Effects of the $t\bar{t}$ Threshold

in $e^+e^- \to t\bar{t}H$

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(. . . on the arXive soon)

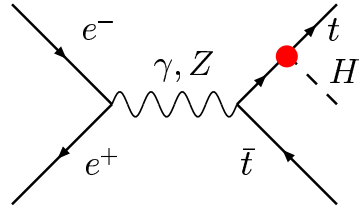
Outline

- Introduction & Status of $\sigma(e^+e^- \to t\bar{t}H)$
- Region of large Higgs energie $\Leftrightarrow$ $t\bar{t}$ threshold
- Effective Theory $\to$ vNRQCD
- $\left(\frac{d\sigma}{dE_H}\right)_{E_H \approx E_{H,max}}$ at NLL order
- Numerics
- Conclusions

Top Yukawa Coupling

- Massgeneration $\Leftrightarrow$ SSB $\Leftrightarrow$ Higgsmechanism

  $m_i = \lambda_i \langle H \rangle$ $\longrightarrow$ top quark physics



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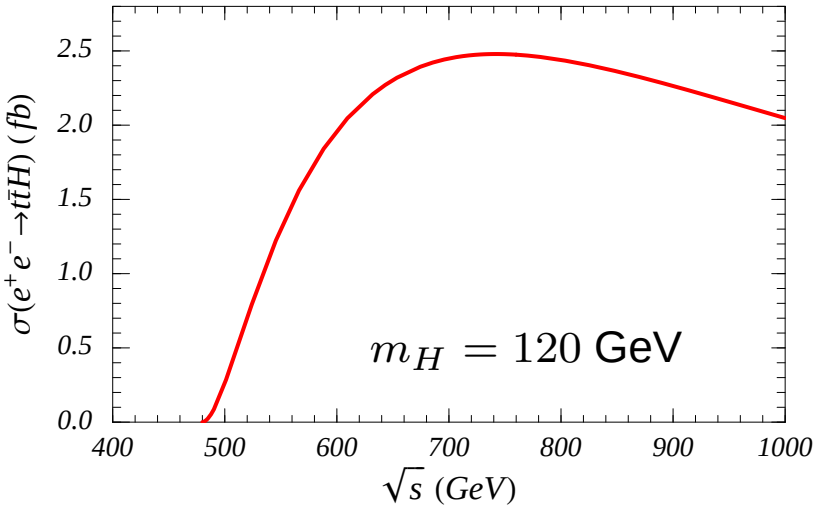
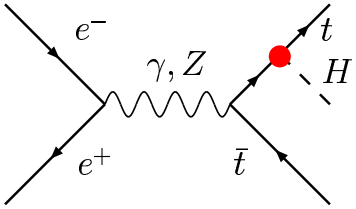
- ILC: $e^+e^- \to t\bar{t}H$





$\to$ $\delta\lambda_t/\lambda_t \simeq 5\%$ ($m_H = 120$ GeV)

[Gay; Besson; Winter]

$\to$ 2nd phase ($\sqrt{s} > 500$ GeV)

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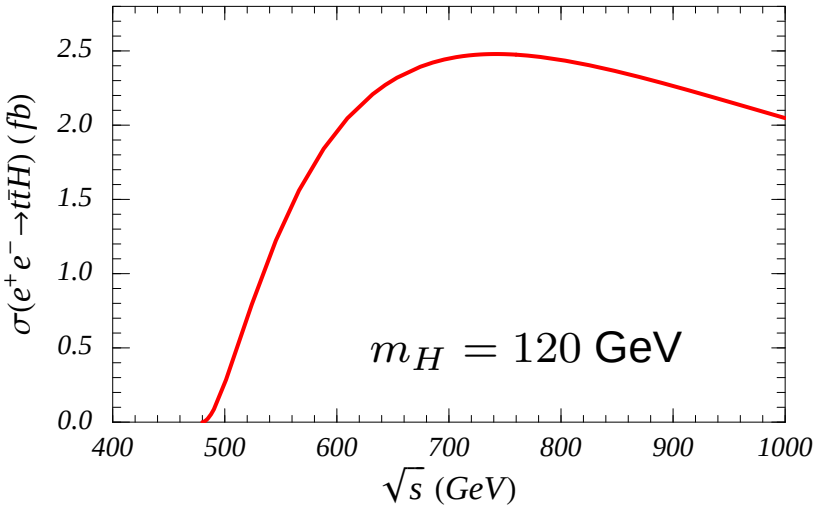

