Effects of Finite Top Lifetime at the $t\bar{t}$ Threshold

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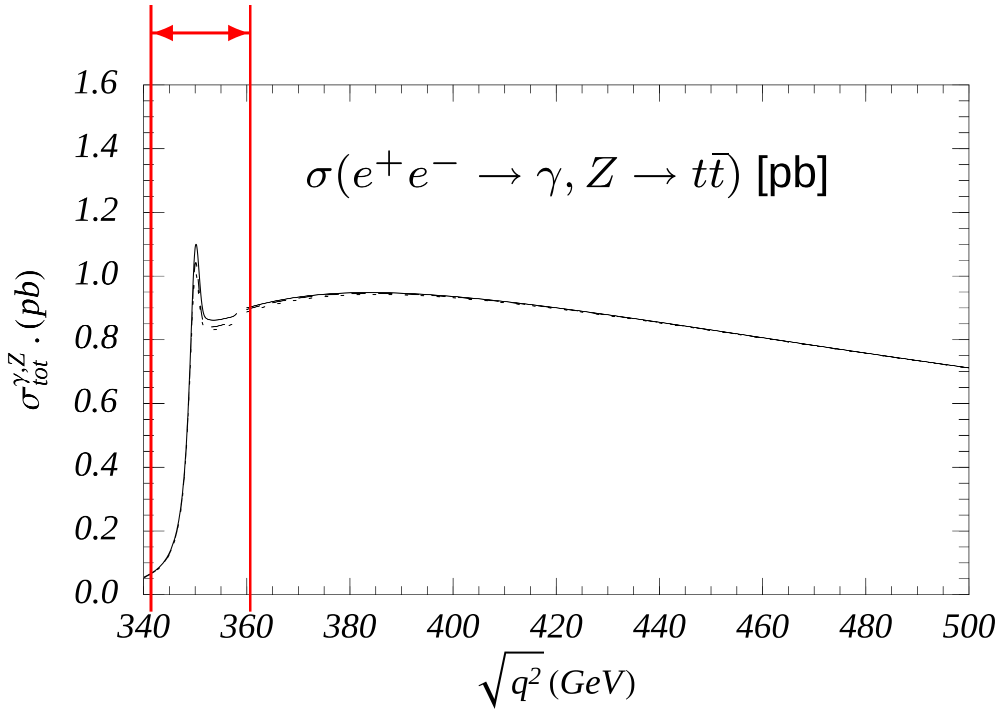
Outline

- Introduction
- Status for $\sigma(e^+e^- \to t\bar{t})$
- Effective Theory (stable tops $\to$ unstable tops)
- NNLL Finite Lifetime Effects
- Numerics
- Conclusion & Outlook

Top Physics at the Threshold

$E_{cm} \approx 340 - 360$ GeV $\Longrightarrow$ $v_{\rm top} = \sqrt{1 - \frac{4m_t^2}{E_{\rm cm}^2}} \ll 1$



Top Physics at the Threshold

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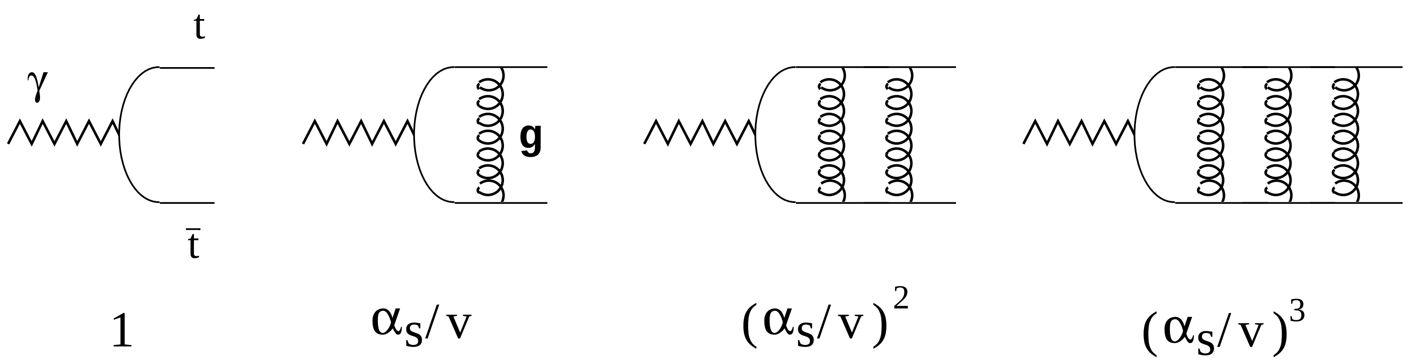
- Top quarks are slow: $\boxed{E_t = \frac{p_t^2}{m_t}, \quad E_t \ll p_t \ll m_t}$



