LCWS05
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Neutrinos in Supersymmetry

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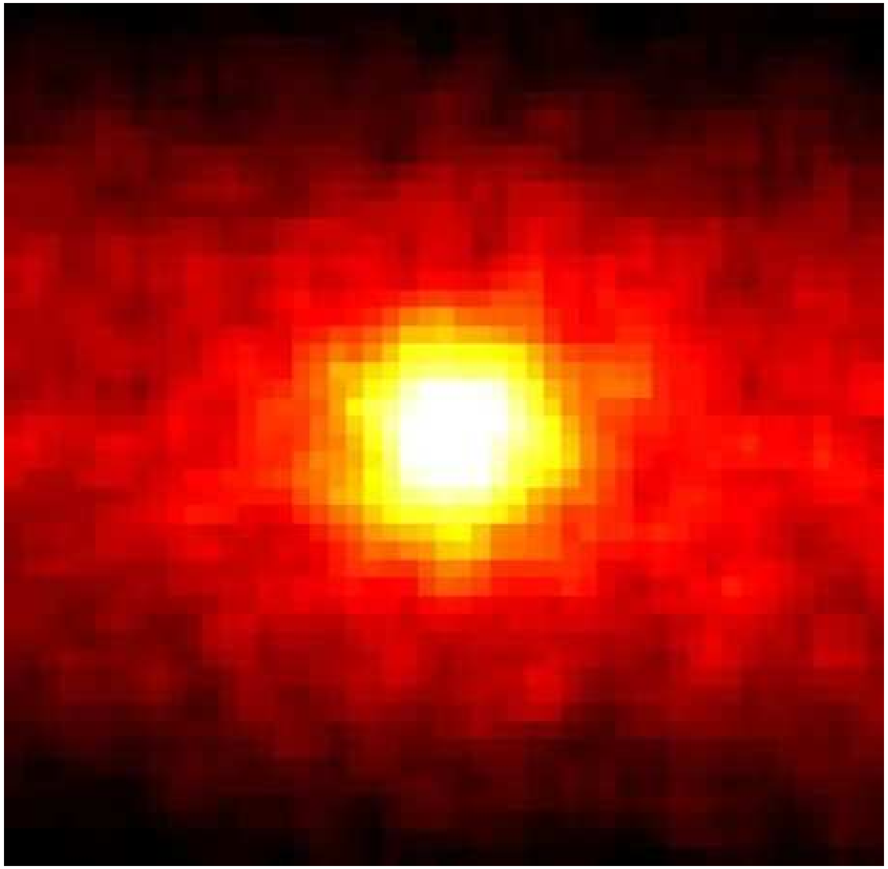
Solar Neutrinos

Nuclear reactions in the core of the sun produce neutrinos via the reaction: $4p \longrightarrow {}^4He + 2e^+ + \gamma + 2\nu_e$

Experiments have measured the flux of electron neutrinos arriving to the Earth from the Sun, and found a much lower flux than expected.

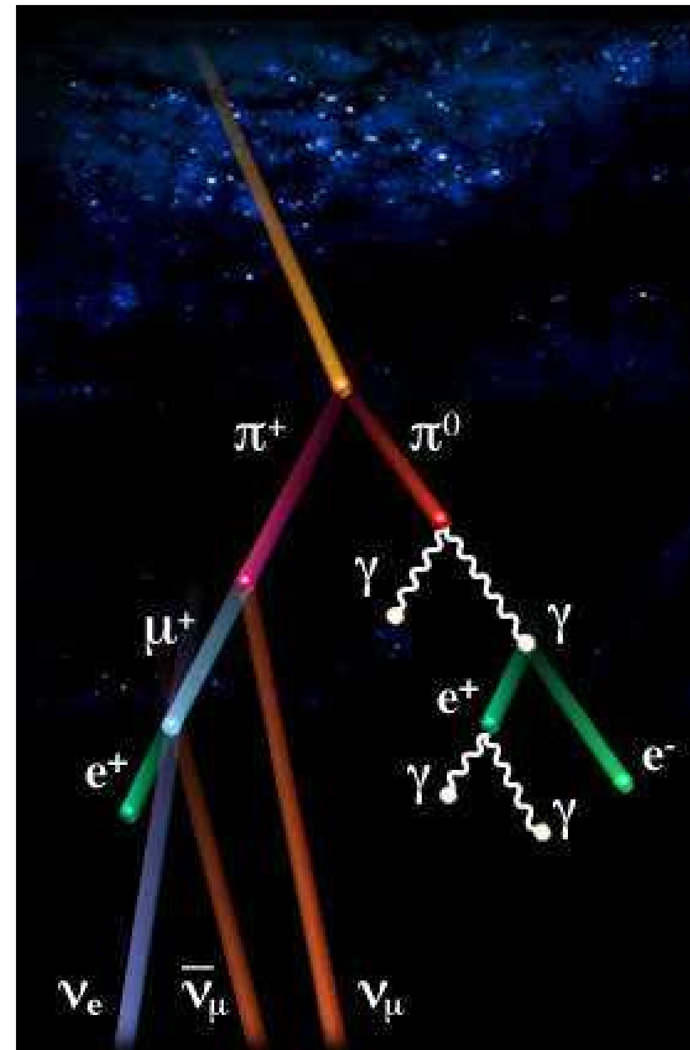
$\longrightarrow$ Solar Neutrino Anomaly



Solution: Neutrinos oscillate $\nu_e \longrightarrow \nu_{\mu/\tau}$

Atmospheric Neutrinos

Atmospheric neutrinos: produced in cascade decays after cosmic rays hit the atmosphere.



pion decays: $\pi^+ \to \mu^+ + \nu_\mu$

muon decay: $\mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu$

Expected: $2\nu_\mu$ per each ν_e. Measurements indicate a strong deficit of muon neutrinos.

$\to$ Atmospheric Neutrino Anomaly

Solution: Neutrinos oscillate $\nu_\mu \to \nu_\tau$

Neutrino Oscillations

The neutrino mass matrix is diagonalized by the rotation matrix V_{PMNS}, such that the mass eigenstates evolve in time as

$$\psi_i = \sum_j e^{-iE_j t} V_{PMNS}^{ij} \psi_j$$

Calculating transition probabilities in the ultra-relativistic limit, where

$$E_i \approx |\vec{p}| + \frac{m_i^2}{2|\vec{p}|}$$

leads to results (in the two neutrino approximation) like

$$P_{\nu_i \to \nu_j} = \sin^2(2\theta) \sin^2\left(1.27 \frac{\Delta m^2 L}{E}\right)$$

where θ is the mixing angle.

Three Neutrinos

A general 3×3 neutrino mass matrix is diagonalize by a Pontecorvo-Maki-Nakagawa-Sakata matrix of the type

$$V_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- θ_{23}: atmospheric angle
- θ_{13}: reactor angle
- θ_{12}: solar angle
- Δm^2_{23}: atmospheric squared mass difference
- Δm^2_{12}: solar squared mass difference

Experimental Constraints

Experimental results are consistent with the following values of the mixing angles and masses:

$$
\begin{aligned}
0.52 < \quad & \tan^2\theta_{23} & < 2.1 & \\
& \tan^2\theta_{13} & < 0.049 & \\
0.30 < \quad & \tan^2\theta_{12} & < 0.61 & \\
1.4\times 10^{-3} < \quad & \Delta m^2_{23} & < 3.3\times 10^{-3} & \quad \mathrm{eV}^2 \\
7.2\times 10^{-5} < \quad & \Delta m^2_{12} & < 9.1\times 10^{-5} & \quad \mathrm{eV}^2 \\
& m_{ee} & < 0.84 & \quad \mathrm{eV}
\end{aligned}
$$

There is no direct measurement of the scale of neutrino masses.

From Table 1 (3 σ values) hep-ph/0405172, M. Maltoni, T. Schwetz, M. Tortola, J. W. F. Valle

Bilinear R-Parity Violation

R-Parity and Lepton Number are violated by bilinear terms in the superpotential. The three parameters $\epsilon_\tau, \epsilon_\mu, \epsilon_e$ have units of mass:

$$W = W_{MSSM} + \epsilon_i \widehat{L}_i \widehat{H}_u$$

These terms induce sneutrino vacuum expectation values $< \tilde{\nu}_i >= v_i$, which contribute to the gauge boson masses:

$$v_u^2 + v_d^2 + v_1^2 + v_2^2 + v_3^3 = v^2 \sim (246\,\mathrm{GeV})^2$$

In the soft supersymmetry breaking potential the following terms are added:

$$V^{soft} = V^{soft}_{MSSM} + B_i \epsilon_i \widetilde{L}_i H_u$$

Neutralinos and Three Neutrinos

In basis $(\psi^0)^T = (-i\lambda', -i\lambda^3, \widetilde{H}_1^1, \widetilde{H}_2^2, \nu_e, \nu_\mu, \nu_\tau)$ the neutralino/neutrino mass matrix is

$$\mathbf{M}_N \;=$$

$$\begin{bmatrix} M_1 & 0 & -\frac{1}{2}g'v_d & \frac{1}{2}g'v_u & -\frac{1}{2}g'v_1 & -\frac{1}{2}g'v_2 & -\frac{1}{2}g'v_3 \\ 0 & M_2 & \frac{1}{2}gv_d & -\frac{1}{2}gv_u & \frac{1}{2}gv_1 & \frac{1}{2}gv_2 & \frac{1}{2}gv_3 \\ -\frac{1}{2}g'v_d & \frac{1}{2}gv_d & 0 & -\mu & 0 & 0 & 0 \\ \frac{1}{2}g'v_u & -\frac{1}{2}gv_u & -\mu & 0 & \epsilon_1 & \epsilon_2 & \epsilon_3 \\ -\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & \epsilon_1 & 0 & 0 & 0 \\ -\frac{1}{2}g'v_2 & \frac{1}{2}gv_2 & 0 & \epsilon_2 & 0 & 0 & 0 \\ -\frac{1}{2}g'v_3 & \frac{1}{2}gv_3 & 0 & \epsilon_3 & 0 & 0 & 0 \end{bmatrix}$$

and a neutrino 3×3 mass matrix is induced.

Low Energy See-Saw

Low energy see-saw mechanism with three neutrinos

$$\mathbf{M}_N = \begin{bmatrix} \mathcal{M}_{\chi^0} & m^T \\ m & 0 \end{bmatrix} \Longrightarrow m_{eff} = -m \cdot \mathcal{M}_{\chi^0}^{-1} \cdot m^T$$

defining $\Lambda_i = \mu v_i + \epsilon_i v_d$, which are proportional to the sneutrino vev's v'_i, the effective mass matrix is

$$m_{eff} = \frac{M_1 g^2 + M_2 g'^2}{4\,det(\mathcal{M}_{\chi^0})} \begin{bmatrix} \Lambda_e^2 & \Lambda_e\Lambda_\mu & \Lambda_e\Lambda_\tau \\ \Lambda_e\Lambda_\mu & \Lambda_\mu^2 & \Lambda_\mu\Lambda_\tau \\ \Lambda_e\Lambda_\tau & \Lambda_\mu\Lambda_\tau & \Lambda_\tau^2 \end{bmatrix} \sim A^{(0)}\Lambda_i\Lambda_j$$

and only one neutrino acquire mass at tree level

$$m_{\nu_3} = Tr(m_{eff}) = \frac{M_1 g^2 + M_2 g'^2}{4\,det(\mathcal{M}_{\chi^0})} |\vec{\Lambda}|^2$$

Neutrino Angles at Tree Level

The diagonalization $V_\nu^T m_{eff} V_\nu = \mathrm{diag}(0,0,m_\nu)$ is performed with two rotations:

$$V_\nu = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & -\sin\theta_{23} \\ 0 & \sin\theta_{23} & \cos\theta_{23} \end{bmatrix} \times \begin{bmatrix} \cos\theta_{13} & 0 & -\sin\theta_{13} \\ 0 & 1 & 0 \\ \sin\theta_{13} & 0 & \cos\theta_{13} \end{bmatrix}$$

The atmospheric angle θ_{23} and the reactor angle θ_{13} are simple functions of the Λ_i

$$\tan\theta_{23} = -\frac{\Lambda_\mu}{\Lambda_\tau} \qquad \tan\theta_{13} = -\frac{\Lambda_e}{\sqrt{\Lambda_\mu^2 + \Lambda_\tau^2}}$$

and the solar angle is undefined at tree level.

