The impact of beamstrahlung on precision measurements of new physics at very high energy e^+e^- colliders

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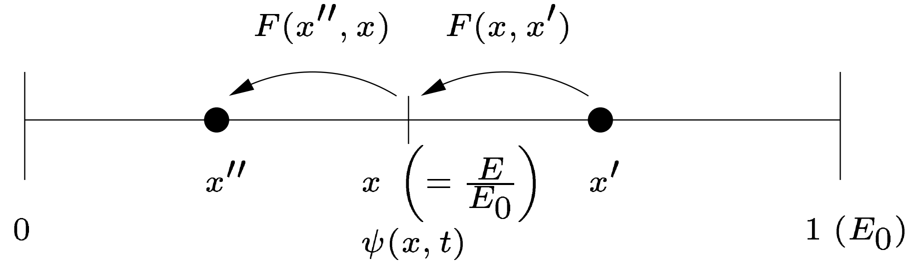
Plan

- Beamstrahlung
- Bremstrahlung and convolution
- Approximations and their limitations
  - Yokoya-Chen approximation
  - Consistent YC by Peskin
  - Applicability of the parametrizations
  - Numerical solution
  - Impact on physics
- Summary

Beamstrahlung

(Sokolov and Ternov 1986)

- Rate equation



  - $\frac{\partial\psi}{\partial t} = -\underbrace{\int_0^x dx'' F(x'', x)}_{\equiv\ \nu(x)} \psi(x,t) + \int_x^1 dx' F(x, x')\psi(x', t)$

  - $\frac{\partial\psi}{\partial t} = \nu(x)\psi(x,t) + \int_x^1 dx' F(x, x')\psi(x', t)$

- $F(x, x') = \frac{\nu_{cl}\kappa}{xx'} f(\xi, \eta)$

  $f(\xi, \eta) = \frac{3}{5\pi}\frac{1}{1+\xi\eta}\left[\int_\eta^\infty du K_{5/3}(u) + \frac{\xi^2\eta^2}{1+\xi\eta} K_{2/3}(\eta)\right]$

  - $\xi = 3x'\Upsilon$, $\eta = \kappa\left(\frac{1}{x} - \frac{1}{x'}\right)$, $\kappa = \frac{2}{3\Upsilon}$

  - $\Upsilon = \frac{5}{6}\frac{r_e^2\gamma_0 N}{\alpha\sigma_z(\sigma_x + \sigma_y)}$

  - $\nu_{cl} = \frac{5}{2\sqrt{3}}\frac{\alpha^2}{r_e\gamma_0}\Upsilon =$ constant

Yokoya-Chen approximation

Yokoya and Chen, IEEE 1989, SLAC-PUB-4935

Chen, Phys. Rev. D46,1186(1992)

- Can't solve rate equation analytically
  $\longrightarrow$ Need approximations to solve
- Most LCs have $\Upsilon \ll 1$ or $\Upsilon < 1$ $\left(\Upsilon \sim \frac{\gamma_0 N}{\sigma_z(\sigma_x+\sigma_y)}\right)$
  - SLC, $\Upsilon \sim 0.004$ (Chen, Phys. Rev. D46,1186(1992))
  - NLC500, $\Upsilon = 0.104$ (Peskin LCC-0010)
  - NLC1000, $\Upsilon = 0.299$ (Peskin LCC-0010)
  - CLIC3000, $\Upsilon = 5$
  - CLIC5000, $\Upsilon = 10$
- Two assumptions (valid for $\Upsilon \ll 1$)
  - $f(\xi,\eta) = \frac{3}{5\pi}\frac{1}{1+\xi\eta}\left[\int_\eta^\infty du K_{5/3}(u) + \frac{\xi^2\eta^2}{1+\xi\eta}K_{2/3}(\eta)\right]$
    $\approx \frac{1}{\Gamma(1/3)}\eta^{-2/3}e^{-\eta}$
  - $\nu(x) = \int_0^x dx'' F(x,x'') \approx \nu_{cl} =$ constant
- Analytic solution is possible by Laplace transformation
  - $\psi(x) = \frac{1}{N_{cl}}\left[(1-e^{-N_{cl}})\delta(1-x) + \frac{e^{-\eta x}}{1-x}\bar{h}(x)\right]$
- Good agreement with simulation data (ABEL) up to $\Upsilon \sim 0.44$ (Chen, Phys. Rev. D46,1186(1992))