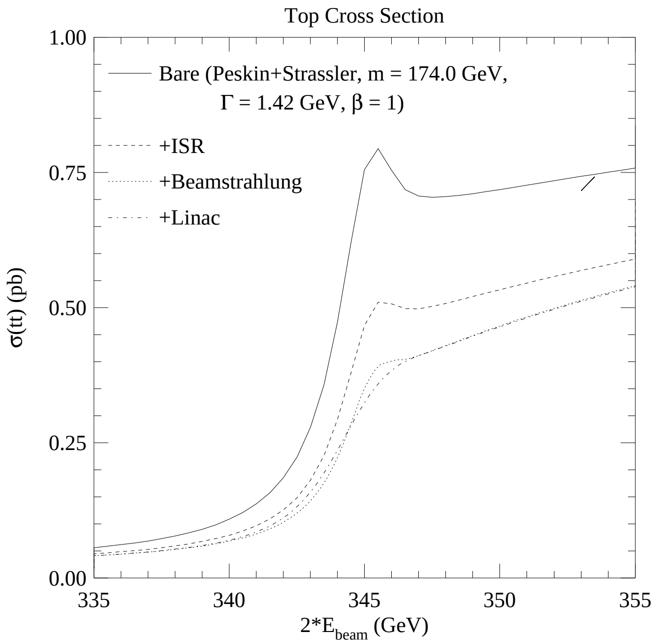
▷ perturbation theory in α_s breaks down

▷ Bound state dynamics $\Rightarrow$ "Schrödinger field theory"

Top Physics at the Threshold

$$E_{cm} \approx 340 - 360\ \text{GeV} \implies v_{\text{top}} = \sqrt{1 - \frac{4m_t^2}{E_{\text{cm}}^2}} \ll 1$$

- Top quarks are slow: $\boxed{E_t = \frac{p_t^2}{m_t}, \quad E_t \ll p_t \ll m_t}$
- Top quarks decay fast: $\boxed{\Gamma_t \approx 1.5\ \text{GeV} \gg \Lambda_{\text{QCD}}}$
  - ▷ no toponium bound states
  - ▷ smooth line-shape
  - ▷ perturbative QCD



Measurements

- Top quark mass $\longrightarrow$ threshold scan

$$(\delta m_t)^{\mathrm{exp+theo}}_{thr.mass} \sim 100 \text{ MeV}$$

- Top Yukawa coupling

$$(\delta y_t / y_t)^{\mathrm{exp}} \sim 0.15 - 0.5 \qquad \text{(light Higgs)}$$

- Strong coupling

$$(\delta \alpha_s(M_Z))^{\mathrm{exp}} \sim 0.001$$

- Top quark total width

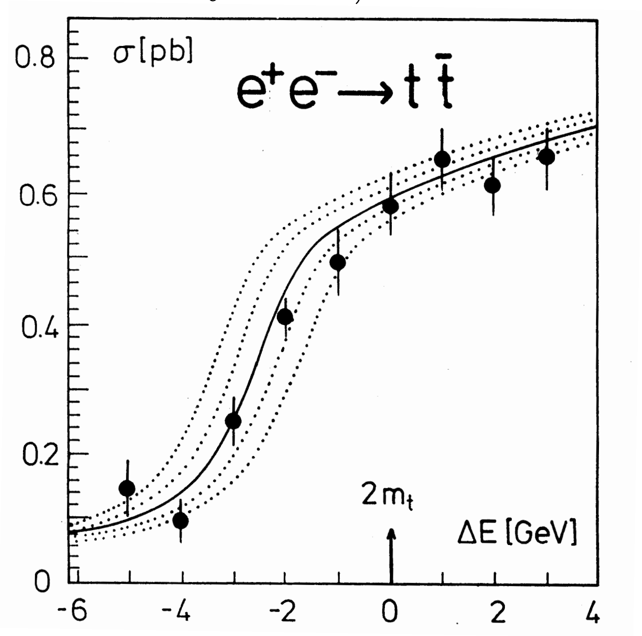
$$(\delta \Gamma_t)^{\mathrm{exp}} \sim 50 \text{ MeV}$$

