Why Do We Need a Top Mass Measurement with ILC Precision?

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1. Introduction

2. Electroweak Precision Physics

3. Higgs Physics in the MSSM

4. Renormalization Group Running and Dark Matter

5. Conclusions

1. Introduction

Top-quark mass is a fundamental parameter of the electroweak theory

By far the largest quark mass,
largest mass of all known fundamental particles

Window to new physics?

Large coupling to the Higgs boson; physics of flavor;
prediction of m_t from underlying theory?

Radiative corrections

$\Rightarrow$ non-decoupling effects proportional to powers of m_t

$\Rightarrow$ Need to know m_t very precisely in order to have sensitivity to effects of new physics

What is the top mass?

Particle masses are not observables
one can only measure cross sections, decay rates, . . .

Additional problem for the top mass:

what is the mass of a colored object?

Top pole mass is not IR safe (affected by large long-distance contributions), cannot be determined to better than $\mathcal{O}(\Lambda_{\mathrm{QCD}})$

Measurement of m_t:

- At Tevatron, LHC:
  kinematic reconstruction, fit to invariant mass distribution
  $\Rightarrow$ "pole" mass

- At the LC:
  mainly from threshold behavior $\Rightarrow$ threshold mass

Experimental accuracy of m_t:

Measurement $\Leftrightarrow$ comparison data from Monte Carlo

$\rightarrow$ you measure the mass that is implemented in your MC

$\Rightarrow$ measured mass is not strictly model independent

$\Rightarrow$ 'Threshold mass' at the LC: accuracy $\lesssim$ 20 MeV

[*M. Martinez, R. Miquel '03*]

Transition to $\overline{\text{MS}}$ mass: [*A. Hoang et al. '00*]

$$\boxed{\delta m_t \lesssim 100 \text{ MeV} \quad \text{(LC)}}$$

Are the uncertainties from unknown higher orders (QCD, electroweak, mixed) under control? $\rightarrow$ not discussed here

Situation at the LHC:

$$\boxed{\delta m_t = 1\text{–}2 \text{ GeV} \quad \text{(LHC)}}$$

2. Electroweak Precision Observables

Precision observables: M_W, $\sin^2\theta_{\mathrm{eff}}$, m_h, $(g-2)_\mu$, b physics, ...

Theoretical prediction for M_W in terms of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}\left(\frac{1}{1-\Delta r}\right)$$

$\Updownarrow$

loop corrections

Theoretical prediction for the effective mixing angle:

$$\sin^2\theta_{\mathrm{eff}}=\frac{1}{4\,|Q_f|}\left(1-\frac{\mathrm{Re}\,g_V^f}{\mathrm{Re}\,g_A^f}\right)$$

Higher order contributions:

$$g_V^f\to g_V^f+\Delta g_V^f,\qquad g_A^f\to g_A^f+\Delta g_A^f$$

Leading m_t contributions:

One-loop result for M_W in the SM:

[*A. Sirlin '80*] , [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\text{loop}} = \quad \Delta\alpha \quad - \quad \frac{c_W^2}{s_W^2}\Delta\rho \quad + \quad \Delta r_{\text{rem}}(M_H)$$

$$\sim \log\frac{M_Z}{m_f} \qquad \sim m_t^2$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

Leading m_t contribution to $\sin^2\theta_{\text{eff}}$:

$$\Delta \sin^2\theta_{\text{eff}} \approx -\frac{c_W^2 s_W^2}{c_W^2 - s_W^2}\Delta\rho \sim m_t^2$$

Example: effect of new m_t value on global fit to all EWPO data:

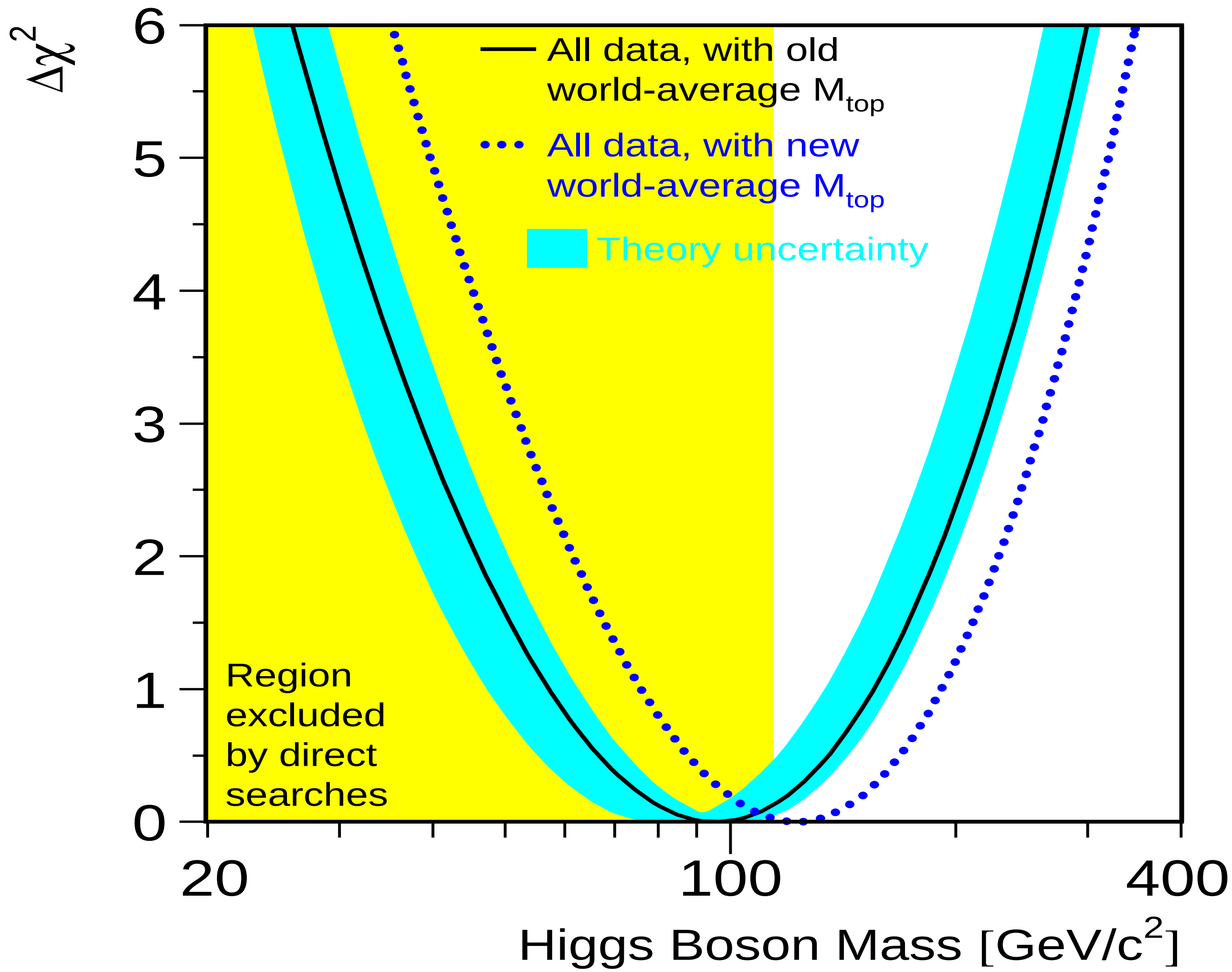
Old m_t value:

$m_t = 174.3 \pm 5.1$ GeV

New m_t value:

$m_t = 178.0 \pm 4.3$ GeV

[*Tevatron EWWG '04*]



[*LEPEWWG '04*]

⇒ best fit value of M_H shifted by more than 20 GeV

Experimental errors:

	today	Tev./LHC	LC	GigaZ
$\delta \sin^2\theta_{\rm eff} (\times 10^5)$	17	17	–	1.3
δM_W [MeV]	34	15	10	7

Relevant SM parametric errors: $\delta(\Delta\alpha_{\rm had}) = 5 \times 10^{-5}$, $\delta M_Z = 2.1$ MeV