Neutrinos at One Loop

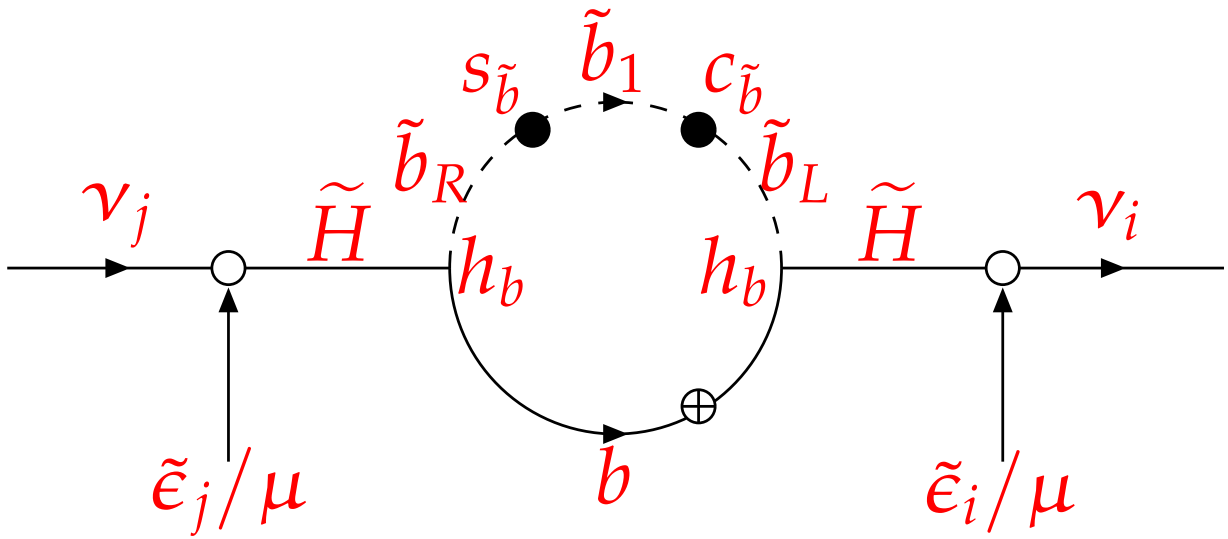
Based on work by,

M.A. Díaz, M. Hirsch, W. Porod, J. Romao, J.W.F. Valle

Phys. Rev. D$\mathbf{68}$, 013009 (2003)
Phys. Rev. D$\mathbf{62}$, 113008 (2000), *E: D*$\mathbf{65}$, 119901 (2002)
Phys. Rev. D$\mathbf{61}$, 071703 (2000)

Simplified Bottom Sbottom Loop

$$\Delta\widetilde{\Pi}_{ij}(p^2) =$$

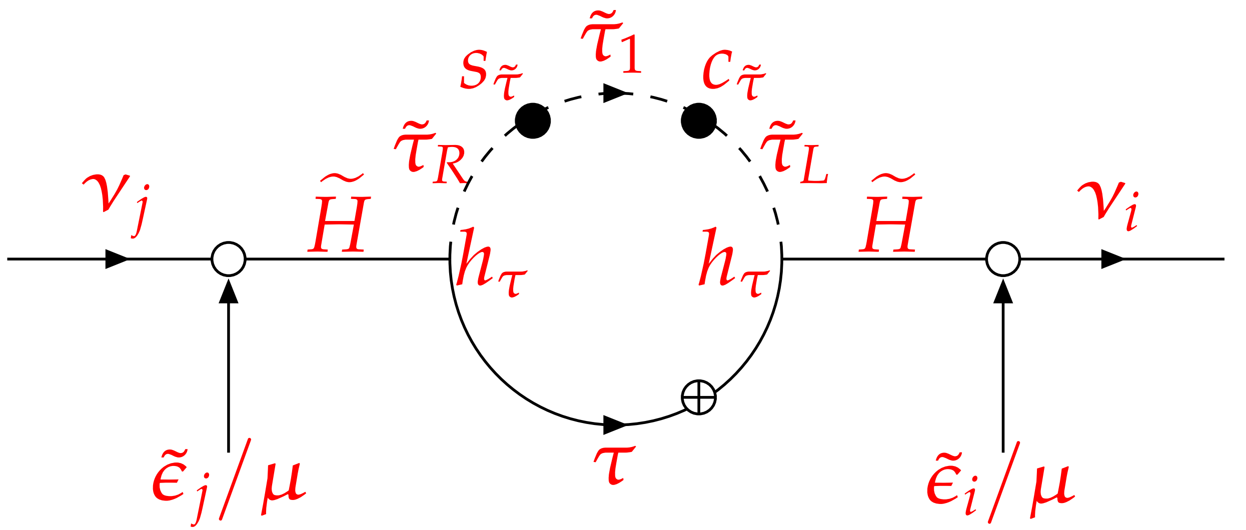


$$\approx -\frac{N_c m_b}{16\pi^2}\left\{2 s_{\tilde{b}} c_{\tilde{b}} h_b^2 \frac{\tilde{\epsilon}_i \tilde{\epsilon}_j}{\mu^2}\left[B_0(p^2, m_{\tilde{b}_1}^2, m_b^2) - B_0(p^2, m_{\tilde{b}_2}^2, m_b^2)\right]\right\}$$

- Proportional to the bottom quark Yukawa squared
- Quadratic in the ϵ parameters
- Finite
- Approximately proportional to $\log(m_{\tilde{b}_1}^2/m_{\tilde{b}_2}^2)$

Simplified Tau Stau Loop

$$\Delta\widetilde{\Pi}_{ij}(p^2) =$$

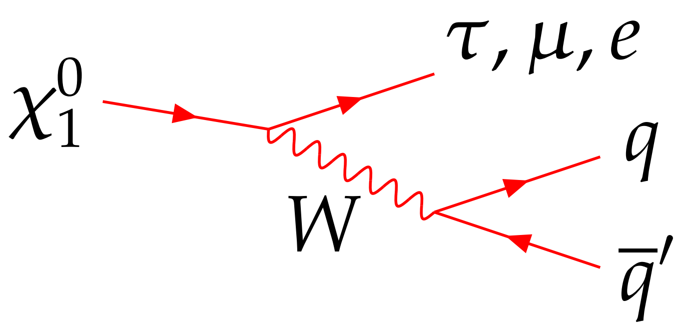


$$\approx -\frac{m_\tau}{16\pi^2}\left\{2 s_{\tilde{\tau}} c_{\tilde{\tau}} h_\tau^2 \frac{\tilde{\epsilon}_i \tilde{\epsilon}_j}{\mu^2}\left[B_0(p^2, m_{\tilde{\tau}_1}^2, m_\tau^2) - B_0(p^2, m_{\tilde{\tau}_2}^2, m_\tau^2)\right]\right\}$$

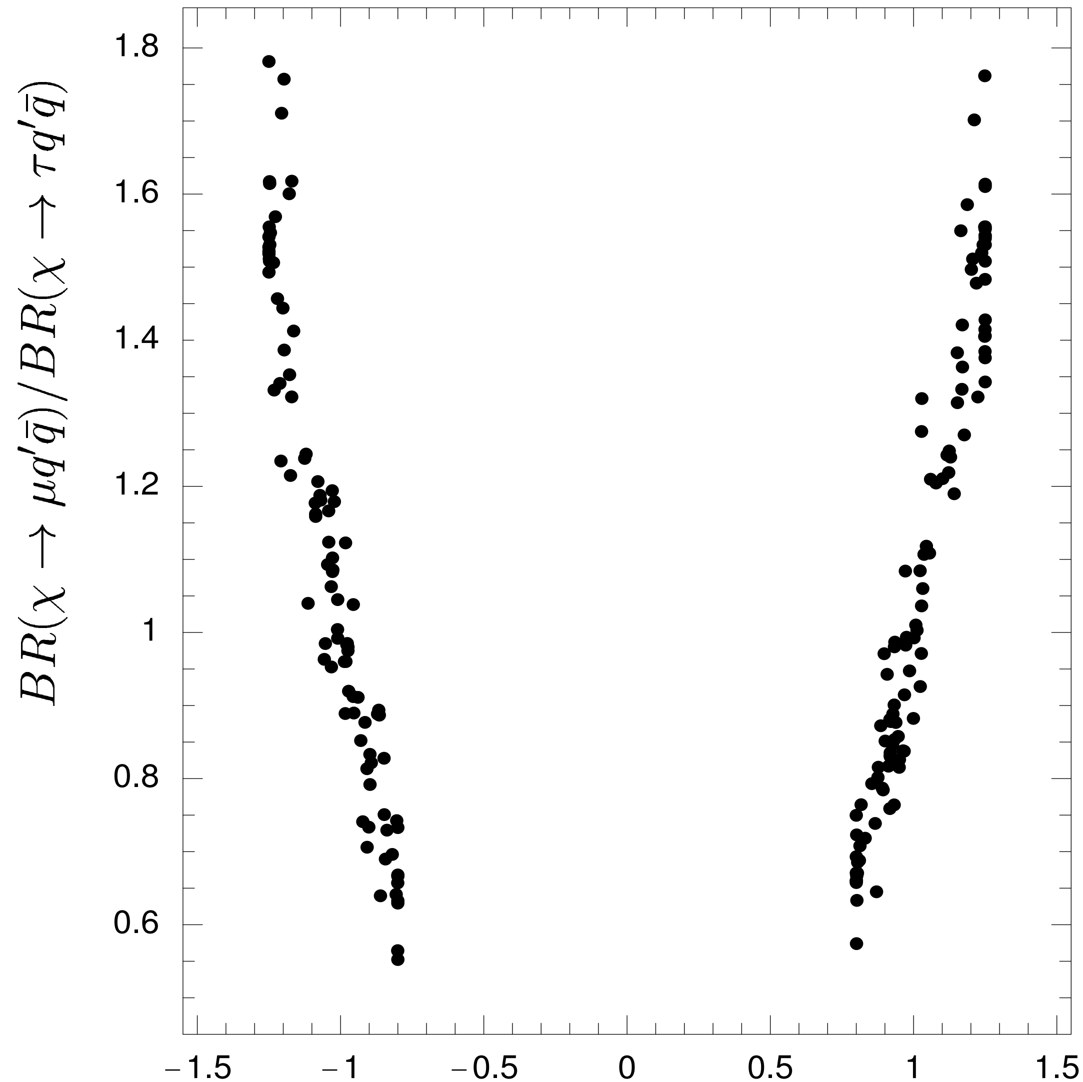
- Proportional to the tau lepton Yukawa squared
- Quadratic in the ϵ parameters
- Finite
- Approximately proportional to $\log(m_{\tilde{\tau}_1}^2/m_{\tilde{\tau}_2}^2)$

Neutralino Decays

In the presence of BRpV a neutralino LSP is not stable:



Ratios of BR are closely related to the Λ_i parameters.