$\Rightarrow$ Requirements to theory

m_t: threshold masses

y_t, α_s, Γ_t: $\boxed{\delta\sigma_{t\bar t}/\sigma_{t\bar t} \leq 3\%}$

$m_t \pm 200, 400$ MeV



Theory Status

- QCD effects (fixed order) $\longrightarrow\ v \sim \alpha_s$

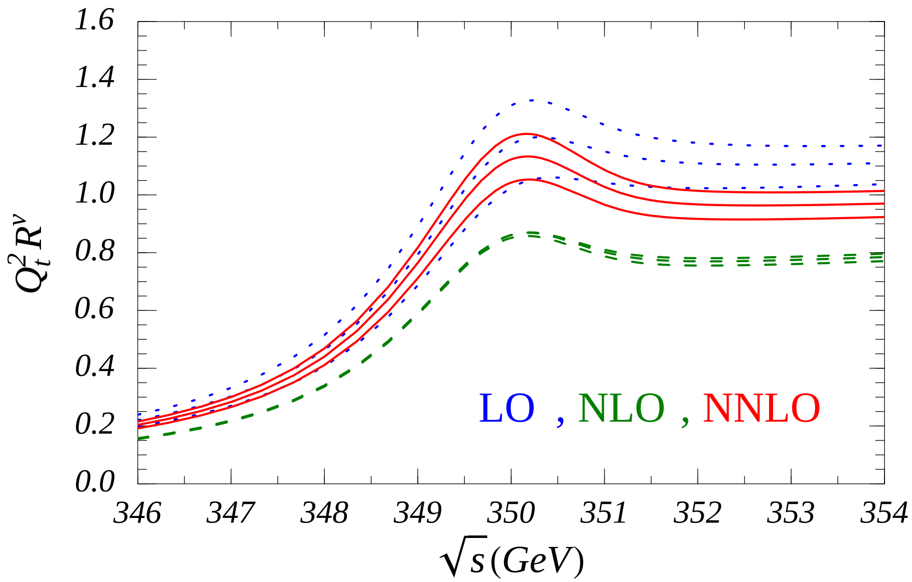
LO ✓ LO $\sim \left(\frac{\alpha_s}{v}\right)^n$

NLO ✓ NLO $\sim \alpha_s \left(\frac{\alpha_s}{v}\right)^n$

NNLO ✓ [→ NNNLO: Steinhauser's talk] NNLO $\sim \alpha_s^2 \left(\frac{\alpha_s}{v}\right)^n$

Teubner, AH; Penin etal; Melnikov etal;
Beneke etal; Sumino etal; Yakovlev etal;

$m_t^{1S} = 175$ GeV



$\longrightarrow \quad \frac{d\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \sim 20\%$

Theory Status

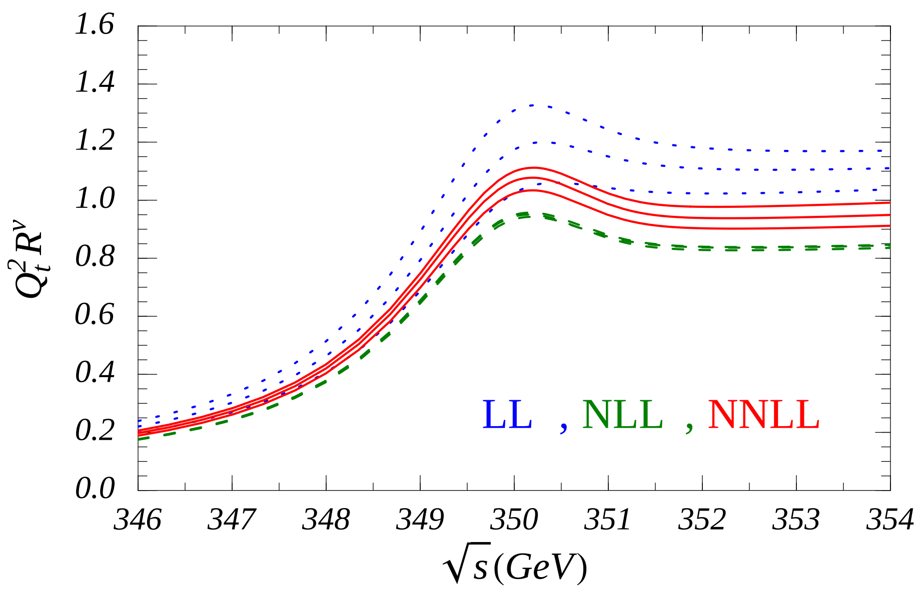
- QCD effects (RGE-improved) $\rightarrow$ $v \sim \alpha_s$, $\alpha_s \ln v \sim 1$

LL ✓   LL $\sim \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \ln v)^m$

NLL ✓   NLL $\sim \alpha_s \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \ln v)^m$

NNLL (almost) $\rightarrow \left(\frac{\delta\sigma}{\sigma}\right) \sim \pm 6\,\%$ NNLL $\sim \alpha_s^2 \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \ln v)^m$

$m_t^{1\mathrm{S}} = 175$ GeV   Manohar, Stewart, Teubner, AHH



$\rightarrow \quad \frac{d\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \sim 6\%$

Theory Status

- QCD effects (RGE-improved) $\quad \rightarrow \; v \sim \alpha_s\,, \quad \alpha_s \ln v \sim 1$

LL	✓	LL	$\sim \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \ln v)^m$
NLL	✓	NLL	$\sim \alpha_s \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \ln v)^m$
NNLL	(almost)	NNLL	$\sim \alpha_s^2 \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \ln v)^m$

- ew. effects $\quad \rightarrow \; g \sim g' \sim v \sim \alpha_s$

LL ✓

NLL (✓) [Khoze et al., Melnikov et al.]

