Determining super-particle masses

based on: B. K. Gjelsten, D. J. Miller, P. Osland

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LCWS 05

Per Osland, CERN / University of Bergen

Outline

- Introduction
- Graphic overview of parameter space
- Kinematical endpoints (squark chain)
- Fitting MC data: precision
- LC input (LSP mass): resolving ambiguities
- gluino chain (b-tagging)
- LC input: higher precision

Supersymmetry may be realized at the LHC/ILC

LHC: initial-state energy undetermined
LSP not seen

ILC: lower energy reach
precise energy, LSP mass determined

LHC: determine mass *differences* up to high values

ILC: can determine LSP mass with high precision

ILC: resolve ambiguities in LHC mass measurements

At the LHC, unstable particles produced copiously, cascade decays, e.g.

$$\boxed{\tilde{q} \to \tilde{\chi}_2^0 q \to \tilde{l} l q \to \tilde{\chi}_1^0 l l q}$$

Challenge: determine masses with high precision

Refs: Baer et al, hep-ph/9512383; Hinchliffe et al, hep-ph/9610544; Bachacou et al, hep-ph/9907518; Polesello, ATLAS Int Note 1997; Allanach et al, hep-ph/0007009; Gjelsten et al, ATLAS Note 2004; Chiorboli, Tricomi, CMS Note 2004

mSUGRA (CMSSM)

- Unification at high energies, fewer parameters
  $m_0 \quad m_{1/2} \quad A_0 \quad \tan\beta \quad \mathrm{sign}\,\mu$
- Snowmass Points and Slopes: Allanach et al, hep-ph/0202233: SPS 1a, SPS 1b, SPS 3, SPS 5.
- WMAP constraints: Bennett et al, astro-ph/0302207; Spergel et al, astro-ph/0302209

footnote: Benchmark Points

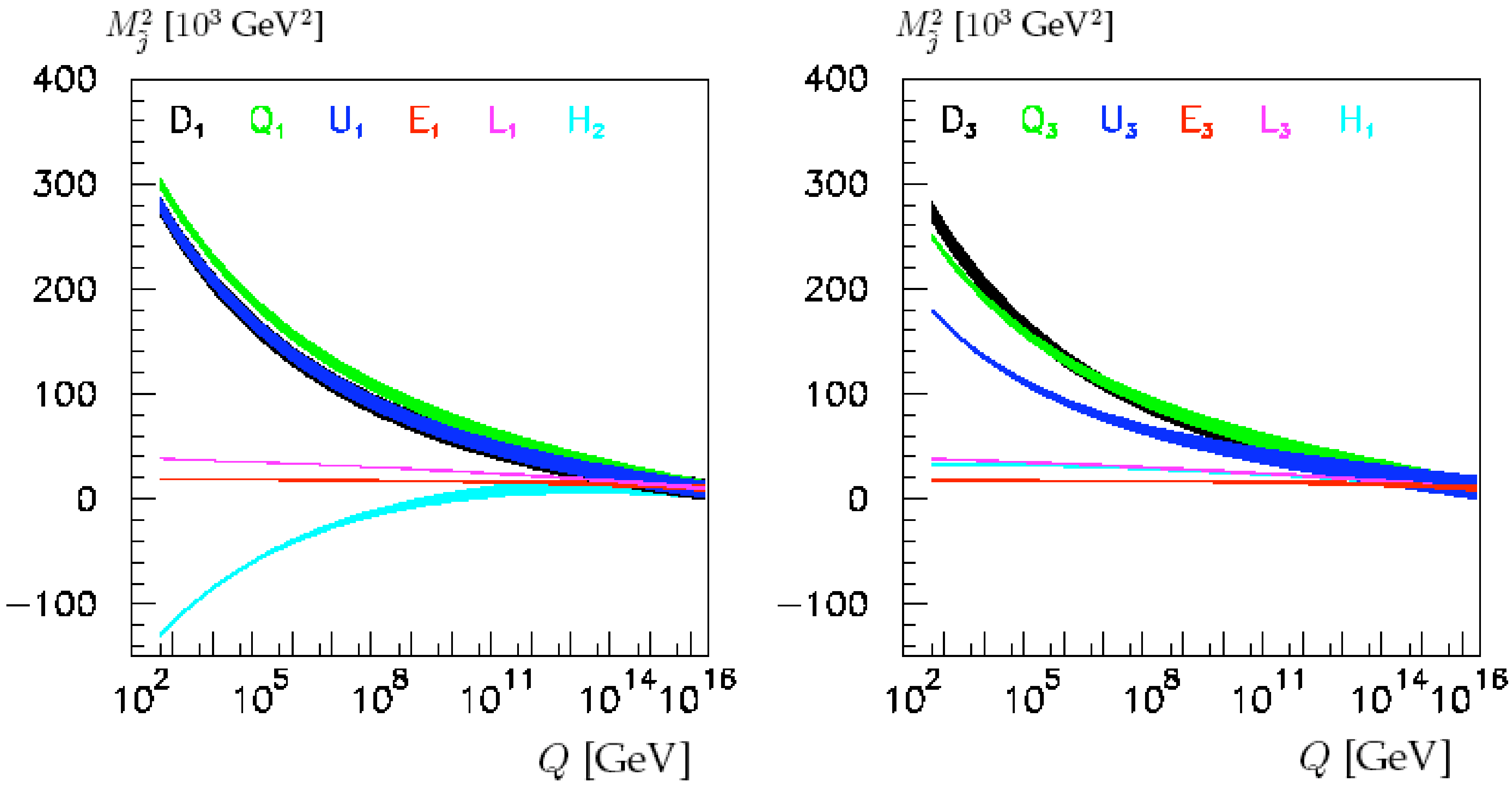
- LHC Points ('SUGRA'): Hinchliffe et al, hep-ph/9610544: Point 1, Point 2, Point 3, Point 4, Point 5
- Post-LEP Benchmarks ('CMSSM'): Battaglia et al, hep-ph/0106204: A, B, C, ..., M
- Snowmass Points and Slopes ('mSUGRA'): Allanach et al, hep-ph/0202233: SPS 1a, SPS 1b, SPS 2, SPS 3, SPS 4, SPS 5, SPS 6, .., SPS 9
- Post-WMAP Benchmarks ('CMSSM'): Ellis et al, hep-ph/0303043, Battaglia et al hep-ph/0306219: A', B', C', ..., M'

Mutations: Point 5 → B → SPS 1a → B'

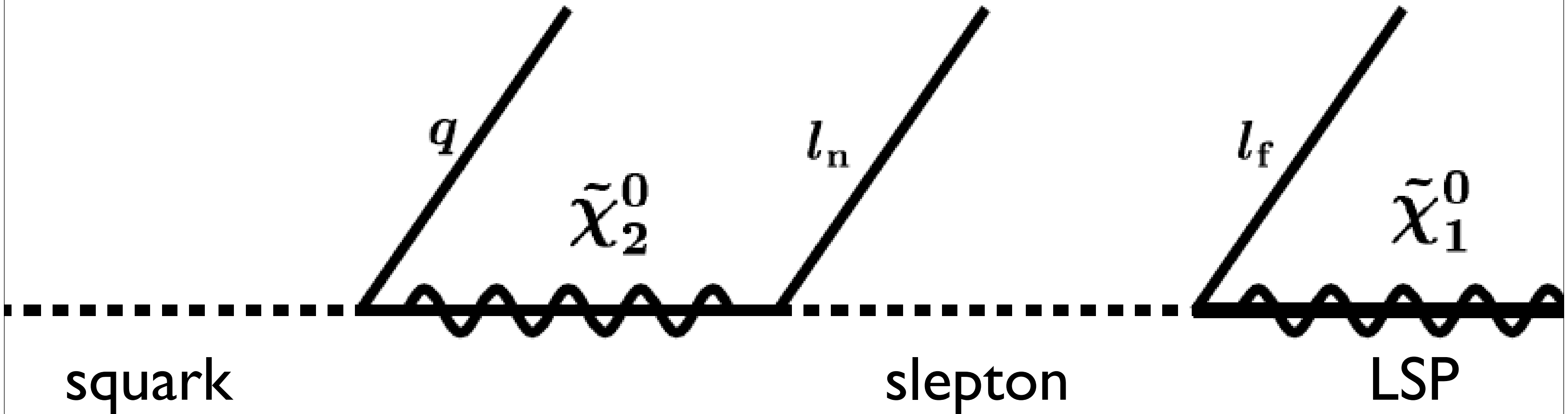
Precision in masses allows extrapolation to Unification scale