Collider Physics $\Leftrightarrow$ Neutrino Physics

AMSB: Three Neutrinos

Based on work by,

F. de Campos, M.A. Díaz, O.J.P. Eboli, R.A. Lineros, M.B. Magro, and P.G. Mercadante

hep-ph/0409043

AMSB Reference Scenario

We work in the following AMSB scenario

$$m_{3/2} = 35\text{TeV}\,, m_0 = 250\text{GeV}\,, \tan\beta = 15\,, \text{sign}(\mu) < 0\,.$$

and spectrum part of the spectrum is

- $m_{\chi_1^0} = 108.17$ GeV, lightest neutralino (LSP)
- $m_{\chi_1^\pm} = 108.18$ GeV, lightest chargino
- $m_{\tilde{\tau}_1} = 155$ GeV, lightest charged slepton
- $m_{\tilde{\nu}_\tau} = 191$ GeV, lightest sneutrino
- $m_{\tilde{t}_1} =$ GeV, lightest squark
- $m_h = 112$ GeV, lightest Higgs boson
- $m_{H^\pm} = 600$ GeV, charged Higgs boson

BRpV Reference Scenario

We work in the following BRpV scenario

$$\epsilon_1 = -0.015 \text{ GeV}\,, \qquad \Lambda_1 = -0.03 \text{ GeV}^2\,,$$
$$\epsilon_2 = -0.018 \text{ GeV}\,, \qquad \Lambda_2 = -0.09 \text{ GeV}^2\,,$$
$$\epsilon_3 = 0.011 \text{ GeV}\,, \qquad \Lambda_3 = -0.09 \text{ GeV}^2\,.$$

predicting the following neutrino physics parameters at one loop

$$\Delta m^2_{\mathrm{atm}} = 2.5 \times 10^{-3}\,\mathrm{eV}^2\,, \qquad \tan^2\theta_{\mathrm{atm}} = 0.73\,,$$
$$\Delta m^2_{\mathrm{sol}} = 7.8 \times 10^{-5}\,\mathrm{eV}^2\,, \qquad \tan^2\theta_{\mathrm{sol}} = 0.47\,,$$
$$m_{ee} = 0.0043\,\mathrm{eV}\,, \qquad \tan^2\theta_{13} = 0.033\,,$$

within experimental bounds.

Texture: Numerical View

The one-loop neutrino mass matrix has the form

$$\mathbf{M}^{\rm eff} \approx \begin{bmatrix} \lambda & 2\lambda & \lambda \\ 2\lambda & a & b \\ \lambda & b & m \end{bmatrix}$$

$$m = 0.031\text{eV}\,, \;\; \lambda \approx 0.12\,, \;\; a/m = 0.74\,, \;\; b/m = 0.66\,.$$

The two heavy neutrinos have an approximated mass

$$m_{\nu_{2,3}} \approx \frac{1}{2}\left[m + a \pm \sqrt{(m-a)^2 + 4b^2}\right]$$

leading to $\Delta m^2_{\rm atm} \approx 2.3 \times 10^{-3}\text{eV}^2$. Atm mixing angle is

$$\tan 2\theta_{\rm atm} \approx \frac{2b}{m-a}$$

leading to $\tan^2\theta_{\rm atm} \approx 0.68$, in agreement with complete results.

Texture: Theoretical View

The neutrino mass matrix can be approximated to

$$\mathbf{M}^{\rm eff}_{ij} = A\Lambda_i\Lambda_j + B(\epsilon_i\Lambda_j + \epsilon_j\Lambda_i) + C\epsilon_i\epsilon_j \,,$$

with $A \approx 3 \text{ eV}/\text{GeV}^4$, $B \approx -2 \text{ eV}/\text{GeV}^3$, and $C \approx 15 \text{ eV}/\text{GeV}^2$. Neglecting further small terms,

$$\mathbf{M}^{\rm eff} \approx \begin{bmatrix} A\Lambda_1^2 + 2B\Lambda_1\epsilon_1 + C\epsilon_1^2 & \sim & \sim \\ A\Lambda_1\Lambda_2 + B(\epsilon_1\Lambda_2 + \epsilon_2\Lambda_1) + C\epsilon_1\epsilon_2 & A\Lambda_2^2 + 2B\epsilon_2\Lambda_2 & \sim \\ A\Lambda_1\Lambda_3 + C\epsilon_1\epsilon_3 & A\Lambda_2\Lambda_3 & A\Lambda_3^2 + 2B\epsilon_3\Lambda_3 \end{bmatrix}$$

- One-loop genereted parameters B and C are very important.
- Small values of ϵ_i and Λ_1 imply small λ, which imply small solar and reactor angle.

Approximations

Neglecting terms in the first column and row we find for the atmospheric angle

$$\tan 2\theta_{\rm atm} = \frac{2A\Lambda_2\Lambda_3}{A(\Lambda_3^2 - \Lambda_2^2) + 2B(\epsilon_3\Lambda_3 - \epsilon_2\Lambda_2)} \,.$$

and the heavier neutrino masses

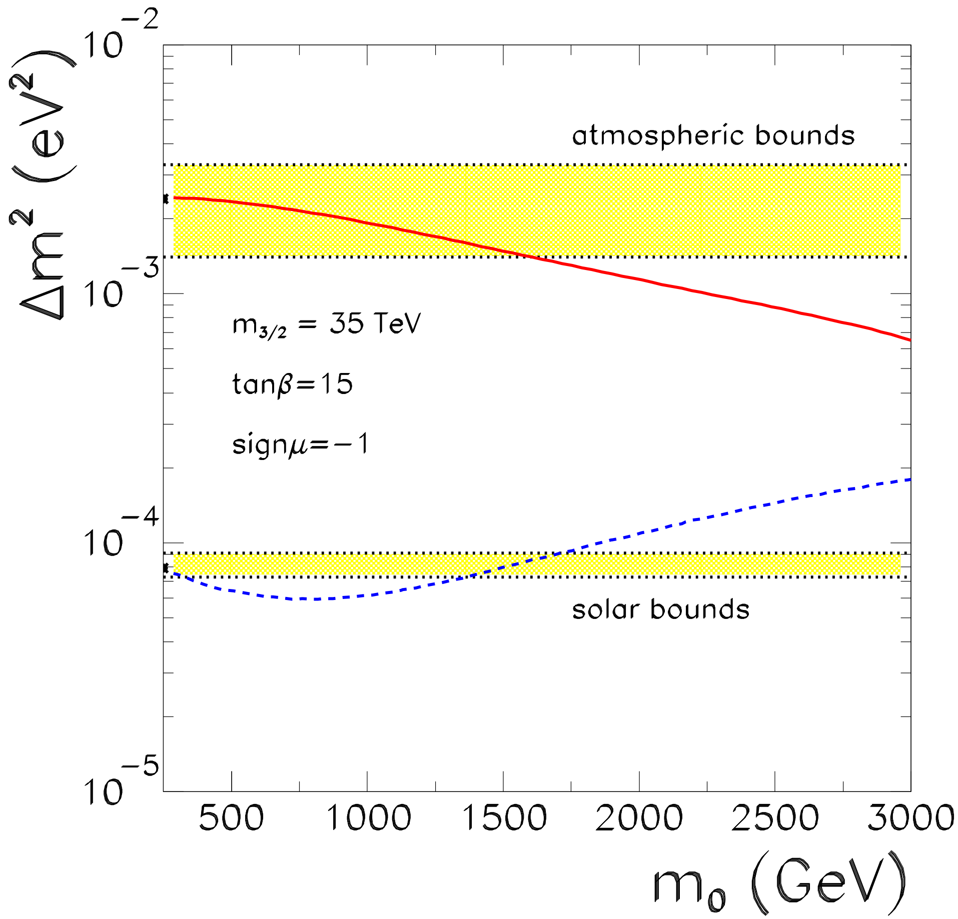
$$\begin{aligned} m_{\nu_{2,3}} \; &= \; \tfrac{1}{2}A(\Lambda_3^2 + \Lambda_2^2) + B(\epsilon_3\Lambda_3 + \epsilon_2\Lambda_2) \\ &\pm \sqrt{\left[\tfrac{1}{2}A(\Lambda_3^2 - \Lambda_2^2) + B(\epsilon_3\Lambda_3 - \epsilon_2\Lambda_2)\right]^2 + A^2\Lambda_2^2\Lambda_3^2} \;, \end{aligned}$$

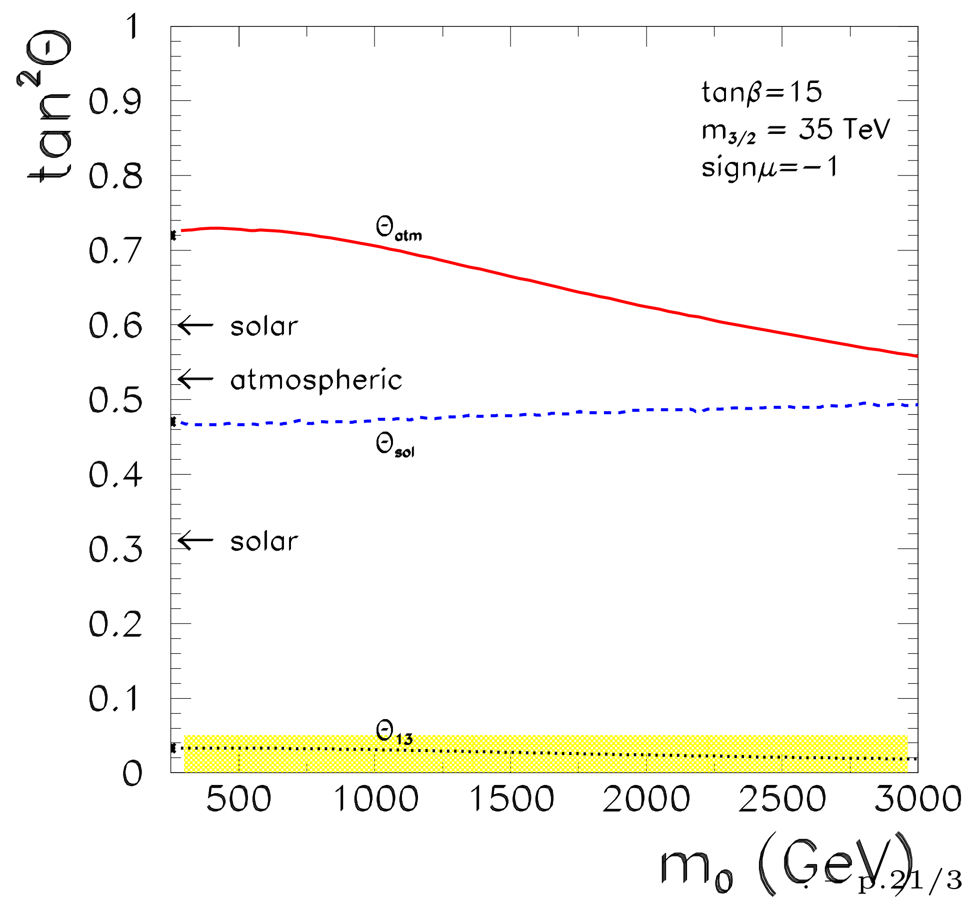
with

$$\Delta m^2_{atm} = m^2_{\nu_3} - m^2_{\nu_2} \,, \qquad \Delta m^2_{sol} \approx m^2_{\nu_2}$$

Scalar Mass Dependence

- At tree level only ν_3 has non zero mass.
- Radiative corrections are important.
- Strong dependence of atmospheric and solar mass squared differences on the scalar mass m_0.