NNLL ? $\rightarrow$ QED, hard weak, phase space effects, finite lifetime

$\rightarrow$ NNLL finite life time corrections

$\rightarrow$ new parametric NLL corrections

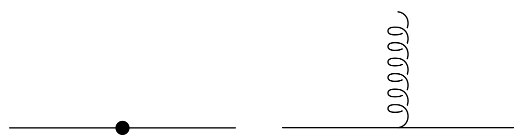
vNRQCD (stable quarks)

$$m_t \text{ (hard)} \quad \gg \quad \mathbf{p} \sim m_t v \text{ (soft)} \quad \gg \quad E \sim m_t v^2 \text{ (ultrasoft)}$$

$$\underline{\mathcal{L} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm potential} + \mathcal{L}_{\rm soft}}$$

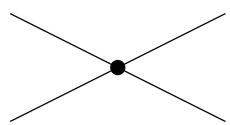
Luke, Manohar, Rothstein, Stewart, A.H.

$\mathcal{L}_{\rm usoft}$:



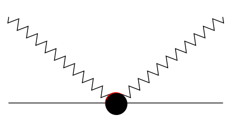
$$\psi^\dagger_{\mathbf{p}}(x) \left\{ iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m_t} - \delta m_t \right\} \psi_{\mathbf{p}}(x)$$

$\mathcal{L}_{\rm potential}$:



$$\left\{ \frac{V_c(\nu)}{(\mathbf{p}-\mathbf{p}')^2} + \cdots \right\} \psi^\dagger_{\mathbf{p}'} \psi_{\mathbf{p}} \chi^\dagger_{-\mathbf{p}'} \chi_{-\mathbf{p}}$$

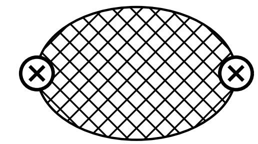
$\mathcal{L}_{\rm soft}$:



vNRQCD (stable quarks)

Currents: $\longrightarrow$ production & annihilation of $t\bar{t}$ pairs

$$\mathbf{O_p} = C(\mu) \cdot (\psi^\dagger_{\mathbf{p}}\, \boldsymbol{\sigma}\, \tilde{\chi}^*_{-\mathbf{p}}) + \cdots \qquad t\bar{t}\ (^3S_1)$$



$$\sigma_{\rm tot} \quad \propto \quad \mathrm{Im}\left[\int \mathrm{d}^4\mathrm{x}\, \mathrm{e}^{-\mathrm{i}\hat{\mathrm{q}}\mathrm{x}} \left\langle 0 \,\middle|\, \mathrm{T}\, \mathbf{O}^\dagger_{\mathbf{p}}(0)\, \mathbf{O}_{\mathbf{p}'}(\mathrm{x}) \,\middle|\, 0 \right\rangle\right]$$

$$\propto \quad \mathrm{Im}\left[\mathrm{C}(\mu)^2\, \mathrm{G}(0,0,\sqrt{\mathrm{s}})\right]$$

$$G^{\rm LL}(0,0,\sqrt{s}) = \frac{m_t^2}{4\pi}\left\{ i\,v - C_F\alpha_s\left[\frac{1}{4\epsilon} + \ln\left(\frac{-i\,m_t v}{\mu}\right) + \psi\left(1 - \frac{i\,C_F\alpha_s}{2\,v}\right)\right]\right\}$$

$$\mathrm{Re}[G(0,0,\sqrt{s})] \sim \frac{m_t^2}{4\pi}\frac{1}{4\epsilon}$$

vNRQCD (unstable quarks)

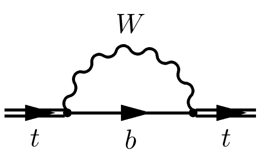
$\Rightarrow$ Optical Theory: effective complex indices of refraction for absorptive processes

$\Rightarrow$ vNRQCD: contributions from real Wb final states included in EFT matching conditions to QCD+ew. theory (=SM)

- complex matching conditions
- effective Lagrangian non-hermitian
- total rates through the optical theorem
- powercounting from stable theory still valid
- (gauge) symmetries constrain effective interactions

vNRQCD (unstable quarks)