Consistent YC approximation

Peskin LCC-0010, SLAC-TN-04-032

- Incompatibility of two assumptions
  - $\tilde{f}(\eta) = \frac{1}{\Gamma(1/3)}\eta^{-2/3}e^{-\eta} \longleftrightarrow \nu(x) = \text{constant}$
- Normalization condition
  - $\int_0^1 dx \psi(x,t) = 1$
- Need better $\tilde{f}$
  - Peskin : $\tilde{f}(\eta) = \frac{x'}{x}\frac{1}{\Gamma(1/3)}\eta^{-2/3}e^{-\eta}$
- Preserves probability sum rule
- Analytic solution
  - $\psi(x) = e^{-N}\left(\delta(x-1) + \frac{e^{-\eta x}}{x(1-x)}h(N\eta_x^{1/3})\right)$
- Numerically not very different from previous solution ($\Upsilon < 1$)
- Good agreement with Guinea Pig simulation for $\Upsilon = 0.104$ (NLC500) and $\Upsilon = 0.299$ (NCL1000)

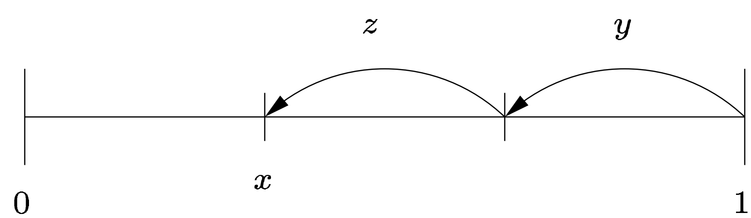
Bremstrahlung and convolution

- Bremstrahlung (ISR)

  Kuraev/Fadin (1985), Jadach/Skrzypek (1991), Jadach/Ward (1990)

  - $f_{ISR}(x) = \frac{1}{2}\beta(1 - x^{\frac{\beta}{2}} - 1)(1 + \frac{3}{8}\beta) - \frac{1}{4}\beta(1 + x)$
  - $\beta = \frac{2\alpha}{\pi}\left(\log\frac{Q^2}{m_e^2} - 1\right)$