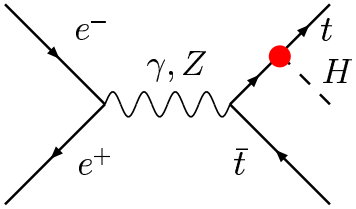

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- Theory Status: $\sigma_{\rm tot}(e^+e^- \to t\bar{t}H)$

  Born ✓ [Gaemers et al., Djouadi et al.]

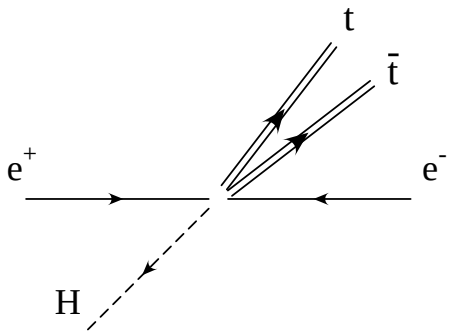
  $\mathcal{O}(\alpha_s)$ ✓ [Dittmaier et al., Dawson et al.]

  1-loop ew. ✓ [Denner et al., Belanger et al., You et al.]

  fully differential partonic $\mathcal{O}(\alpha_s)$: ✓ [Denner et al.]

Large Higge Energy Region

$\longrightarrow$ region of large Higgs energy



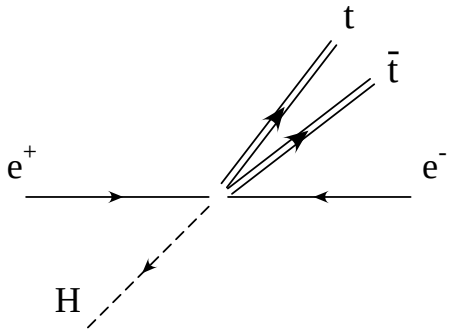
$\longrightarrow$ $t\bar{t}$ collinear

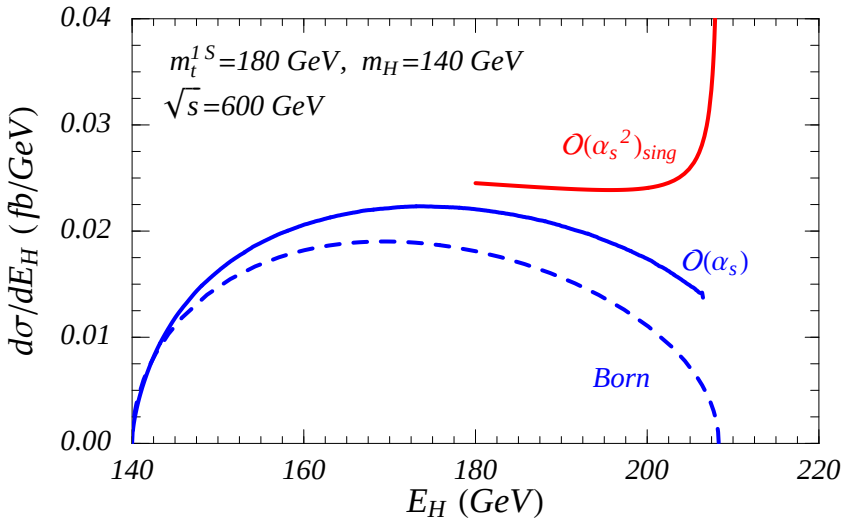
$\longrightarrow$ QCD effects localized in $t\bar{t}$ system

$\Rightarrow$ $t\bar{t}$ dynamics non-relativistic

Large Higge Energy Region

→ region of large Higgs energy





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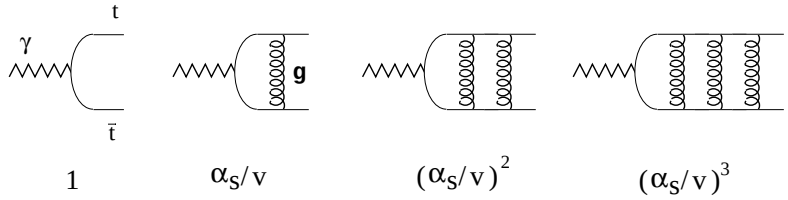