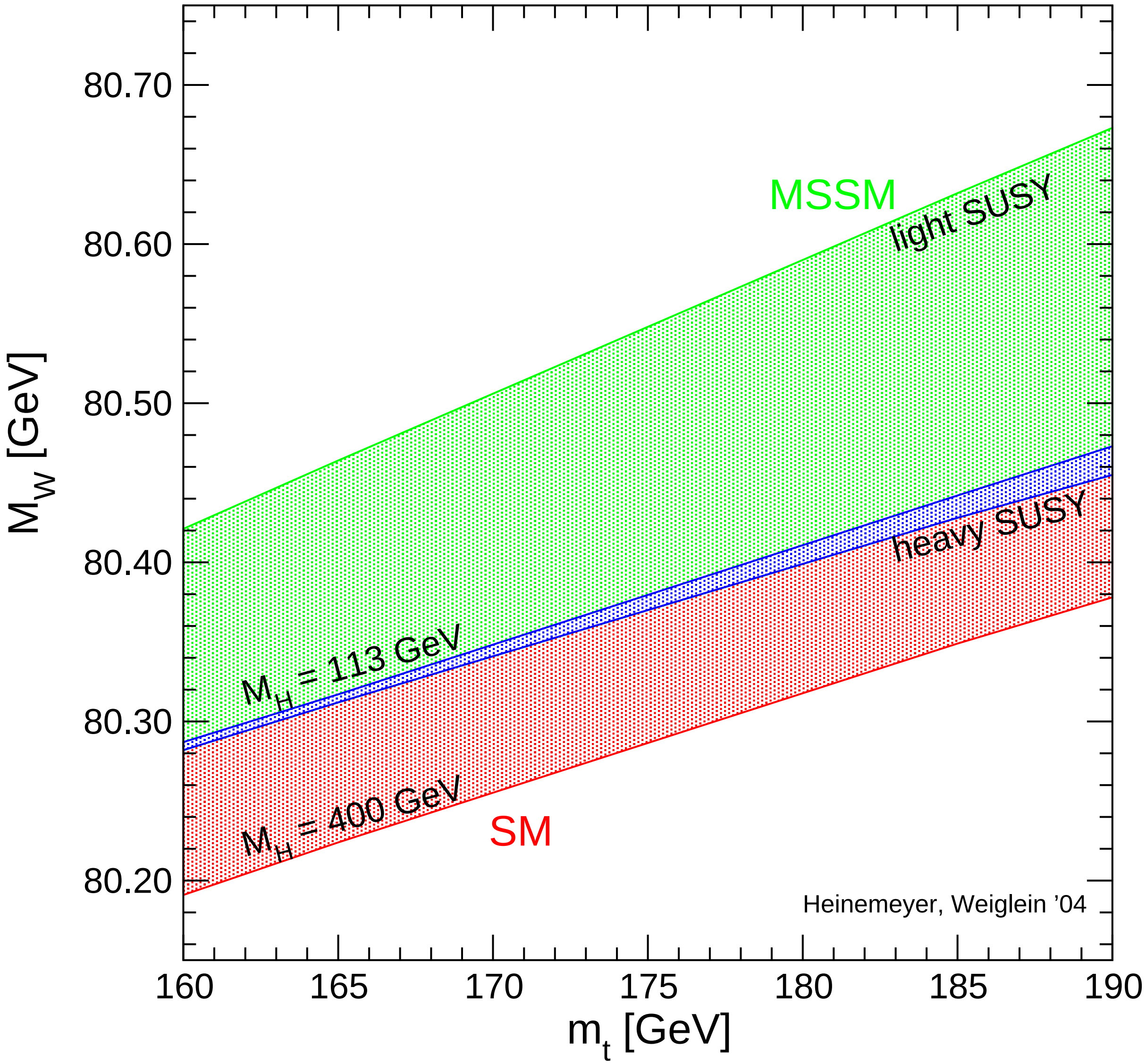
	$\delta m_t = 2$	$\delta m_t = 1$	$\delta m_t = 0.1$	$\delta(\Delta\alpha_{\rm had})$	δM_Z
$\delta \sin^2\theta_{\rm eff}$ [10^{-5}]	6	3	0.3	1.8	1.4
ΔM_W [MeV]	12	6	1	1	2.5

To keep the parametric error induced by m_t at/below the level of other uncertainties:

$\Rightarrow$ $\delta m_t \lesssim 0.2$ GeV for M_W

$\Rightarrow$ $\delta m_t \lesssim 0.5$ GeV for $\sin^2\theta_{\rm eff}$

Example I: general scan: Prediction for M_W in the SM and the MSSM :



MSSM uncertainty:

unknown masses of SUSY particles

SM uncertainty:

unknown Higgs mass

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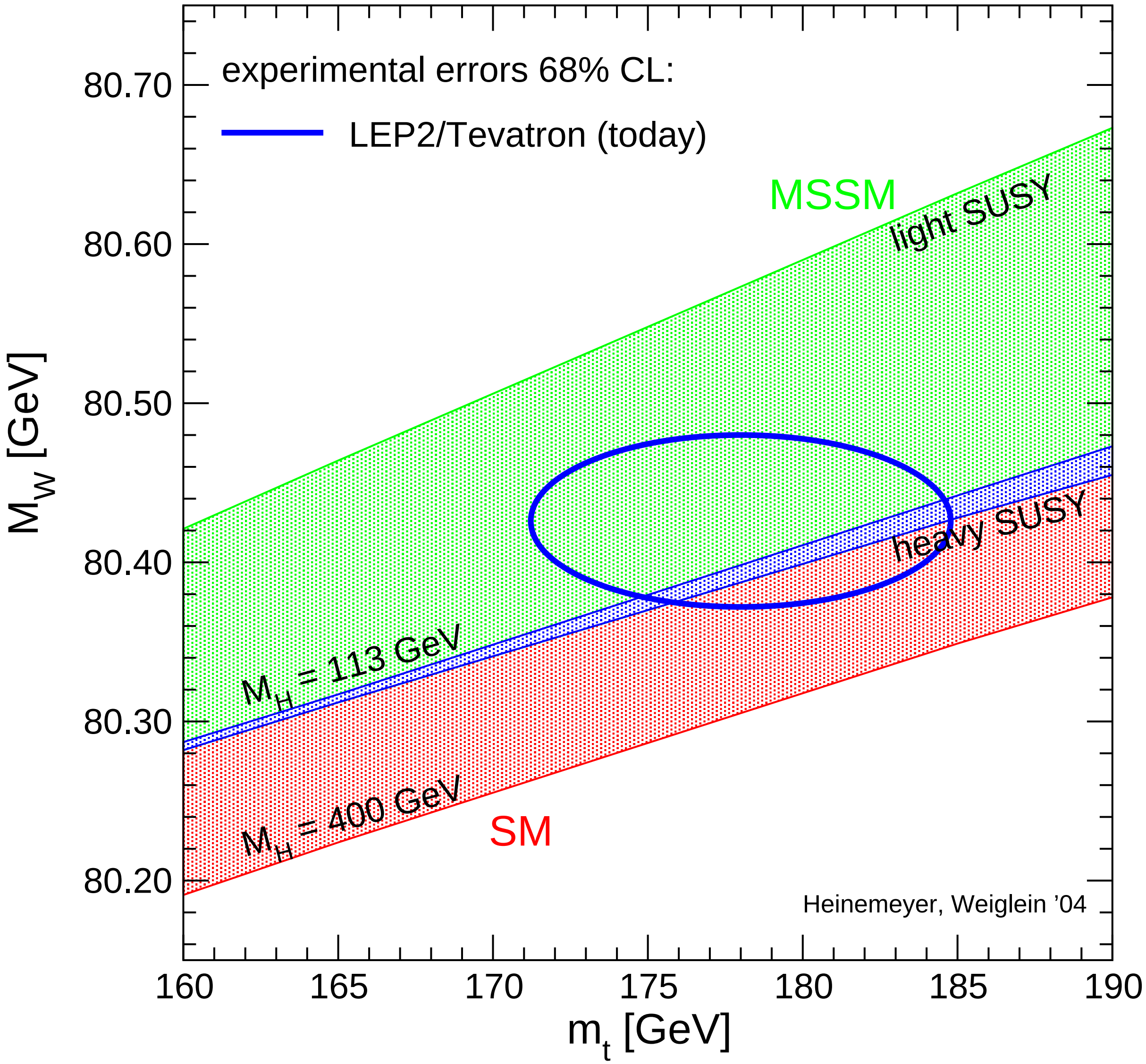