- Convolution

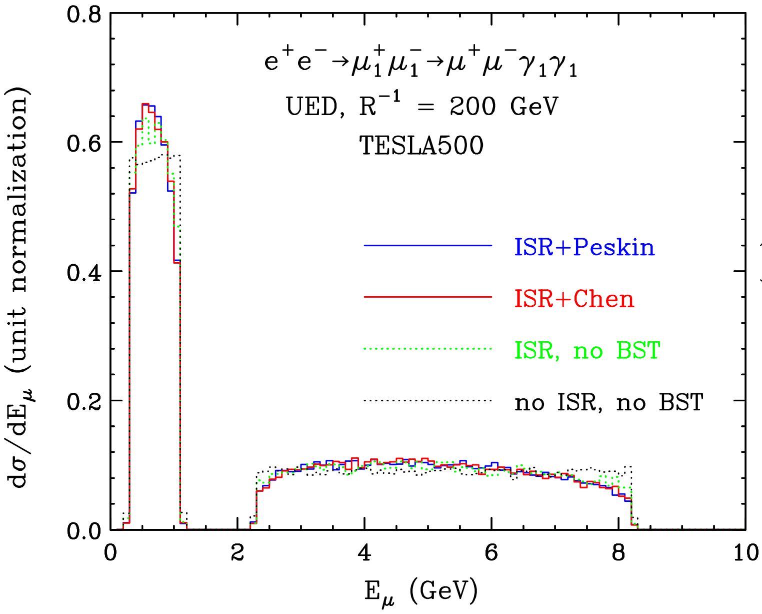


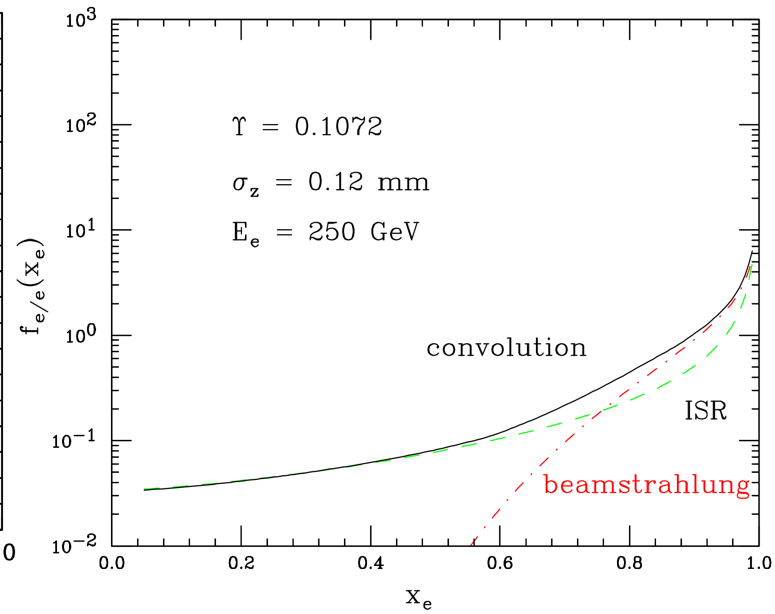
  - $f_1 * f_2 = \int_0^1 dydz f_1(y) f_2(z) \delta(x - yz)$
  - $f_{ISR} * f_{beam} = f_{beam} * f_{ISR}$

$$= \int_x^1 \frac{dz}{z} f_{ISR}\left(\frac{x}{z}\right) f_{beam}(x)$$

E_μ distiribution at colliders with $\Upsilon < 1$

Datta, Kong, Matchev (preliminary)