Example: Sfermion mass parameters

Allanach et al, hep-ph/0403133



"Easy" SPS 1a squark cascade



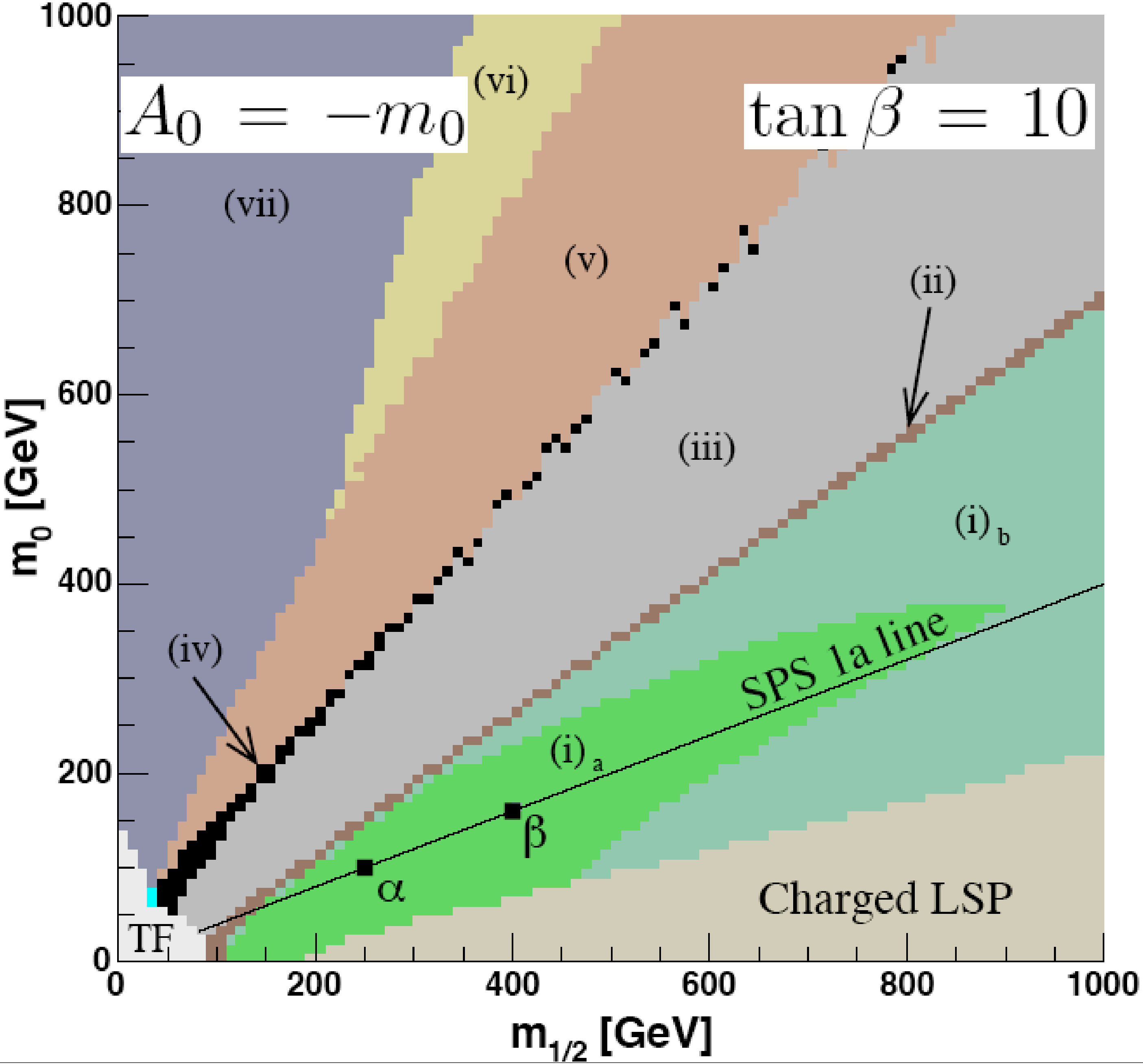
Detect: quark jet and two leptons

Aim: determine squark, slepton and neutralino masses

Question: Is this mass hierarchy "typical"?

Want "heavy" gluino and "heavy" neutralino $\tilde{\chi}_2^0$

Hierarchies:



heavy gluino

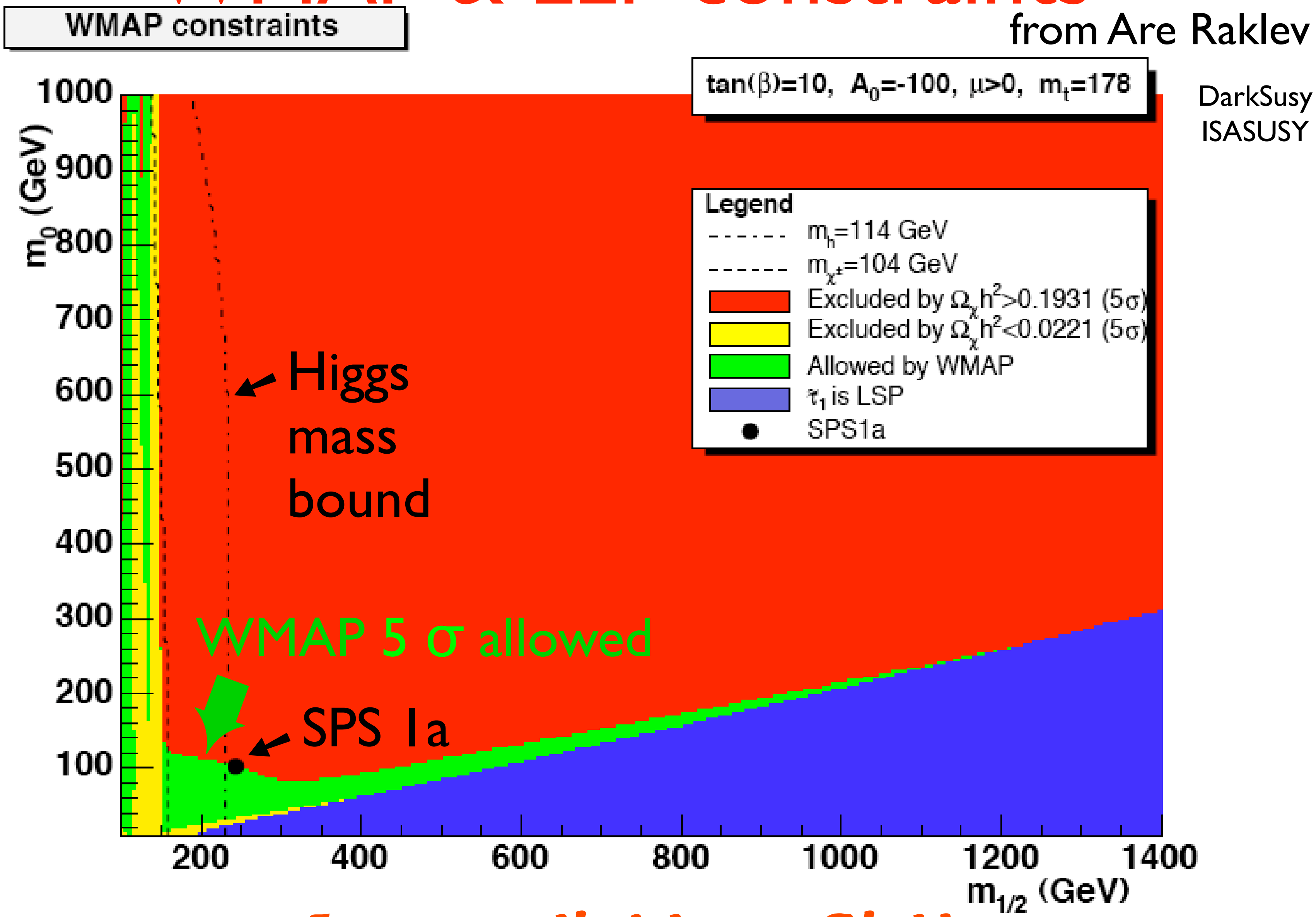
heavy neutralino

(i)	$\boxed{\tilde{g} >} \max(\tilde{d}_L, \tilde{u}_L, \tilde{b}_1, \tilde{t}_1)$	and	$\boxed{\tilde{\chi}_2^0 >} \max(\tilde{l}_R, \tilde{\tau}_1)$
(ii)	$\boxed{\tilde{g} >} \max(\tilde{d}_L, \tilde{u}_L, \tilde{b}_1, \tilde{t}_1)$	and	$\tilde{l}_R > \tilde{\chi}_2^0 > \tilde{\tau}_1$
(iii)	$\boxed{\tilde{g} >} \max(\tilde{d}_L, \tilde{u}_L, \tilde{b}_1, \tilde{t}_1)$	and	$\min(\tilde{l}_R, \tilde{\tau}_1) > \tilde{\chi}_2^0$
(iv)	$\tilde{d}_L > \tilde{g} > \max(\tilde{u}_L, \tilde{b}_1)$	and	$\min(\tilde{l}_R, \tilde{\tau}_1) > \tilde{\chi}_2^0$
(v)	$\min(\tilde{d}_L, \tilde{u}_L) > \tilde{g} > \tilde{b}_1$	and	$\min(\tilde{l}_R, \tilde{\tau}_1) > \tilde{\chi}_2^0$
(vi)	$\min(\tilde{d}_L, \tilde{u}_L, \tilde{b}_1) > \tilde{g} > \tilde{t}_1$	and	$\min(\tilde{l}_R, \tilde{\tau}_1) > \tilde{\chi}_2^0$
(vii)	$\min(\tilde{d}_L, \tilde{u}_L, \tilde{b}_1, \tilde{t}_1) > \tilde{g}$	and	$\min(\tilde{l}_R, \tilde{\tau}_1) > \tilde{\chi}_2^0$

heavy gauginos, $\tilde{g}, \tilde{\chi}_2^0$ lower right

WMAP & LEP constraints

from Are Raklev



Ignore slight conflict!