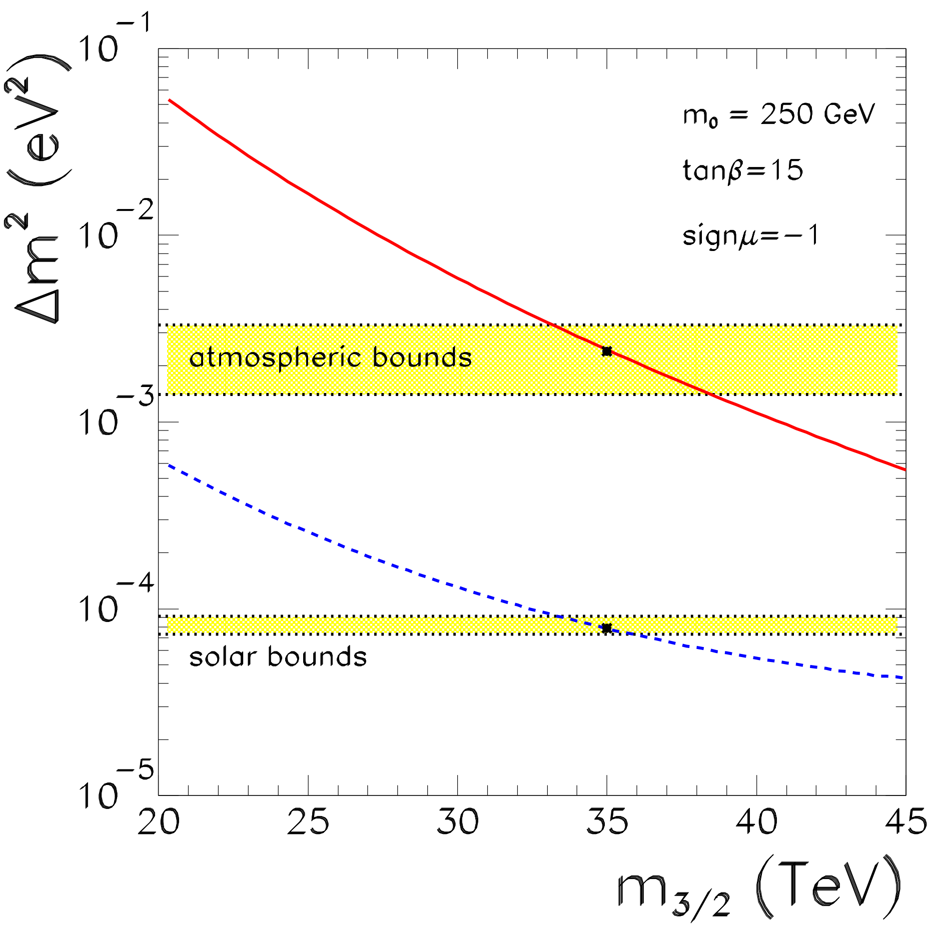




Gravitino Mass Dependence

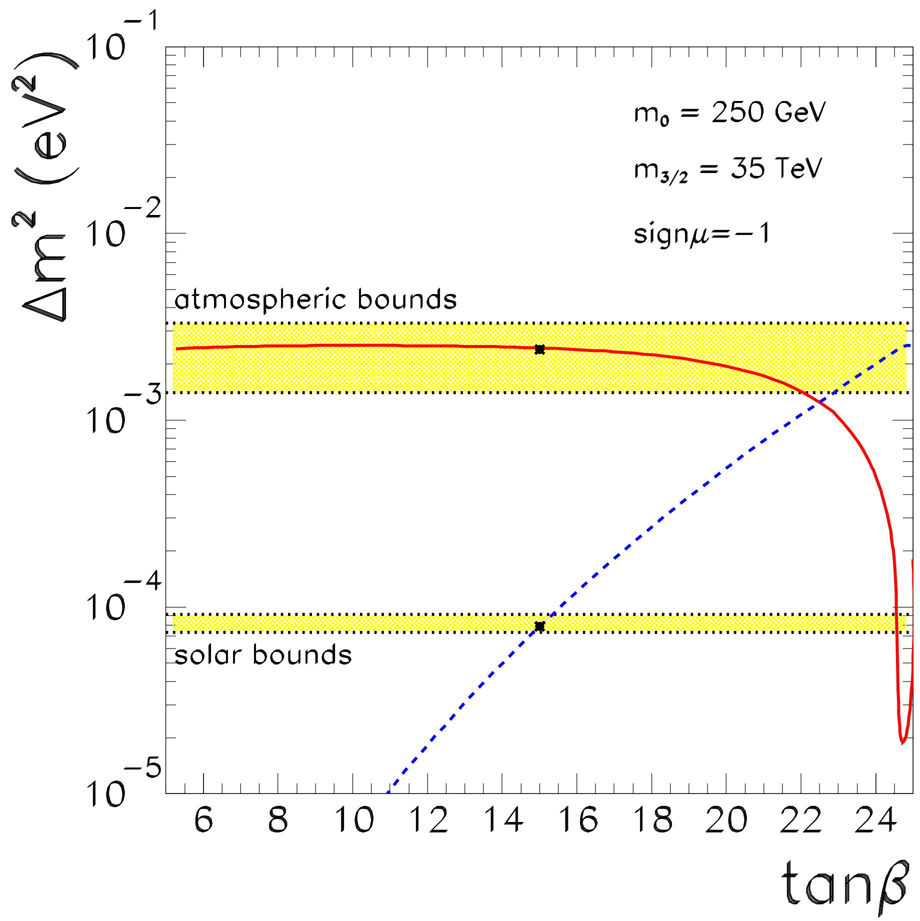
- Very strong monotonically descendent dependence of atmospheric and solar mass squared differences on the gravitino mass $M_{3/2}$.

- Solutions in a small band near $M_{3/2} = 35$ TeV.
- This band depends on the values of $\tan\beta$ and m_0.



Tan(beta) Dependence

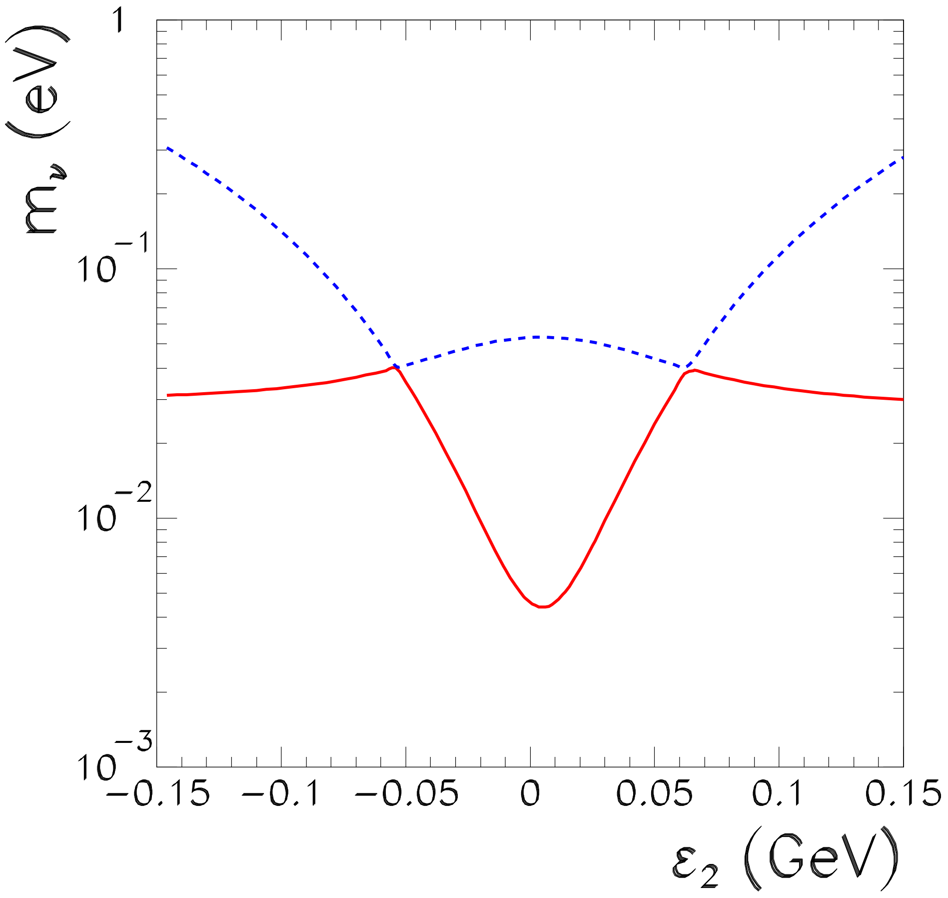
- Very strong dependence of atmospheric and solar mass squared differences on $\tan\beta$.
  - Solutions in a small band near $M_{3/2} = 35$ TeV.
  - This band depends on the values of $\tan\beta$ and m_0.
  - At large $\tan\beta$ radiative corrections become very important.



Epsilon2 dependence

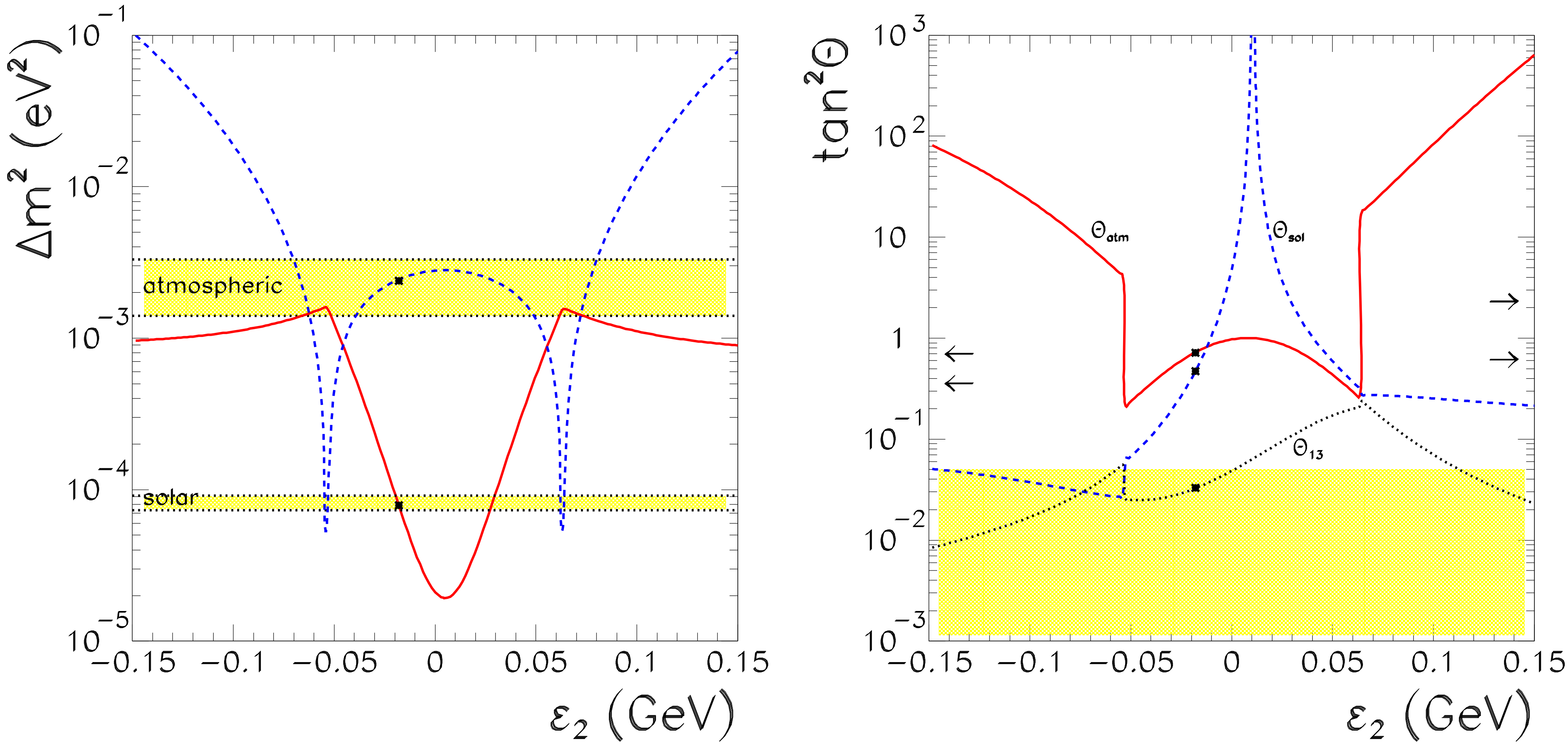
Masses of the two heavy neutrinos as a function of ϵ_2.

