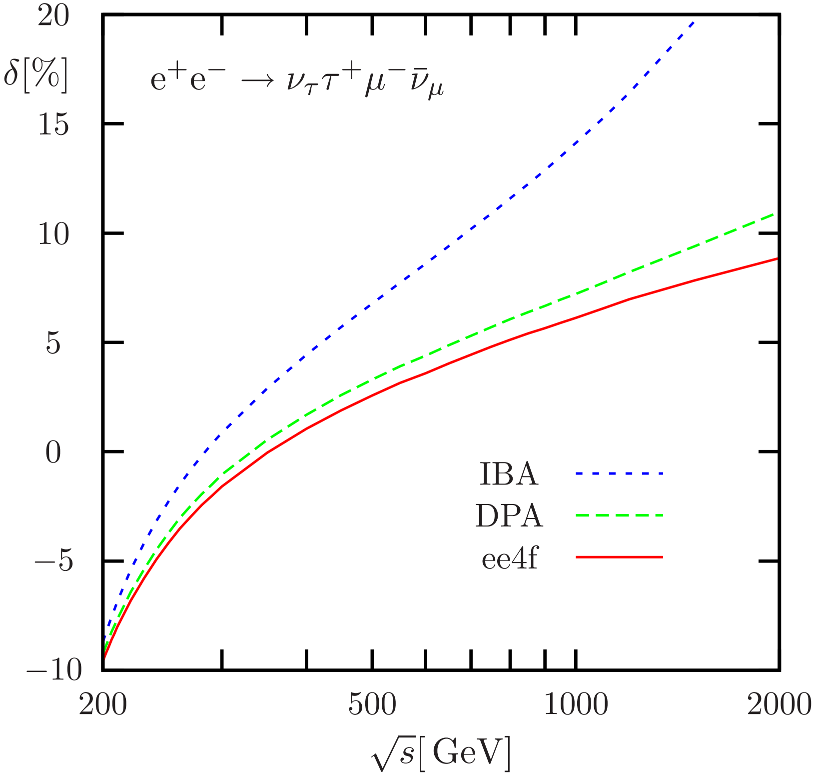
- $|\mathrm{ee4f} - \mathrm{DPA}| \sim 0.5\%$ for $170\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 210\,\mathrm{GeV}$
- $|\mathrm{ee4f} - \mathrm{IBA}| \sim 2\%$ for $\sqrt{s} \lesssim 170\,\mathrm{GeV}$

$\hookrightarrow$ agreement with error estimates of DPA and IBA

Complete $\mathcal{O}(\alpha)$ corrections to the total cross section – ILC energies