Next question:

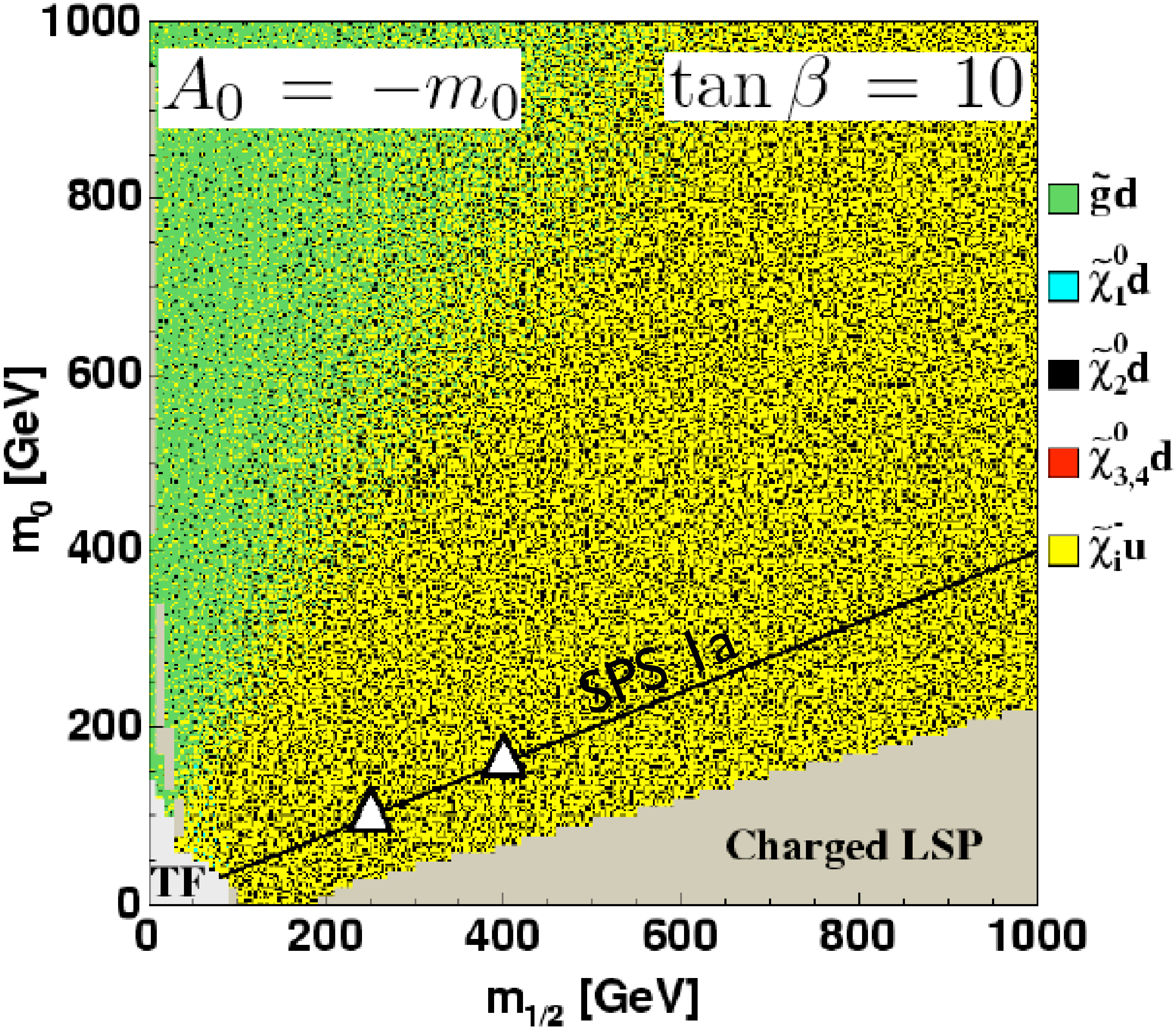
Given "correct" hierarchy,

$$m_{\tilde{g}} > m_{\tilde{q}_L} > m_{\tilde{\chi}_2^0} > m_{\tilde{l}_R} > m_{\tilde{\chi}_1^0}$$

is there enough BR?

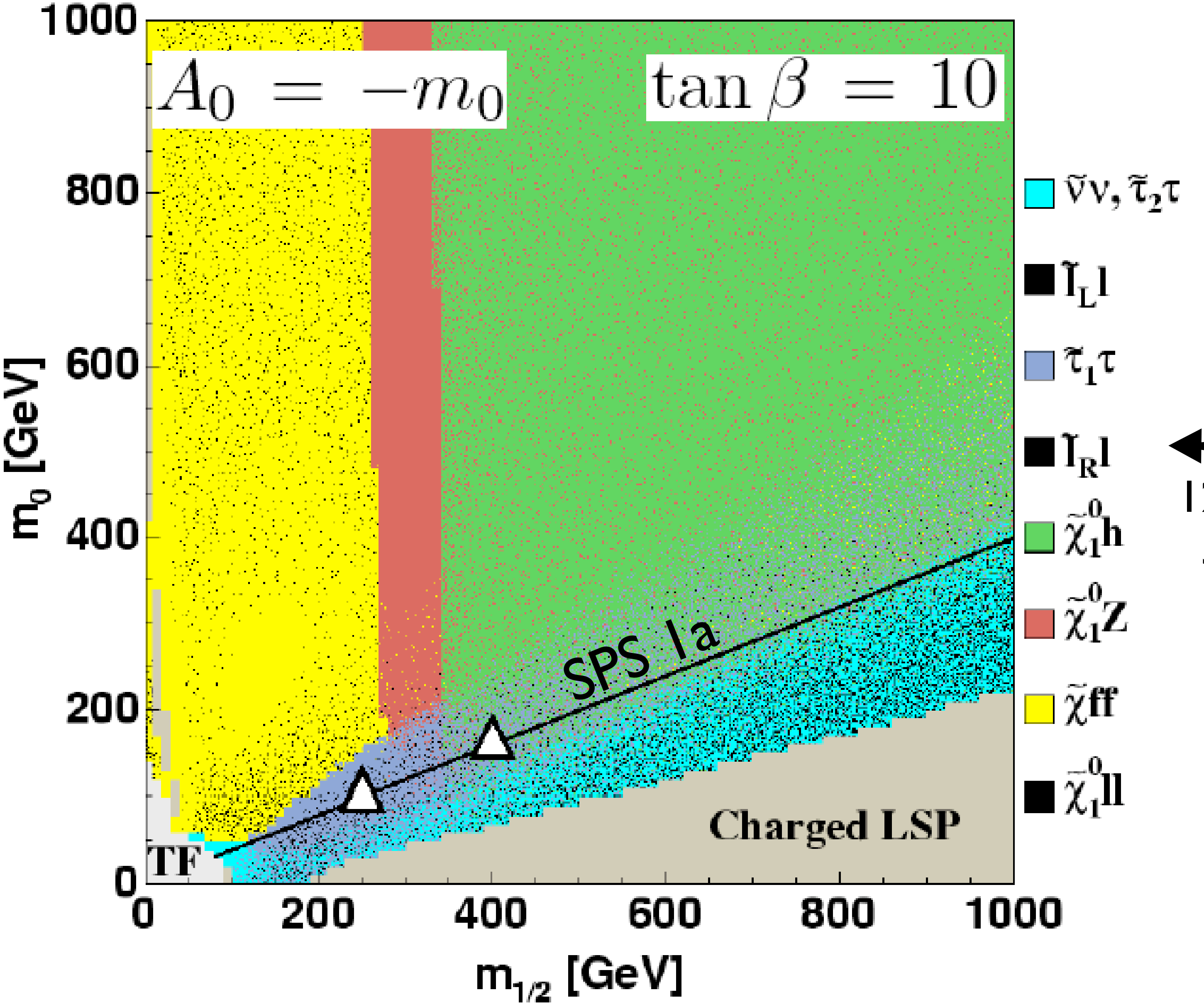
- Does the squark have significant BR to neutralino and quark?
- Does the neutralino have significant BR to slepton and lepton?

Squark Branching Ratios ($\tilde{u}_L$)



31% (α)
32% (β)

Neutralino Branching Ratios ($\tilde{\chi}_2^0$)



12% (α)
5% (β)

SPS 1a (line)

$$\boxed{\begin{array}{l} m_0 = -A_0 = 0.4\, m_{1/2} \\ \tan\beta = 10\,, \qquad \mu > 0 \end{array}}$$