→ singularities: $\sim (\alpha_s/v)^n$,

$\sim (\alpha_s \ln v)^n$

→ fixed order expansion breaks down

⇒ summation of singular terms

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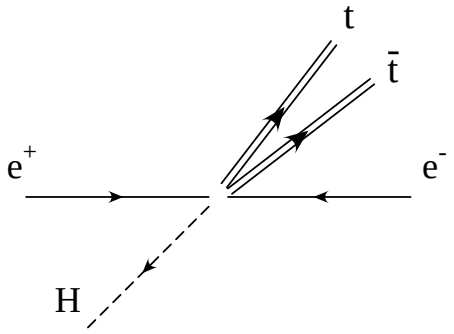

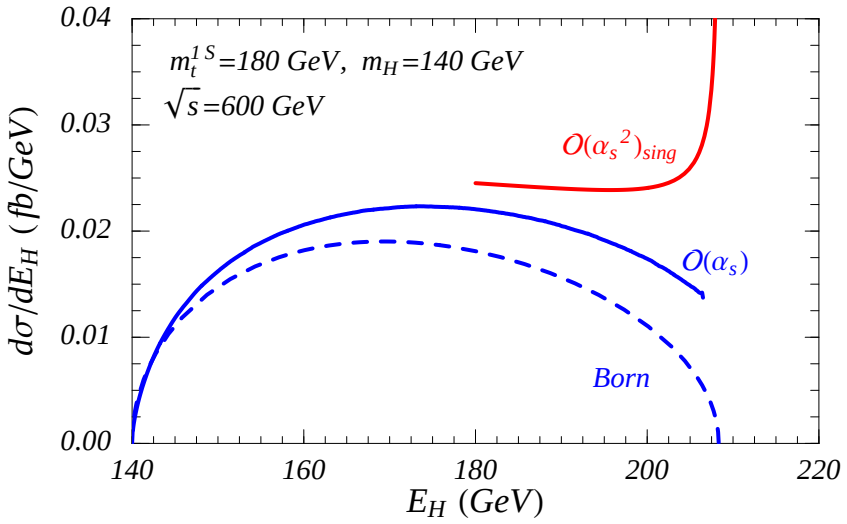

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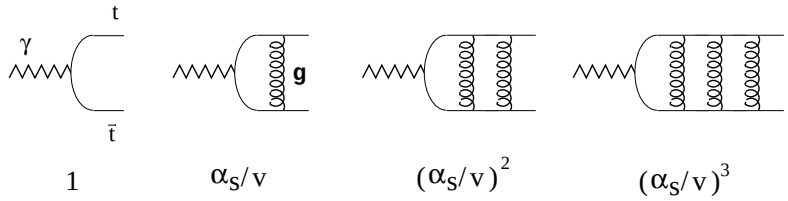

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⇒ summation of singular terms

(A) impose cut: $E_H > E_H^{\max}$

(B) summation via vNRQCD

} ⇒ control of singular QCD effects desired

→ computation at NLL order

vNRQCD in a Nutshell

$$m_t \quad \gg \quad \mathbf{p} \sim m_t v \quad \gg \quad E \sim m_t v^2 \sim \Gamma_t \quad > \quad \Lambda_{\rm QCD}$$

(hard) (soft) (ultrasoft)

- scale hierarchy $\longrightarrow$ factorization
- non-perturbative effects suppressed

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(hard) (soft) (ultrasoft)

- scale hierarchy $\rightarrow$ factorization
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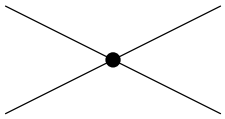
$$\mathcal{L} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm potential} + \mathcal{L}_{\rm soft}$$

Luke, Manohar, Rothstein, Stewart, A.H.