- The eigenvalue which is less dependent on ϵ_2 has a large tree level component.
- Radiative corrections are very important for large values of ϵ_2.
- Lightest neutrino mass is smaller than 5×10^{-5} eV.



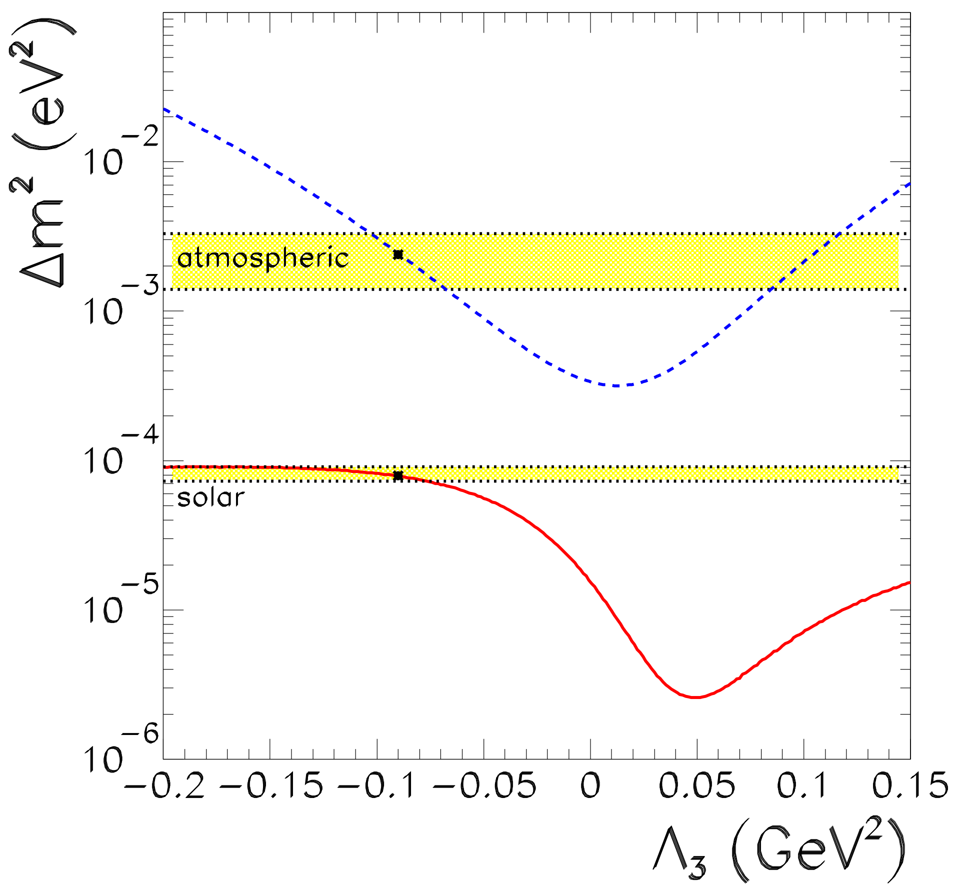
Epsilon2 Dependence

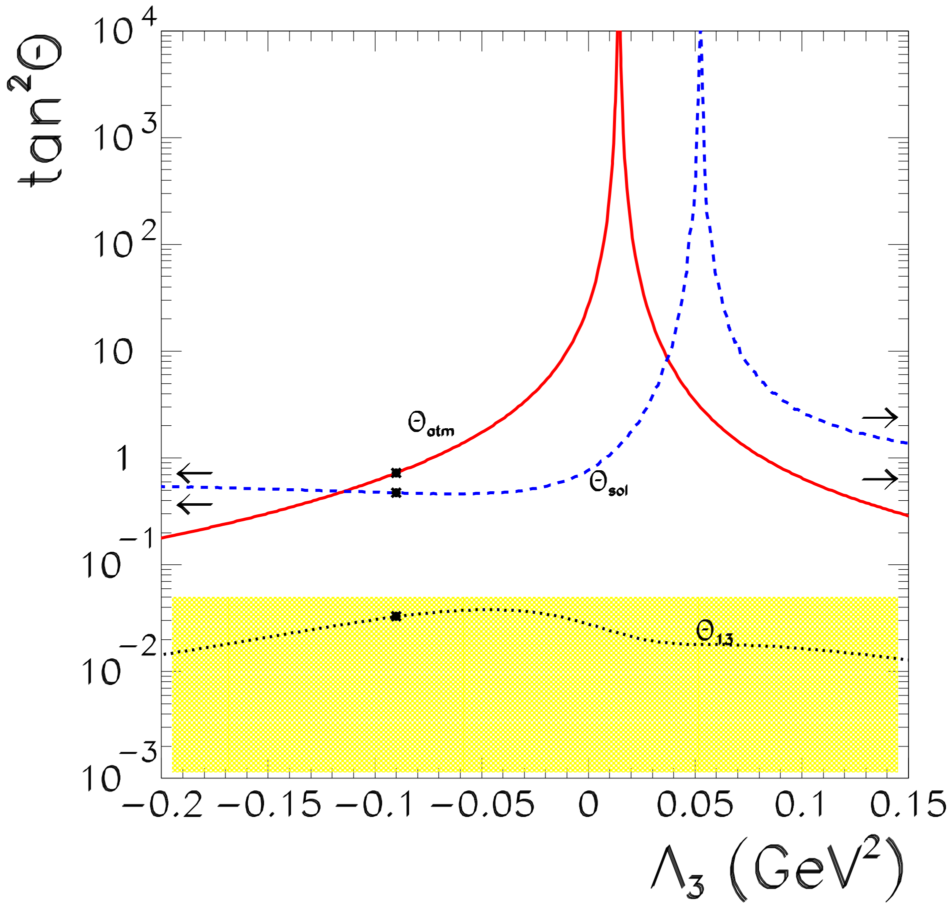
- There is a very large dependence of the mass differences and mixing angles on ϵ_2.
- Spikes are due to eigenvalue crossing.



Lambda3 Dependence

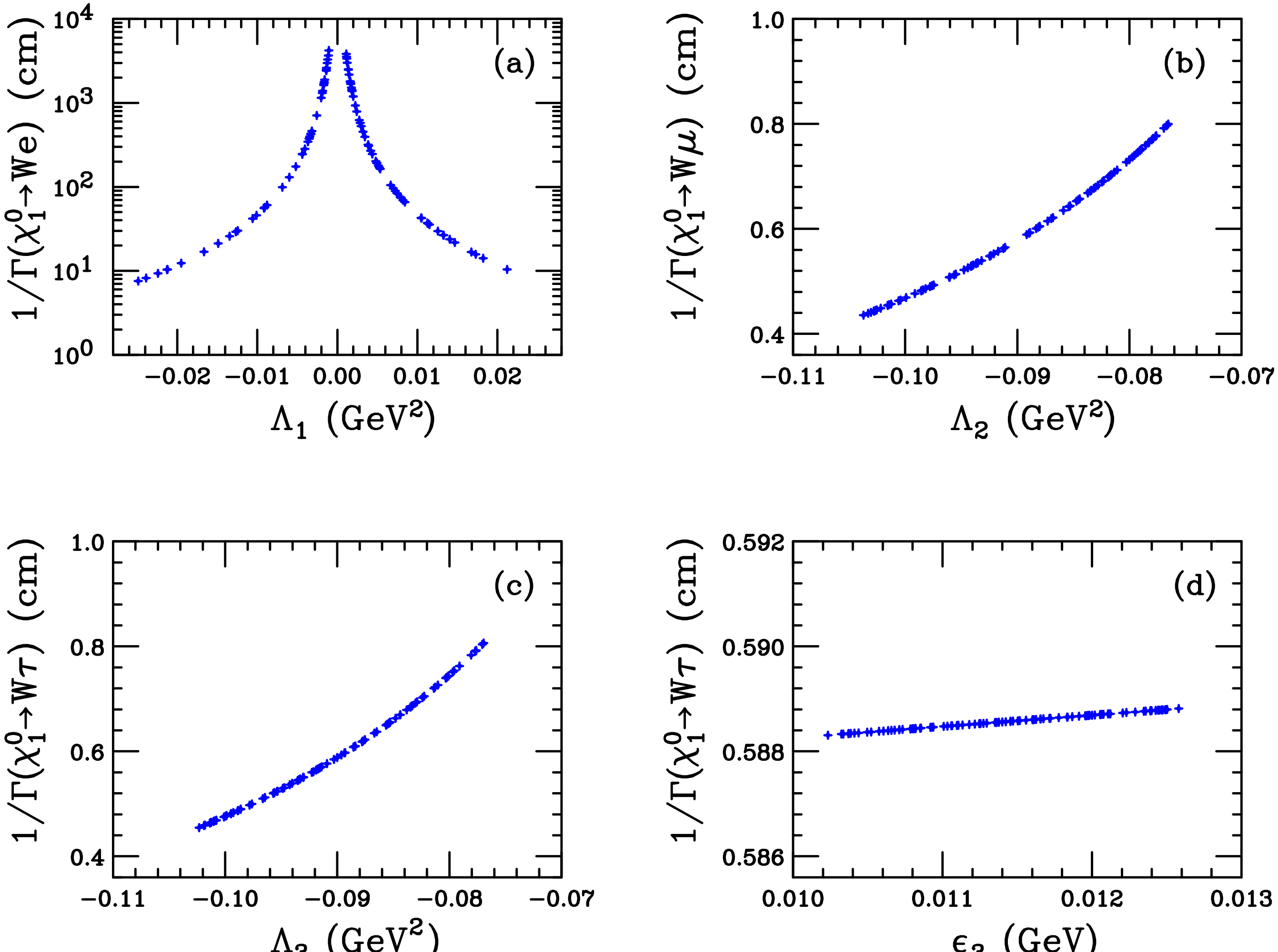
- There is a large dependence of the mass differences and mixing angles on Λ_3.





Neutralino Decays

- $\Gamma(\chi_1^0 \to We)$ depends only on Λ_1.
- $\Gamma(\chi_1^0 \to W\mu)$ depends only on Λ_2.
- $\Gamma(\chi_1^0 \to W\tau)$ depends on Λ_3 and only weakly on ϵ_3.
- The LSP decays inside the detector.
- $\Lambda_1 = 0$ also is a good solution.



