Updated Constraints on the Minimal Supergravity Model and Prospects for Sparticle Production at the ILC

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based on collaboration with A. Djouadi, M. Drees and P. Slavich

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1. Introduction, Motivations

2. Reminder of MSSM and minimal SUGRA features

3. (Updated) Theoretical and Experimental Constraints

-Consistent radiative electroweak symmetry breaking (ESWB)

-More refined Higgs mass calculation
(include dominant two-loop contributions)

-LEP Higgs and sparticle limits

Updated top mass range; + uncertainties effects

-$b \to s\gamma$ (updated), + $b \to s l^+ l^-$

-$g_\mu - 2$ (updated)

-Dark matter constraints (neutralino relic density) (WMAP,..)

4. Scanning of mSUGRA parameter space:

deriving lower bounds on Sparticle masses in mSUGRA

5. Summary

1. Introduction, Motivations

mSUGRA most widely studied of MSSM, with virtues:

-only a handful of parameters;

-stable gauge hierarchy (for $m_{sparticles}$ not $\gg$ 1 TeV);

-radiative EWSB, not much FCNC problems, etc;

-plausible Dark Matter candidate;

Yet, recently a perception that mSUGRA parameter space got "squeezed" (specially after WMAP)

However, the fact that it can accommodate WMAP is a further success..

Anyway, time to re-assess mSUGRA, taking into account recent Th developments and updated Exp constraints.

NB Our "scans" of parameter space results qualitatively consistent with other similar analysis,
(see e.g. G. Weiglein, H. Baer,..talks in this LCWS05)

but we put emphasize on "minimal allowed sparticle masses"

from various set of constraints.

2. Minimal Supersymmetric Standard Model/SUGRA reminder

(Super)field content: vector: G^a, W^a, B

chiral: L, Q, E^c, U^c, D^c, H_d, H_u

MSSM Superpotential ($SU(3) \times SU(2) \times U(1)$ +R-parity):

$$W = \mu H_d H_u + Y_E H_d L E^c + Y_d H_d Q D^c + Y_u H_u Q U^c$$

• Supersymmetry must be broken, without reintroducing quadratic divergences = Soft SUSY-breaking Lagrangian:

$$-\mathcal{L}_{soft} = \frac{1}{2} \sum_A M_A \bar{\lambda}_A \lambda_A + \sum_i m_i^2 |\phi_i|^2$$
$$+ B H_d H_u + Y_E A_E H_d L E^c + Y_d A_d H_d Q D^c + Y_u A_u H_u Q U^c$$

m_i^2, A_E, A_u, A_d are 3×3 matrices in generation space

→ the general MSSM contains 105 parameters...

Instead, minimal SUGRA proposes simplifying assumptions: universal high energy (GUT) boundary conditions:

$$m_i^2(Q_{GUT}) = m_0^2$$
$$M_A(Q_{GUT}) = m_{1/2}$$
$$A_i(Q_{GUT}) = A_0$$

Main steps for MSSM model spectrum calculation:

here performed by SuSpect 2.3 (Djouadi, JLK, Moultaka)

- Defining model at High scale (boundary condition)
  e.g. in mSUGRA: m_0, $m_{1/2}$, A_0

  (alternatively, can deal with many models of SUSY-breaking: GMSB, AMSB, general MSSM,..(not considered here))

- Renorm. Group Evolution (RGE) from Q_{GUT} to Q_{EWSB}

- Electroweak Symmetry Breaking (EWSB):
  determines consistently μ, B, and Higgs masses

- Electroweak scale ($Q = m_Z$) boundary conditions
  fix $g_1, g_2, g_3, Y^E, Y^D, Y^U$; m_Z, $\tan\beta$

- Physical spectrum calculation:
  from the RG parameters (defined in the $\overline{DR}$ scheme)
  to physical masses (including radiative corrections (R.C.))

NB Many improvements in R.C. implementation in the past year in most public spectrum calculation codes (see below)

3. (Updated) Theoretical and Experimental Constraints

Th. constraint: consistent EWSB

Electroweak symmetry breaking (EWSB) triggered by the soft breaking terms $m^2_{H_u}$, $m^2_{H_d}$
Consistent EWSB in MSSM: typically $m^2_{H_u}(Q)$ driven < 0 by RG evolution $\propto Y_t^2$ from Q_{GUT} to Q_{EWSB}

with $|\mu|$ determined by minimization of scalar potential:

$$2\mu^2 = \tan(2\beta)(\hat{m}^2_{H_u} \tan\beta - \hat{m}^2_{H_d} cot\beta - M_Z^2)$$
$$2B\mu = \sin 2\beta(\hat{m}^2_{H_u} + \hat{m}^2_{H_d} + 2\mu^2)$$

where $\hat{m}^2_{H_i} = m^2_{H_i} + \partial_{v_i} V^{eff}_{loop}$
and V^{eff}_{loop} depends on all sparticle masses, thus on μ

$\rightarrow$ not always consistent sol. for μ
Remember: μ very sensitive to Rad. Corr., m_t,.. via RGE:

e.g. $d\,(m^2_{H_u}) \propto Y_t^2\, d\,(\ln Q)$;

and $\mu \sim -m^2_{H_u} - m^2_Z/2$ for $\tan\beta \gg 1$

Moreover CCB (Charge and/or Color breaking) minima of V_{eff}, deeper than true EWSB minimum, may occur. (relevant for large A_0 in mSUGRA)

Recent detailed comparisons (e.g. S. Kraml et al '04; Allanach, Djouadi, JLK, Porod, Slavich '04) of latest versions of the public codes:

–SuSpect 2.3, (A. Djouadi, JLK, G. Moultaka)
–SoftSusy 1.8.7 (B. Allanach)
–SPheno 2.2.1 (W. Porod)
–Isajet/sugra 7.71 (H. Baer, F. Paige, S. Protopescu, X. Tata)

• in particular, the first three (latest version) codes include a two-loop computation of Higgs masses and EWSB conditions performed in the $\overline{DR}$ renormalization scheme

• The full one-loop corrections are taken from

Pierce-Bagger-Matchev- Zhang (PBMZ) 1996

• The leading two-loop corrections (in the limit of zero external momentum) in the self-energies are taken from

Brignole-Dedes-Degrassi-Slavich-Zwirner (BDDSZ) 2001–03

• Note: scale and scheme (comparing with OS scheme FeynHiggs results) dependences measure higher order theoretical uncertainties.

Results for the Higgs masses

- Light CP-even Higgs boson mass m_h:

	SPS1a	SPS2	SPS4	SPS5	SPS8	SPS9
SoftSusy	112.1	116.8	114.1	116.3	115.4	117.4
SPheno	112.2	117.1	114.3	116.5	115.8	117.8
SuSpect	112.1	116.8	114.1	116.1	115.5	117.5

- Heavy CP-even Higgs boson mass m_H:

	SPS1a	SPS2	SPS4	SPS5	SPS8	SPS9
SoftSusy	406.5	1553.0	335.8	686.8	550.4	1056.9
SPheno	406.0	1554.6	360.5	686.5	552.4	1051.1
SuSpect	406.5	1552.1	355.3	686.9	550.6	1056.6

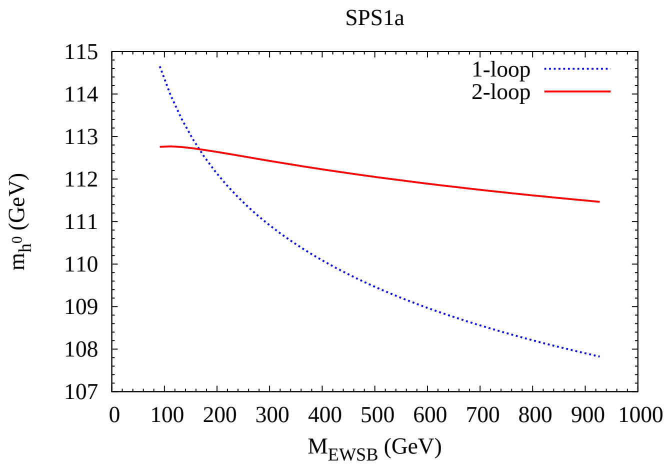
- CP–odd Higgs boson mass m_A:

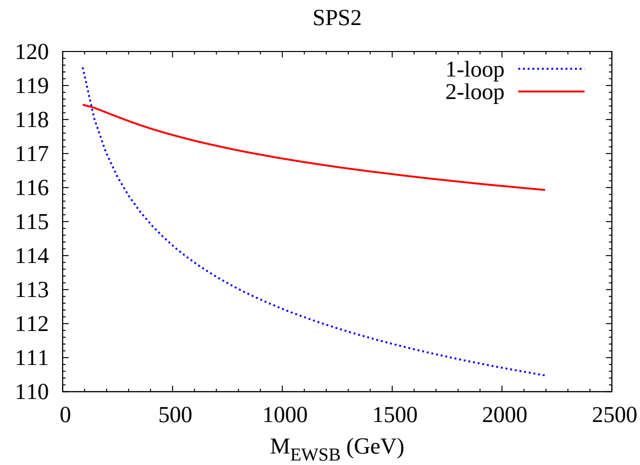
	SPS1a	SPS2	SPS4	SPS5	SPS8	SPS9
SoftSusy	406.2	1552.9	355.8	687.0	550.1	1056.8
SPheno	405.7	1554.5	360.5	686.9	552.1	1051.0
SuSpect	406.1	1552.0	355.3	687.2	550.3	1056.5

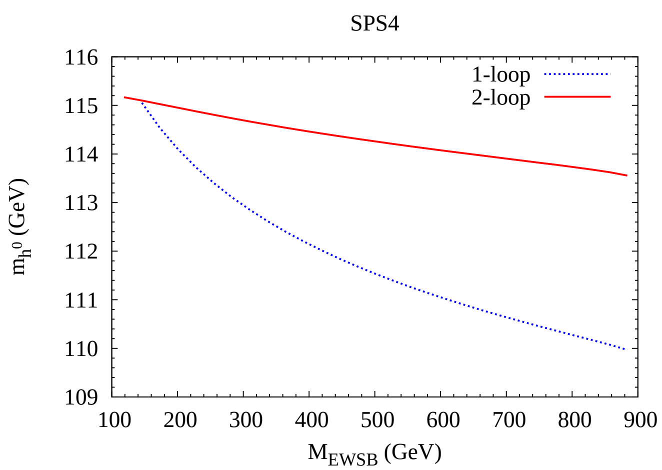
- Superpotential Higgs mass parameter μ:

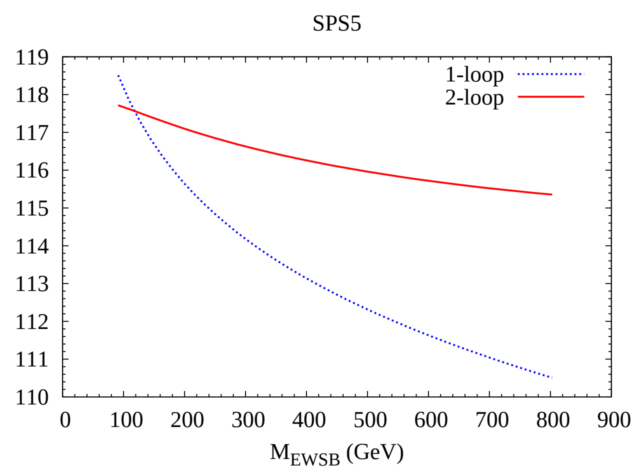
	SPS1a	SPS2	SPS4	SPS5	SPS8	SPS9
SoftSusy	364.8	586.5	413.8	631.2	440.1	1011.8
SPheno	364.3	588.2	414.7	631.2	442.2	1005.9
SuSpect	364.7	583.6	413.6	631.3	440.3	1011.1