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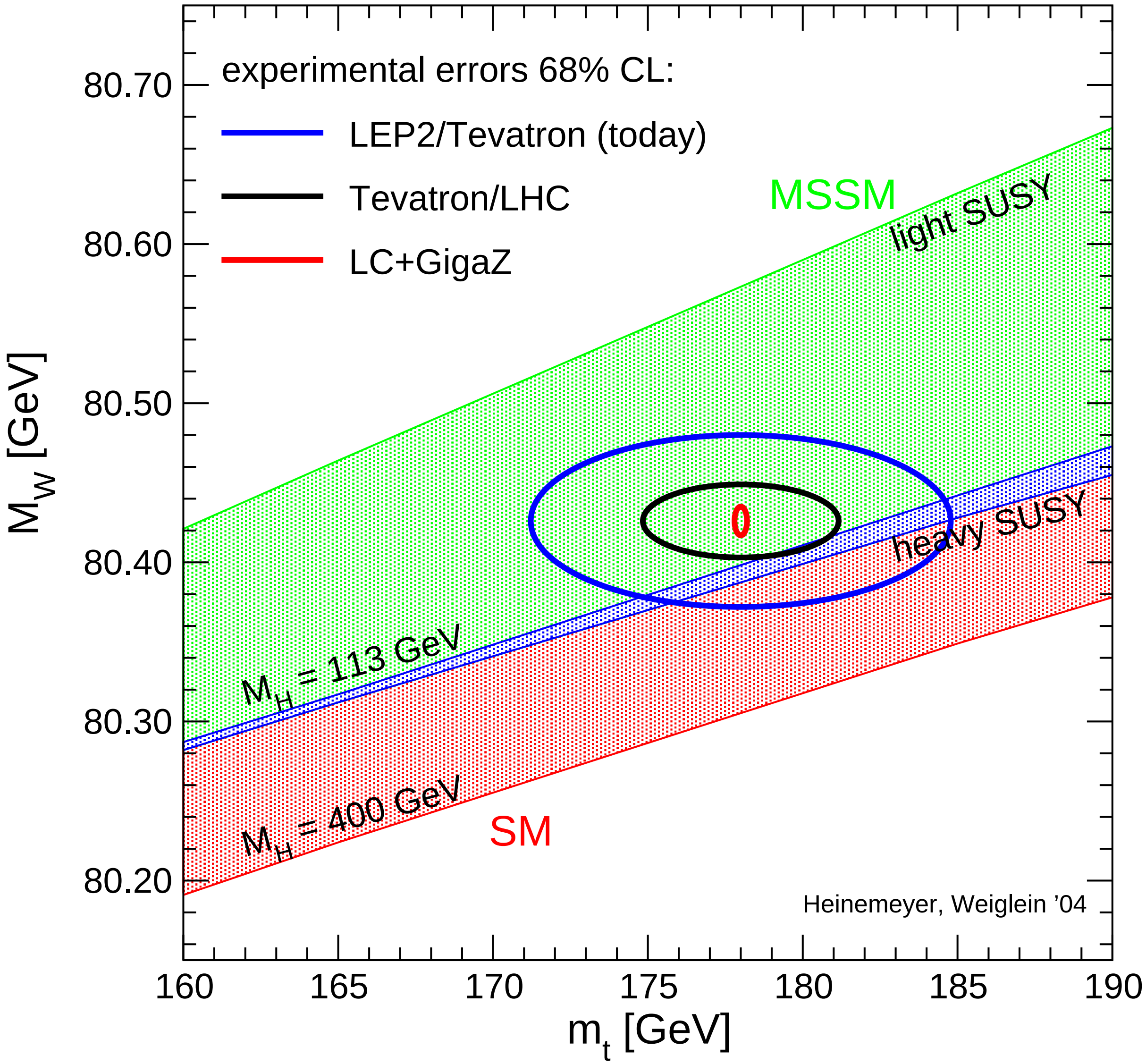


MSSM uncertainty:
unknown masses
of SUSY particles

SM uncertainty:
unknown Higgs mass

Example II: possible future scenario:

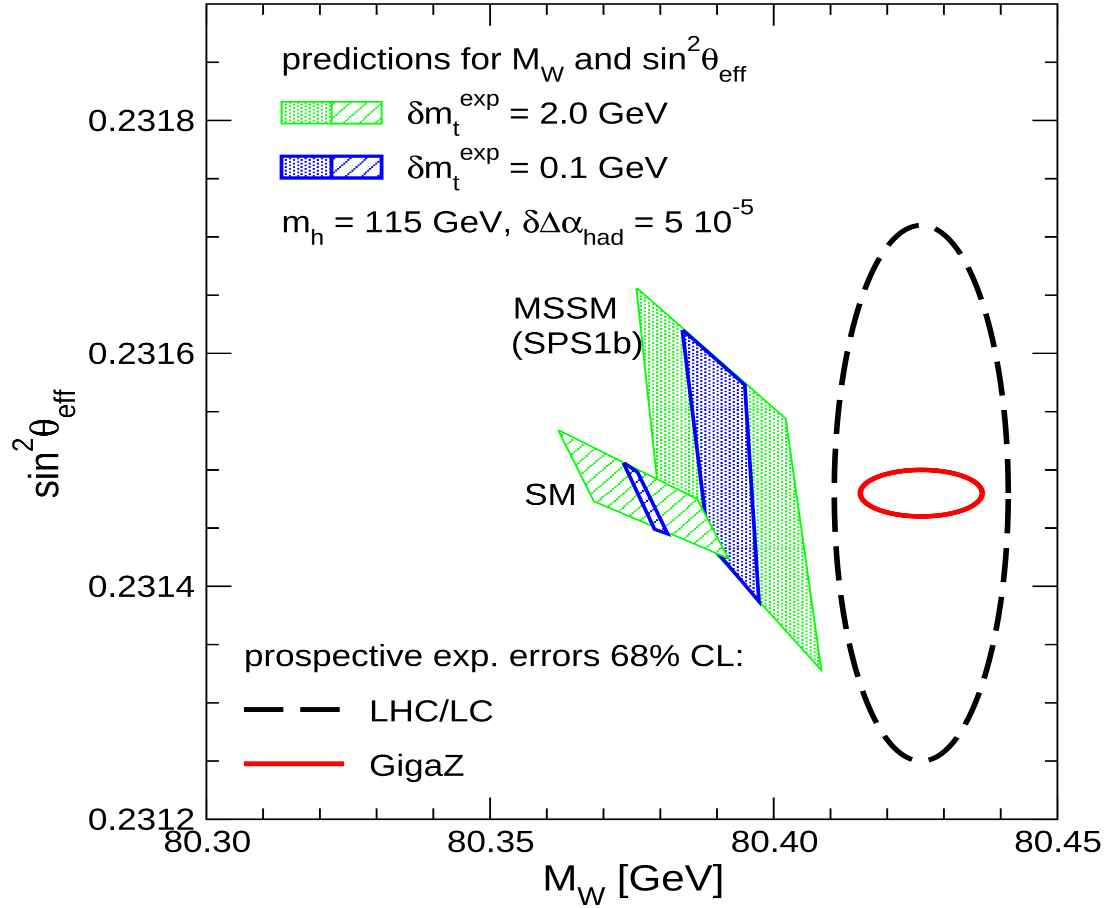
[*S.H., S. Kraml, W. Porod, G. Weiglein '03*]

SM: $M_H = 115$ GeV

MSSM: SPS 1b

all SUSY parameters varied

within realistic errors



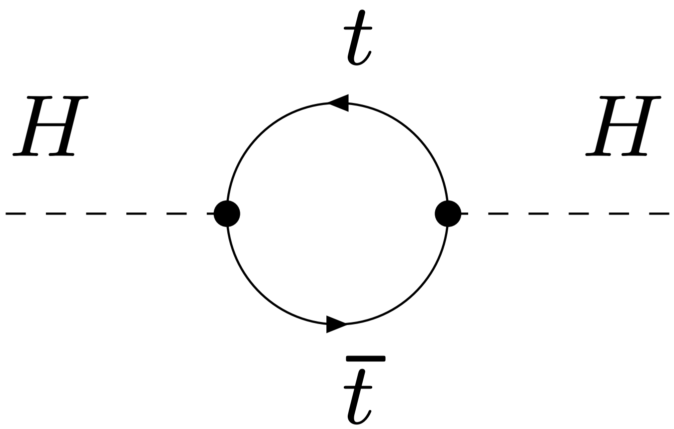
$\delta m_t = 0.1$ GeV vs. $\delta m_t = 2$ GeV

$\Rightarrow$ SM: improvement by a factor ~ 10

$\Rightarrow$ MSSM: improvement by a factor $\sim 2 - 3$

3. Higgs Physics in the MSSM

Nearly any model: large coupling of the Higgs to the top quark:



$\Rightarrow$ one-loop corrections $\Delta m_h^2 \sim G_\mu m_t^4$

$\Rightarrow$ M_H depends sensitively on m_t in all models where M_H can be predicted (SM: M_H is free parameter)

SUSY as an example: $\Delta m_t \approx \pm 4$ GeV $\Rightarrow$ $\Delta m_h \approx \pm 4$ GeV

$\Rightarrow$ Precision Higgs physics needs precision top physics (ILC: $\Delta m_t \lesssim 0.1$ GeV)

MSSM Higgs sector:

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + \frac{\phi_1 + i\chi_1}{\sqrt{2}} \\ \phi_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{\phi_2 + i\chi_2}{\sqrt{2}} \end{pmatrix}$$

$$\begin{aligned} V &= m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2(\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\ &+ \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2 \end{aligned}$$

gauge couplings, in contrast to SM $\Rightarrow m_h^{\text{tree}} \leq M_Z$

physical states: $h^0, H^0, A^0, H^\pm$ Goldstone bosons: $G^0, G^\pm$

Input parameters: $\tan\beta = \frac{v_2}{v_1}$, $M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$

Large radiative corrections (Yukawa sector, . . .):

Yukawa couplings: $\frac{e\, m_t}{2 M_W s_W}$, $\frac{e\, m_t^2}{M_W s_W}$, . . .