Denner, S.D., Roth, Wieders '05





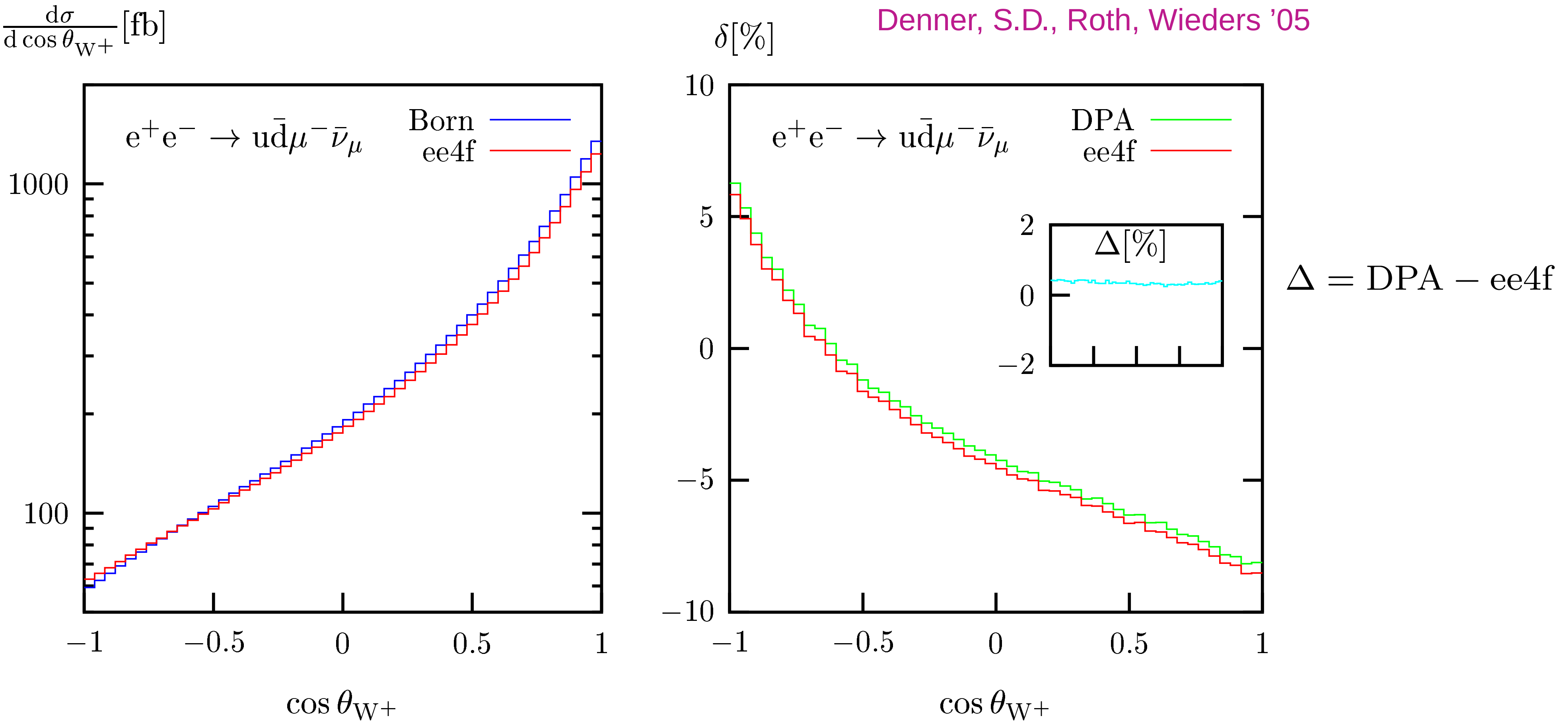
- $|\mathrm{ee4f} - \mathrm{DPA}| \;\sim\; 0.7\%$ for $200\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 500\,\mathrm{GeV}$

  $\hookrightarrow$ agreement with error estimate of DPA

- $|\mathrm{ee4f} - \mathrm{DPA}| \;\sim\; 1{-}2\%$ for $500\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 1{-}2\,\mathrm{TeV}$

