Heavy Quark Fragmentation Function at NNLO

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Outline

I. Heavy Flavor Production and not-completely inclusive observables.

II. Perturbative Fragmentation Function Formalism or how to deal with the problem of mass singularities

- Features
- Applications

III. Derivation of the initial condition: a process independent approach

IV. Conclusions

Based on:

- K. Melnikov and A.M.: hep-ph/0404143
- A.M.: hep-ph/0410205

Heavy Flavor Production

We are interested in processes of the following type:

$$A + B \longrightarrow H(\mathcal{Q}, m) + X$$

where:

- A, B - Initial state particles: e^+e^-, $p, \bar{p}$, etc.,
- H - Heavy flavored hadron (for example B-meson),
- $\mathcal{Q}$ - heavy quark (b or c) with mass m.

We are also interested in measuring certain properties of the produced hadron H:

- Transverse momentum,
- Energy distribution, etc.

Example: B-production in e^+e^-, $p, \bar{p}$, top-decay, etc.

The processes of this kind are characterized by two scales:

- Q - hard scale,
- m - quark mass.

with $m >> \Lambda$. For example, for b, c quarks $m_{b,c} \sim \mathcal{O}(10)\Lambda$.

For the present presentation we will consider processes for which:

$$Q >> m$$

This assumption is:

I. experimentally relevant for colliders where (typically) $Q \geq m_Z$.

II. guarantees the factorization of long- and short-distance physics,

III. makes perturbative treatment possible since $m >> \Lambda$ is a natural cut-off for collinear radiation.

IV. The hadron H is produced at a scale $\mu \sim m$ from the fragmentation of the heavy quark $\mathcal{Q}$:

$$\mathcal{Q}\,(+\bar{q}) \longrightarrow H$$

What happens if the assumption $Q >> m$ is not quite applicable ?

Example is the transverse momentum distribution of hadrons. In that process the hard scale is $Q \sim p_T$ (for large p_T).

- One has to account for power corrections of m.
- In this formalism one can consistently treat the asymptotic regime $Q >> m$ and can extend it to include power corrections from FO calculations if needed.

Such study was successfully applied at NLO + NLL for b-production at the Tevatron (Cacciari et al. hep-ph/0312132).

In the asymptotic regime $Q >> m$ one applies the Factorization Th. to the process $A + B \to H + X$:

$$d\sigma_H = (f_{A\to a} f_{B\to b}) \otimes d\sigma_{ab\to \mathcal{Q}} \otimes D^{\rm n.p.}_{\mathcal{Q}\to H}(z)$$

where:

- $d\sigma_H$ - hadronic observable ($p_T,\ E$-distribution) ,
- $d\sigma_{\mathcal{Q}}$ - partonic "observable",
- $D^{\rm n.p.}_{\mathcal{Q}\to H}(z)$ - Non-Perturbative fragmentation function,
- z - energy or momentum fraction; $0 \le z \le 1$.

Note: The presence of pdf's is irrelevant for our discussion.

Note: $D^{\rm n.p.}_{\mathcal{Q}\to H}(z)$ is:

- a universal, process-independent (but not unambiguous) quantity ,
- extracted from experimental data (much like pdf's),
- its extraction from one process (typically e^+e^-) must be consistent with its application to another process.

Examples:

- energy spectrum ($E_H = zE_{\mathcal{Q}}$):

$$\frac{d\sigma_H}{dz} = \frac{d\sigma_{ab\to\mathcal{Q}}}{dz} \otimes D^{\rm n.p.}_{\mathcal{Q}\to H}(z)$$

- p_T-distribution ($p_{T,H} = zp_{T,\mathcal{Q}}$):

$$\frac{d\sigma_H}{dp_T} = \frac{d\sigma_{ab\to\mathcal{Q}}}{dp_T} \otimes D^{\rm n.p.}_{\mathcal{Q}\to H}(z)$$

However: Since what we measure is $d\sigma_H$ then $D^{\rm n.p.}$ is scheme dependent because it depends on the treatment of the perturbative part!

Perturbative part

Let's turn our attention to $d\sigma_{ab\to\mathcal{Q}}(Q,m,z)$:

Since $m > 0$, $d\sigma_{ab\to\mathcal{Q}}$ is finite. However that function contains large quasi-collinear logs:

$$\alpha_s^n \ln^k\left(\frac{m^2}{Q^2}\right) \quad ; \quad k \leq n$$

to all orders in α_s.