$\Rightarrow$ Dominant one-loop corrections: $G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$, $\mathcal{O}(100\%)$!

$\tilde{t}$ sector of the MSSM: (scalar partner of the top quark)

Mass matrix for $\tilde{t}_L, \tilde{t}_R$:

$$(\tilde{t}_L, \tilde{t}_R) \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{1t} & m_t X_t \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{2t} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

$$\Bigg\Downarrow \quad \leftarrow \text{Diagonalization}, \theta_{\tilde{t}}$$

$$(\tilde{t}_1, \tilde{t}_2) \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix} \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}$$

$X_t = A_t - \mu \cot\beta$; large mixing possible

$\Rightarrow$ Physical parameters: $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$

$\Rightarrow$ Soft SUSY-breaking parameters: $M_{\tilde{t}_L}$, $M_{\tilde{t}_R}$, A_t

$\Rightarrow$ Soft SUSY-breaking parameters determine SUSY mass patterns

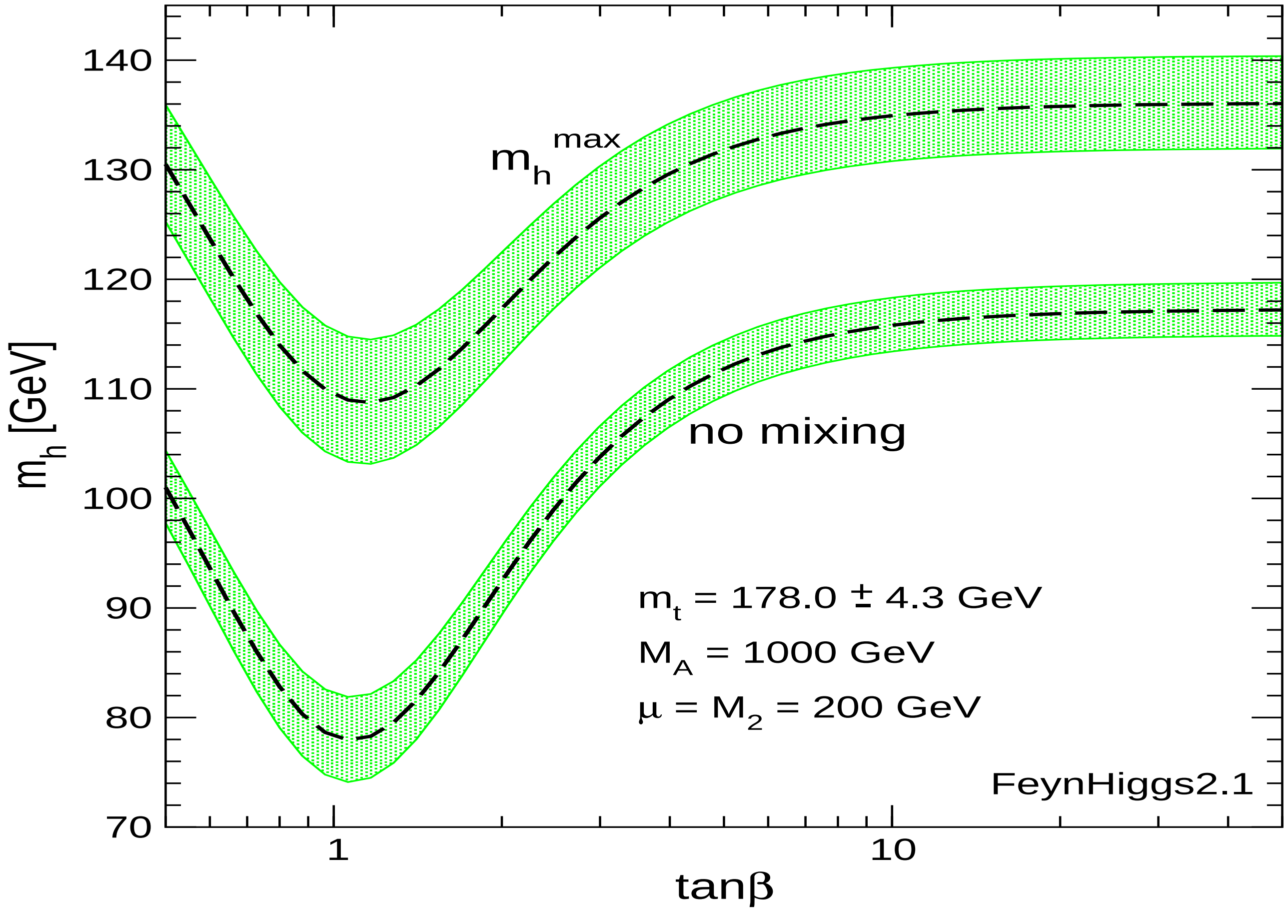
Theory uncertainties in m_h prediction:

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weigelein '02*]

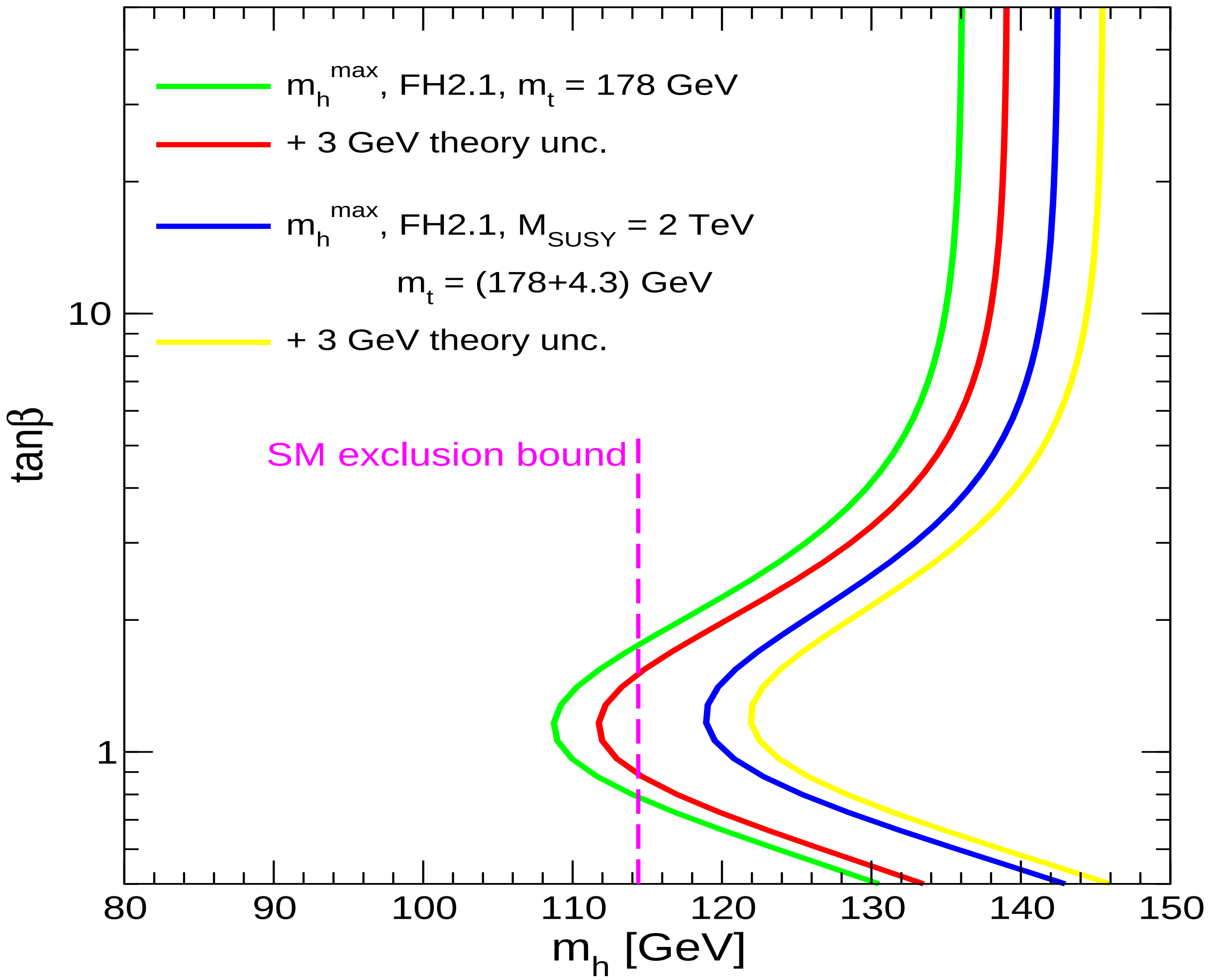
– From unknown higher order corrections $\Rightarrow \Delta m_h \approx 3$ GeV
(if $b/\tilde{b}$ sector is under control [*S.H., W. Hollik, H. Rzehak, G. Weiglein '04*])