$\mathcal{L}_{\rm usoft}$:

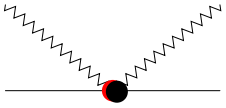
$$\psi^\dagger_{\mathbf{p}}(x)\left\{iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m_t} + i\frac{\Gamma_t}{2} - \delta m_t\right\}\psi_{\mathbf{p}}(x)$$

$\mathcal{L}_{\rm potential}$:



$$\left\{\frac{V_c(\nu)}{(\mathbf{p}-\mathbf{p}')^2} + \cdots\right\}\psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}}$$

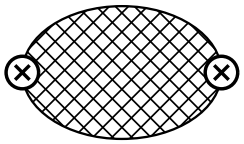
$\mathcal{L}_{\rm soft}$:



vNRQCD in a Nutshell

Currents: $\rightarrow$ production & annihilation of $t\bar{t}$ pairs

$$\mathbf{O}_{\mathbf{p}}^{\mathrm{trip}} = C_1(\mu) \cdot (\psi_{\mathbf{p}}^{\dagger}\, \boldsymbol{\sigma}\, \tilde{\chi}_{-\mathbf{p}}^{*}) + \cdots \qquad ({}^3S_1)$$



$$\mathbf{O}_{\mathbf{p}}^{\mathrm{sing}} = C_0(\mu) \cdot (\psi_{\mathbf{p}}^{\dagger}\, \tilde{\chi}_{-\mathbf{p}}^{*}) + \cdots \qquad ({}^1S_0)$$

$\rightarrow$ $t\bar{t}$ production rate:

$$R_{t\bar{t}} \propto \mathrm{Im}\left[\int \mathrm{d}^4\mathrm{x}\, \mathrm{e}^{-\mathrm{i}\hat{\mathrm{q}}\mathrm{x}} \left\langle 0 \left| \mathrm{T}\, \mathbf{O}_{\mathbf{p}}^{\dagger}(0)\, \mathbf{O}_{\mathbf{p}'}(\mathrm{x}) \right| 0 \right\rangle\right]$$

$$\propto C(\mu)^2\, \mathrm{Im}\left[\mathrm{G}(0,0,\mathrm{v},\mu)\right]$$

vNRQCD in a Nutshell

Currents: → production & annihilation of $t\bar{t}$ pairs

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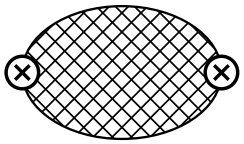

→ $t\bar{t}$ production rate:

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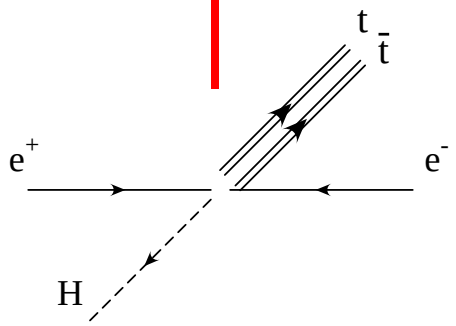
$$\propto C(\mu)^2\, \mathrm{Im}\left[\mathrm{G}(0,0,\mathrm{v},\mu)\right]$$

→ known from $\sigma(e^+e^- \to t\bar{t})$:

- NLL zero-distance Green function ✓ [Manohar et al., Teubner, AHH] [Beneke et al., Penin et al.]
- NLL running of $C^1(\mu)$ and $C^0(\mu)$ ✓ [Luke et al.; Pineda; Manohar et al., AHH]
- matching conditions $C^1(m_t)$ and $C^0(m_t)$ → new

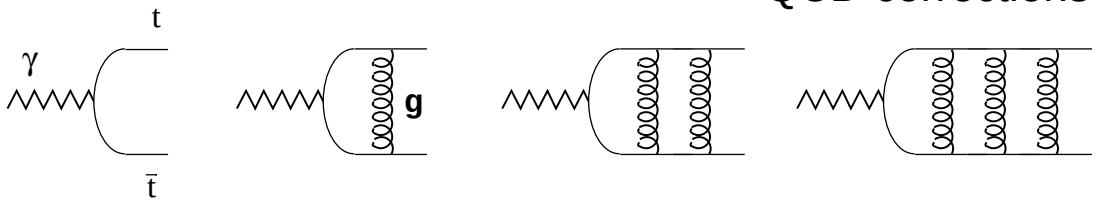
NLL Result

$$\left(\frac{d\sigma}{dE_H}\right)_{E_H \approx E_H^{max}} \sim \left[C_0^2(\mu, \sqrt{s}, m_t, m_H) + C_1^2(\mu, \sqrt{s}, m_t, m_H)\right] \mathrm{Im}[G(0,0,v,\mu)]$$