<u>quark bilinears:</u> $(\rightarrow \mathcal{L}_{\rm usoft})$



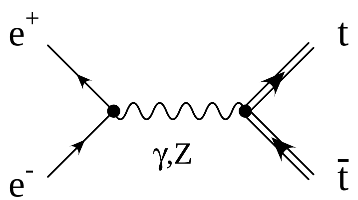
$$= i\Sigma_t \quad \overset{\mathrm{Im}\Sigma_t = \frac{1}{2}\Gamma_t}{\Longrightarrow} \quad \delta\mathcal{L} = \psi_{\mathbf{p}}^\dagger \left[i\frac{\Gamma_t}{2}\left(1 - \frac{\mathbf{p}^2}{2m_t^2}\right)\right]\psi_{\mathbf{p}}$$

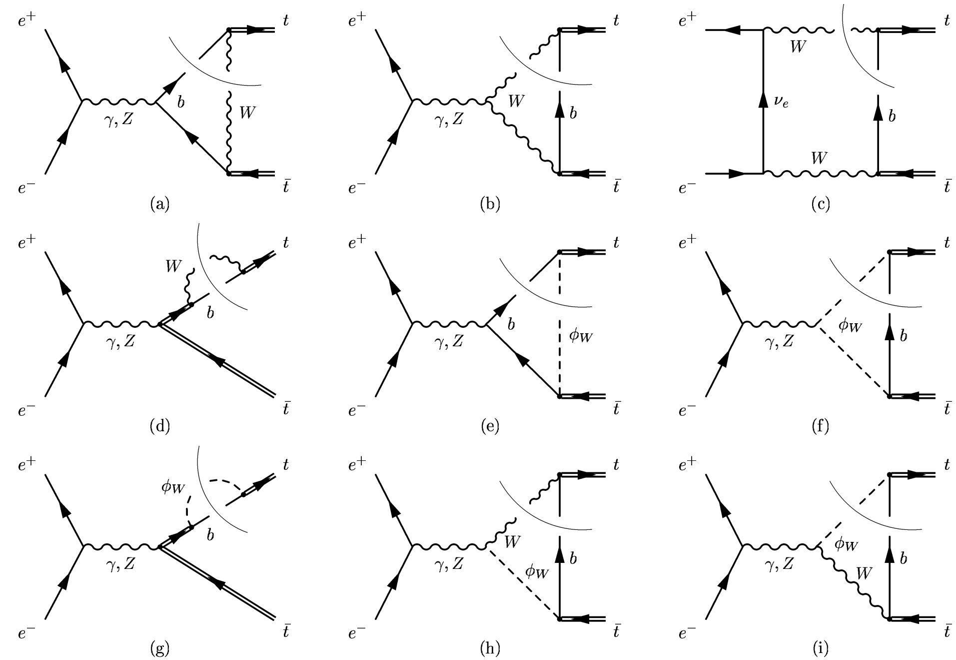
- power counting: $\Gamma_t \sim m_t g^2 \sim m_t v^2 \quad \Rightarrow \quad \boxed{g \sim g' \sim v \sim \alpha_s}$
- finite lifetime is LL order, $\quad E \rightarrow E + i\Gamma_t$ Fadin,Khoze
- NNLL time dilatiation effect
- $E \rightarrow E + i\Gamma_t$ prescription does not work beyond LL order

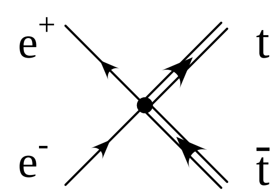
vNRQCD (unstable quarks)

Currents:

- non-bW-cuts not included
- bW-cuts gauge invariant



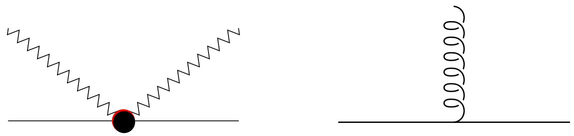


$$\mathbf{O_p} = \Big[\, C^{\rm LL} + C^{\rm NLL} + C^{\rm NNLL} + i C^{\rm NNLL}_{\rm abs} \cdots \Big] \cdot \Big($$  $$\Big) + \cdots$$

Re$[C^{\rm NNLL}_{\rm ew}]$ (Kühn, Guth 1992)