– From uncertainties in input parameters $\Delta m_t \approx \pm 4$ GeV
$\Rightarrow \Delta m_h \approx \pm 4$ GeV

Upper bound:
$m_h \lesssim 140$ GeV



Effect on $\tan\beta$ exclusion bounds



m_h^{max} scenario, *FeynHiggs*2.1,
$m_t = 178$ GeV

m_h^{max} scenario, *FeynHiggs*2.1,
3 GeV theory unc.

$\Rightarrow$ $\tan\beta$ bound considerably weakened by theory unc.

Exp. error in m_t:

m_h^{max} scenario with
$m_t = 182.3$, $M_{\text{SUSY}} = 2$ TeV

m_h^{max} scenario with
$m_t = 182.3$, $M_{\text{SUSY}} = 2$ TeV
$+$ 3 GeV theory unc.

$\Rightarrow$ no $\tan\beta$ exclusion bound

$\Rightarrow$ considerable improvement needed in both: δm_t^{exp} and δm_h^{theo}

Future experimental error vs. parametric/intrinsic uncertainties:

- LHC: $\delta m_h^{\text{exp}} \approx 0.2$ GeV (if $\gamma\gamma$ decay mode or $ZZ \to 4l$ mode accessible)

  LC: $\delta m_h^{\text{exp}} \approx 0.05$ GeV

  $+$ many more precision measurements in the Higgs sector

- Parametric uncertainty:

  $\Delta m_t \approx 2$ GeV $\Rightarrow$ $\Delta m_h \approx 2$ GeV

  $\Delta m_t \approx 0.1$ GeV $\Rightarrow$ $\Delta m_h \approx 0.1$ GeV

- All other future parametric uncertainties: $\Delta m_h \lesssim 0.2$ GeV

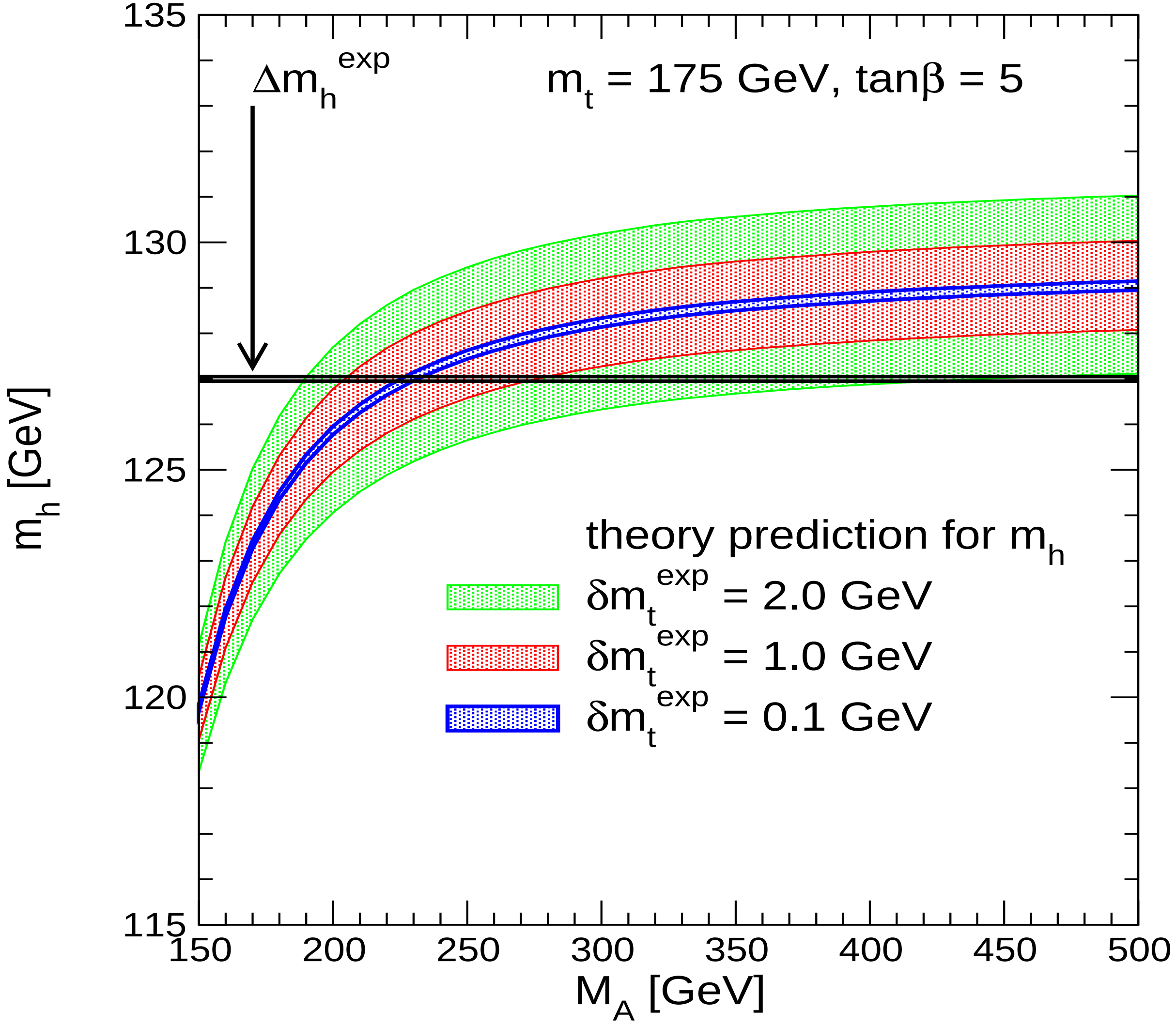
$\Rightarrow$ Need LC precision on m_t in order to exploit LHC (and LC) precision on Higgs sector measurements

Intrinsic uncertainties: $\Delta m_h \approx 3$ GeV

$\Rightarrow$ need to improve uncertainty from unknown higher-order corrections by more than factor 10!

Example: parametric uncertainty vs. experimental error:

[*S.H., S. Kraml, W. Porod, G. Weiglein '03*]