+ hard QCD corrections

soft & ultrasoft QCD corrections



1 α_s/v $(\alpha_s/v)^2$ $(\alpha_s/v)^3$

NLL Result

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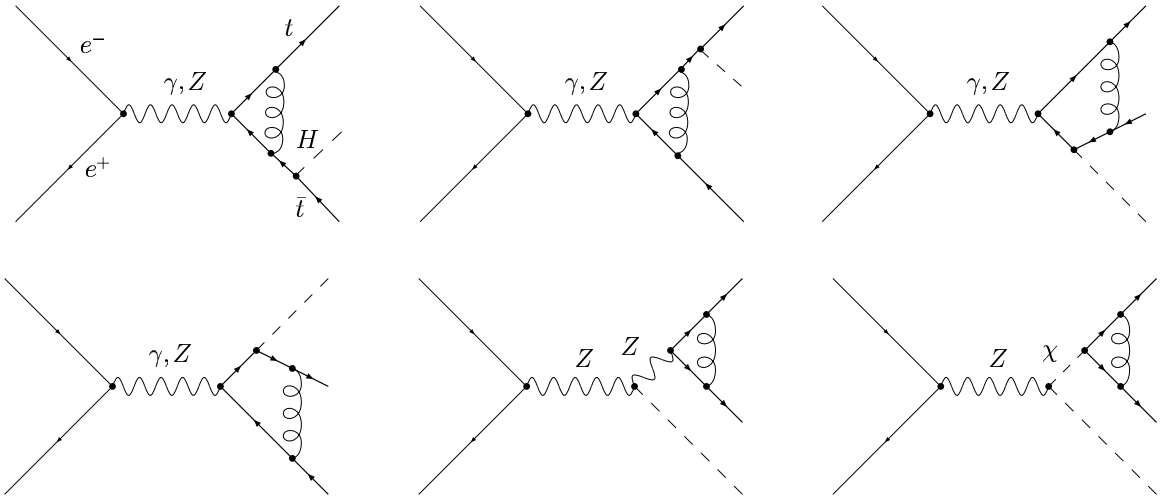
→ matching computation: $(\mu = m_t)$

EFT result at NLL (expanded to $\mathcal{O}(\alpha_s)$)

$\stackrel{!}{=}$ $\mathcal{O}(\alpha_s)$ result (full theory) for $E_H \approx E_H^{\max}$

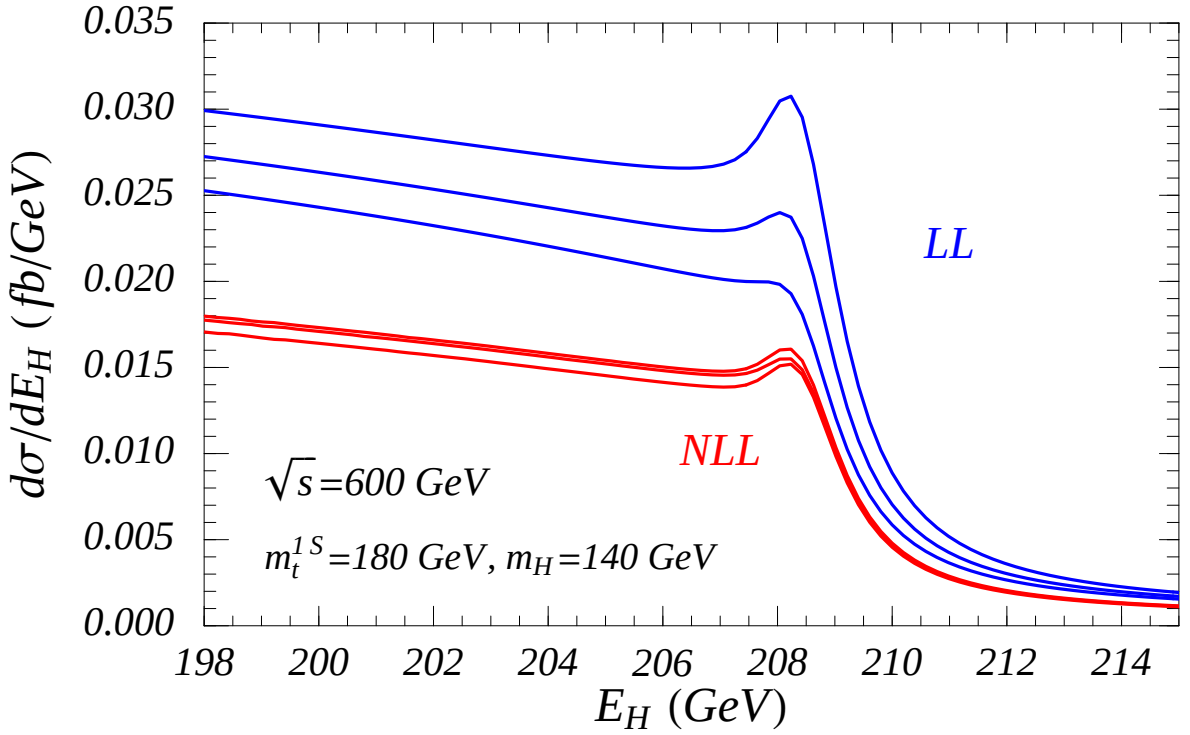
$\Rightarrow C_{0,1}(m_t,\sqrt{s},m_t,m_H)$