Two particular points on the line:

$$(\alpha): \quad m_0 = 100\ \mathrm{GeV}\,, \qquad m_{1/2} = 250\ \mathrm{GeV}$$

$$(\beta): \quad m_0 = 160\ \mathrm{GeV}\,, \qquad m_{1/2} = 400\ \mathrm{GeV}$$

Spectrum

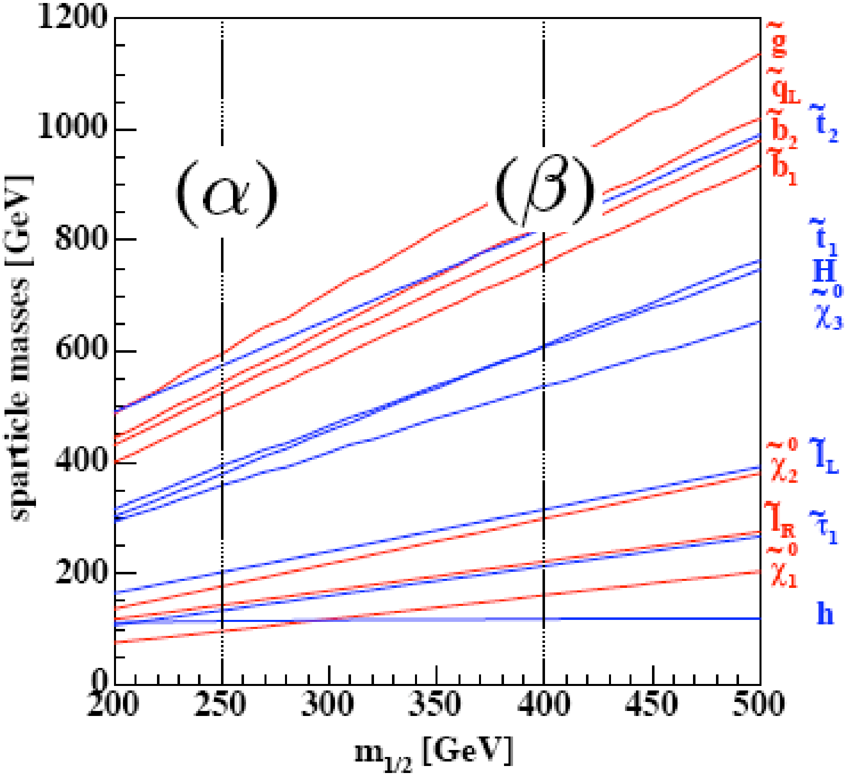
Point	$\tilde{g}$	$\tilde{d}_L$	$\tilde{d}_R$	$\tilde{u}_L$	$\tilde{u}_R$	$\tilde{b}_2$	$\tilde{b}_1$	$\tilde{t}_2$	$\tilde{t}_1$
(α)	**595.2**	**543.0**	520.1	**537.2**	520.5	**524.6**	**491.9**	574.6	379.1
(β)	**915.5**	**830.1**	799.5	**826.3**	797.3	**800.2**	**759.4**	823.8	610.4
	$\tilde{e}_L$	$\tilde{e}_R$	$\tilde{\tau}_2$	$\tilde{\tau}_1$	$\tilde{\nu}_{e_L}$	$\tilde{\nu}_{\tau_L}$		$H^{\pm}$	A
(α)	202.1	**143.0**	206.0	133.4	185.1	185.1		401.8	393.6
(β)	315.6	**221.9**	317.3	213.4	304.1	304.1		613.9	608.3
	$\tilde{\chi}_4^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^{\pm}$	$\tilde{\chi}_1^{\pm}$		H	h
(α)	377.8	358.8	**176.8**	**96.1**	378.2	176.4		394.2	114.0
(β)	553.3	538.4	**299.1**	**161.0**	553.3	299.0		608.9	117.9

as determined by ISASUSY 7.58 by integrating RGE's

in **bold**: particles used in study

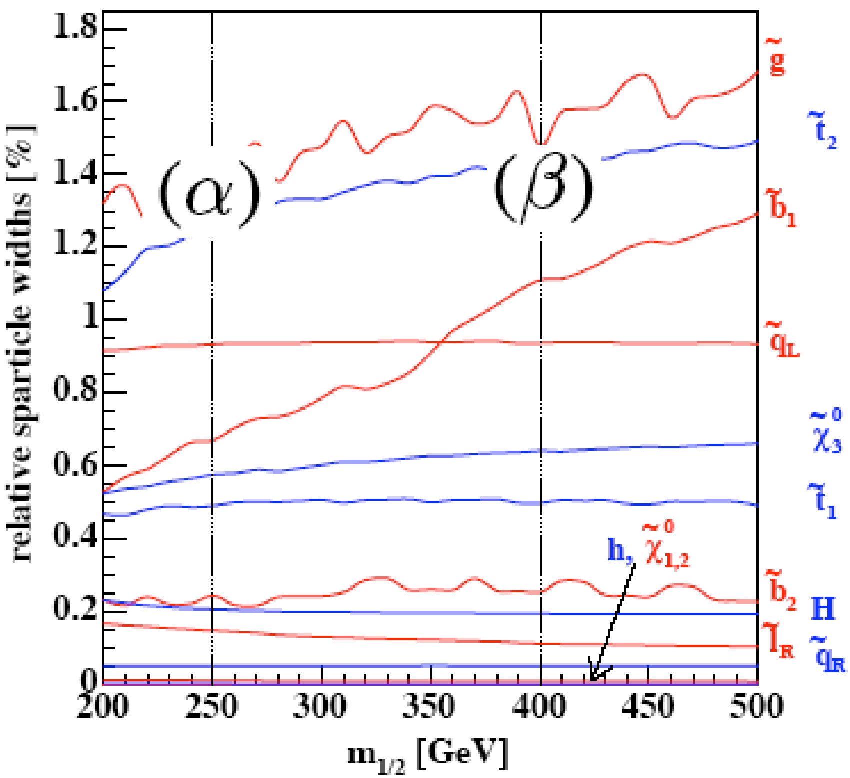
SPS 1a line

Masses



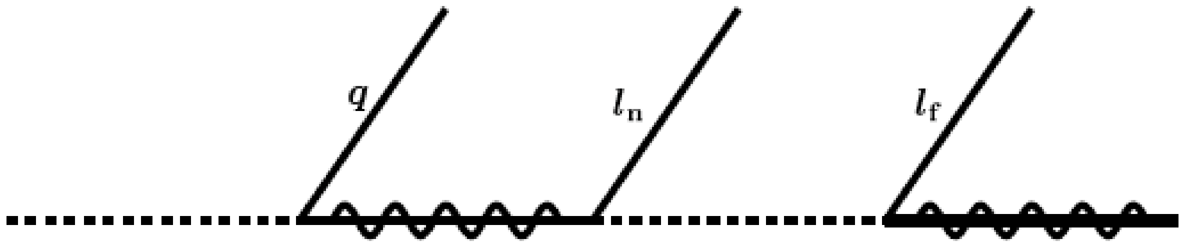
Widths

ISASUSY 7.58



~1% of mass

Quantifying the cascade:



	$\sigma = $ 32.8 pb	$\tilde{q}_L$	31.4%						1245 fb
(α)	$\sigma = $ 7.7 pb	$\tilde{b}_1$	35.5%	$\tilde{\chi}_2^0$	12.1%	$\tilde{l}_R$	100%	$\tilde{\chi}_1^0$	329 fb
	$\sigma = $ 4.3 pb	$\tilde{b}_2$	18.0%						94 fb
									1669 fb
	$\sigma = $ 3.21 pb	$\tilde{q}_L$	31.9%						54.3 fb
(β)	$\sigma = $ 0.50 pb	$\tilde{b}_1$	23.6%	$\tilde{\chi}_2^0$	5.3%	$\tilde{l}_R$	100%	$\tilde{\chi}_1^0$	6.3 fb
	$\sigma = $ 0.31 pb	$\tilde{b}_2$	8.8%						1.5 fb
									62.0 fb

Reduction (rate and BR) from (α) to (β)

Maximum di-lepton mass:

Back-to-back in $\tilde{l}_R$ Rest Frame:

$$(m_{ll}^{\max})^2 = 4|\mathbf{p}_{l_\mathrm{n}}||\mathbf{p}_{l_\mathrm{f}}| = \frac{(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{l}_R}^2}$$

Prototype of "endpoint formulas"

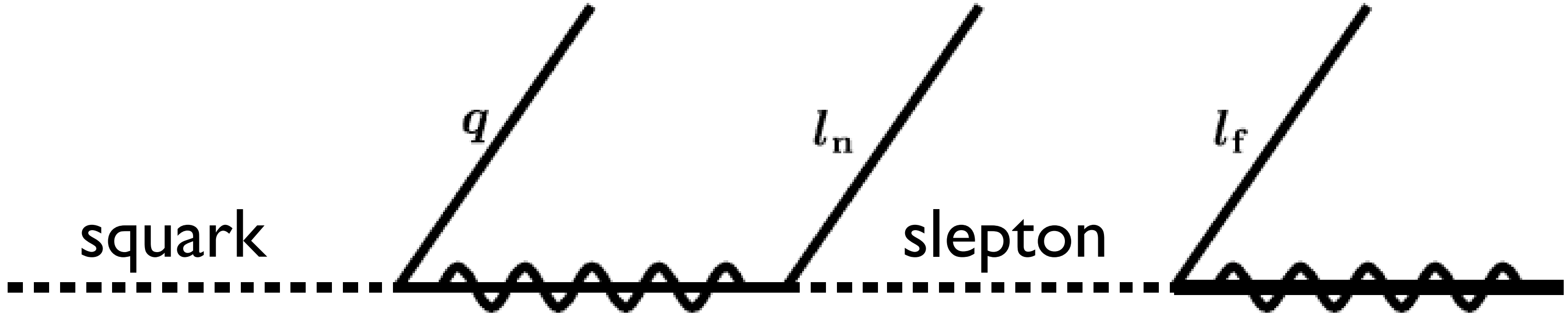
One kinematical endpt is related to various (3) masses of unstable particles:

$$m_{\tilde{\chi}_2^0} \quad m_{\tilde{l}_R} \quad m_{\tilde{\chi}_1^0}$$

Need more such formulas!