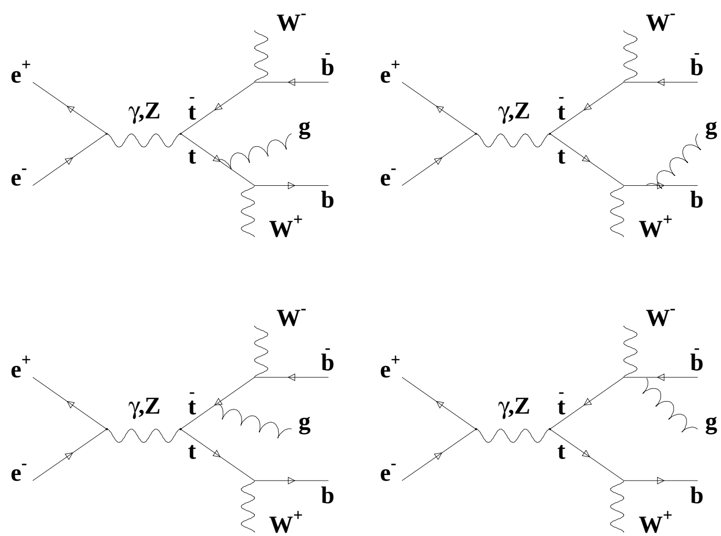
vNRQCD (unstable quarks)

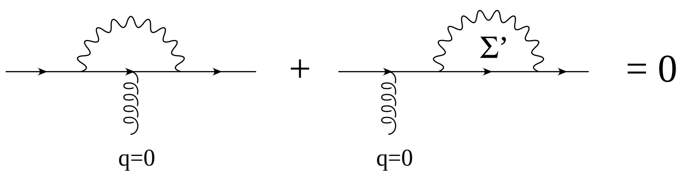
<u>soft & ultrasoft interactions:</u> $(\rightarrow \mathcal{L}_{\rm usoft}, \mathcal{L}_{\rm soft})$



$\rightarrow$ (LL), NLL and NNLL for stable quarks

- electroweak matching conditions ($g^2 \sim v^2$) are beyond NNLL
- no interference (=non-factorizable) effects from ultrasoft gluon exchange at NLL (Khoze etal., Melnikov etal.) <u>and</u> NNLL

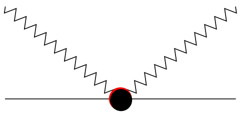


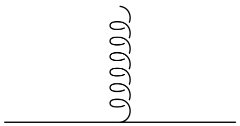


Watch the power of powercounting and gauge invariance !

vNRQCD (unstable quarks)

soft & ultrasoft interactions: $(\to \mathcal{L}_{\mathrm{usoft}}, \mathcal{L}_{\mathrm{soft}})$





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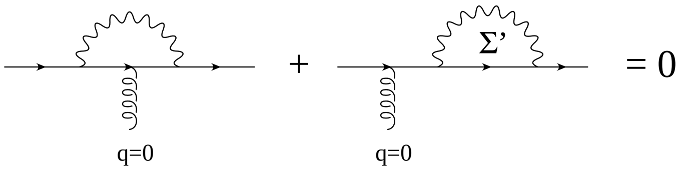
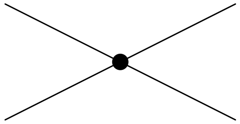