→ virtual corrections sufficient

Dittmaier, Roth, et al.
[Dawson et al.]

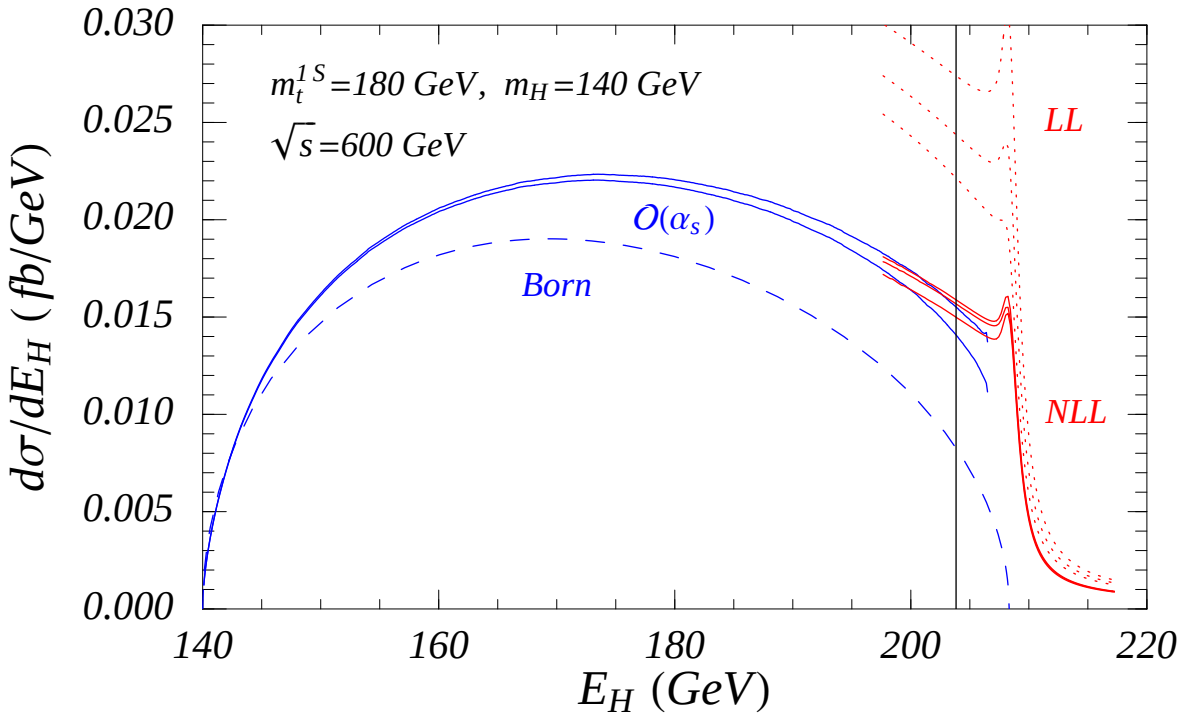
Numerics



$\mu_{\rm soft} = 18 - 72$ GeV

- signifi cant reduction of scale variation: LL $\rightarrow$ NLL
- generic feature for all cases