Add the squark:

$$\tilde{q}_L \to \tilde{\chi}_2^0\, q \to \tilde{l}_R\, q\, l_{\rm n} \to \tilde{\chi}_1^0\, q\, l_{\rm n}\, l_{\rm f}$$



More invariants and endpoints:

$$m_{qll} \quad m_{ql_{\rm n}} \quad m_{ql_{\rm f}} \quad m_{ll}$$

Four endpoints and *four* masses:

$$m_{\tilde{q}_L} \quad m_{\tilde{\chi}_2^0} \quad m_{\tilde{l}_R} \quad m_{\tilde{\chi}_1^0}$$

B.C. Allanach et al, hep-ph/0007009 (conditions rephrased):

$$(m_{ll}^{\max})^2 = \frac{(m_{\tilde\chi_2^0}^2 - m_{\tilde l_R}^2)(m_{\tilde l_R}^2 - m_{\tilde\chi_1^0}^2)}{m_{\tilde l_R}^2}$$

one case

mass ratios of adjacent sparticles in chain

$$(m_{qll}^{\max})^2 = \begin{cases} \frac{\left(m_{\tilde q_L}^2 - m_{\tilde\chi_2^0}^2\right)\left(m_{\tilde\chi_2^0}^2 - m_{\tilde\chi_1^0}^2\right)}{m_{\tilde\chi_2^0}^2} & \text{for} \quad \frac{m_{\tilde q_L}}{m_{\tilde\chi_2^0}} > \frac{m_{\tilde\chi_2^0}}{m_{\tilde l_R}} \frac{m_{\tilde l_R}}{m_{\tilde\chi_1^0}} \quad (1) \\ \frac{\left(m_{\tilde q_L}^2 m_{\tilde l_R}^2 - m_{\tilde\chi_2^0}^2 m_{\tilde\chi_1^0}^2\right)\left(m_{\tilde\chi_2^0}^2 - m_{\tilde l_R}^2\right)}{m_{\tilde\chi_2^0}^2 m_{\tilde l_R}^2} & \text{for} \quad \frac{m_{\tilde\chi_2^0}}{m_{\tilde l_R}} > \frac{m_{\tilde l_R}}{m_{\tilde\chi_1^0}} \frac{m_{\tilde q_L}}{m_{\tilde\chi_2^0}} \quad (2) \\ \frac{\left(m_{\tilde q_L}^2 - m_{\tilde l_R}^2\right)\left(m_{\tilde l_R}^2 - m_{\tilde\chi_1^0}^2\right)}{m_{\tilde l_R}^2} & \text{for} \quad \frac{m_{\tilde l_R}}{m_{\tilde\chi_1^0}} > \frac{m_{\tilde q_L}}{m_{\tilde\chi_2^0}} \frac{m_{\tilde\chi_2^0}}{m_{\tilde l_R}} \quad (3) \\ (m_{\tilde q_L} - m_{\tilde\chi_1^0})^2 & \text{otherwise} \quad (4) \end{cases}$$

four cases

$$(m_{ql(\mathrm{low})}^{\max}, m_{ql(\mathrm{high})}^{\max}) = \begin{cases} (m_{ql_\mathrm{n}}^{\max}, m_{ql_\mathrm{f}}^{\max}) & \text{for} \quad 2m_{\tilde l_R}^2 > m_{\tilde\chi_1^0}^2 + m_{\tilde\chi_2^0}^2 > 2m_{\tilde\chi_1^0} m_{\tilde\chi_2^0} \quad (1) \\ (m_{ql(\mathrm{eq})}^{\max}, m_{ql_\mathrm{f}}^{\max}) & \text{for} \quad m_{\tilde\chi_1^0}^2 + m_{\tilde\chi_2^0}^2 > 2m_{\tilde l_R}^2 > 2m_{\tilde\chi_1^0} m_{\tilde\chi_2^0} \quad (2) \\ (m_{ql(\mathrm{eq})}^{\max}, m_{ql_\mathrm{n}}^{\max}) & \text{for} \quad m_{\tilde\chi_1^0}^2 + m_{\tilde\chi_2^0}^2 > 2m_{\tilde\chi_1^0} m_{\tilde\chi_2^0} > 2m_{\tilde l_R}^2 \quad (3) \end{cases}$$

three cases

where
$$\left(m_{ql_{\rm n}}^{\max}\right)^2 = \frac{\left(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2\right)\left(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2\right)}{m_{\tilde{\chi}_2^0}^2}$$
$$\left(m_{ql_{\rm f}}^{\max}\right)^2 = \frac{\left(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2\right)\left(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2\right)}{m_{\tilde{l}_R}^2}$$
$$\left(m_{ql(\rm eq)}^{\max}\right)^2 = \frac{\left(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2\right)\left(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2\right)}{\left(2m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2\right)}$$

Finally:

$$\begin{aligned}\left(m_{qll(\theta>\frac{\pi}{2})}^{\min}\right)^2 = \Big[&\left(m_{\tilde{q}_L}^2 + m_{\tilde{\chi}_2^0}^2\right)\left(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2\right)\left(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2\right) \\ &- \left(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2\right)\sqrt{\left(m_{\tilde{\chi}_2^0}^2 + m_{\tilde{l}_R}^2\right)^2\left(m_{\tilde{l}_R}^2 + m_{\tilde{\chi}_1^0}^2\right)^2 - 16 m_{\tilde{\chi}_2^0}^2 m_{\tilde{l}_R}^4 m_{\tilde{\chi}_1^0}^2} + \\ &+ 2m_{\tilde{l}_R}^2\left(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2\right)\left(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2\right)\Big]\left(4 m_{\tilde{l}_R}^2 m_{\tilde{\chi}_2^0}^2\right)^{-1}\end{aligned}$$

one case

θ is opening angle between leptons in $\tilde{l}_R$ rest frame

Over-all: 4×3 cases, denoted (1,1), (1,2), etc.

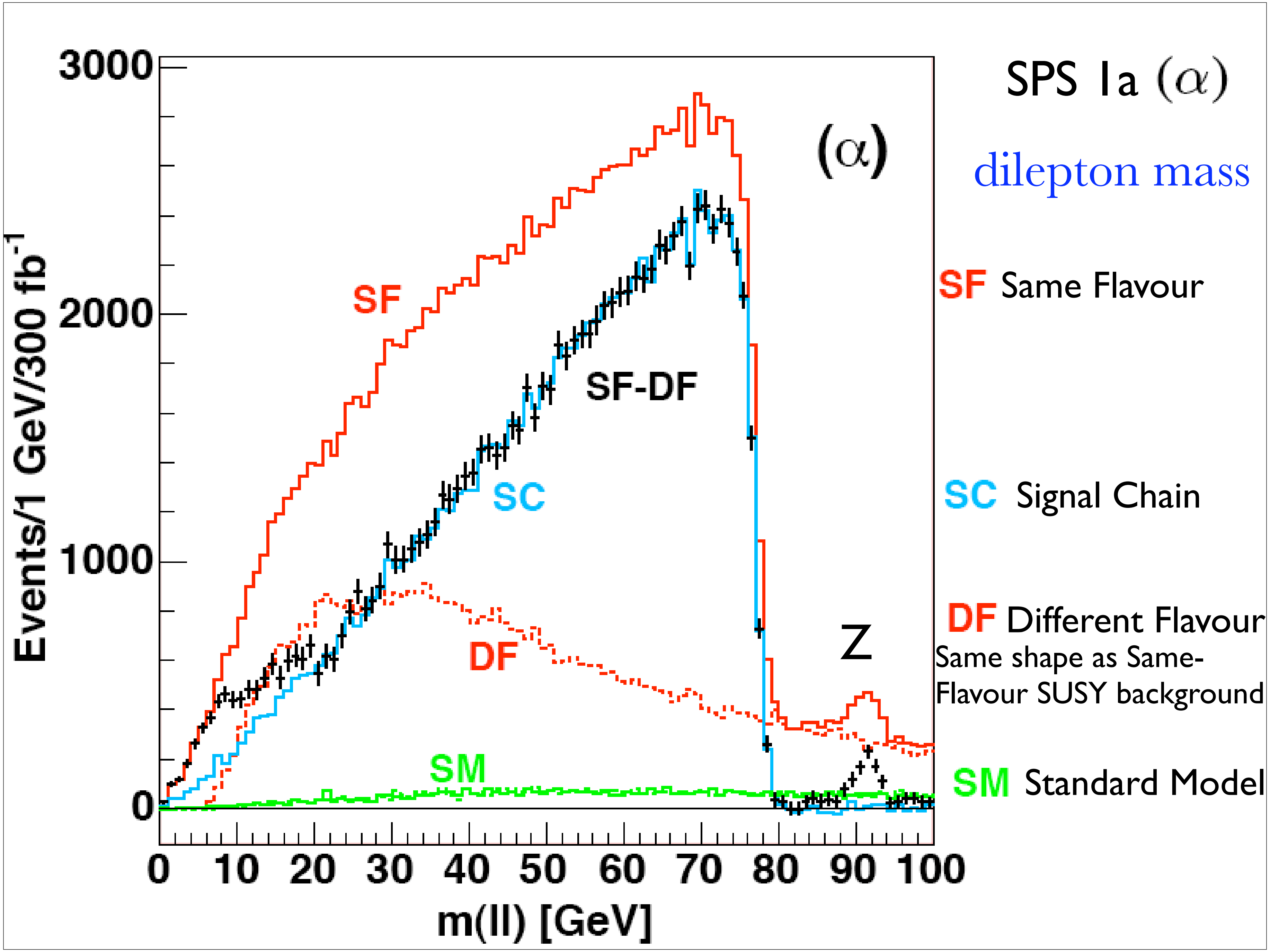
(9 of 12 are realized)