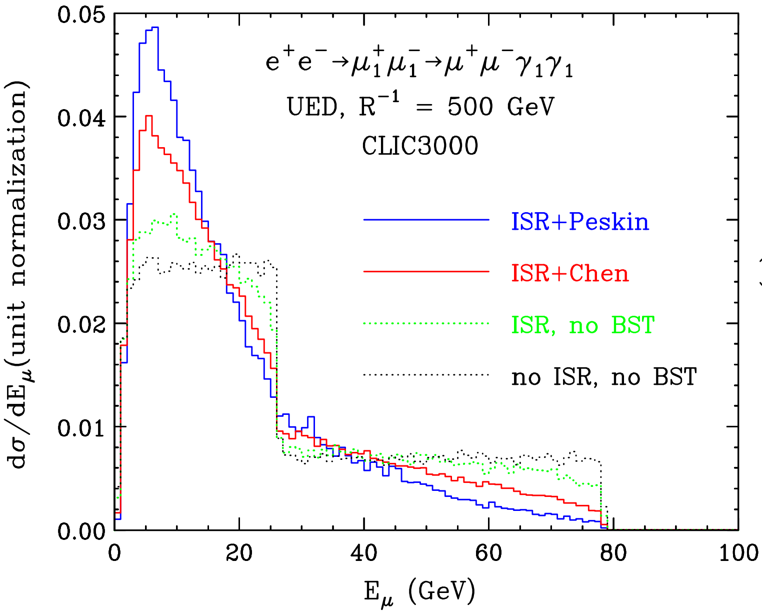


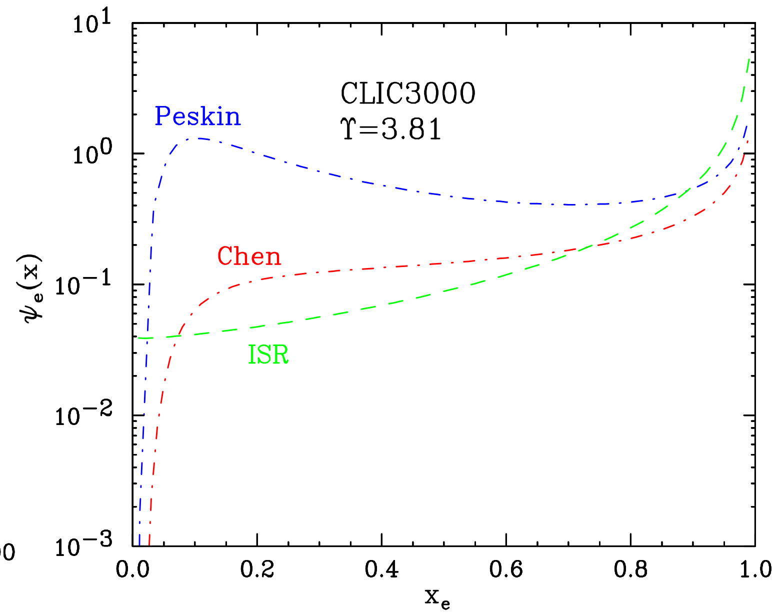


- End points are determined by the kinematics
  - $E_{max/min} = \frac{1}{2} M_{\tilde{\mu}} \left(1 - \frac{M^2_{\tilde{\chi}^0_1}}{M^2_{\tilde{\mu}}} \right) \gamma(1 \pm \beta)$
  - $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = \sqrt{1 - M^2_{\tilde{\mu}}/E^2_{beam}}$
  - Sensitive to masses and energy
- Exactly flat distribution is expected if no energy loss
- ISR and beamstrahlung distribute the energies of incoming electrons $\to$ smear the distribution $\to$ endpoints OK
- ISR is more important at low x since beamstrahlung drops fast

E_μ distiribution at colliders with $\Upsilon > 1$

Datta, Kong, Matchev (preliminary)



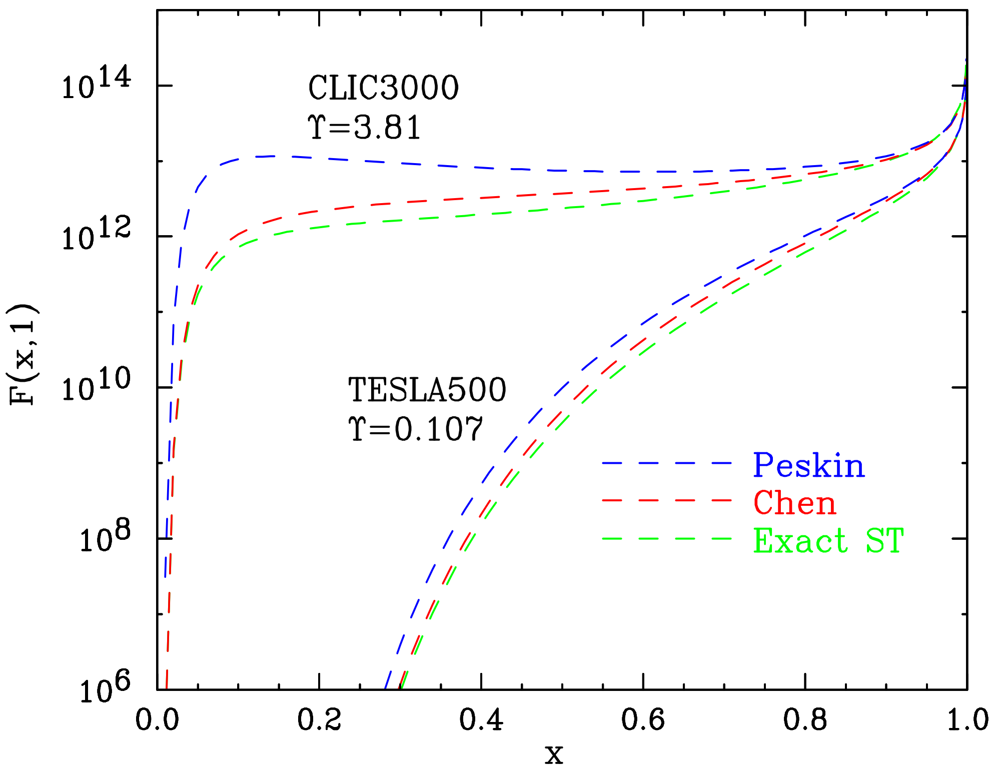


- Can't see nice flat distribution $\longrightarrow$ distorted a lot
- Consistent YC deviates more since it has a peak at low x
- Beamstrahlung is more important at low x
- Flat tail or peak at low x (soft photons) affect physical distribution
- Is there a peak ? Which is the right answer ?

Applicability of approximations for $\Upsilon > 1$

- $\tilde{f}(\eta) = \frac{x'}{x}\frac{1}{\Gamma(1/3)}\eta^{-2/3}e^{-\eta}$ is only valid for $\Upsilon \ll 1$
- Original approximation is better