Sugra: Three Neutrinos

Based on work by,

M.A. Díaz, Clemencia Mora, and Alfonso Zerwekh

hep-ph/0410285

Sugra Parameters at the GUT Scale

Sugra is characterized by the following parameters, all defined at the GUT scale, exept fot $\tan\beta$ which is defined at the SUSY scale:

- m_0: Universal scalar mass.
- $M_{1/2}$: Universal gaugino mass.
- $\tan\beta$: Ratio between vev's.
- A_0: Common trilinear coupling.
- $\mathrm{sign}(\mu)$: Sign of higgsino mass.

In BRpV we add ϵ_i and Λ_i as input at the SUSY scale.

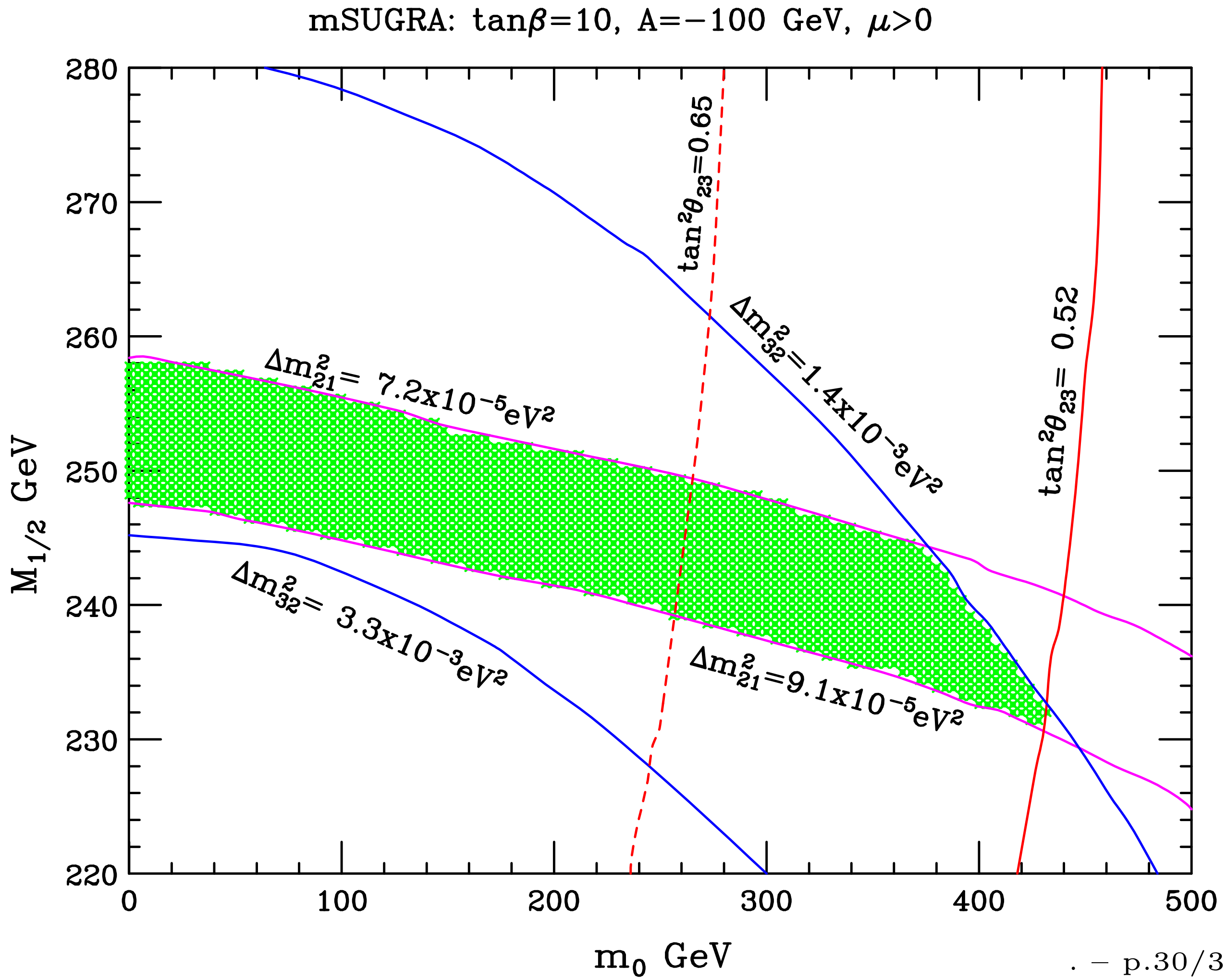
Neutrinos in Supergravity

Solutions to neutrino physics in a Sugra model with universal soft terms at the GUT scale, exept for ϵ_i and $B_i (\Rightarrow \Lambda_i)$, which are free at the weak scale.

Input:

$$\epsilon_1 = -0.0004\,\text{GeV}$$
$$\epsilon_2 = 0.052\,\text{GeV}$$
$$\epsilon_3 = 0.051\,\text{GeV}$$

$$\Lambda_1 = 0.022\,\text{GeV}^2$$
$$\Lambda_2 = 0.0003\,\text{GeV}^2$$
$$\Lambda_3 = 0.039\,\text{GeV}^2$$



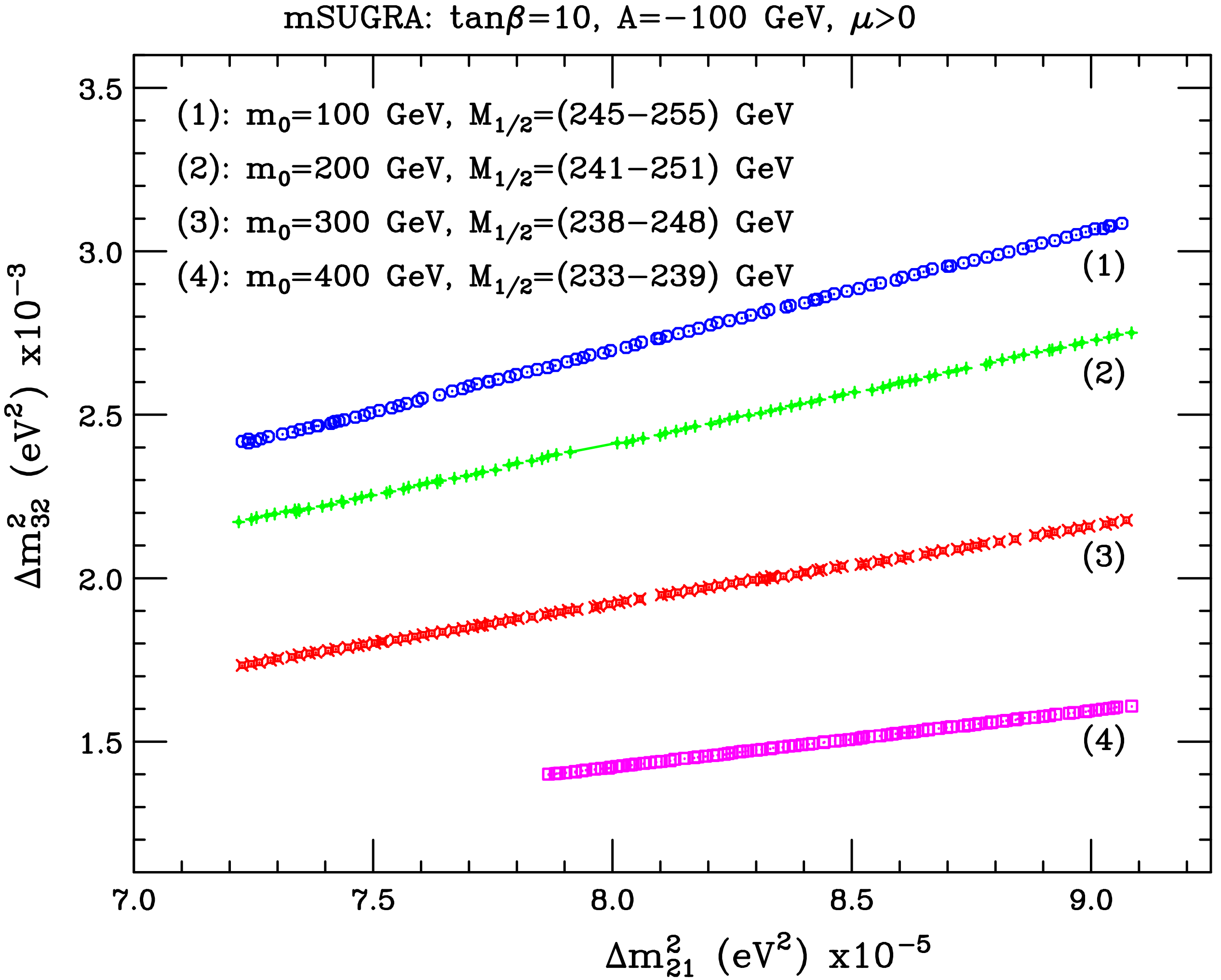
Atmospheric Neutrinos in Sugra

Influence of universal scalar mass m_0 and universal gaugino mass $M_{1/2}$ on the atmospheric mass and angle.

Input:

$|\vec{\epsilon}| = 0.073\,\mathrm{GeV}$

$\sqrt{|\vec{\Lambda}|} = 0.21\,\mathrm{GeV}$



Sugra: scan on neutrino parameters

For a fixed sugra point in parameter space, ϵ_i and Λ_i are randomly varied, accepting solutions with good masses and mixing angles.

Spectrum:

$m(\chi_1^0) = 99\,\mathrm{GeV}$

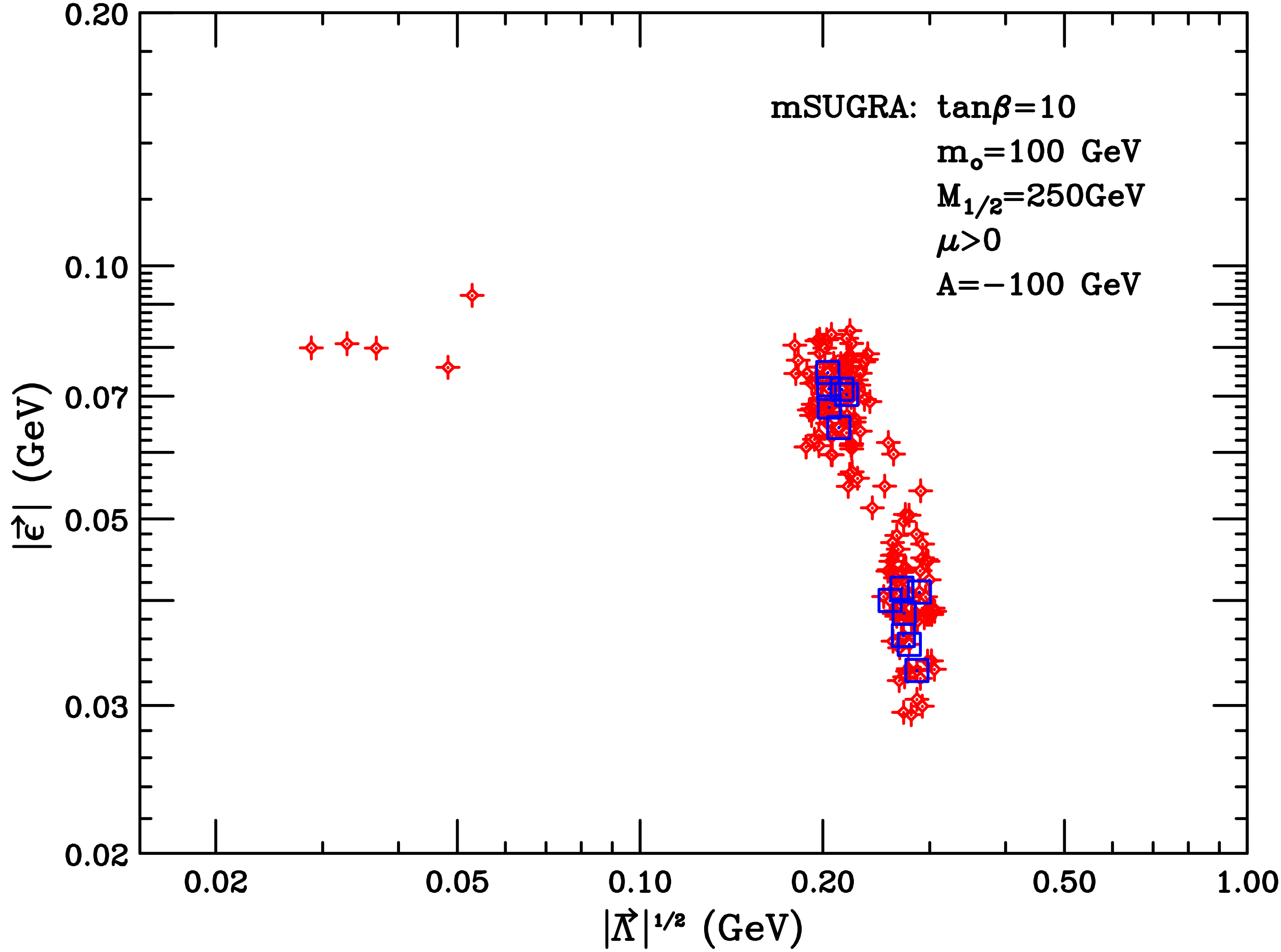
$m(\chi_1^\pm) = 175\,\mathrm{GeV}$

$m(\tilde{t}_1) = 376\,\mathrm{GeV}$

$m(\tilde{b}_1) = 492\,\mathrm{GeV}$

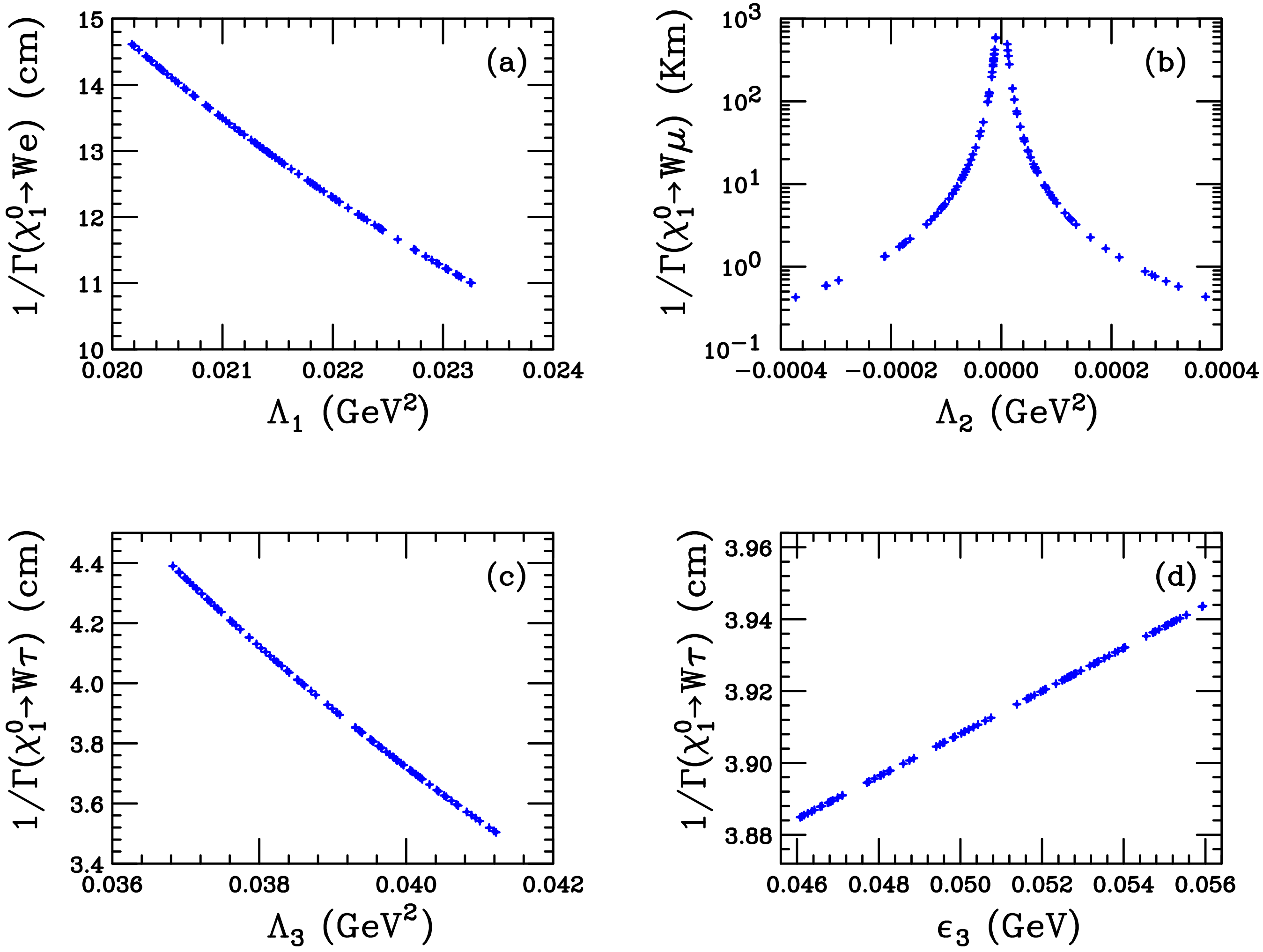
$m(h) = 111\,\mathrm{GeV}$

$m(H^\pm) = 408\,\mathrm{GeV}$



Neutralino Decays

The lighest neutralino χ_1^0 decays into a W boson and a charged lepton.

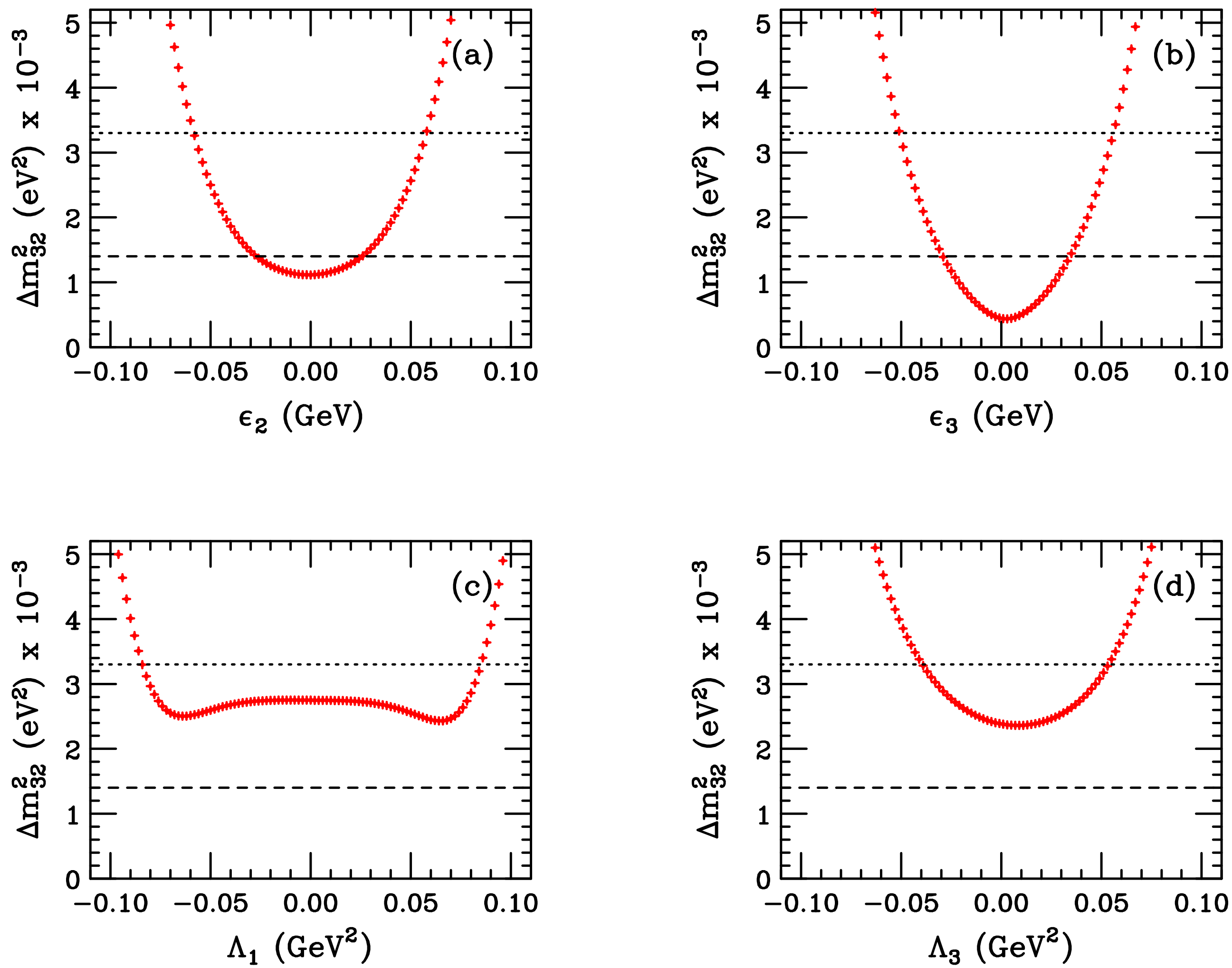


Atmospheric Mass

The atmospheric mass can be approximated as

$$\Delta m_{32}^2 \approx \tfrac{3}{2}\sqrt{5}(A\Lambda_3^2 + C\epsilon_3^2)C\epsilon_2^2$$

explaining the quadratic dependence of Δm_{32}^2 on ϵ_2, ϵ_3, and Λ_3, and the mild dependence on Λ_1



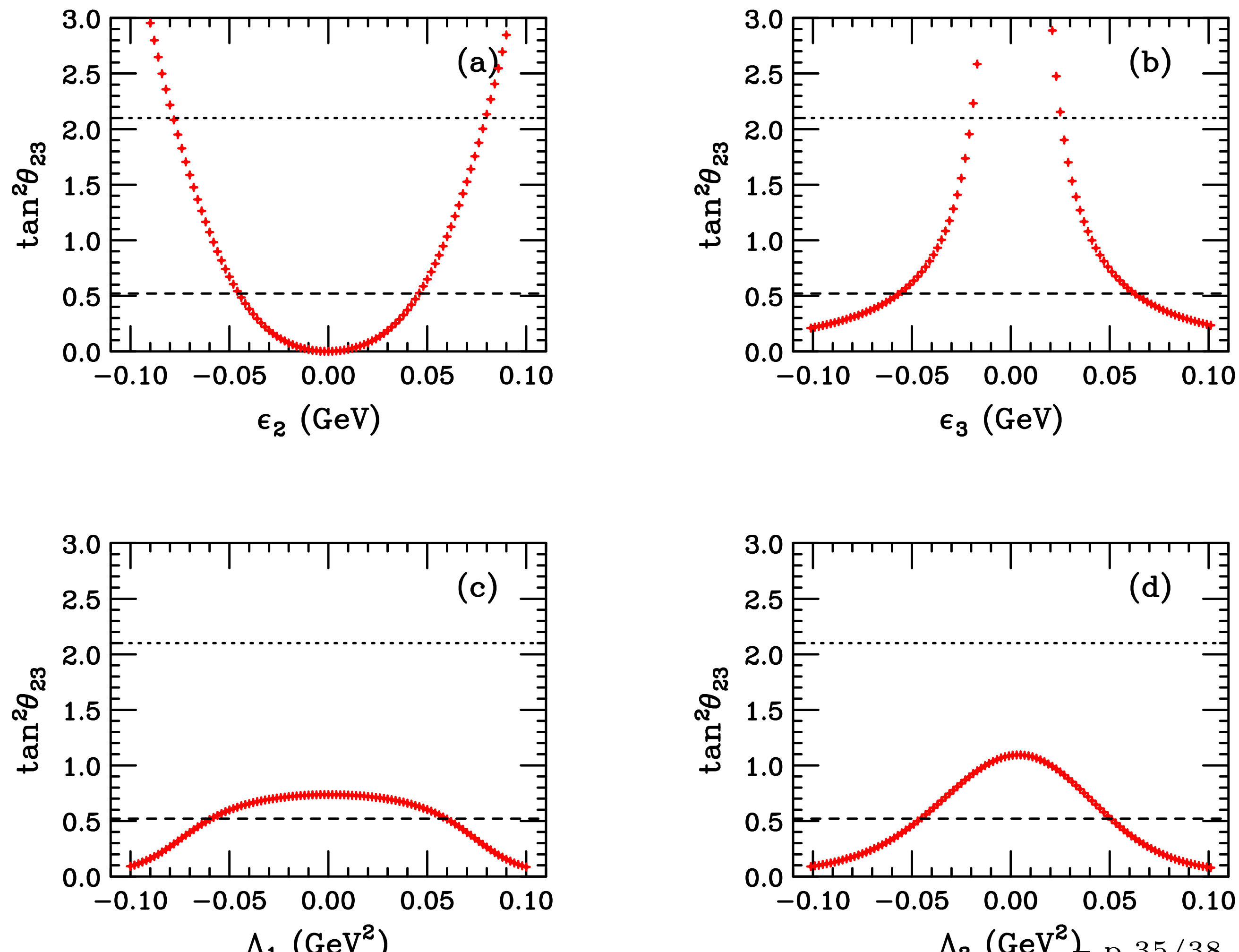
Atmospheric Angle

The atmospheric angle can be approximated as

$$\tan 2\theta_{23} \approx \frac{2C\epsilon_2\epsilon_3}{A\Lambda_3^2 + C(\epsilon_3^2 - \epsilon_2^2)} \left[\neq \tan 2\theta_{23}^{(0)} = \frac{2\Lambda_2\Lambda_3}{\Lambda_3^2 - \Lambda_2^2} \right]$$

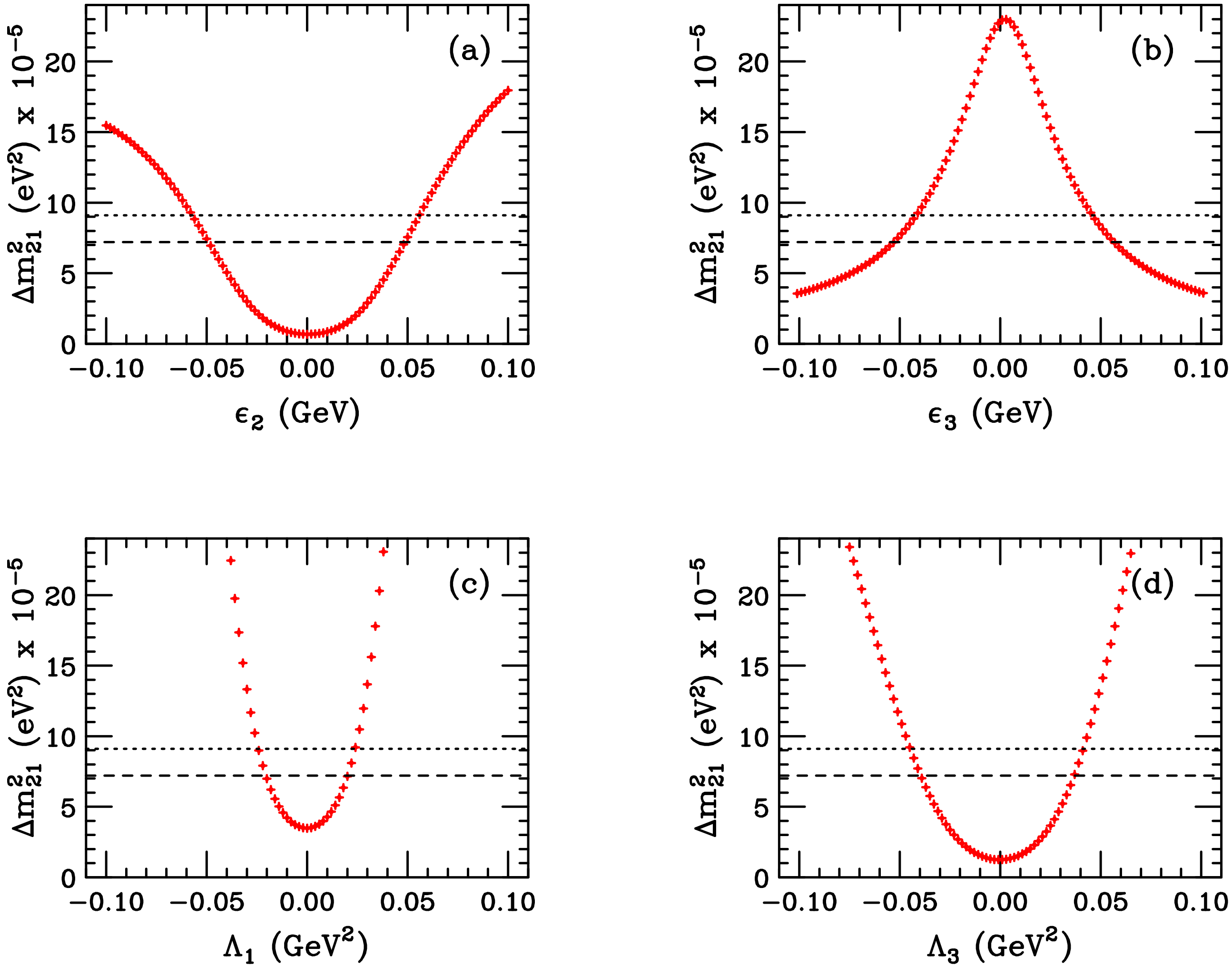
If $\epsilon_2 \to 0$ then $\tan^2\theta_{23} \to 0$, as seen in frame (a).

If $\epsilon_3 \to 0$ then $\tan^2\theta_{23} \to \pi/2$, as seen in frame (b).



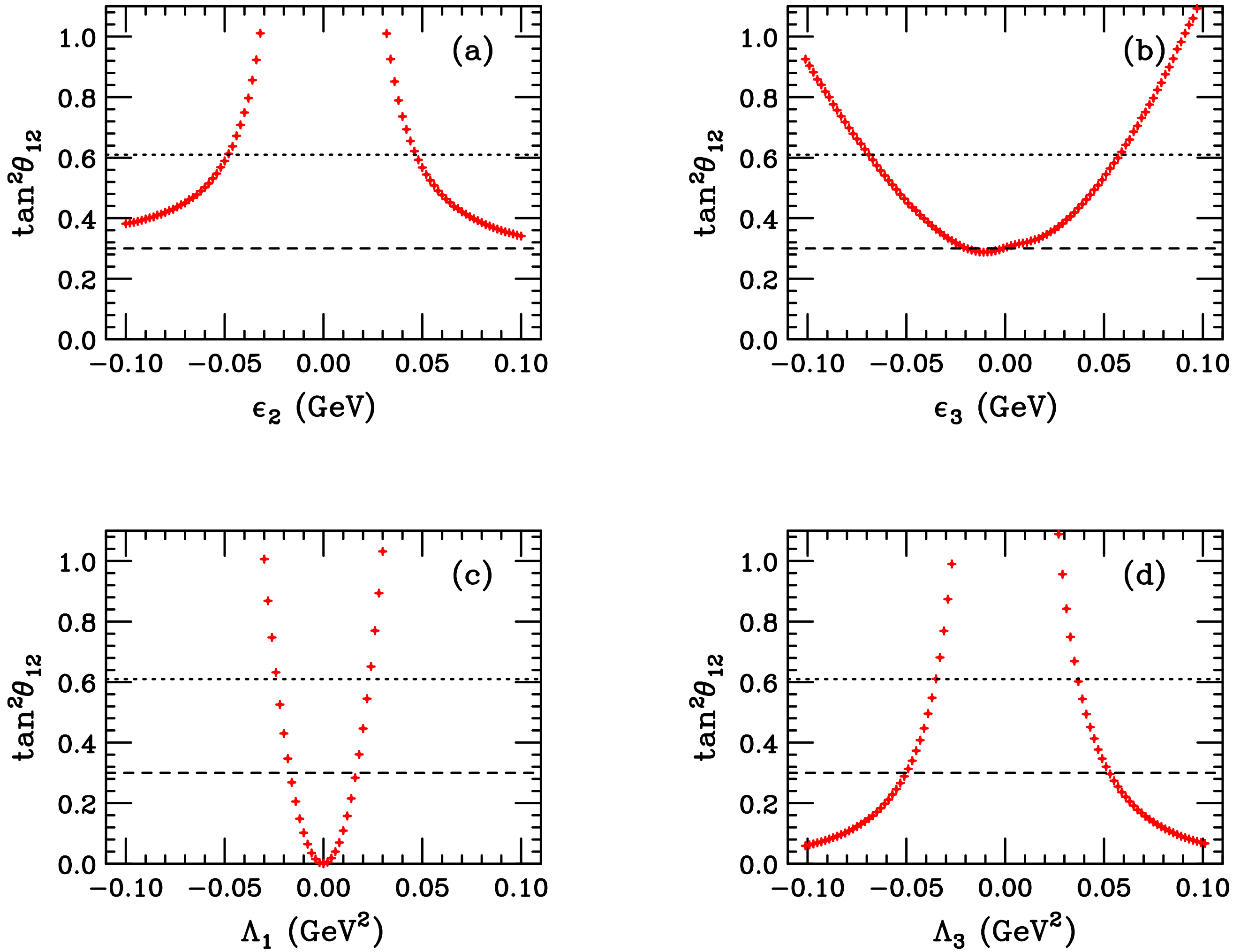
Solar Mass

The allowed region is a very narrow band. The four BRpV parameters have a strong influence on the solar mass.



Solar Angle

The solar angle is not maximal, and depends on all four BRpV parameters.



Conclusions

- Supersymmetry with Bilinear R-Parity Violation provides a framework for neutrino masses and mixing angles compatible with experiments.

- At tree level, a low energy see saw mechanism gives mass to one neutrino. One loop corrections complete the three neutrino masses and mixing angles.

- Neutrino parameters can be extracted from collider physics, and can help to differentiate supersymmetric models.