LHC simulation

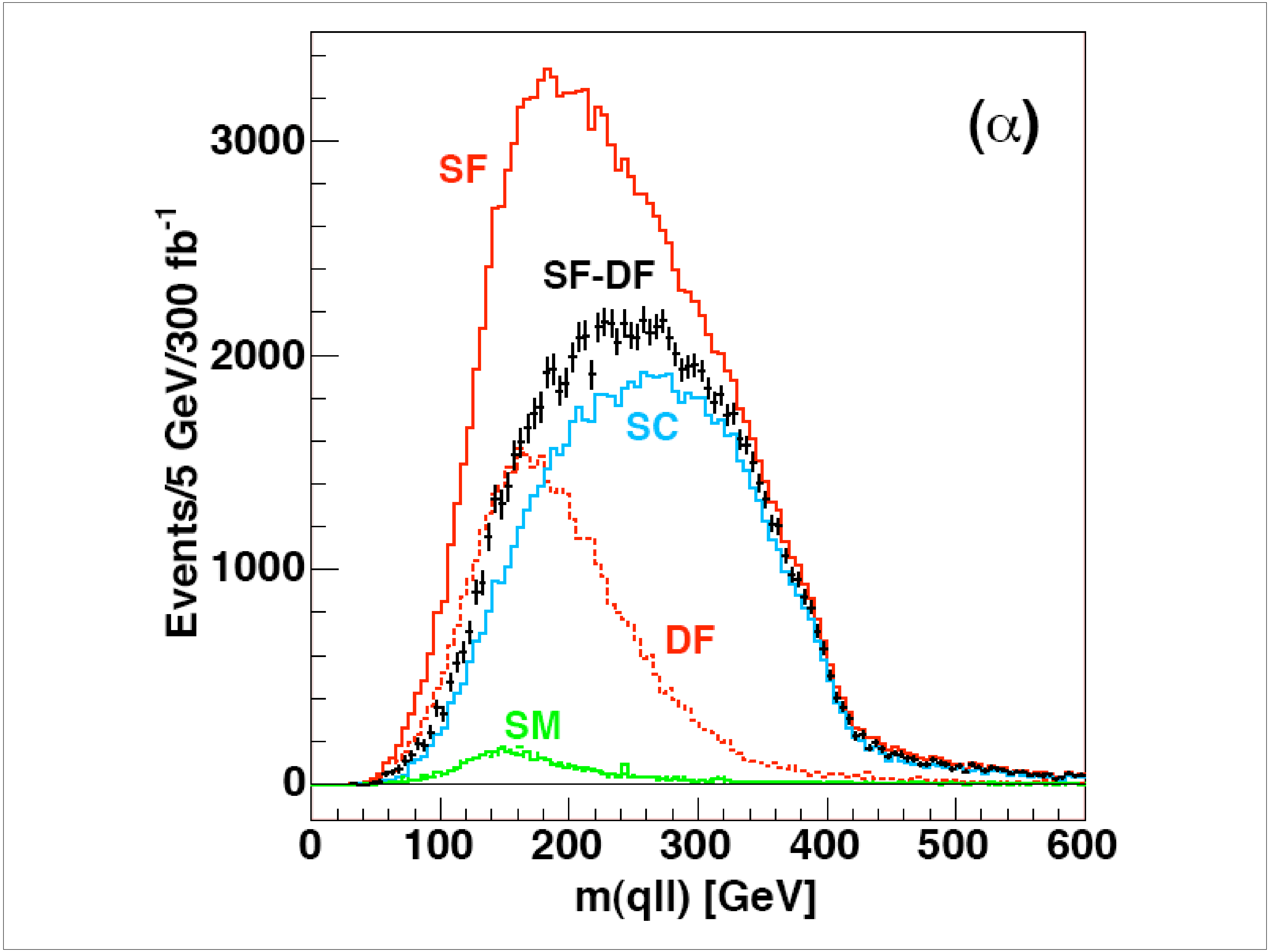
- ISAJET 7.58 defines low-energy model
- PYTHIA 6.2 with CTEQ 5L: Monte Carlo sample
- ATLFAST 2.60 simulates ATLAS detector
- precuts:
  - At least three jets, satisfying: $p_T^{\text{jet}} > 150, 100, 50$ GeV
  - $E_{T,\text{miss}} > \max(100 \text{ GeV}, 0.2 M_{\text{eff}})$
    with $M_{\text{eff}} \equiv E_{T,\text{miss}} + \sum_{i=1}^{3} p_{T,i}^{\text{jet}}$
  - Two isolated opposite-sign same-flavour leptons (e or μ),
    satisfying $p_T^{\text{lep}} > 20, 10$ GeV

SM background: 95% $t\bar{t}$

Aim: determine/study expected accuracy



SPS 1a (α)
(α)
dilepton mass
Events/1 GeV/300 fb^{-1}
3000
2000
1000
0
0 10 20 30 40 50 60 70 80 90 100
m(ll) [GeV]
SF
SF-DF
SC
DF
SM
Z
SF Same Flavour
SC Signal Chain
DF Different Flavour
Same shape as Same-Flavour SUSY background
SM Standard Model



(α)
SF
SF-DF
SC
DF
SM
Events/5 GeV/300 fb^{-1}
m(qll) [GeV]
0
1000
2000
3000
0
100
200
300
400
500
600

Extraction of masses

- simulate 10,000 ATLAS ‘experiments’
- focus on statistical uncertainty
- each endpoint: gaussian distribution
- invert endpoint formulas, fit masses
- what is chance of finding correct minimum?

Following Allanach et al, each endpt $E_i^{\exp}$ taken as:

$$\boxed{E_i^{\exp} = E_i^{\text{nom}} + A_i \sigma_i^{\text{stat}} + B \sigma_i^{\text{scale}}}$$

A, B picked from gaussian distribution, mean 0, width 1

One A for each endpoint,
one B for m_{ll}, other B for endpoints involving jets

determine masses

Minimize:

$$\boxed{\Sigma = [\mathbf{E}^{\exp} - \mathbf{E}^{\text{th}}(\mathbf{m})]^T \mathbf{W} [\mathbf{E}^{\exp} - \mathbf{E}^{\text{th}}(\mathbf{m})]}$$