potentials: $(\to \mathcal{L}_{\mathrm{potential}})$



$\longrightarrow$ V_{Coulomb} is the only LL interaction

- but no electroweak corrections due to gauge invariance

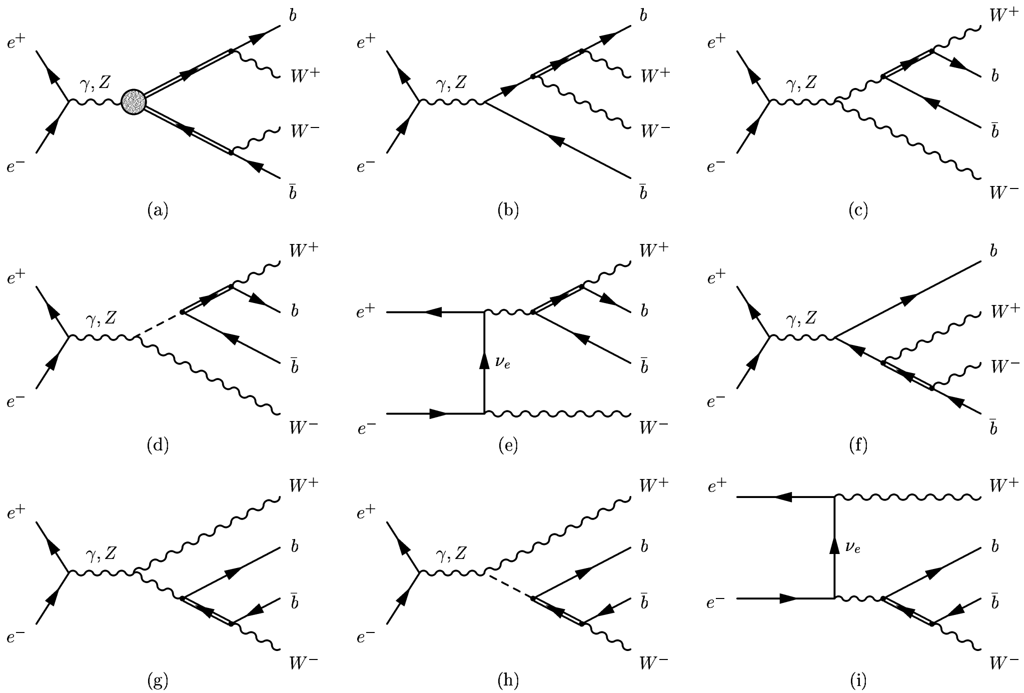
Total Cross Section

$$\Delta\sigma_{\rm tot}^{\Gamma,1} \,=\, 2N_c\,\mbox{Im}\Big[\,2\,C^{\rm LL}\,i\,C_{\rm abs}^{\rm NNLL}\,G_{\rm coul} + (C^{\rm LL})^2\,\delta G_{\rm coul}\,\Big]$$

Total Cross Section

$$\Delta\sigma^{\Gamma,1}_{\rm tot} \;=\; 2N_c\ \mathrm{Im}\Big[\,2\,C^{\rm LL}\,i\,C^{\rm NNLL}_{\rm abs}\,G_{\rm coul} + (C^{\rm LL})^2\,\delta G_{\rm coul}\Big]$$

- accounts for interferences: $\rightarrow$ irreducible
  resonant vs. non-resonant $W^+W^-b\bar{b}$ final states



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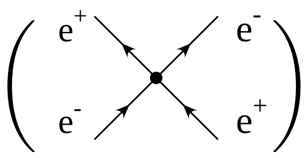
- accounts for interferences: $\rightarrow$ irreducible

  resonant vs. non-resonant $W^+W^-b\bar{b}$ final states

- $(\Delta\sigma_{\rm tot}^{\Gamma}) \sim \alpha_s\Gamma_t\,\frac{1}{\epsilon}$ $\Rightarrow$ logarithmic phase space UV divergences

  $\rightarrow$ anom. dim. for $(e^+e^-)(e^+e^-)$ operator $\rightarrow i\tilde{C}(\mu)\cdot$ 

  $\rightarrow$ additional correction: $\Delta\sigma_{\rm tot}^{\Gamma,2} \sim \tilde{C}(\mu)$ (optical theorem)

Total Cross Section

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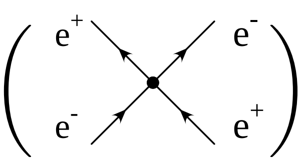
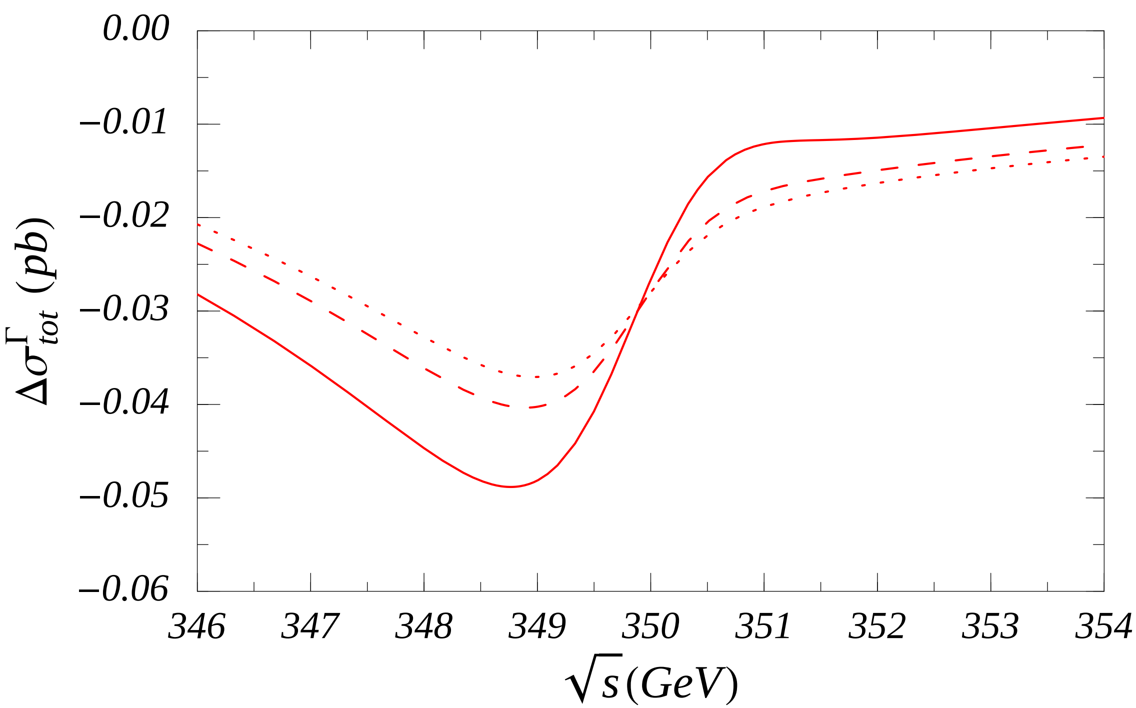
  $\rightarrow$ anom. dim. for $(e^+e^-)(e^+e^-)$ operator $\rightarrow i\tilde{C}(\mu)\cdot$ 