The presence of those logs indicates the breakdown of perturbation series in the asymptotic regime $Q >> m$. One has to resum (classes of) such logs.

How do we resum?

Step one: Factorization $\Longrightarrow$ Evolution. This is well known; μ_F emerge and from:

$$\frac{d}{d\mu_F}\text{Observable} = 0,$$

we get the DGLAP equation.

Physical interpretation of H-production:

1) we first produce parton "a" at large scale Q with $p_T >> m$; the parton behaves as massless.

2) The parton "a" radiates p_T down to scale $p_T \sim m$ and perturbatively fragments into massive quark $\mathcal{Q}$.

3) Non-perturbative hadronization $\mathcal{Q} \longrightarrow H$ at a scale set by m.

Therefore we write:

$$d\sigma_{\mathcal{Q}}(Q, m, z) = \sum_a \widehat{d\sigma}_a(Q, \mu, z) \otimes D_{a\to\mathcal{Q}}(\mu, m, z)$$

where:

- $\widehat{d\sigma}_a$ - usual coefficient function for producing parton a. The collinear divergences are subtracted in the $\overline{\mathrm{MS}}$ scheme:

$$d\sigma_a(Q, \epsilon, z) = \Gamma_{ab}(\epsilon, \mu) \otimes \widehat{d\sigma}_b(Q, \mu, z)$$

- $\widehat{d\sigma}_a$ contains all process dependence. Insensitive to low energy.
- $D_{a\to\mathcal{Q}}(\mu, m, z)$ - Perturbative Fragmentation Function (PFF).
- PFF is a process independent solution of the DGLAP equation that describes the transition:
$$a(\mu) \longrightarrow \mathcal{Q}(m)$$

The PFF can be decomposed as:

$$D_{a\to\mathcal{Q}}(\mu, m, z) = E_{ab}(\mu, \mu_0, z) \otimes D^{\rm ini}_{b\to\mathcal{Q}}(\mu_0, m, z)$$

with $E_{ab}(\mu_0, \mu_0, z) = \delta_{ab}\delta(1 - z)$.

Note: Given $D^{\rm ini}(\mu_0)$ one can completely specify the PFF !

Step two: Evolution $\Longrightarrow$ Resummation.

Let us choose $\mu \sim Q$; $\mu_0 \sim m$. Then:

- $\widehat{d\sigma}(Q, \mu)$ and $D^{\rm ini}(\mu_0, m)$ cannot contain large logs,
- Can be computed perturbatively.

This way, all large logs are absorbed in the function $E_{ab}(\mu, \mu_0, z)$ and are resumed with the DGLAP equation to all orders in α_S.

Therefore to achieve resummation up to logarithmic order n, one needs the initial condition to order n and the splitting functions to the same order.

$$\begin{aligned} D^{\rm ini}_{a\to\mathcal{Q}}(\mu_0, m, z) &= \delta_{a\mathcal{Q}}\delta(1-z) + \frac{\alpha_s}{2\pi} d^{(1)}_{a\to\mathcal{Q}} + \left(\frac{\alpha_s}{2\pi}\right)^2 d^{(2)}_{a\to\mathcal{Q}} + \dots \\ &= \text{LL} \quad + \quad \text{NLL} \quad + \quad \text{NNLL} \quad + \dots \end{aligned}$$