$\mathbf{W}$ inverse error/correlation matrix

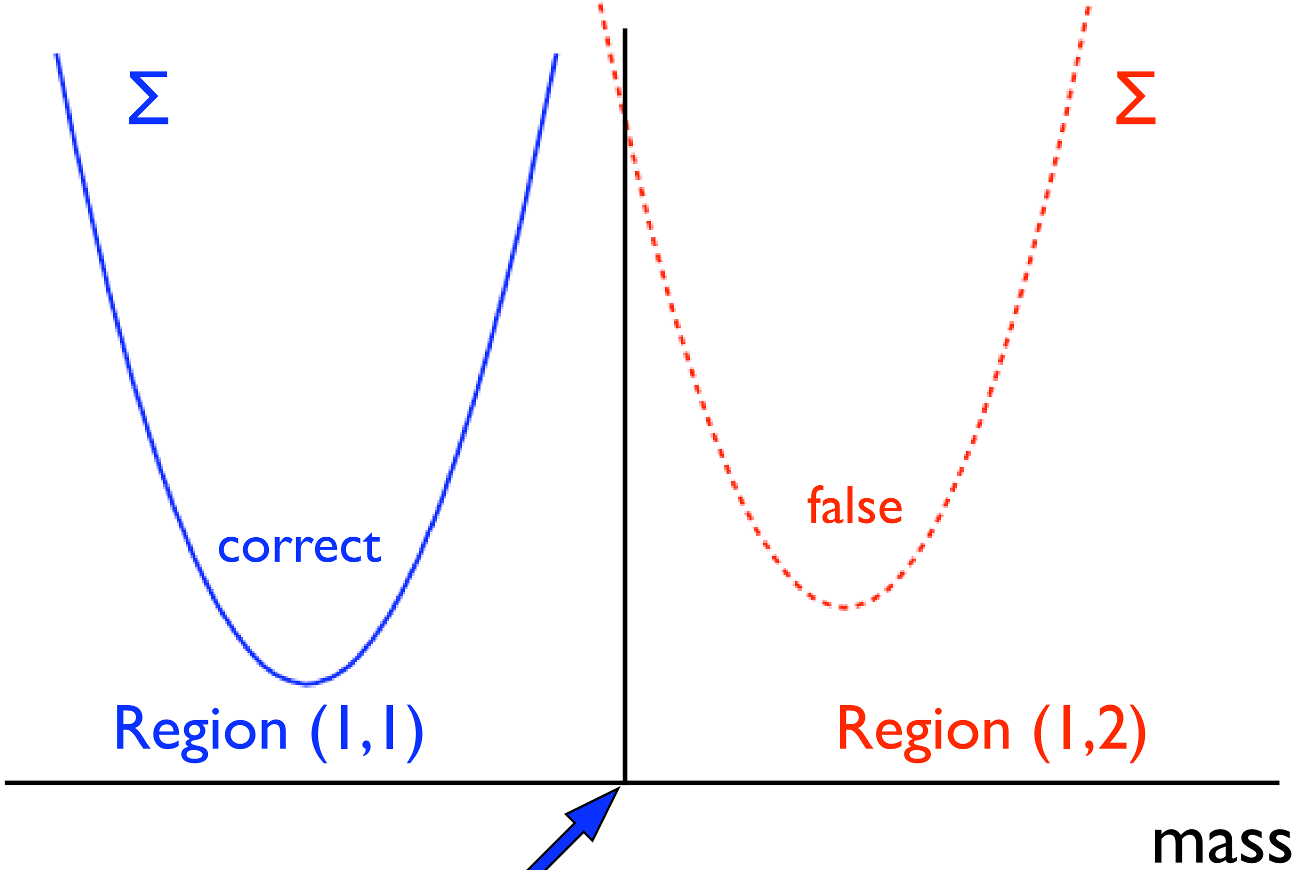
SPS 1a (α) $\Delta\Sigma \leq 1$

	nominal	correct fit			false fit		
		(1,1)			(1,2)		
	Nom	$\langle m \rangle$	σ	γ_1	$\langle m \rangle$	σ	γ_1
$m_{\tilde{\chi}_1^0}$	96.1	96.3	3.8	0.2	85.3	3.4	0.1
$m_{\tilde{l}_R}$	143.0	143.2	3.8	0.2	130.4	3.7	0.1
$m_{\tilde{\chi}_2^0}$	176.8	177.0	3.7	0.2	165.5	3.4	0.1
$m_{\tilde{q}_L}$	537.2	537.5	6.1	0.1	523.2	5.1	0.1
$m_{\tilde{b}_1}$	491.9	492.4	13.4	0.0	469.6	13.3	0.1
$m_{\tilde{l}_R} - m_{\tilde{\chi}_1^0}$	46.9	46.9	0.3	0.0	45.1	0.7	-0.2
$m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$	80.8	80.8	0.2	0.0	80.2	0.3	-0.1
$m_{\tilde{q}_L} - m_{\tilde{\chi}_1^0}$	441.2	441.3	3.1	0.0	438.0	2.7	0.0
$m_{\tilde{b}_1} - m_{\tilde{\chi}_1^0}$	395.9	396.2	12.0	0.0	384.4	12.0	0.1

Note: Three lightest masses are very correlated

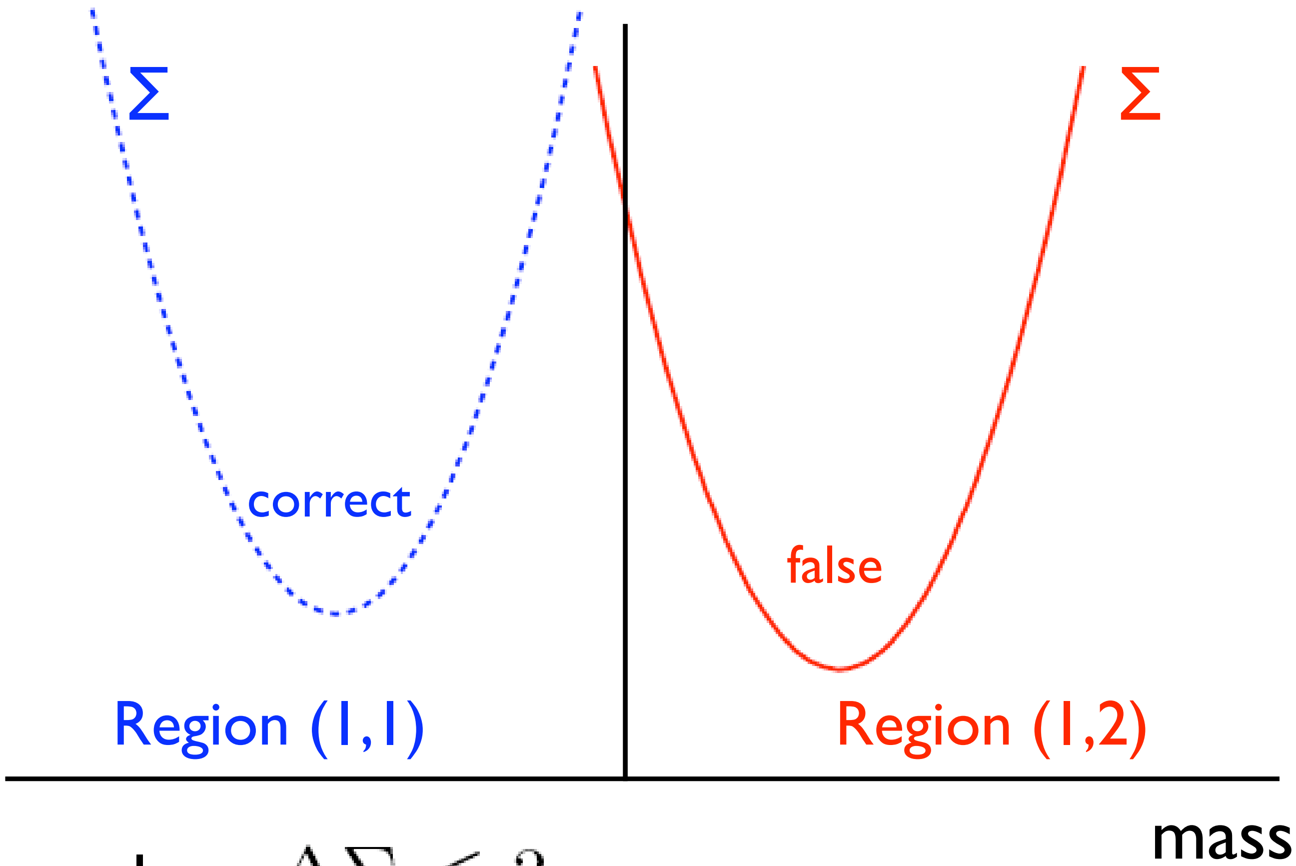
Problem due to compositeness of formulas:

If masses are close to border of ‘region’, may find a similar-quality or better minimum in ‘other’ region



$$m^2_{\tilde{l}_R} = (m^2_{\tilde{\chi}^0_1} + m^2_{\tilde{\chi}^0_2})/2$$

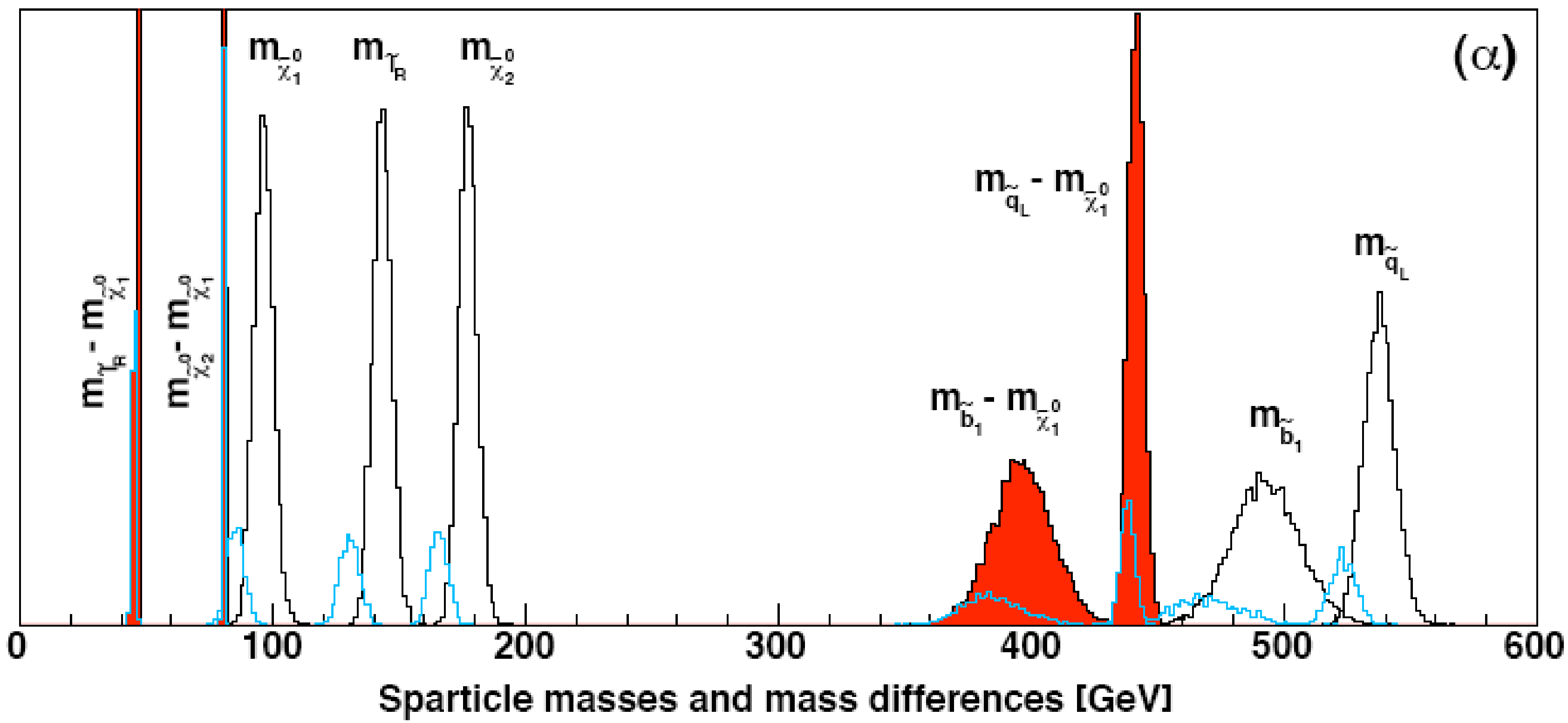
Measurement ‘error’ may interchange minima



Example: $\Delta\Sigma \leq 3$

30% chance of finding *two* minima

Masses and mass differences



black: correct solution

red: mass differences

blue-green: false solution (area prop to probability)

LC input ("fixing" LSP mass)

SPS 1a (α)

