Dimensional Reduction or $\overline{DR}$-scheme: some new results

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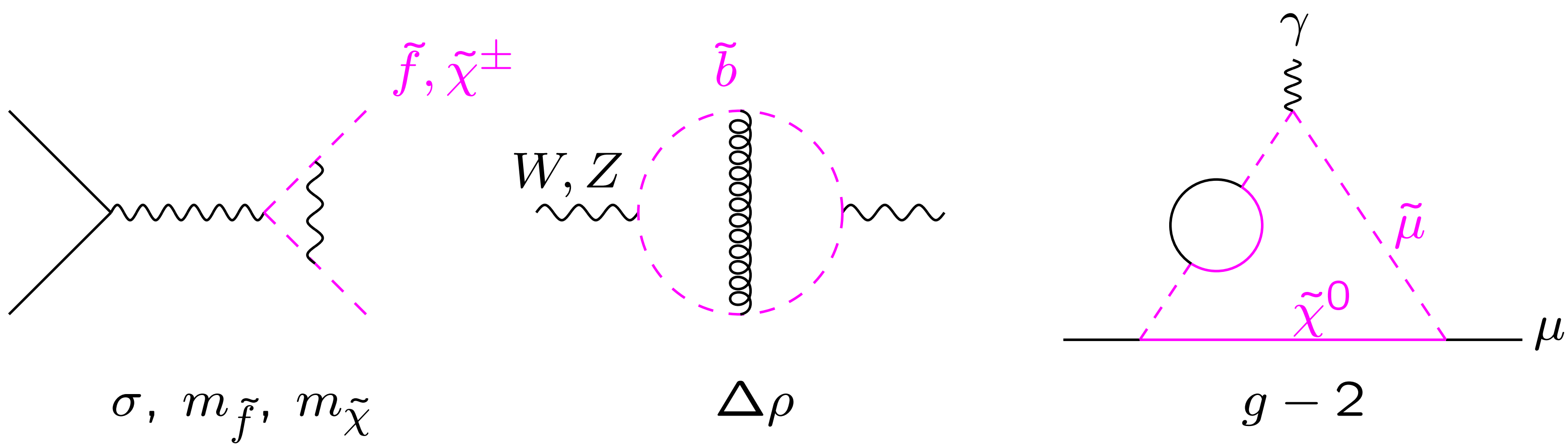
based on hep-ph/0503129

1. Motivation: Why consider regularization? What are the problems?
2. Mathematically consistent DRED
3. Symmetries in DRED: Quantum action principle
4. Applications: Verify SUSY-identities in DRED at 1-, 2-Loop level

Motivation

Precise measurement of SUSY observables



justifies/necessitates SUSY loop calculations. ($\rightarrow$ SPA project)

These require

- Regularization: intermediate steps in calculation
- Renormalization scheme: defines theory/parameters

Motivation

Regularization: intermediate steps in calculation

Dim. Regularization (DREG) : breaks SUSY $\Rightarrow$ complicated in practice

Dim. Reduction (DRED) : doesn't (?) break SUSY $\Rightarrow$ usually applied

Renormalization scheme: defines theory/parameters

SPA-convention: $\overline{DR}$ scheme $\Leftrightarrow$ DRED $\oplus$ prescription to throw away divergent parts

$\overline{DR}$ scheme and DRED are central in SPA project and in many SUSY calculations

Two problems of DRED

1. DRED is mathematically inconsistent and there is no full proof that SUSY is preserved

   (main topic of this talk)

2. There seems to be a problem with factorization of hadron processes

Remarks on factorization problem

Factorization of hadron cross sections: $\sigma_{\text{had}} = f \otimes \sigma_{\text{partonic}}$

$\longrightarrow$ It seems that this structure works in DREG but not in DRED
[Beenakker,Kuijf,van Neerven,Smith '88, '04]

Why? Is there a way around this conclusion?

$\longrightarrow$ Interesting QFT problem, more work needed

The practical consequences are not disastrous

- Anyway, parton distribution functions f are known in $\overline{MS}$ scheme based on DREG $\Rightarrow$ one has to convert a DRED-calculation to DREG
- Problem only discovered in real (gluon radiation) graphs, but for us, DRED is only necessary for virtual (loop) graphs
- Many calculations/observables do not involve hadrons and are not affected

Problem of DRED

DREG: *all* 4-vectors D-dimensional $\Rightarrow$ Photon: D d.o.f., Photino: 4 d.o.f.
$\Rightarrow$ mismatch, SUSY breaking

DRED: momenta D-dimensional; Photon, γ matrices 4-dimensional
$\Rightarrow$ no apparent mismatch

However, in DRED the following relation is required: ${g^{(4)}}_{\mu\nu}{g^{(D)}}_{\rho}{}^{\nu} = {g^{(D)}}_{\mu}{}^{\rho}$
D-dimensional space is a subspace of 4-dimensional space

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One can then calculate $\epsilon^{\mu\nu\rho\sigma}\,\epsilon_{\alpha\beta\gamma\delta}\,\epsilon_{\mu\nu\rho\sigma}\,\epsilon^{\alpha\beta\gamma\delta}$ in two different ways

$$\Rightarrow 0 = D(D-1)^2(D-2)^2(D-3)^2(D-4)$$

different calculational steps lead to different results,
mathematical inconsistency!!! *[Siegel'80]*

$\Rightarrow$ Difficult to prove general statements in DRED

Particularly important problem

To what extent is DRED supersymmetric?

- in many checks DRED behaves supersymmetric
- however, no general proof
- not all cases of interest have been checked

main checks of SUSY relations:

one-loop propagator Ward identities *[Capper,Jones,van Nieuwenhuizen'80]*
β-functions *[Martin, Vaughn '93] [Jack, Jones, North '96]*
one-loop S-matrix relation $S(qqG) = S(q\tilde{q}\tilde{g})$ *[Beenakker,Höpker,Zerwas'96]*
one-loop Slavnov-Taylor identities *[Hollik,Kraus,DS'99] [Hollik,DS'01]*
[Fischer,Hollik,Roth,DS'03]

missing: e.g. four-point interactions $S(\tilde{q}\tilde{q}\tilde{q}\tilde{q})$, 2-loop relations,...

Aims:

1. Define DRED without mathematical inconsistency

2. Find general ways to analyze SUSY-properties of DRED

3. Check that DRED preserves SUSY in some interesting cases

Interlude: Why / What is regularization??

Regularization seems to be a technical subject. . .

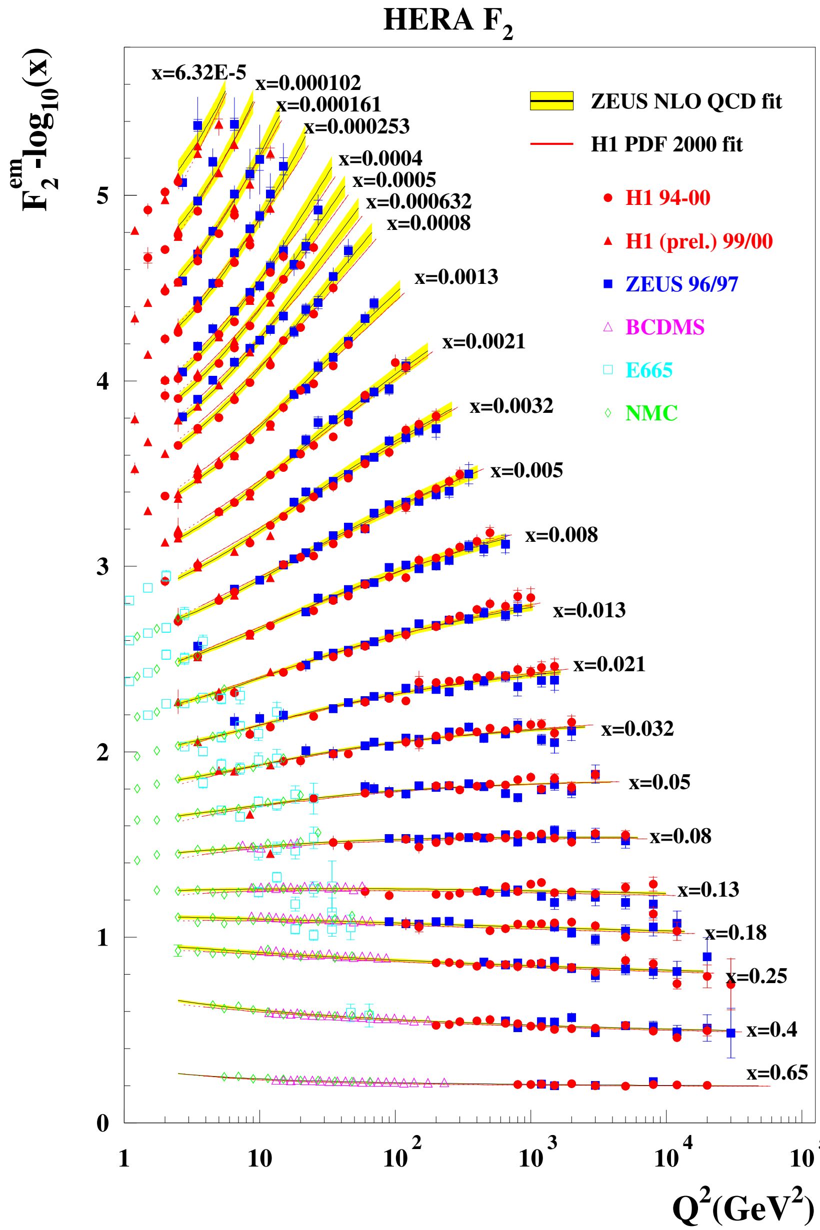
but regularization is fundamentally necessary (divergent loop integrals)
and it has physical consequences

Interlude: Why / What is regularization??

Regularization $\Leftrightarrow$ Modification of theory,
e.g. dimensions: $4 \to D$

Unregularized QCD $\Rightarrow$ scaling invariance

Regularized QCD $\Rightarrow$ violation of scaling invariance as seen in experiment



Interlude: Why / What is regularization??

Regularization $\Leftrightarrow$ Modification of theory, some modifications are physical effects!

but in practice the regularization should preserve as many features of the theory as possible

In particular, for DRED: consider mathematical consistency, supersymmetry

Where does the inconsistency come from (I) ?

In 4 dimensions, one can count indices: $\mu = 0, 1, 2, 3$.

- e.g. five indices μ_1, μ_2, μ_3, μ_4, μ_5: two of them equal $\mu_i = \mu_j$
- similarly: Fierz relations for γ matrices
- applying such arguments one arrives at the inconsistency

Idea: replace 4-dim space with "quasi-4-dim space", where one *cannot* count indices *[Avdeev, Chochia, Vladimirov '81]*

How does that work in detail?

Where does the inconsistency come from (II) ?

DREG: "D-dimensional space" can be consistently defined as a truly ∞-dimensional space with some D-dim characteristics:

[Wilson'73],[Collins]

$$\mu = 0, 1, 2, \dots \infty, \quad g^{(D)\mu}{}_{\mu} = D$$

DRED: "D-dimensional space" should be a subspace of 4-dim space

$$g^{(4)}{}_{\mu\nu} g^{(D)}{}_{\rho}{}^{\nu} = g^{(D)}{}_{\mu}{}^{\rho} \qquad (*)$$

- "D-dimensional space" or 4-dimensional space alone: no problem
- requirement $(*)$ cannot be satisfied

$$\Downarrow$$

origin of inconsistency

Way out

"D-dim space" should be ∞-dimensional but subspace of 4-dim space

$\Rightarrow$ Replace ordinary 4-dim space by yet another ∞-dimensional space with some 4-dim characteristics $\longrightarrow$ "quasi-4-dim space"

$$D\text{-dim space} \subset \text{quasi-4-dim space}$$

$$g^{(D)\mu}{}_{\mu} = D, \quad g^{(4)\mu}{}_{\mu} = 4, \quad \mu = 0, 1, 2, \ldots \infty$$

- true 4-dim space: μ_1, μ_2, μ_3, μ_4, μ_5: two of them equal $\mu_i = \mu_j$
- quasi-4-dim space: not true

quasi-4-dim space can be explicitly constructed $\Rightarrow$ no mathematical problems, no inconsistency, unique results for calculations

Practical consequences

In practice one can forget that the "D-dim" and quasi-4-dim spaces are in reality ∞-dimensional

One can simply apply all usual calculational rules for index contractions and γ matrices etc

Only exception: one cannot rely on index counting or Fierz identities

These rules will never lead to inconsistent results

DRED and SUSY

Consistent formulation $\Rightarrow$ prove quantum action principle!

Quantum action principle: $i\,\delta_{\mathsf{SUSY}}\langle T\phi_1\ldots\phi_n\rangle \;=\; \langle T\phi_1\ldots\phi_n\Delta\rangle$

SUSY Ward/ST identities: $i\,\delta_{\mathsf{SUSY}}\langle T\phi_1\ldots\phi_n\rangle \;\stackrel{?}{=}\; 0$

- $\Delta \equiv \delta_{\mathsf{SUSY}}\mathcal{L}$ in D dimensions
- if $\Delta = 0$, all SUSY Ward and Slavnov-Taylor identities are satisfied

Very useful, but this has to be established as a theorem in DRED!

Quantum action principle in DRED

$$i\,\delta_{\mathsf{SUSY}}\langle T\phi_1\ldots\phi_n\rangle = \langle T\phi_1\ldots\phi_n\Delta\rangle$$

Proof has to be carried out for each regularization,

known for BPHZ *[Lowenstein et al '71]*

and DREG *[Breitenlohner, Maison '77]*

Using the consistent formulation of DRED, proof is possible by studying properties of Feynman diagrams in DRED

Similar to proof in DREG

Application: SUSY of DRED

General SUSY relation:

$$i\,\delta_{\mathsf{SUSY}}\langle T\phi_1\ldots\phi_n\rangle=0$$

Quantum action principle:

$$i\,\delta_{\mathsf{SUSY}}\langle T\phi_1\ldots\phi_n\rangle=\langle T\phi_1\ldots\phi_n\Delta\rangle$$

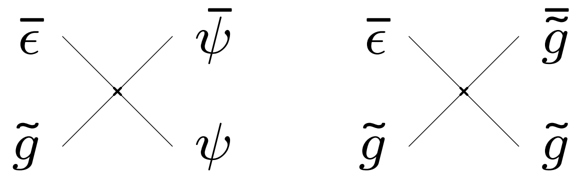
with $\Delta = \delta_{\mathsf{SUSY}}\mathcal{L}$

Two tasks:

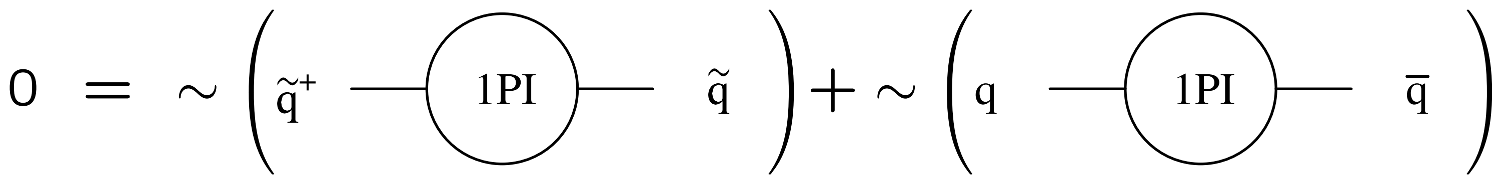
- Determine Δ in DRED: $\Delta \neq 0$!
- Determine insertion of Δ:
  if it vanishes $\Rightarrow$ DRED preserves SUSY

Application: SUSY of DRED

Some details: $\Delta = \delta_{\mathsf{SUSY}}\mathcal{L}$ contains 4-fermion operators that would vanish if Fierz identities were valid

e.g. 

Example Slavnov-Taylor identity for propagators: $\delta\langle T\tilde{q}^\dagger q\rangle = 0$



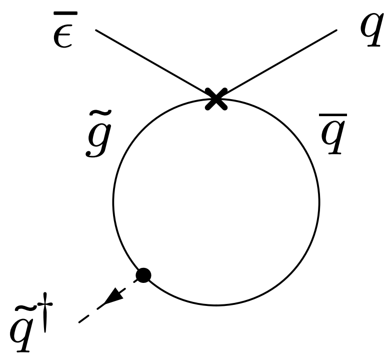
Studied and verified at one-loop in *[Hollik, DS '01]*

$\rightarrow$ full evaluation of all contributing diagrams necessary

Application: SUSY of DRED

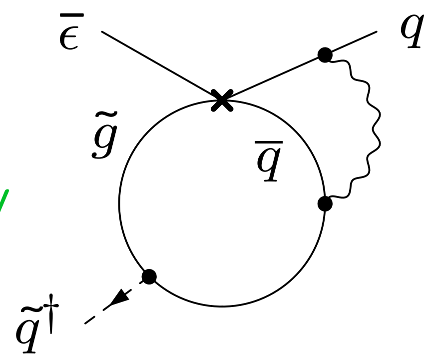
Now: one-loop violation of this identity is simply given by

$\langle T\tilde{q}^\dagger q\Delta\rangle =$



$=0$ $\longrightarrow$ Proof much simpler!

two-loop violation is given by



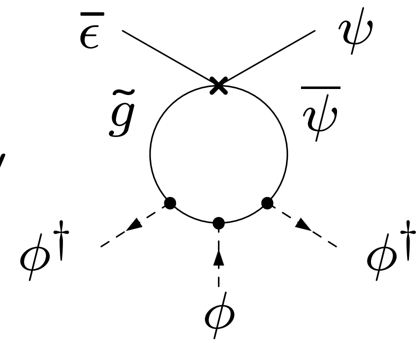
$+\ldots=0$

Hence, it is easy to see that the propagator-identity is valid in DRED up to the two-loop level

Application: SUSY of DRED

Another example: Slavnov-Taylor identity for ϕ^4 interaction

$\rightarrow$ SUSY-relation between ϕ^4 and gauge/Yukawa couplings

violation of this identity is given by 

$=0$

$\Rightarrow$ This identity is also valid in DRED (at one-loop)

Summary:

1. Define DRED without mathematical inconsistency

   - quasi-4-dimensional space: like 4-dim space but no Fierz identities

2. Find general ways to analyze SUSY-properties of DRED

   - Quantum action principle: $\Delta = \delta_{\mathsf{SUSY}}\mathcal{L} \neq 0$, possible violation of SUSY identities given by Feynman diagrams with insertion of Δ

3. Check that DRED preserves SUSY in some interesting cases

   - Propagator identities up to two-loop level, identity for ϕ^4 interaction at one-loop

Conclusions

Status of DRED:

- DRED might have a problem with factorization — but it is certainly good for SUSY loop calculations

- Improvements: consistency, quantum action principle, SUSY of DRED
  - more SUSY identities can be checked; the checks are drastically simplified

- $\Delta \neq 0 \Rightarrow$ there will be SUSY-violations of DRED at high orders

- SUSY identities should be checked at least to the level required for loop calculations of observables at the LHC/ILC
  - this is achievable