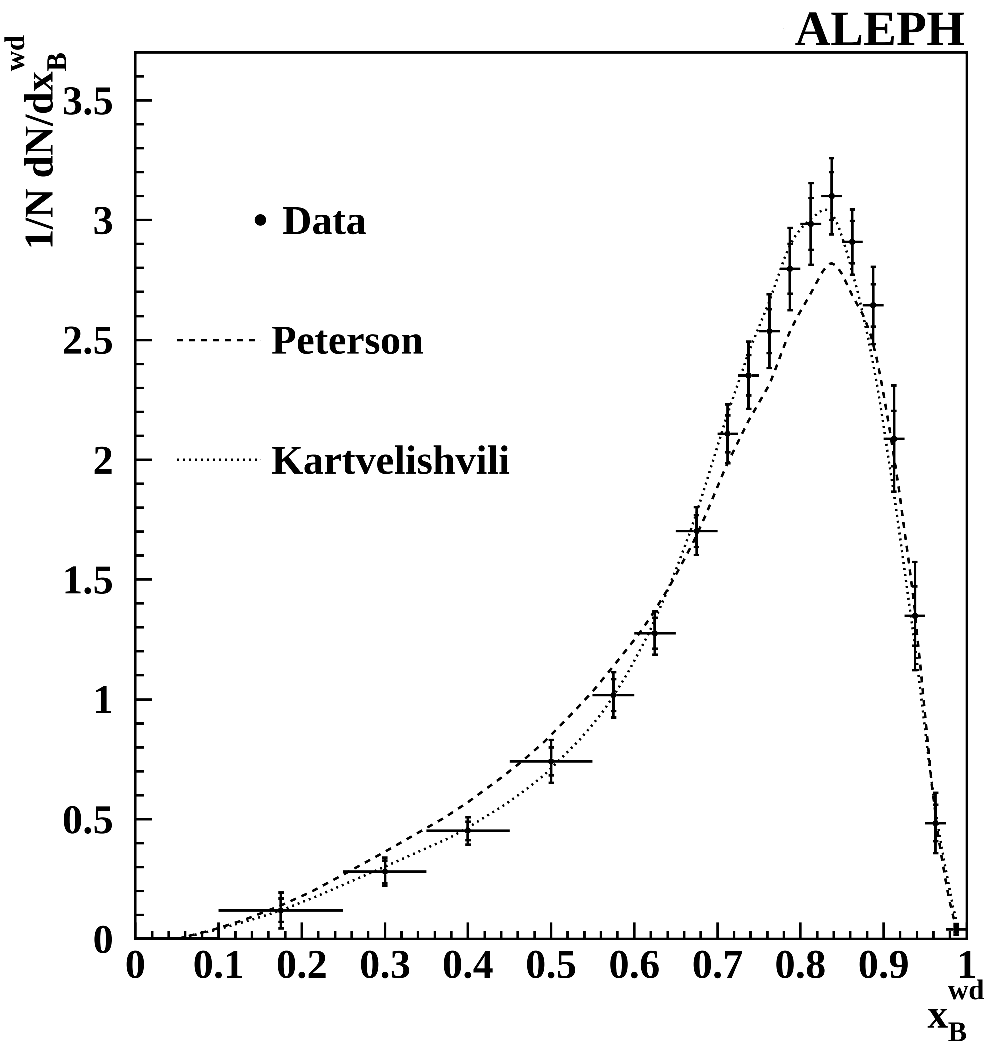
- $d^{(1)}$ - computed by Mele and Nason (1991).
- We have evaluated $d^{(2)}_{a\to\mathcal{Q}}$ for $a = \mathcal{Q},\ \overline{\mathcal{Q}},\ q, \overline{q},$ gluon.

Collect all pieces:

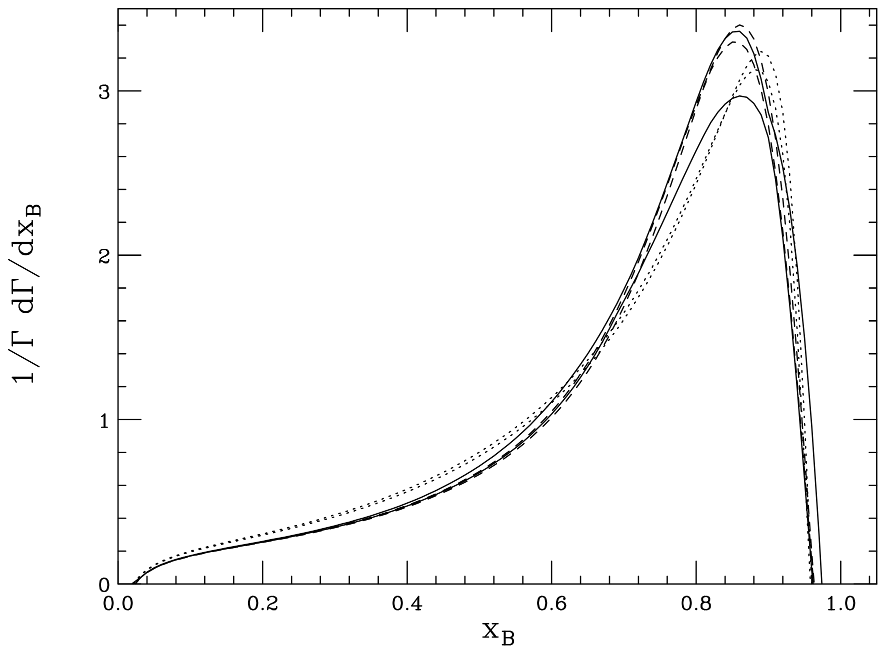
$$d\sigma_H = (f_{\dots}) \otimes \widehat{d\sigma}_a(Q,\mu) \otimes E_{ab}(\mu,\mu_0) \otimes D^{\rm ini}_{b\to\mathcal{Q}}(\mu_0, m) \otimes D^{\rm n.p.}_{\mathcal{Q}\to H} + \mathcal{O}(m/Q)^p$$

At present, $D^{n.p.}$ has been extracted with NLO+NLL accuracy.

The Aleph data (hep-ex/0106051):



... and the result from the NLO+NLL extraction (M. Cacciari, G. Corcella and A.M.; hep-ph/0209204):



- Solid line: power law $\sim x^a(1-x)^b$.
- Dashes: Kartvelishvili et al.
- Dots: Peterson.

Note: the above figure represents the convolution of $D^{\rm n.p.}$ with the perturbative part for top decay.

Can $D^{\rm n.p.}$ be extracted at NNLO + NNLL at present from e^+e^-?

- The missing ingredient are the NNLO time-like splitting functions.
- The coefficient function is known,
- $D^{\rm ini}$ is available,
- Note that our result for $D^{\rm ini}$ at order α_S^2 can be used to extract constants needed to promote the soft-gluon resummation of $D^{\rm ini}$ (for large z) with NNLL accuracy.

Other options:

- The largest uncertainty is from the fits to the existing LEP data.
- A run of ILC in GigaZ regime is expected to improve the LEP data (and particularly the one on b-fragmentation) 2-3 times.

Is NNLO+NNLL needed?

- Reduced scale dependence,
- Improved $z \to 1$ behavior,
- Extraction of $D^{\mathrm{n.p.}}$ with smaller sensitivity to non-perturbative physics.

However one must also apply such result to other processes that are evaluated with NNLL+NNLO.

Feasible in view of the present advances in NNLO calculations.

Example: A non-trivial application is the b-spectrum in top decay at NNLO+NNLL. Presently known at NLO+NLL.

Another excellent example: recent analysis of b-production at the Tevatron (Cacciari, Nason,...).

Observation: Consistent treatment of $d\sigma_B/dp_T$ as above with:

- NLL resummation,
- NLL+NLO for low p_T,
- Consistent extraction of $D^{\mathrm{n.p.}}$,

leads to dramatic shift in the theoretical prediction.

Applications of the Perturbative Fragmentation Function:

F.O. results

A parton level result is of the form:

$$d\sigma_{\mathcal{Q}}(Q,m) = \widehat{d\sigma}_a(Q,\mu) \otimes E_{ab}(\mu,\mu_0) \otimes D^{\rm ini}_{b\to\mathcal{Q}}(\mu_0,m) + \mathcal{O}(m^2/Q^2)$$

Let us ignore all terms that are beyond given Fixed Order n by (formally) setting $\mu = \mu_0$ above (i.e. $E_{ab} = 1$). We get:

$$d\sigma^{f.o.}_{\mathcal{Q}}(Q,m) = \sum_a \widehat{d\sigma}_a(Q,\mu) \otimes D^{\rm ini}_{a\to\mathcal{Q}}(\mu,m) + \mathcal{O}(m^2/Q^2)$$

- $D^{\rm ini}$ is process independent $\Longrightarrow$ one can compute spectra of massive particles from pure massless calculations. Great simplification beyond NLO.
- $D^{\rm ini}$ (similarly to its time-like counterpart) relates massless $\overline{\rm MS}$ subtractions to massive calculations.

Recall: that relation was used to first derive $D^{\rm ini}$ at NLO.

Example: electron spectrum in μ-decay (QED):

$$\frac{d\Gamma^{(\mu)}}{dz_e} = \cdots + \alpha^2 \left(c_2(z_e) \ln^2\left(\frac{m_\mu}{m_e}\right) + c_1(z_e) \ln\left(\frac{m_\mu}{m_e}\right) + c_0(z_e) + \mathcal{O}(m_e^2/m_\mu^2) \right)$$

- The constants c_2 and c_1 are known (they are fixed from the NLO result and from the evolution equation).
- $D^{\rm ini}$ together with a calculation of that process with massless electron at order α^2 are sufficient to determine the constant $c_0(z)$.