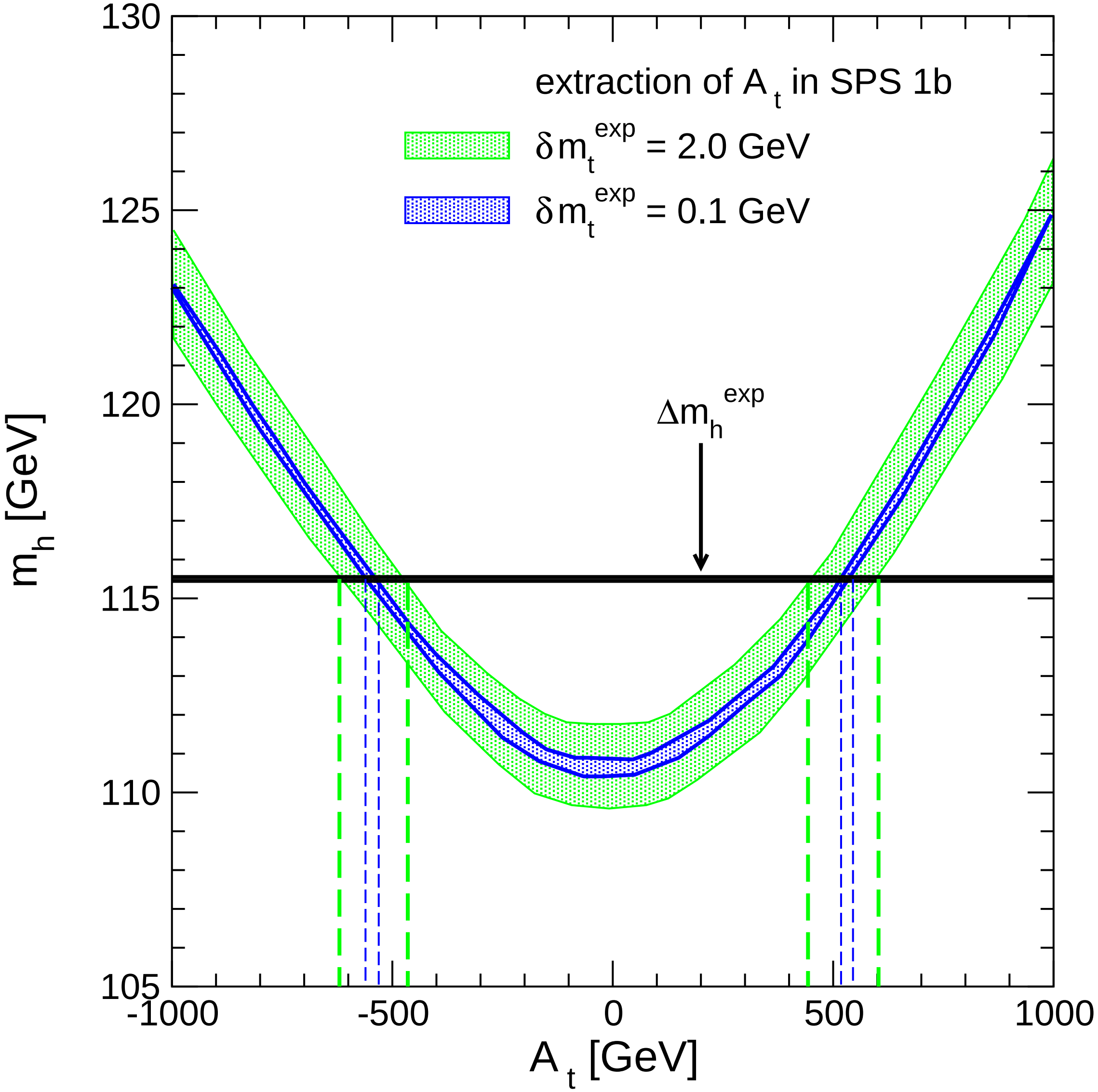


$m_h^{\max}$ benchmark scenario (no experimental errors on SUSY parameters)

$\Rightarrow$ LC precision on m_t needed to match experimental accuracy

Example: determination of $\tilde{t}$ mixing parameter A_t:

[S.H., S. Kraml, W. Porod, G. Weiglein '03]



SPS 1b benchmark scenario

Assume to be known:

$m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$, $\tan\beta$, M_A

Realistic estimate of parametric error!

$\Rightarrow$ Indirect determination of A_t from m_h measurement

(crucial for SUSY fit programs!)

LC accuracy on m_t

Improvement by $\sim 3-4$

However: Theory uncertainty has to be under control!

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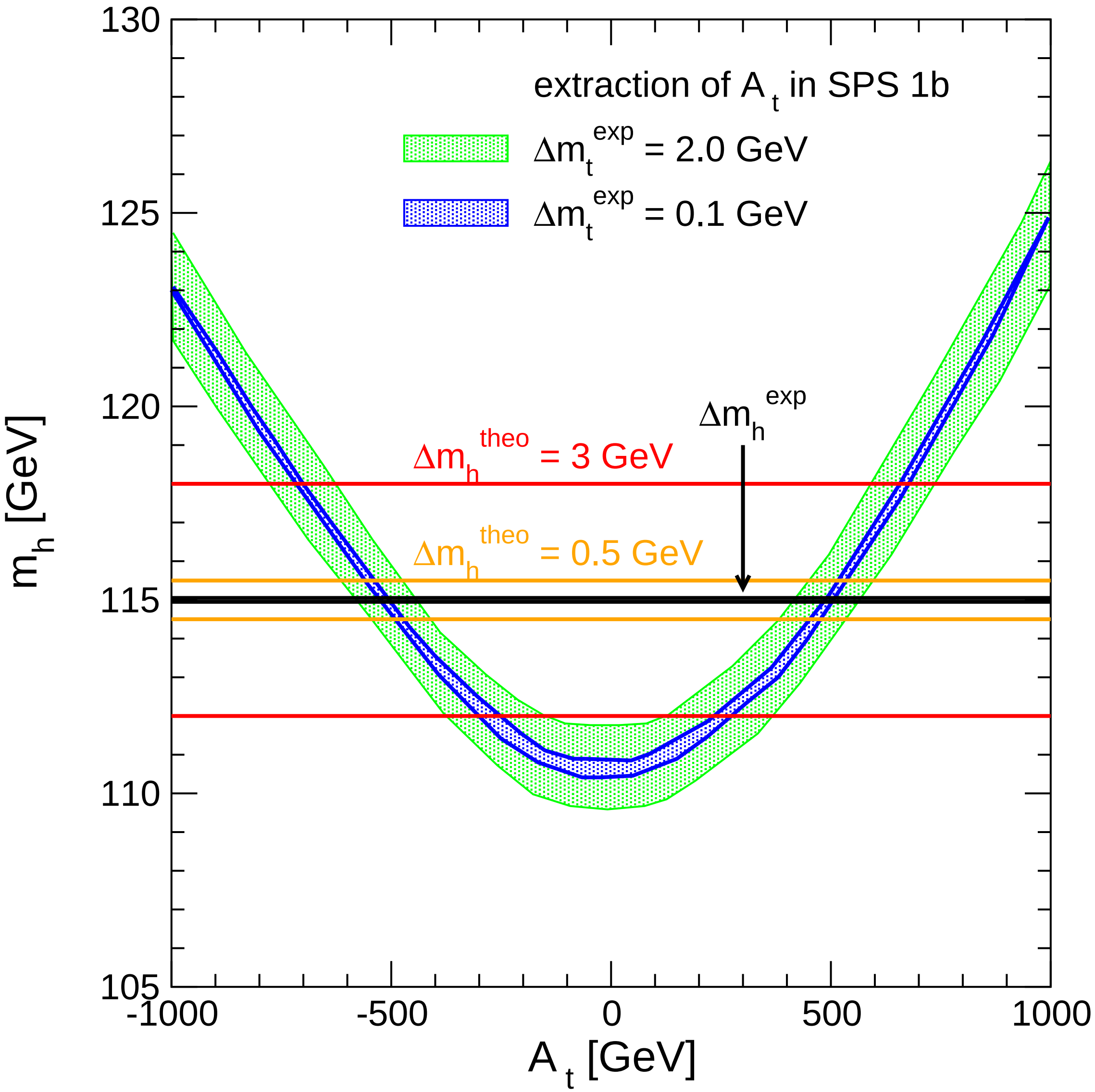


SPS 1b benchmark scenario

Assume to be known:

$m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$, $\tan\beta$, M_A

Realistic estimate of parametric error!

$\Rightarrow$ Indirect determination of A_t from m_h measurement

(crucial for SUSY fit programs!)