Numerics



- good matching of NLL vNRQCD and $\mathcal{O}(\alpha_s)$ full theory results
- matching at $v \approx 0.2$

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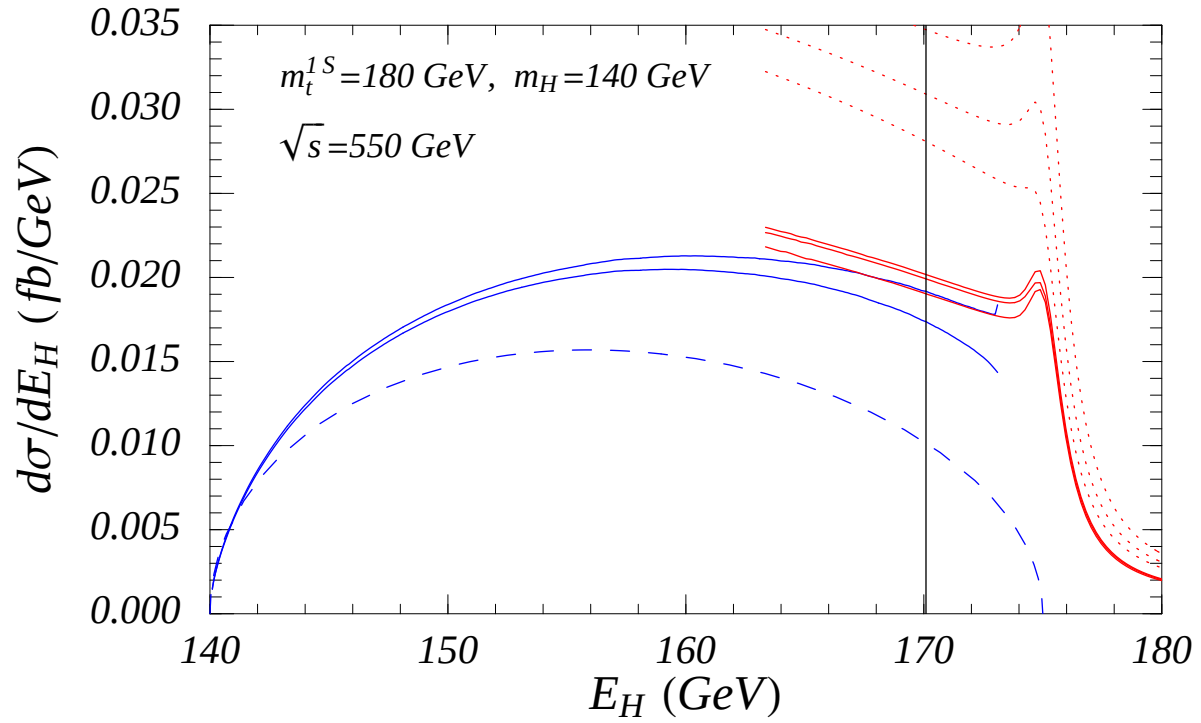