Applications of the Perturbative Fragmentation Function:

All order resummations with NNLL

Consider ,say, b-quark production:

$$\frac{d\sigma_b}{dz} = \frac{\widehat{d\sigma_a}}{dz} \otimes D^{\mathrm{PFF}}_{a\to\mathcal{Q}_b} + \mathcal{O}(m^2/Q^2)$$

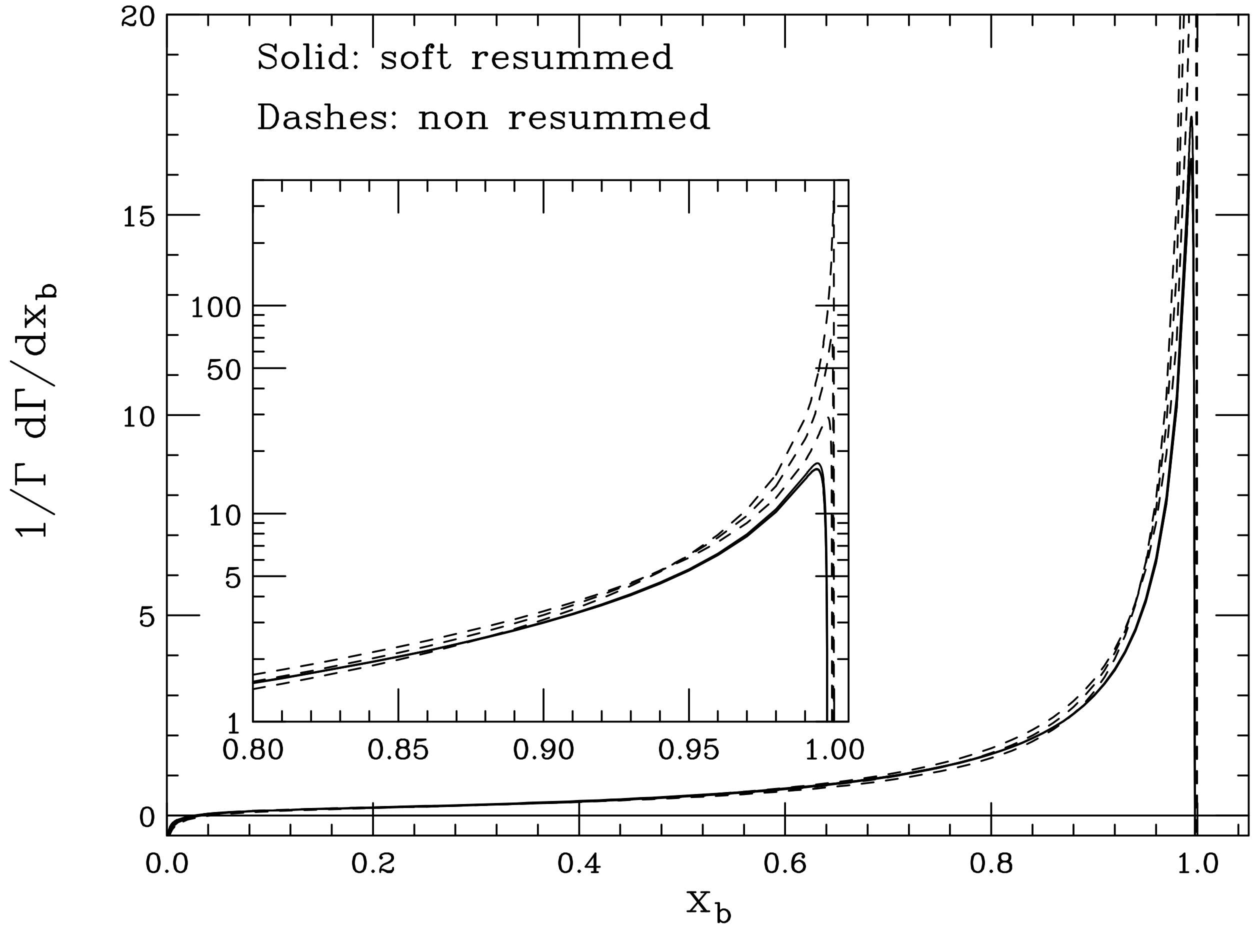
At present done at NLO + NLL.

Note: at large $z \longrightarrow 1$ additional large logs $\sim \ln^k(1-z)/(1-z)$ appear.