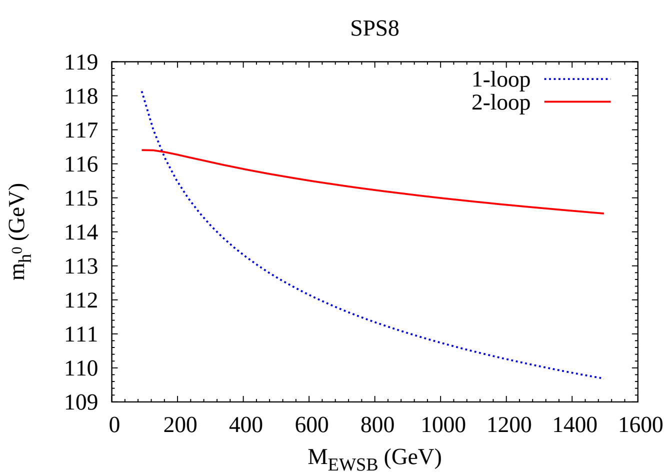
Renormalization scale dependence of m_h

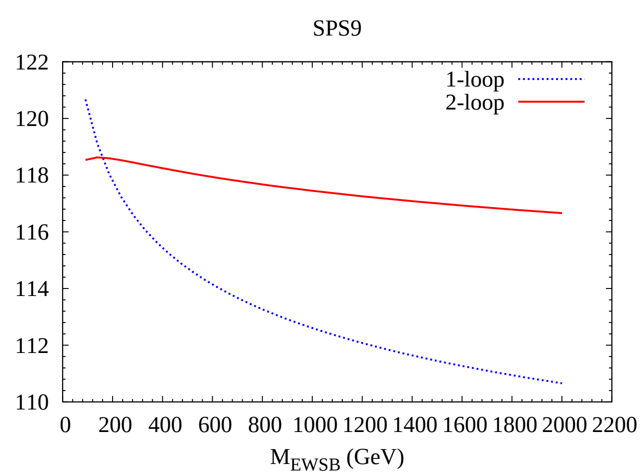












- LEP/Tevatron limits on sparticle masses

$m_{\chi_1^+} \gtrsim 104.5$ GeV
$m_{\tilde{\tau}^\pm} \gtrsim 98$ GeV ($\rightarrow m_{\tilde{g}}, m_{\tilde{q}\neq t,b} \gtrsim 250$ GeV in mSUGRA)
$m_{\tilde{t}_1,\tilde{b}_1} \gtrsim 101.5$ GeV

Higgs mass constraints

- $M_A \gg M_h \rightarrow h =$ SM-like, $\sin^2(\beta-\alpha) \sim 1$
$\rightarrow M_h \gtrsim 114.5$ GeV (LEP-combined)

-light A: $\sin^2(\beta-\alpha) \sim 0 \rightarrow M_{h,A} \gtrsim 90$ GeV $(e^+e^- \rightarrow hA)$

-in between: we fit for $\{\sin^2(\beta-\alpha), M_h\}$ cf. LEP exclusion plot

Electroweak precision data

potentially dangerous: largest R.C contributions to $\Delta\rho \equiv \rho - 1$: IF sparticles with large mass splittings.

$\rightarrow \Delta\rho(\tilde{b},\tilde{t}) + \Delta\rho(\tilde{\tau},\tilde{\nu}) < 2.2 \cdot 10^{-3}$

in fact, safe for most of mSUGRA parameter space
(i.e. small or superseded by other constraints)

SUSY contributions

Charginos+ sneutrino (dominant); Neutralinos + smuon

We take:

$-5.7 \cdot 10^{-10} < \Delta a_\mu^{SUSY} < 47 \cdot 10^{-10}$

(conservative, combined e^+e^-+ τ-decay data, 2 σ)

$10.6 \cdot 10^{-10} <$ $< 43.6 \cdot 10^{-10}$

(more "aggressive": e^+e^- data only)

STILL constraining for mSUGRA ($\mu < 0$ not favored)

• $b \to s\gamma$:

SM contributions (LO): $W^\pm$ and t essentially

SUSY contributions (LO): Charginos + stops; H^+ + top

+potentially large NLO SUSY contributions

IF enhanced by large $\tan\beta$ and/or large $\ln(M_{susy}/M_W)$

(Degrassi, Gambino, Guidice '2000)

NLO SUSY contribution from Gambino et al 2000

we take now: $2.65 \le 10^4 \cdot B.R.(b \to s\gamma) \le 4.45$

+ constraint on amplitude sign! ($\simeq$ constraints on BR($b \to l^+l^-$) (i.e. requires SM sign, cf. Gambino et al '04)

• Dark Matter relic density:

$0.087 < \Omega h^2 < 0.138$ conservative (99% C.L.) WMAP
$0.05 <$ < 0.15 (even more conservative, also shown)

IF LSP = χ_0: $\Omega_\chi h^2 \equiv$ relic density $\sim$

$[\sigma(\chi_0\chi_0 \to all) + co - annihilation\ processes]^{-1}$

$\to \sigma$ large $\to \Omega h^2 \ll .1$ too small;

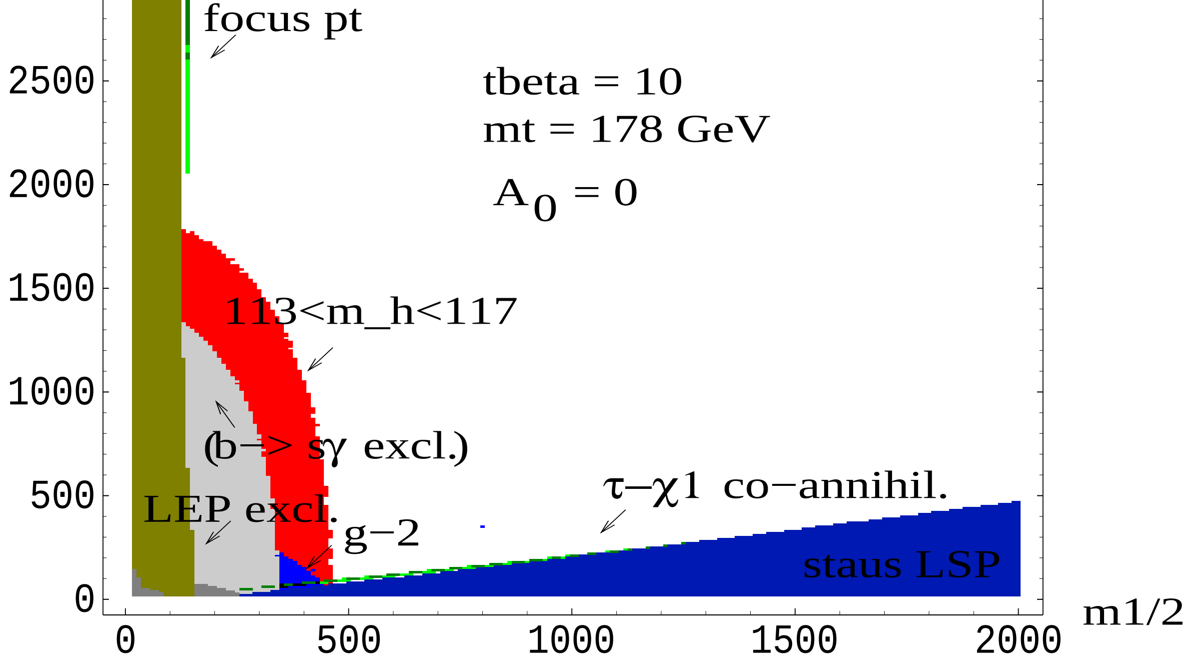
σ small $\to \Omega h^2$ too large

NB Ωh^2 calculation performed by "private" code (M. Drees)

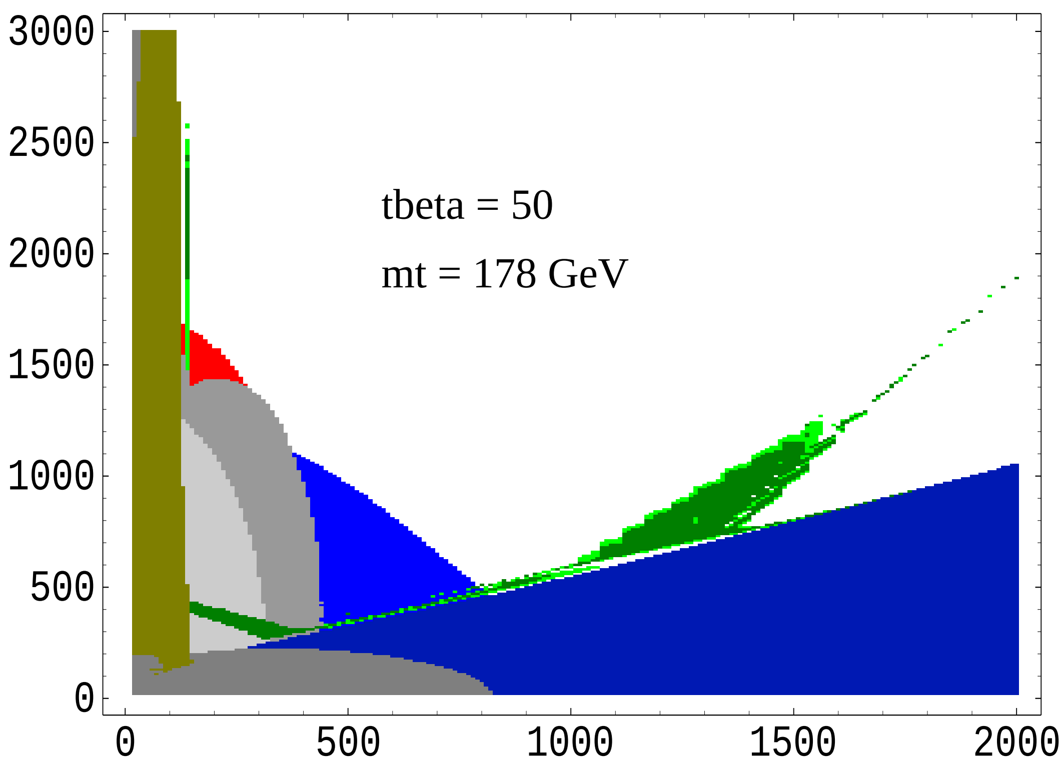
• LEP2 '2000 "evidence" for SM-like h with $M_h \simeq 115$ GeV
Nowadays only $\sim 1.7\sigma$..
Anyway, we indicate regions of mSUGRA /
113 GeV $< M_h <$ 117 GeV
(also : $111 < M_h < 119 \sim$ th. uncertainties)
and $\sin^2(\beta - \alpha) \geq 0.95$

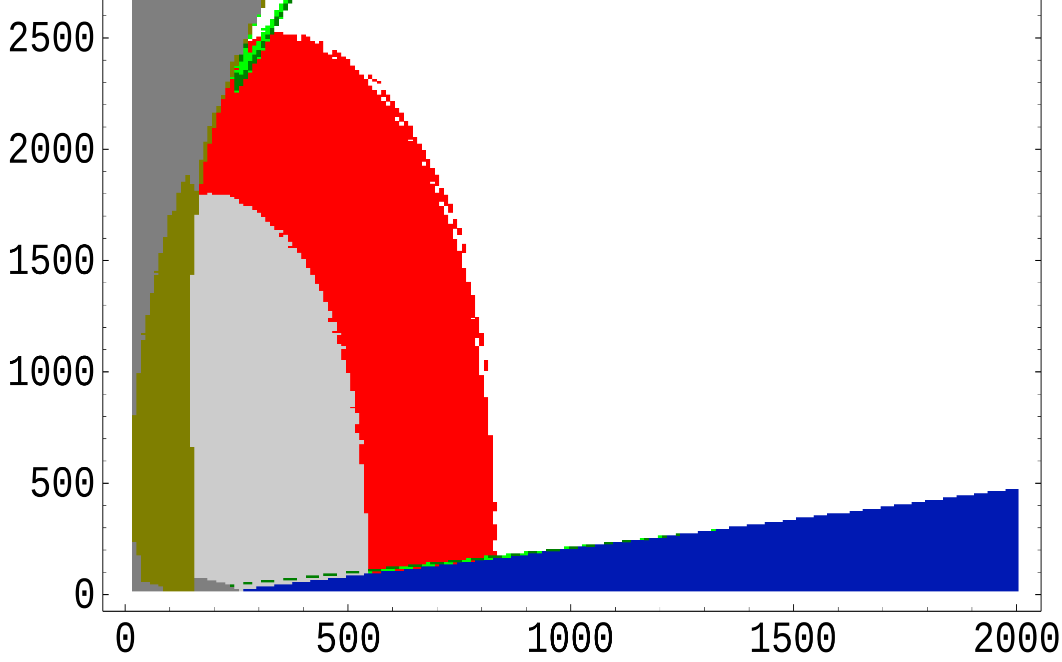
Finally, study effects of top mass uncertainties on all this:

we take 171 GeV $< m_{top} <$ 185 GeV (90 % C.L.)

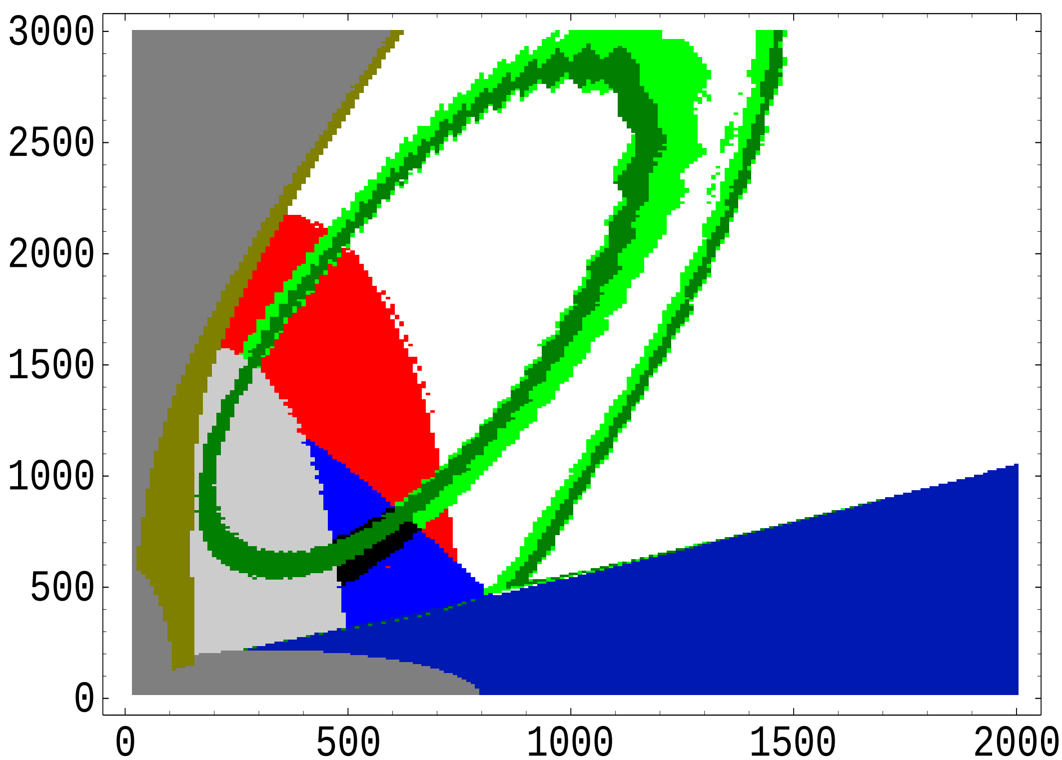


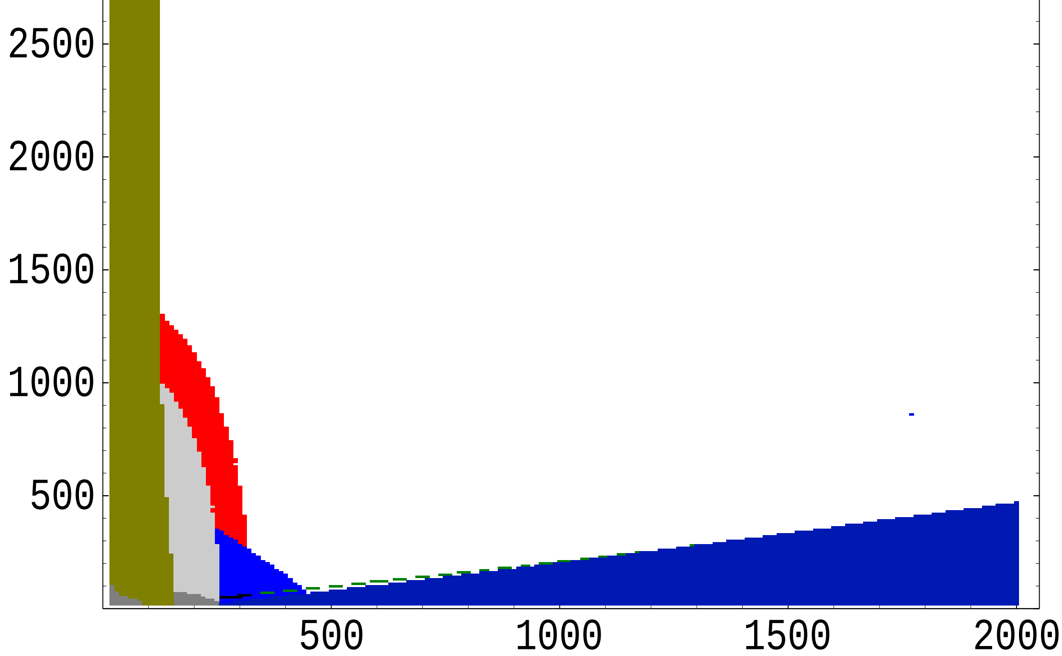
NB: $A_0 = 0$ for all these $(m_0, m_{1/2})$ plots.



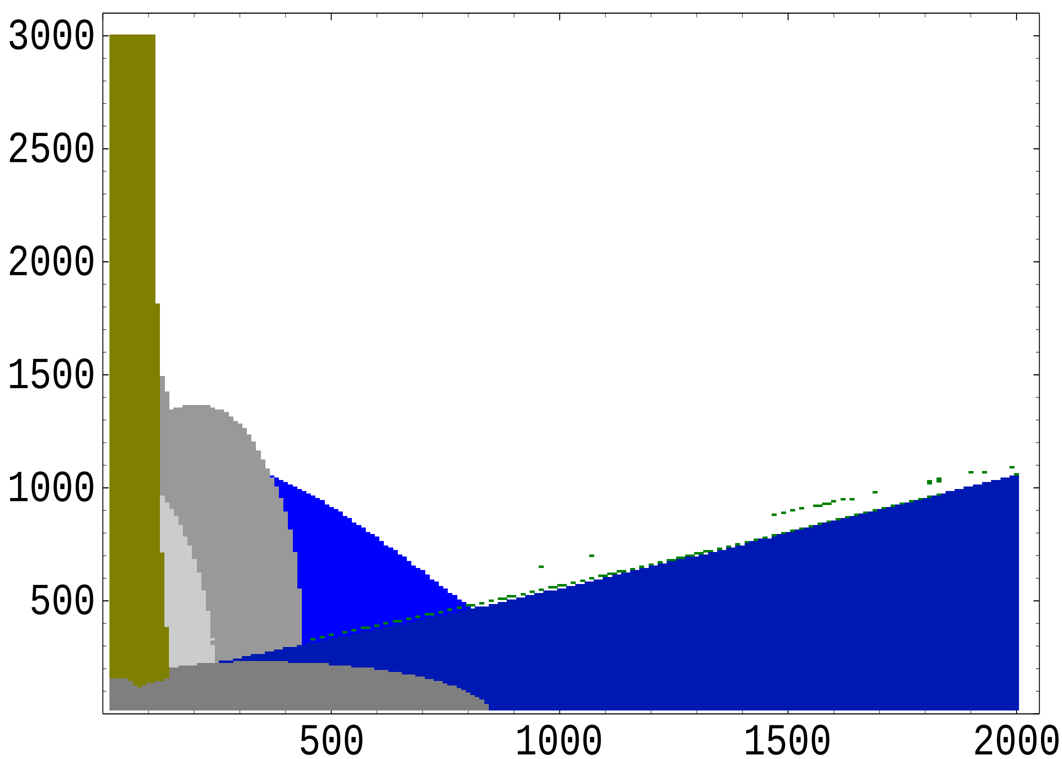


$m_t = 171$ GeV; $\tan\beta$ =10 (top fig.) 50 (bottom fig.)

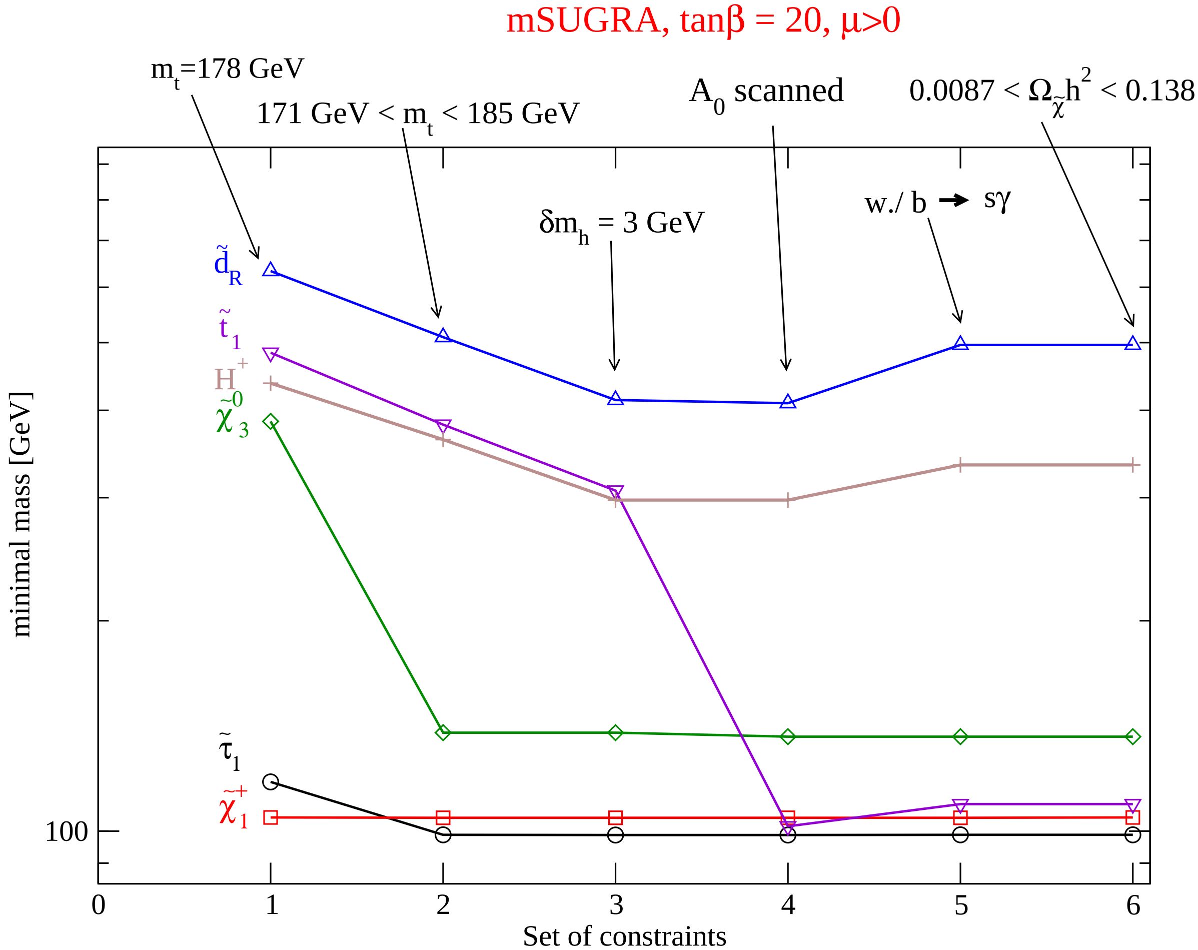




$m_t = 185$ GeV; $\tan\beta$ =10 (top fig.) 50 (bottom fig.)

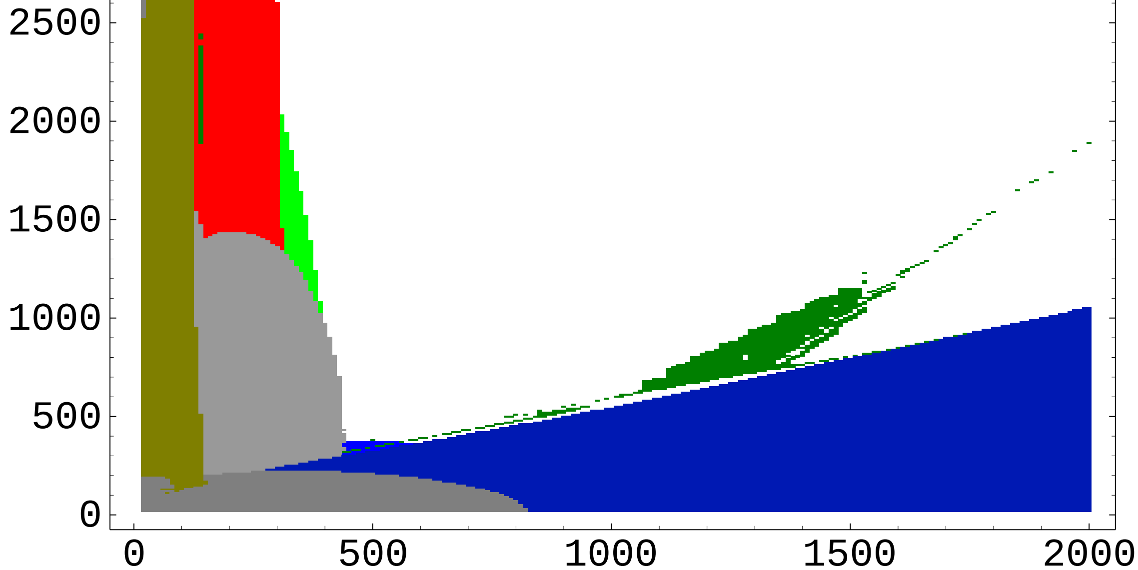


Illustrate the effects (on minimal allowed masses) of:
–scanning parameters,
–theoretical m_{top}, m_h uncertainties,
–switching on the various constraints.



$m_{\tilde{e}_R} = m_{\tilde{\mu}_R}$ [GeV]	105	105	104	105	105	104
$m_{\tilde{e}_L} \simeq m_{\tilde{\mu}_L}$ [GeV]	152	157	157	152	157	157
$m_{\tilde{\tau}_1}$ [GeV]	99	99	99	99	99	99
$m_{\tilde{\tau}_2}$ [GeV]	155	160	160	155	160	160
$m_{\tilde{\nu}_\tau}$ [GeV]	129	136	136	129	136	136
$m_{\tilde{\chi}_1^\pm}$ [GeV]	105	105	105	105	105	105
$m_{\tilde{\chi}_2^\pm}$ [GeV]	218	218	229	218	218	229
$m_{\tilde{\chi}_1^0}$ [GeV]	50	51	53	52	53	53
$m_{\tilde{\chi}_2^0}$ [GeV]	105	105	105	105	105	105
$m_{\tilde{\chi}_3^0}$ [GeV]	136	136	137	159	159	195
$m_{\tilde{\chi}_4^0}$ [GeV]	217	217	227	217	217	227
$m_{\tilde{g}}$ [GeV]	360	374	394	360	394	407
$m_{\tilde{d}_R} \simeq m_{\tilde{s}_R}$ [GeV]	401	444	444	401	444	444
$m_{\tilde{d}_L} \simeq m_{\tilde{s}_L}$ [GeV]	421	466	466	421	466	466
$m_{\tilde{b}_1}$ [GeV]	298	414	414	301	414	414
$m_{\tilde{b}_2}$ [GeV]	393	445	445	397	445	445
$m_{\tilde{t}_1}$ [GeV]	102	102	103	102	220	228
$m_{\tilde{t}_2}$ [GeV]	417	500	500	417	500	500
m_h [GeV]	91	91	91	91	91	91
m_H [GeV]	111	111	111	111	111	111
$m_{H^\pm}$ [GeV]	128	128	128	128	128	128
$m_{\rm HWIP}$ [GeV]	325	332	332	331	332	332
$m_{\rm HSIP}$ [GeV]	427	500	500	428	500	500
$\sigma_{\tilde{\chi}_1^0 p}$ [ab]	8.7	8.7	5.8	8.2	8.2	5.2

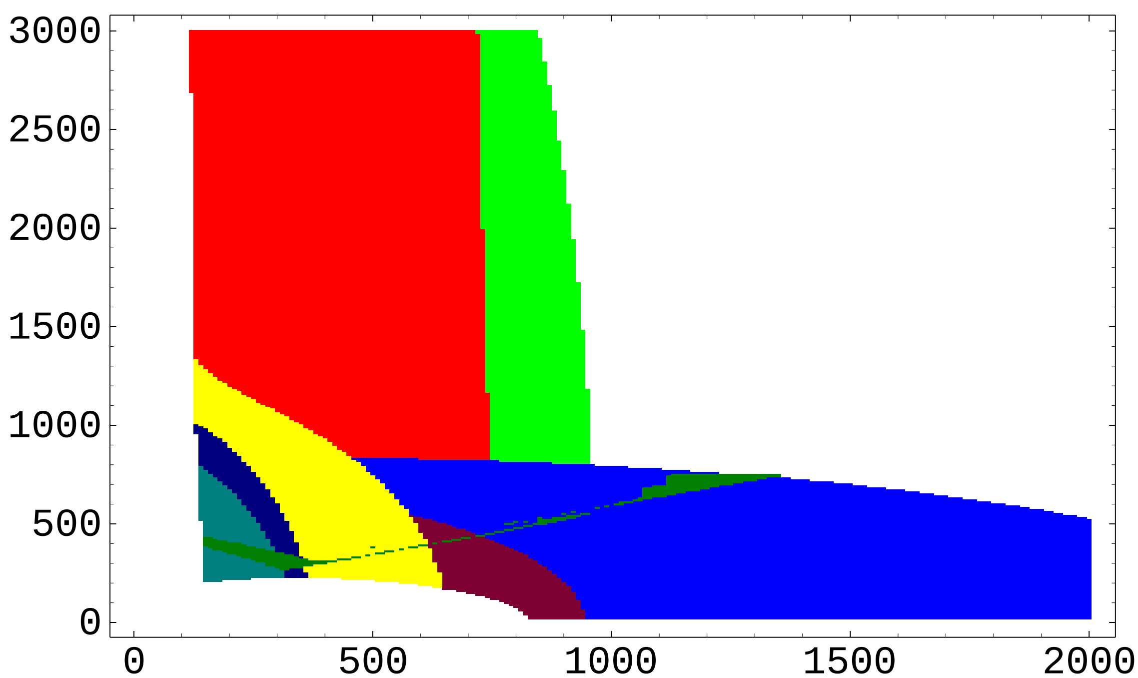
Table 1: Lower bounds on sparticle and Higgs masses and upper bound on the LSP–nucleon scattering cross section, in mSUGRA under six different sets of assumptions. "HWIP" = "heaviest weakly interacting particle"; HSIP = "heaviest strongly interacting particle", respectively. In all cases the minimal set of constraints are imposed. In Set II the constraint from $b \to s\gamma$ decays (including the sign of the decay amplitude) is added. The DM constraint (WMAP) is added in Set III. Sets IV–VI are like Sets I–III, but with the "more aggressive" $g_\mu - 2$ constraint. All limits obtained by scanning the full parameter space for 171 GeV $\leq m_t \leq$ 185 GeV.



Prospects for Sparticle production at ILC

(NB here simply illustrate accessibility, NO refined analysis)

$e^+e^- \to \chi_1^0\chi_2^0$ (green); $\chi_1^+\chi_1^+$ (red); $\tilde{\tau}^+\tilde{\tau}^-$ (blue),
$\tilde{\nu}\tilde{\nu}$ (purple); $\tilde{t}_1\tilde{t}_1, \tilde{b}_1\tilde{b}_1$ (dark blue); $H, A, H^\pm$ (yellow).
m_t =178 GeV, $\tan\beta$ =50;
$\sqrt{s}$ = 500 GeV (top fig.); 1 TeV (bottom fig.)



- We re-assess mSUGRA analysis in the light of new th. developments and updated exp. constraints

- Requirement of correct DM density constrain ANY model..

- Special emphasize on minimal allowed sparticle masses:

(Might be more meaningful than "size of allowed parameter space")

- mSUGRA with thermal χ_1^0 as DM still works fine,

lower bounds on sparticle masses only very mildly affected by DM constraints

- → **Direct experimental limits from colliders**

can still be saturated in many cases, even after WMAP

- → **Possible copious sparticle production at ILC**,

already for option 1 ($\sqrt{s} = 500$ GeV)