  $\rightarrow$ additional correction: $\Delta\sigma^{\Gamma,2}_{\rm tot} \sim \tilde{C}(\mu)$ (optical theorem)

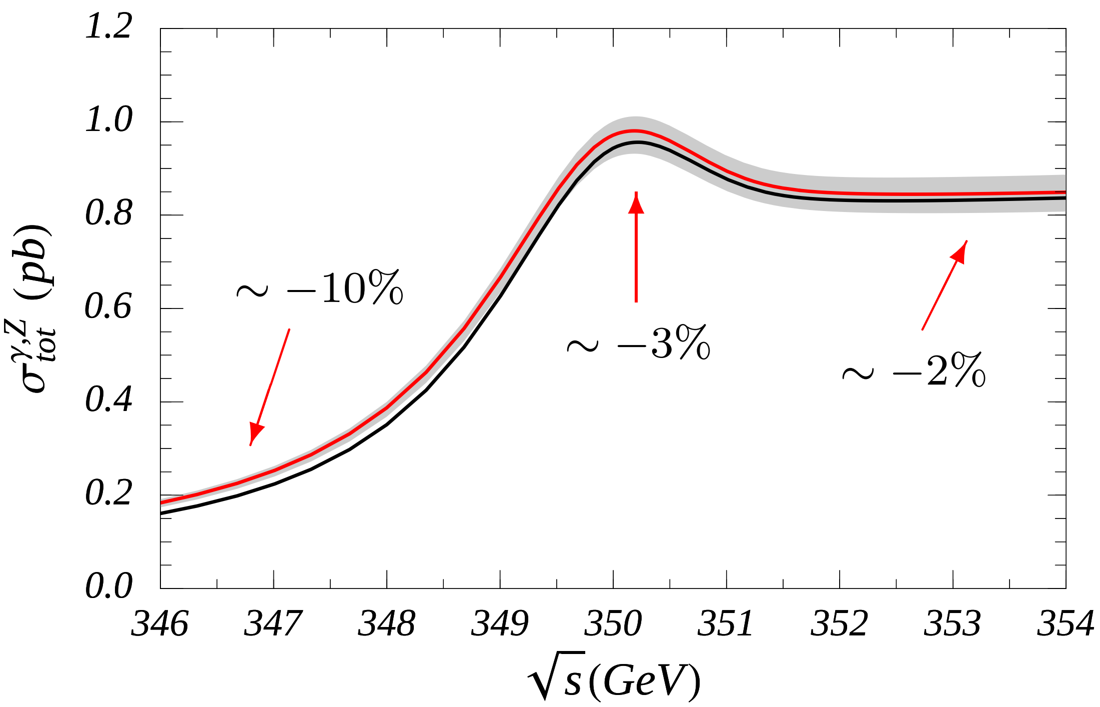
  - $\sqrt{s}$-independent
  - μ-dependent, parametrically NLL order (!)
  - $\tilde{C}(m_t)$ unknown $\rightarrow$ phase space corrections (work i.p.)

Total Cross Section



- finite lifetime corrections comparable to NNLL QCD corrections
- shift in the peak position: $30 - 50$ MeV ($\delta m_t^{\rm ex} \approx 50$ MeV)

Total Cross Section



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What's left to do . . .

NNLL Total Cross Section:

(A) electroweak corrections

- NNLL phase space matching corrections $\longrightarrow \tilde{C}^{eeee}(\mu = m_t) < 0$
- NNLL running of $\tilde{C}^{eeee}(\mu)$
- $\mathcal{O}(\alpha_s)$ corrections to $iC^{eett}_{\rm abs}(\mu = m_t)$ due to mixing

(B) QCD corrections (RGE-improved)

- Current uncertainty: $(\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}})^{\rm QCD} \sim \pm 6\%$

  $\longrightarrow$ full NLL running of potentials

Differential Cross Sections, (multi-quark final states): $\longrightarrow$ efficiencies

$\longrightarrow$ more involved treatment of unstable particles

Colors

This is blue

This is red

This is brown

This is magenta

This is Dark Green

This is Dark Blue

This is Green

This is Cyan

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks

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