- They need to be resummed too. Those logs appear both in the coefficient function and in the initial condition.
- The resummation of those soft logs lead to improvement in the large z part of the spectrum.

Example: The effect of the soft-gluon resummation at large z for b-energy spectrum in top decay:

b-quark spectrum for different μ_F (M. Cacciari, G. Corcella, A.M.)

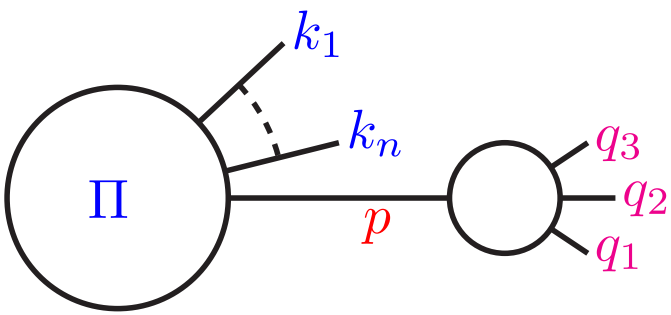


- Three different values of $\mu_F = 2m_t, m_t, m_t/2$.
- Other scales: $\mu = m_t$ and $\mu_{0F} = \mu_0 = m_b$.

A process independent derivation of $D^{\rm ini}$

- Previously applied to NLO by Keller and Laenen; Cacciari and Catani.

- Based on the factorization of short- and long-distance physics in an arbitrary hard scattering process:



The momenta $q_{1,2,3}$ are collinear i.e. $q_1 + q_2 + q_3 = p + \mathcal{O}(q_T)$

We work in physical gauge $A^\mu n_\mu = 0$. In such case no contribution from interference diagrams. As a result the collinear splitting effectively decouples from the rest of the process.

One also uses the factorization of both matrix elements and phase space in the collinear limit :

$$|M^{(n+3)}(k_1,\ldots,k_n,q_1,q_2,q_3)|^2 = |M^{(n+1)}(k_1,\ldots,k_n,p)|^2 W(q_1,q_2,q_3)$$

and

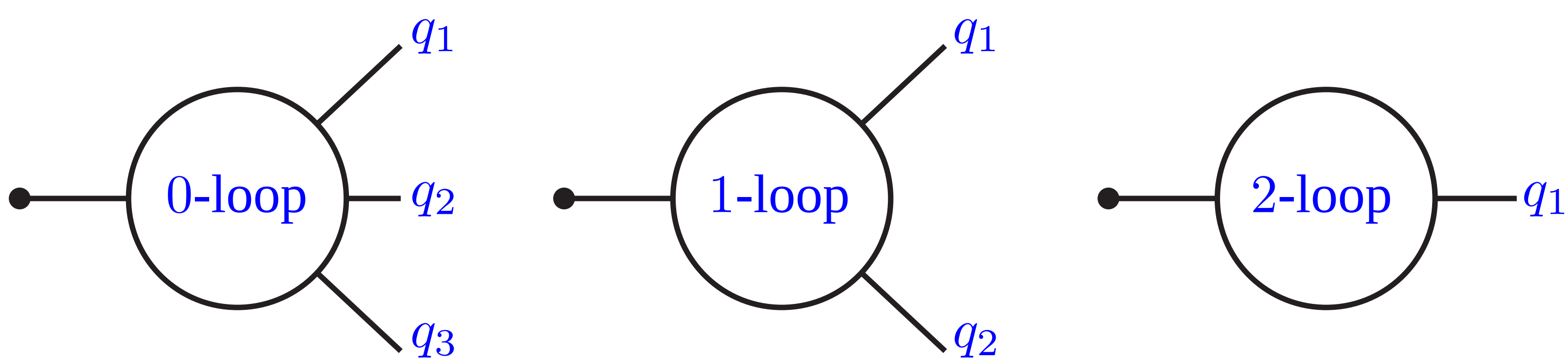
$$\mathrm{dPS}^{(n+3)}(k_1,\ldots,k_n,q_1,q_2,q_3) = \mathrm{dPS}^{(n+1)}(k_1,\ldots,k_n,p)\,\mathrm{d}\Phi^{\mathrm{coll}}(q_2,q_3)$$

The functions D^{ini} can now be obtained by integrating the factor W over the momenta of the unobserved collinear partons, i.e. $\mathrm{d}\Phi^{\mathrm{coll}}(q_2,q_3)$.

Evaluation of the function D^{ini}.