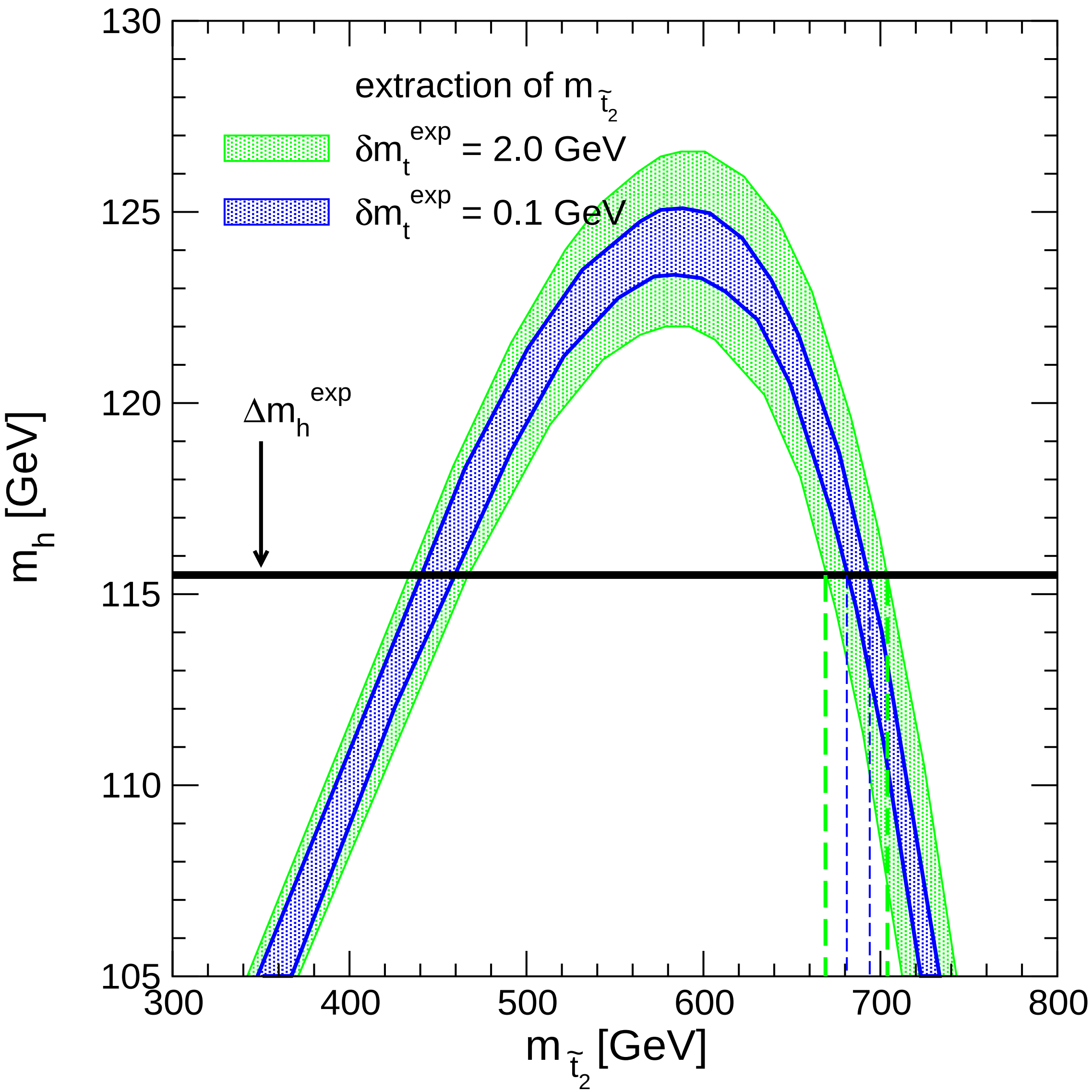
LC accuracy on m_t

Improvement by $\sim 3 - 4$

However: Theory uncertainty has to be under control!

Example: determination heavier $\tilde{t}$ mass $m_{\tilde{t}_2}$:

[S.H., S. Kraml, W. Porod, G. Weiglein '03]



scenario with light $\tilde{t}_1$:

$m_{\tilde{t}_1} \approx 180$ GeV

Assume to be known:

$m_{\tilde{t}_1}$, $\theta_{\tilde{t}}$, $m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$, M_A, $\tan\beta > 10$

Realistic estimate of parametric error!

$\Rightarrow$ Indirect determination of $m_{\tilde{t}_2}$ from m_h measurement

LC accuracy on m_t

Improvement by $\sim 2-3$

Again: Theory uncertainty has to be under control!

4. Renormalization group running and Dark Matter

$\rightarrow$ focus on mSUGRA/CMSSM

mSUGRA/CMSSM:

$$\{m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ \mathrm{sign}(\mu)\}$$

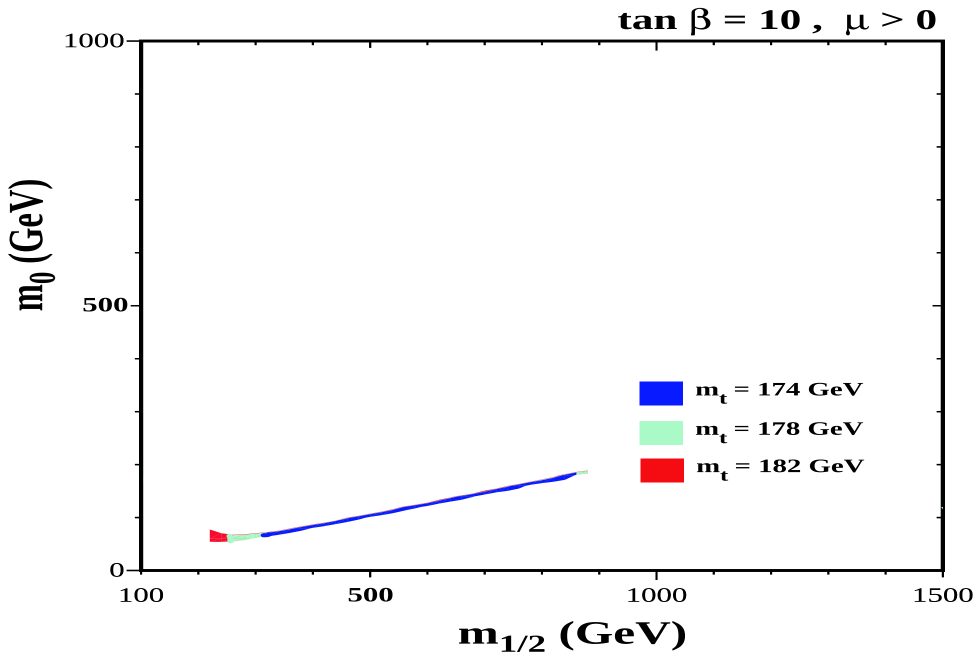
m_0 : common scalar mass at the GUT scale
$m_{1/2}$: common gaugino mass at the GUT scale
A_0 : common trilinear coupling at the GUT scale
$\tan\beta$: low energy value
$\mathrm{sign}(\mu)$: low energy value

$\Rightarrow$ prediction of all SUSY parameters in terms of $4\frac{1}{2}$ input parameters

RGEs connect GUT scale parameters with low-energy parameters

Prediction of Cold Dark Matter:

[*J. Ellis, S.H., K. Olive, G. Weiglein '04*]



CDM prediction in terms of $m_{1/2}$, m_0 and m_t

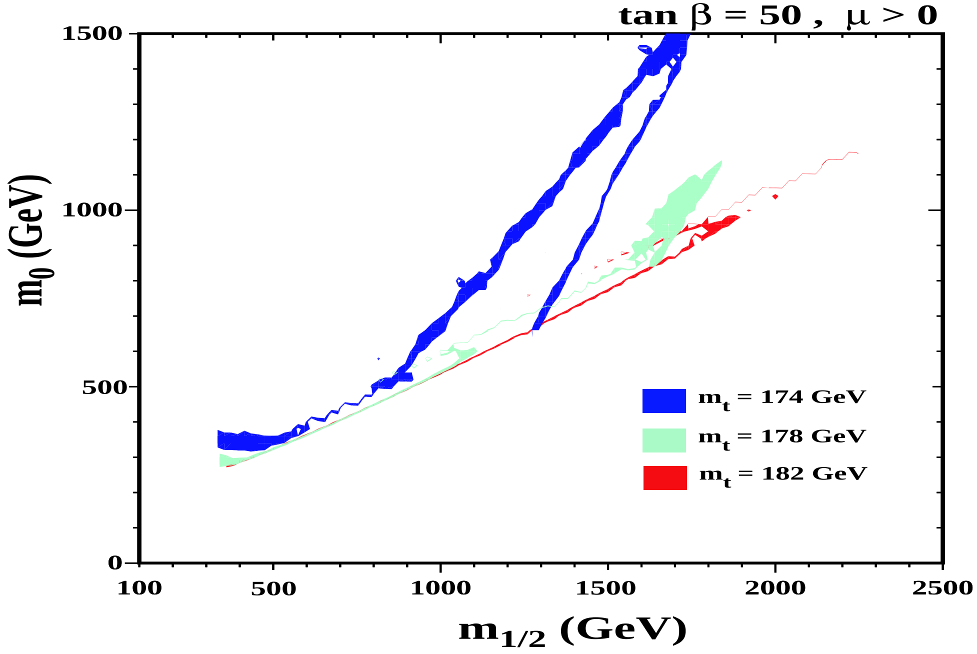
$A_0 = 0$, $\tan\beta = 10, 50$ fixed

$m_t = 178.0 \pm 4.3$ GeV

Shown: parameters allowed by $0.094 \leq \Omega_\chi h^2 \leq 0.129$ [*WMAP '03*]

$\Rightarrow$ strong variation for large $\tan\beta$

$\Rightarrow$ precise m_t value needed for precise CDM predictions!