W-production angle distribution at $\sqrt{s} = 200\,\mathrm{GeV}$

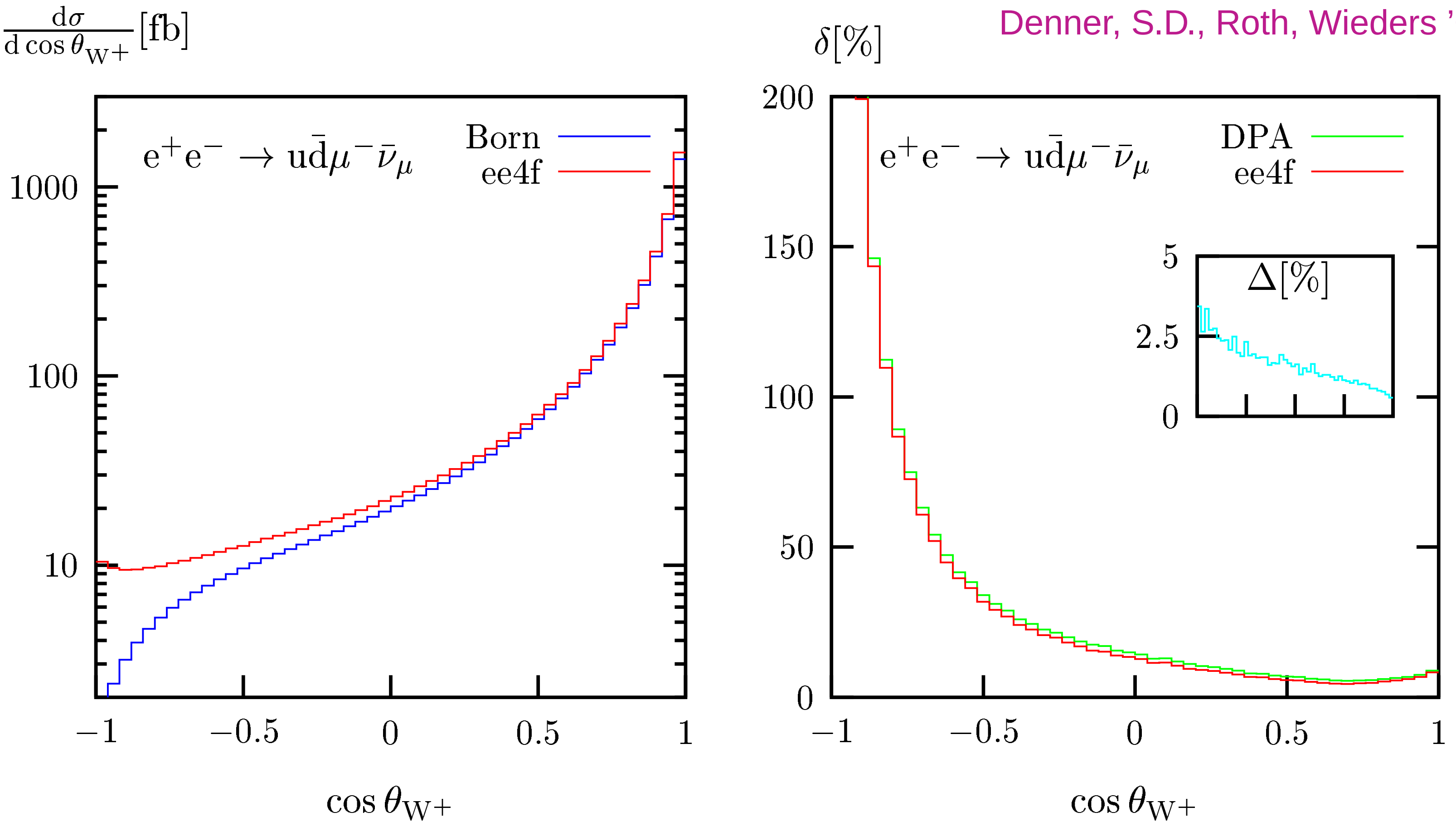
Denner, S.D., Roth, Wieders ’05



No visible distortion of shape w.r.t. DPA at LEP2 energies

W-production angle distribution at $\sqrt{s} = 500\,\text{GeV}$

Denner, S.D., Roth, Wieders '05



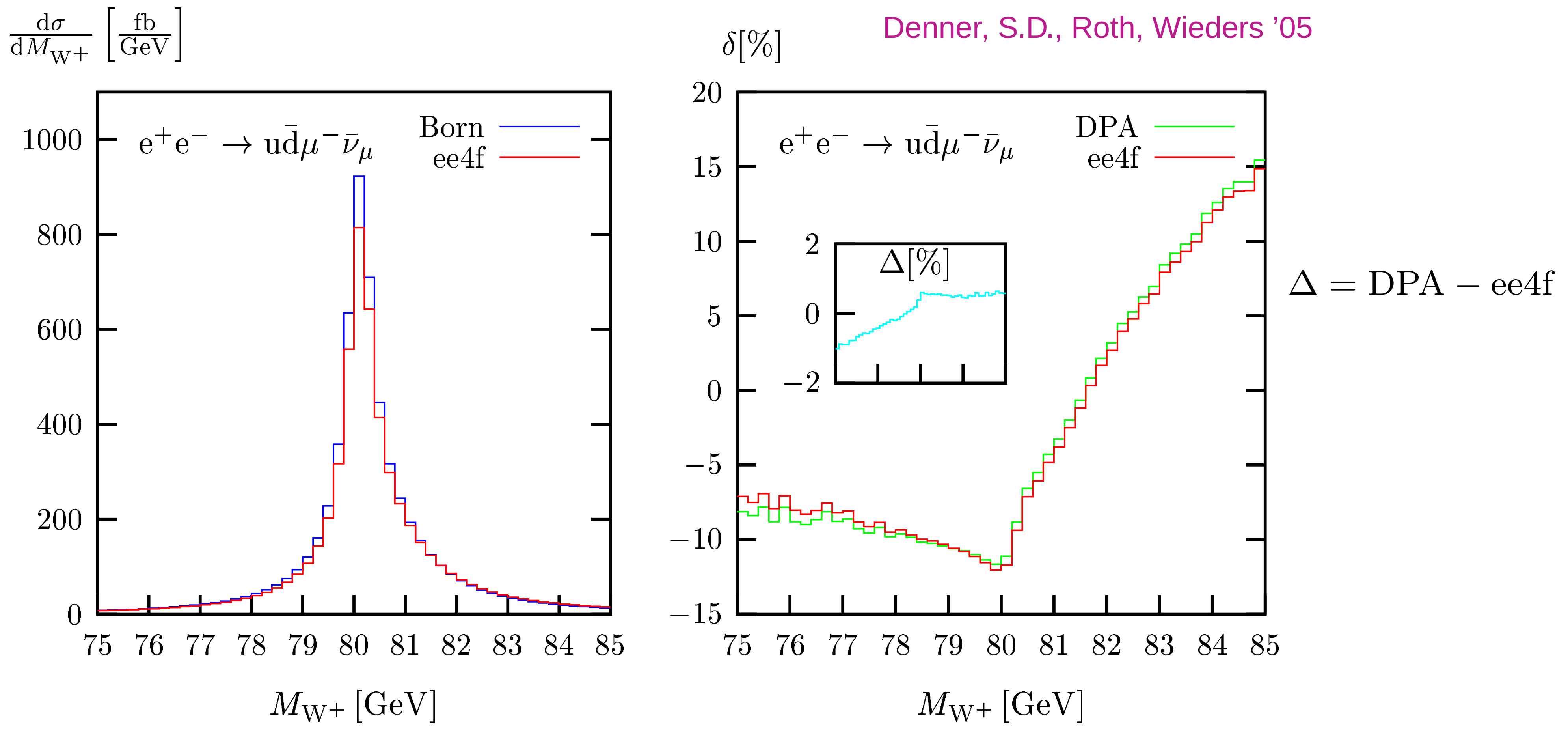
$\Delta = \text{DPA} - \text{ee4f}$

Significant distortion of shape w.r.t. DPA at ILC energies

$\hookrightarrow$ Important for TGC studies at ILC

W-invariant-mass distribution at $\sqrt{s} = 200\,\mathrm{GeV}$

(photon recombination applied)



Small distortion of shape w.r.t. DPA at LEP2 energies

$\hookrightarrow$ Shift in M_W in direct reconstruction ?

4 Conclusions

W-pair production at future ILC requires

- full $\mathcal{O}(\alpha)$ correction for $\mathrm{e^+e^-} \to 4f$
- leading corrections beyond $\mathcal{O}(\alpha)$

Complete $\mathcal{O}(\alpha)$ correction for $\mathrm{e^+e^-} \to \nu_\tau\tau^+\mu^-\bar{\nu}_\mu, \mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu, \mathrm{u\bar{d}s\bar{c}}$ calculated

- new techniques in the calculation:
  - ◇ complex gauge-boson masses
  - ◇ new tensor reductions
- new Monte Carlo generator: RACOONWW $\longrightarrow$ RACOON4F
- theoretical uncertainty at threshold reduced from $\sim 2\%$ to a few 0.1%
- benchmark results published

  $\hookrightarrow$ comparison with future results of Grace/1-loop ?

  (progress reported at Loops&Legs 2004)

Remaining theoretical uncertainties dominated by

- electroweak effects beyond $\mathcal{O}(\alpha)$, e.g. $\left(\frac{\alpha}{\pi}\right)^2 \ln\left(\frac{m_\mathrm{e}^2}{s}\right) \sim 0.1\%$
- QCD effects (matching between matrix elements and parton shower)