- We have modified significantly the original method; the new formulation is suitable for evaluation beyond NLO and for inclusion of the virtual diagrams.

- As a result we have to evaluate "processes" like:



- ... and then to project out the fragmentation component (applying power counting arguments in the collinear limit).

Various components to PFF and the participating sub-processes at tree-level:

I. $D^{\rm ini}_{\mathcal{Q}\to\mathcal{Q}}$:

- $\mathcal{Q} \longrightarrow \mathcal{Q} + g + g,$
- $\mathcal{Q} \longrightarrow \mathcal{Q} + q + \overline{q},$
- $\mathcal{Q} \longrightarrow \mathcal{Q} + \mathcal{Q} + \overline{\mathcal{Q}}.$

II. $D^{\rm ini}_{\overline{\mathcal{Q}}\to\mathcal{Q}}$:

- $\overline{\mathcal{Q}} \longrightarrow \mathcal{Q} + \overline{\mathcal{Q}} + \overline{\mathcal{Q}}.$

III. $D^{\rm ini}_{q(\overline{q})\to\mathcal{Q}}$:

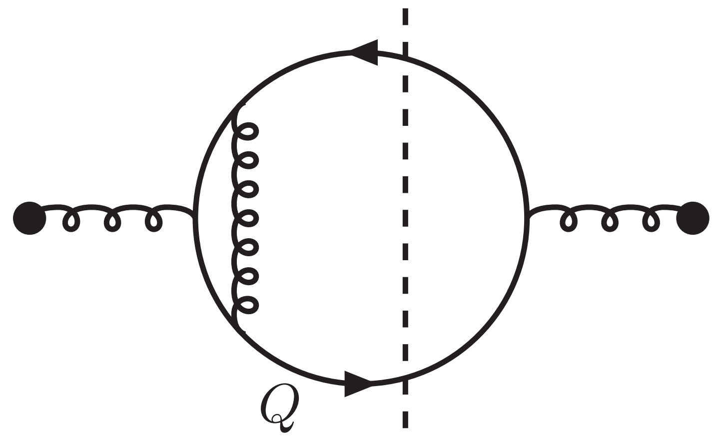
- $q(\overline{q}) \longrightarrow \mathcal{Q} + \overline{\mathcal{Q}} + q(\overline{q}).$

IV. $D^{\rm ini}_{g\to\mathcal{Q}}$:

- $g \longrightarrow \mathcal{Q} + \overline{\mathcal{Q}} + g\ .$

Evaluation of the integrals

- We evaluate the integrals using the IBP identities,
- Algebraically reduce the contributions from all diagrams to ~ 20 Master Integrals, containing single scale (m) and a single variable (z),
- We use light cone gauge: no particular problems at one (virtual) loop
- interesting contributions from the gluon initiated component. Contains terms behaving as $|1-2z|$ that we have interpreted as threshold production of a heavy pair. They originate from the diagram:



Properties of $D^{\rm ini}$ at order $\mathcal{O}(\alpha_S^2)$:

- The result satisfy the fermion number conservation condition (by construction):

$$\int_0^1 dz \left(D^{\rm ini}_{\mathcal{Q}/\mathcal{Q}}(z) - D^{\rm ini}_{\overline{\mathcal{Q}}/Q} \right) = 1.$$

- The Non-Singlet term $\sim n_f$ coinsides with the known result in the large β_0-limit (Cacciari and Gardi).

- The limit $z \to 1$: our result reproduces the NLL "soft-logs" $\alpha_s^2 \ln^k(1-z)/(1-z)$ for $k = 3, 2, 1$ from the known result for the soft-gluon resumed initial condition (Cacciari and Catani). Also from our result one can extract the constant $H^{(2)}$ that is needed to promote the soft-resummation to NNLL accuracy.

Conclusions:

- We have calculated all components of the initial condition for the perturbative fragmentation function at order α_s^2 (NNLO), thus extending the PFF formalism to NNLL level.
- We followed a process independent approach for the computation of D^{ini} that exploits the universal behavior of the collinear radiation.
- To evaluate the two-loop integrals we apply powerful techniques for multi-loop calculations: IBP, reduction to MI's and their solving.
- I discussed the general properties of our result as well as the checks with partial results existing in the literature.
- I discussed some of the many possible applications of our result, like:
- Fixed order spectra for heavy particles from massless results,
- Resummations of quasi-collinear logs with NNLL accuracy and accurate extraction of non-perturbative fragmentation function from data.