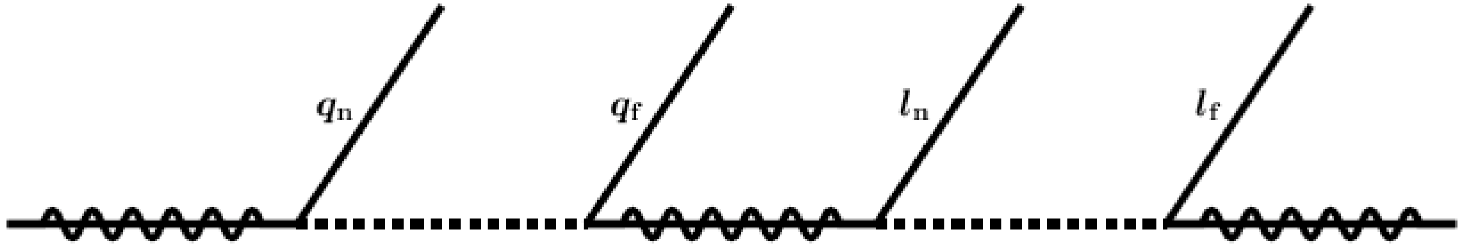
	Nom	(1,1) $\langle m \rangle$	(1,1) σ
$\tilde{\chi}_1^0$	96.05	96.05	0.05
$\tilde{l}_R$	142.97	142.97	0.29
$\tilde{\chi}_2^0$	176.82	176.82	0.17
$\tilde{q}_L$	537.25	537.2	2.5
$\tilde{b}_1$	491.92	492.1	11.7

Masses in GeV

SPS 1a (β)

		1 solution		2 solutions			
		(1,2)/(1,3)/B		(1,2)		(1,3)	
	Nom	$\langle m \rangle$	σ	$\langle m \rangle$	σ	$\langle m \rangle$	σ
$\tilde{\chi}_1^0$	161.02	161.02	0.05	161.02	0.05	161.02	0.05
$\tilde{l}_R$	221.86	221.15	3.26	222.22	1.32	217.48	1.01
$\tilde{\chi}_2^0$	299.05	299.15	0.57	299.11	0.53	299.05	0.52
$\tilde{q}_L$	826.29	826.1	6.3	825.9	5.8	828.6	5.5

Gluino cascade chain



$$\sigma = 35.4\ \text{pb} \qquad \tilde{g} \quad \left\{ \begin{array}{lcl} 23.0\% & \tilde{q}_L & 31.4\% \\ 17.0\% & \tilde{b}_1 & 35.5\% \\ 9.7\% & \tilde{b}_2 & 18.0\% \end{array} \right\} \quad \tilde{\chi}_2^0 \quad 12.1\% \quad \tilde{l}_R \quad 100\% \quad \tilde{\chi}_1^0 \qquad \begin{array}{r} 309\ \text{fb} \\ 259\ \text{fb} \\ 75\ \text{fb} \\ \hline 643\ \text{fb} \end{array}$$

SPS 1a numbers

Several new kinematical edges involving $q_{\rm n}$

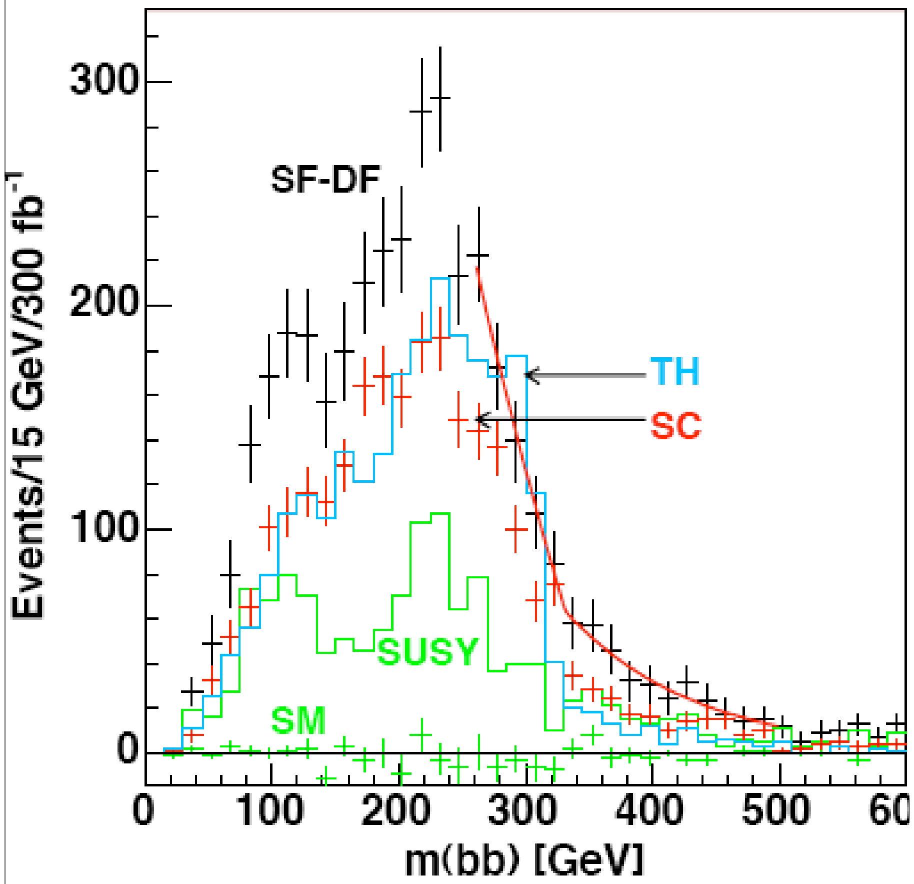
Only *one* new mass, need (minimum) only *one* more edge

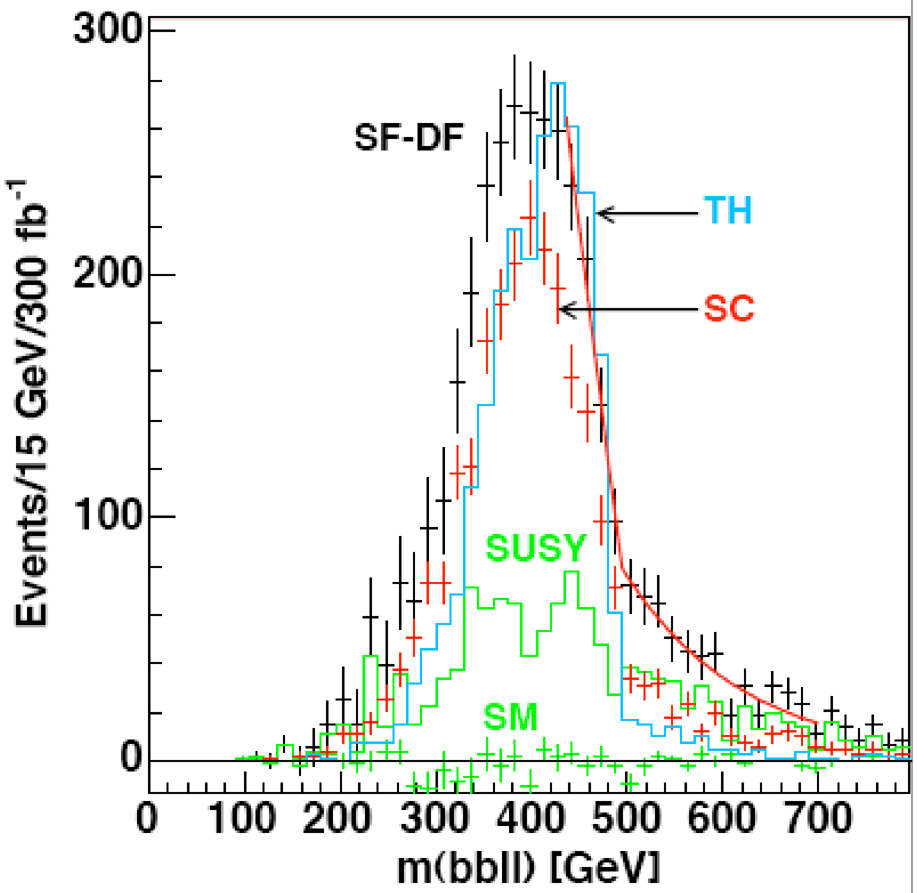
LHC simulation analogous to squark study

To reduce (SUSY) background, require two *b* jets

The majority of $\tilde{b}$'s are produced indirectly from gluino decay

SUSY background reduced from 80% to 35%





	Nom	*(1,1)* Mean	RMS
$m_{\tilde{\chi}_1^0}$	96.05	96.05	0.05
$m_{\tilde{l}_R}$	142.97	142.97	0.29
$m_{\tilde{\chi}_2^0}$	176.82	176.82	0.17
$m_{\tilde{q}_L}$	537.2	537.2	2.5
$m_{\tilde{b}_1}$	491.9	491.9	10.9
$m_{\tilde{g}}$	595.2	595.2	5.5
$m_{\tilde{g}} - m_{\tilde{b}_1}$	103.3	103.3	9.0

Mass values (all in GeV) from LHC+LC. Occurrences of *(1,2)* solutions are reduced to $\sim 1\%$, and left out.

Summary

- SPS 1a SUSY masses can be determined with precision 4-10 GeV
- Non-zero probability of fitting wrong minimum
  (could be off by 10-20 GeV)
- Gluino mass can be obtained using two b jets
- LC input on LSP mass (σ = 50 MeV) removes ambiguity
- LC input increases precision from 6 GeV (~15 GeV if wrong minimum) to 2.5 GeV