Test of unification:

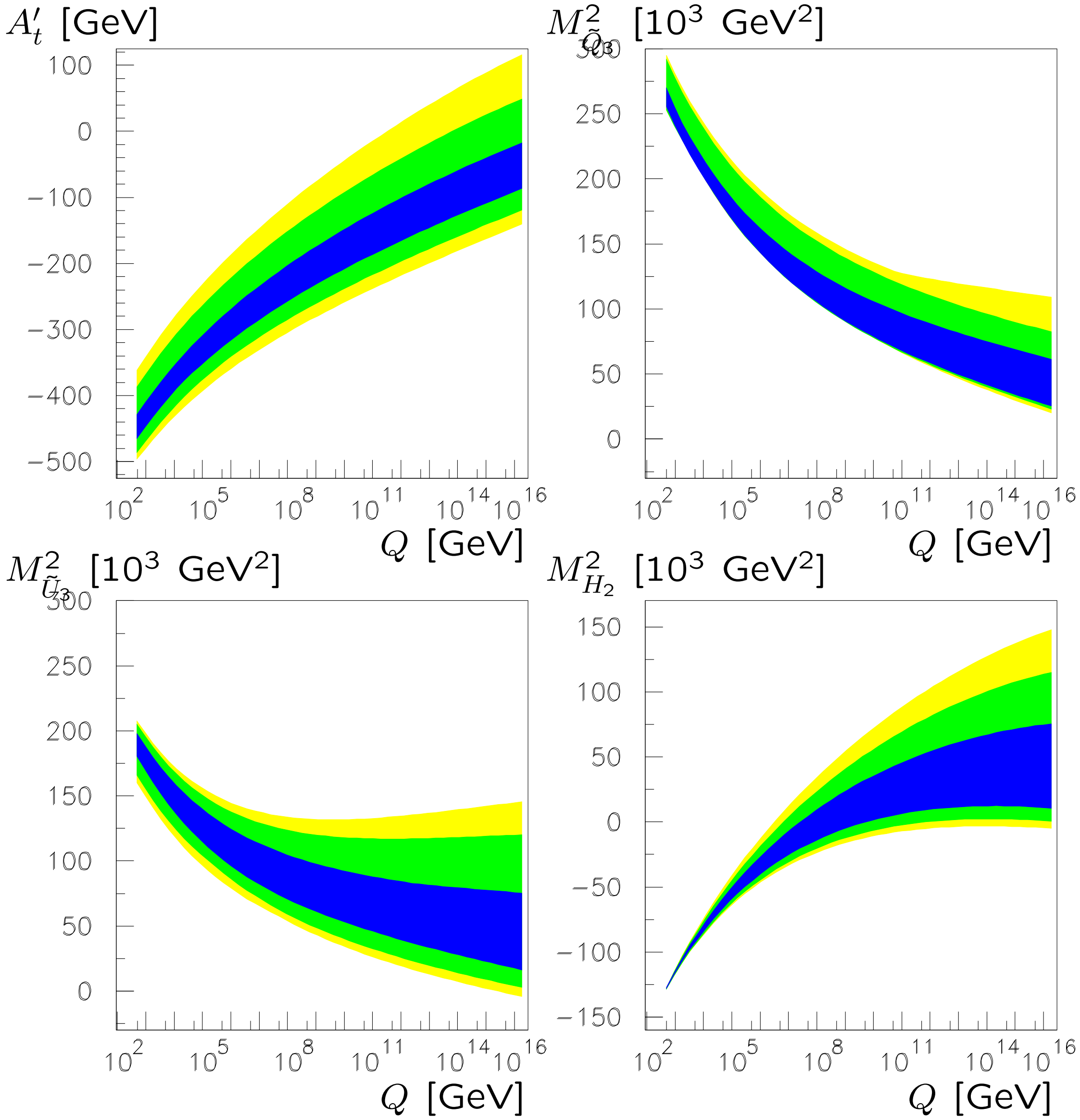
[S.H., S. Kraml, W. Porod, G. Weiglein '03]

Low-energy measurement of SUSY parameters
$\Rightarrow$ extrapolation to GUT scale

m_t enters RGEs
via Yukawa couplings,
stop masses, loop effects

$\delta m_t = 0.1$ GeV
$\delta m_t = 1.0$ GeV
$\delta m_t = 2.0$ GeV

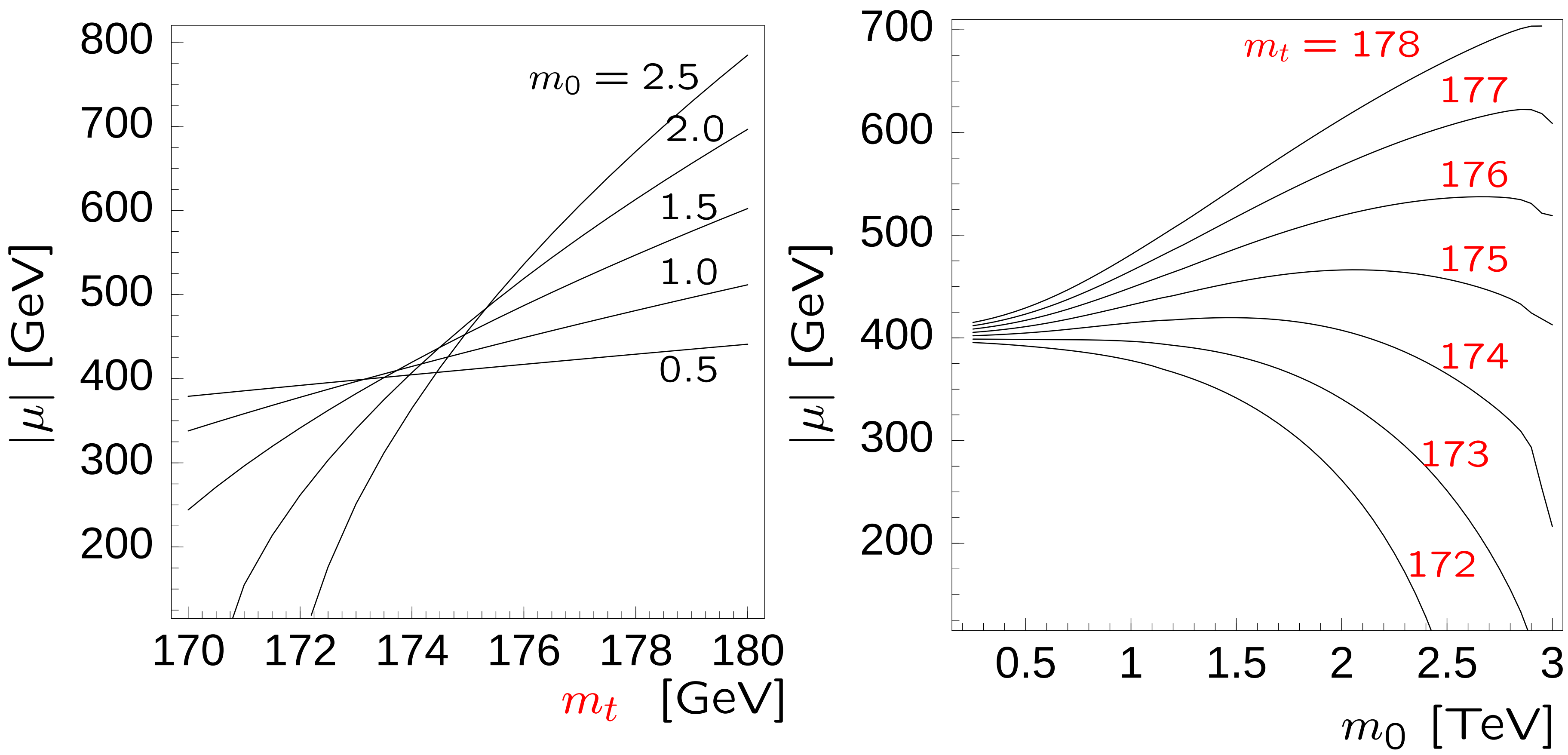
$\Rightarrow$ large improvement in precision of GUT scale parameters



Prediction of neutralino/chargino masses:

[*S.H., S. Kraml, W. Porod, G. Weiglein '03*]

Start with GUT parameters: $m_{1/2}$, m_0, A_0, $\tan\beta = 10$
derive low-energy parameters: μ, . . .



$\Rightarrow$ Precise knowledge of m_t crucial (for large m_0)

5. Conclusinos

- Precise knowledge of m_t is crucial for electroweak physics:

  precision tests, constraints on the Higgs sector,
  sensitivity to new physics, . . .

- Comparison of $\delta m_t = 0.1$ GeV (LC) with $\delta m_t = 1$–2 GeV (LHC)

- LC precision mandatory for exploiting EWPO, SUSY Higgs physics, RG running to high scales, CDM constraints, . . . :

- δm_t^{LHC} is dominant parametric error

  (examples discussed here are not an exclusive list)

  $\Rightarrow$ With LC precision: typical improvements by factors 2–10 (SM)
  factors 2–3 (MSSM)

  Even larger sensitivity to m_t in extreme parameter regions
  (e.g. SUSY focus point scenario)