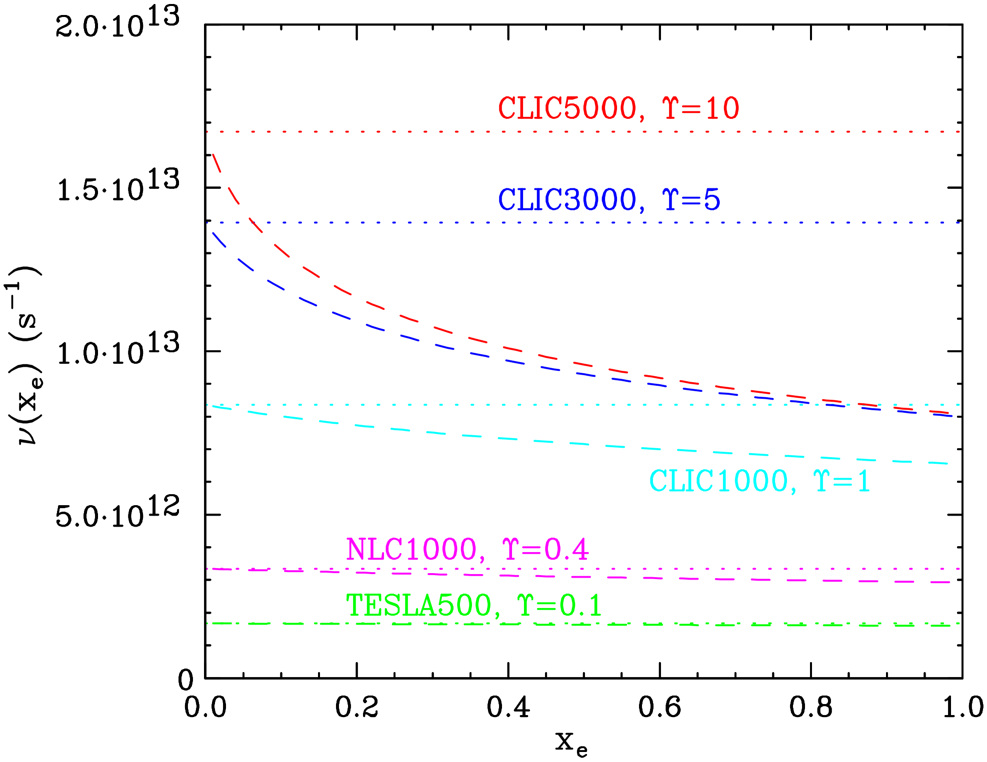
Datta, Kong, Matchev (preliminary)



Applicability of approximations for $\Upsilon > 1$

- $\nu(x) = \int_0^x F(x'', x)dx'' \neq$ constant

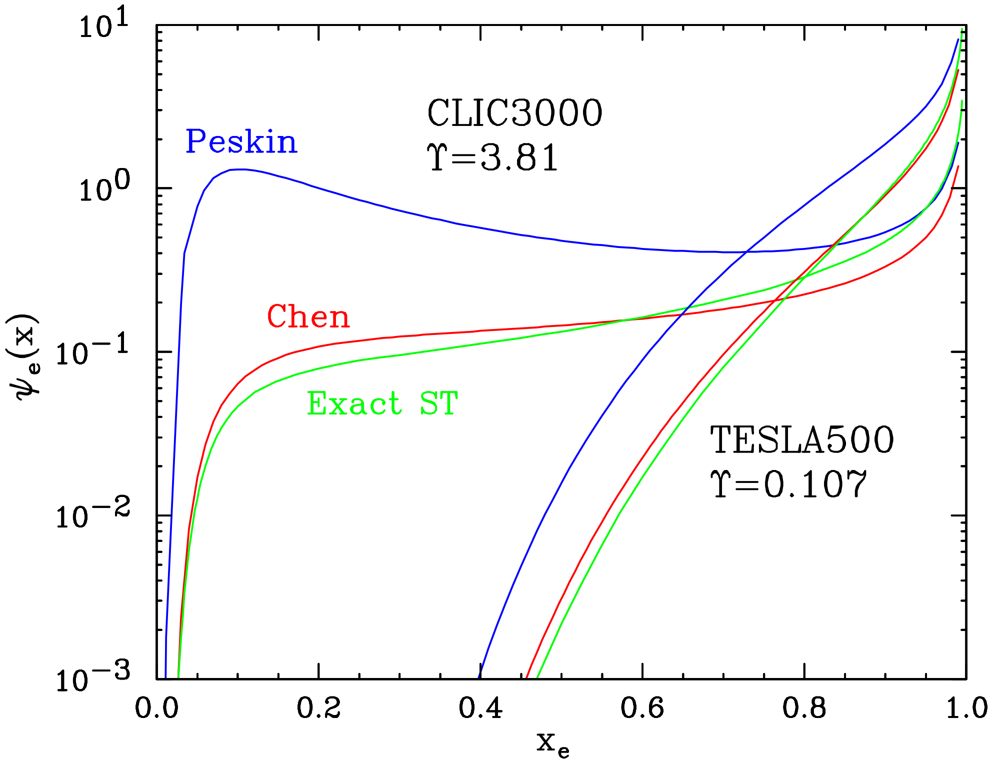
Datta, Kong, Matchev (preliminary)



- For large Υ (CLIC), we can trust neither approximations
- Need new formula/better approximation (couldn't find)

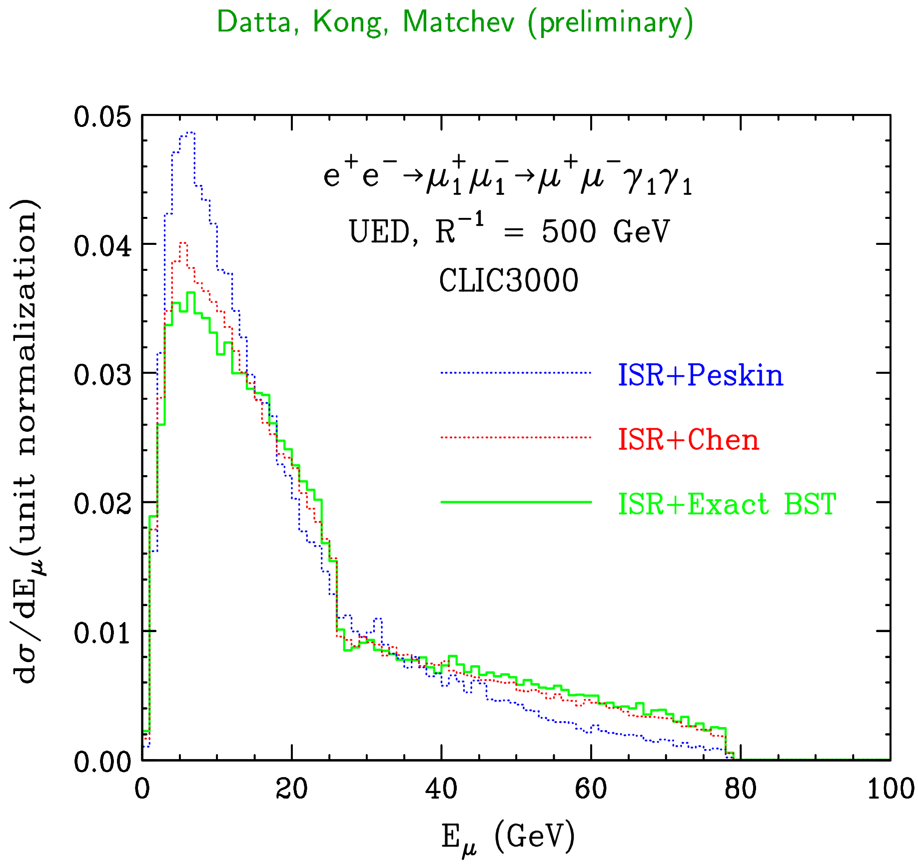
Numerical solution

Datta, Kong, Matchev (preliminary)



- Solve the rate equation directly with Sokolov-Ternov solution
  - Good : valid for arbitrary Υ, normalized
  - Bad : no analytic expression
    $\rightarrow$ need to modify the event generator code
- Chen is closer for identical parametrization
- Roughly agrees with old simulation data.
  We will compare to new data from Schulte

E_μ distiribution at colliders with $\Upsilon > 1$



- Still distorted
- We know the right answer

Conclusion

- Analytic solutions are limited for small Υ only
  - Good agreement with simulation data
  - This is true for LC 500-1000
- Can't use same solution for large Υ
  - Need new approximation $\rightarrow$ No analytic solution for large Υ in the case of high energy e^+e^- colliders such as CLIC
  - Solve rate equation numerically instead or
  - Use simulation data
- Caution : Implementation in event generators
  - Most event generators have one of these two parametrizations
  - Either numerically worse or has normalization problem
  - How to fix the event generator
    * Use old parametrizations and fake parameters
    * Use numerical solution/simulation data and import in the event generator
- A lot of soft photons at high energy e^+e^- colliders distort physical distributions, e.g. E_μ