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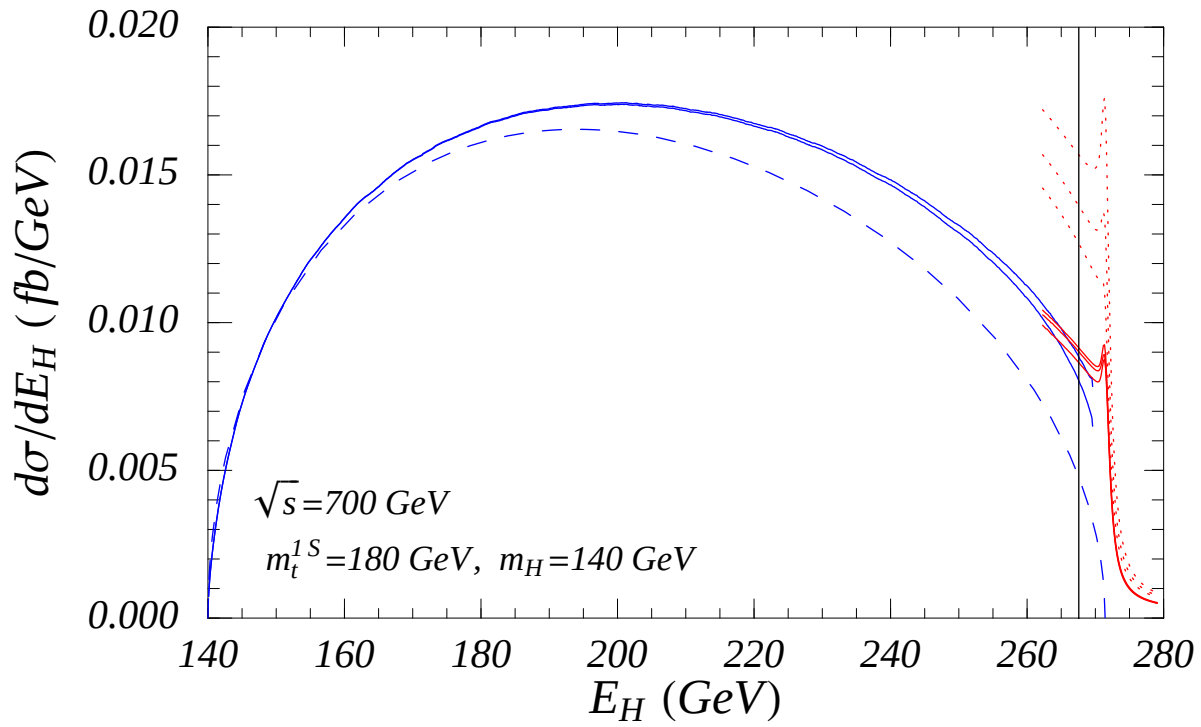

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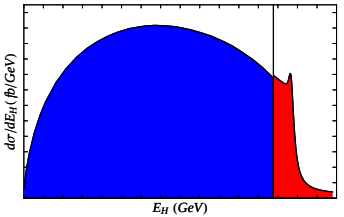
Numerics

$$\frac{\sigma_{t\bar{t}H}(v < 0.2)}{\sigma_{t\bar{t}H}(v > 0.2)}$$



$\sqrt{s}$	$m_H = 120$ GeV	$m_H = 140$ GeV	$m_H = 160$ GeV
500 GeV	1.30	–	–
550 GeV	0.14	0.25	0.62
600 GeV	0.06	0.08	0.12
700 GeV	0.02	0.03	0.03
800 GeV	0.01	0.01	0.02

$\sigma_{\rm tot} < 1$ fb $\quad$ $\sigma_{\rm tot} < 0.5$ fb

- non-relativistic effects relevant up to $\sqrt{s} \lesssim 700$ GeV
- larger for smaller $\sqrt{s}$ and larger m_H

Conclusions & Outlook

- $\sigma(e^+e^- \to t\bar{t}H) \Leftrightarrow$ top Yukawa coupling

  $\to \delta\sigma_{t\bar{t}H}/\sigma_{t\bar{t}H} \sim \%$ precision required

- Region of large $E_{\rm Higgs} \Leftrightarrow t\bar{t}$ system non-relativistic

- Summation of $(\alpha_s/v)^n$ and $(\alpha_s \ln v)^m$ singularities relevant

  for $\sqrt{s} \lesssim 700$ GeV $\to$ probably negligible for LHC: $(\sqrt{s})_{\rm eff} \sim$ TeV

- NLL order computation using vNRQCD

  $\to$ "hard" matching coeffs. from existing $\mathcal{O}(\alpha_s)$ fi xed order results

  $\to$ excellent matching: EFT (NLL) $\leftrightarrow$ fi xed order $\mathcal{O}(\alpha_s)$ at $v = 0.2$

- <u>Outlook:</u>

  $\to$ combination: EFT (NLL) & fi xed order $\mathcal{O}(\alpha_s)$

  $\to$ fully differential treatment

  . . . still many conceptual problems to solve

Colors

This is blue

This is red

This is brown

This is magenta

This is Dark Green

This is Dark Blue

This is Green

This is Cyan

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks