# Dominant Two-Loop Electroweak Correction to $H \to \gamma\gamma^*$

Bernd Kniehl
II. Institut für Theoretische Physik
Universität Hamburg
kniehl@desy.de

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*F. Fugel, B.A. Kniehl, M. Steinhauser, Nucl. Phys. **B702** (2004) 333.

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# 1 Introduction

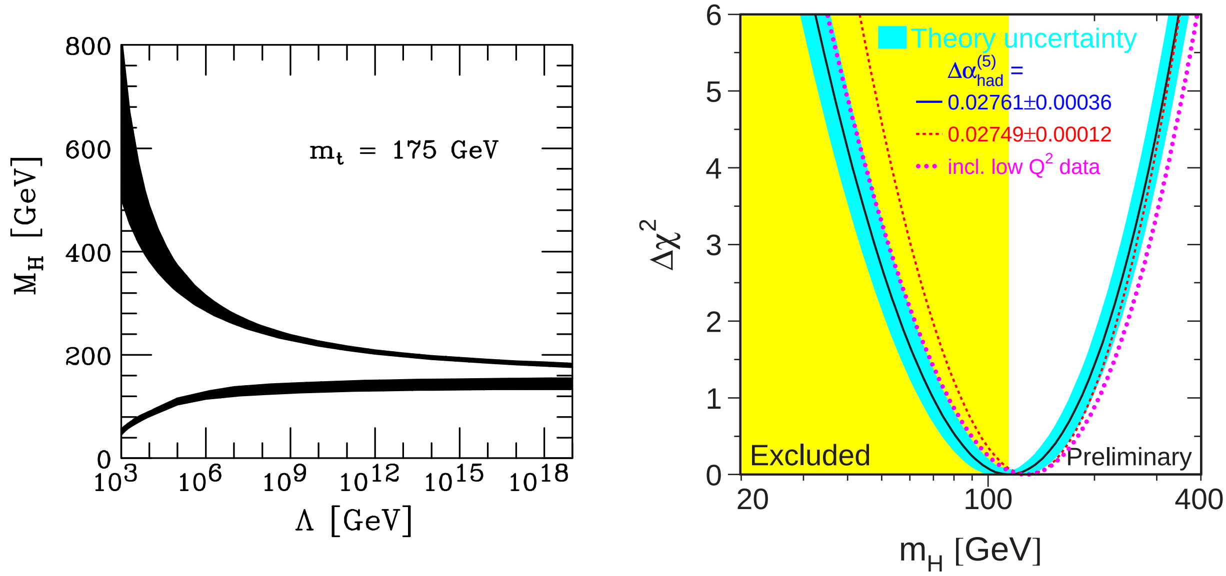


Figure 1: Left: Triviality and vacuum-stability bounds on $M_H$; right: $\Delta\chi^2 = \chi^2 - \chi^2_{\min}$ as a function of $M_H$.

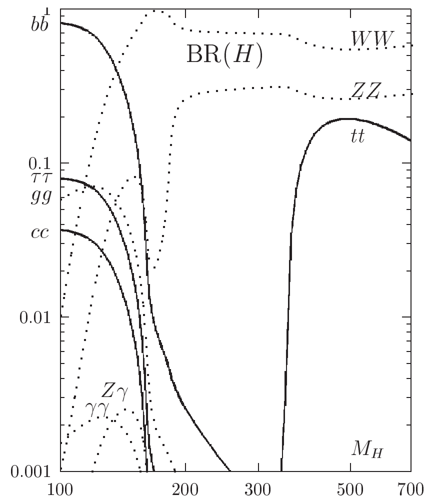


Figure 2: SM Higgs decay branching fractions.

- SM Higgs has intermediate mass: $M_H = 114^{+69}_{-45}$ (EWWG).
- $B(H \to \gamma\gamma) \lesssim 0.3\%$ in this $M_H$ range.
- $H\gamma\gamma$ coupling sensitive to new charged heavy particles.
- At ILC, $H \to \gamma\gamma$ has clear signal.
- At photon collider, $\sigma(\gamma\gamma \to H) \propto \Gamma(H \to \gamma\gamma)$.
- At LHC, $H \to \gamma\gamma$ important discovery mode.

$\leadsto$ Precise knowledge of $\Gamma(H \to \gamma\gamma)$ required for $M_W < M_H < 2M_W$.

## 2 One-Loop Result

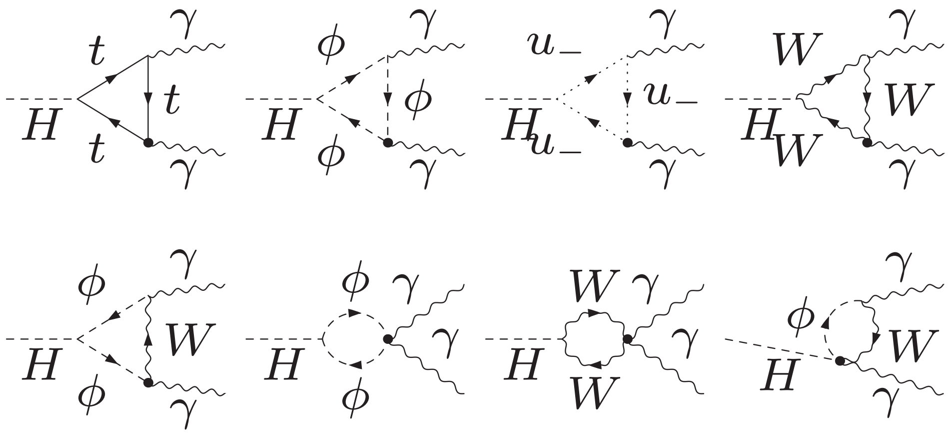


Figure 3: Sample diagrams for $\Gamma(H \to \gamma\gamma)$ at one loop.

Amplitude:

$$\mathcal{T}^{\mu\nu} = (q_1 \cdot q_2\, g^{\mu\nu} - q_1^\nu q_2^\mu)\mathcal{A},$$
$$\mathcal{A} = \mathcal{A}_t^{(0)} + \mathcal{A}_W^{(0)} + \mathcal{A}_{tW}^{(1)} + \cdots$$

Partial decay width:

$$\Gamma(H \to \gamma\gamma) = \frac{M_H^3}{64\pi}|\mathcal{A}|^2.$$

J.R. Ellis, M.K. Gaillard, D.V. Nanopoulos, Nucl. Phys. B 106 (1976) 292;
B.L. Ioffe, V.A. Khoze, Sov. J. Part. Nucl. 9 (1978) 50.

# Method

- Put $M_b = 0$ and $V_{tb} = 1$.
- Exploit (formal) hierarchies $M_H^2 = 2q_1 \cdot q_2 \ll 4M_W^2 \ll M_t^2$.
  $\leadsto$ Use asymptotic expansion.
- Find leading large-$M_t$ term and its expansion in powers of $\tau_W = (M_H/2M_W)^2$.
- Automatize calculation:
  - `QGRAF` Nogueira: generates diagrams
  - q2e Seidensticker: converts output
  - `exp` Seidensticker: performs asymptotic expansion and generates relevant subdiagrams according to hard-mass procedure
  - `MATAD` Steinhauser: calculates diagrams
- Use dimensional regularization.
- Use on-mass-shell renormalization scheme.
- Treat tadpoles properly.
- Perform checks:
  - Compute coefficients of $q_1 \cdot q_2\, g^{\mu\nu}$ and $q_1^\nu q_2^\mu$ separately.
  - Work in $R_\xi$ gauge.
  - Verify UV cancellations.
  - Verify cancellation of $M_t^4$ terms from asymptotic expansion, genuine two-loop tadpoles, and counterterms.
  - Compare with known result for $\Gamma(H \to gg)$ involving only Higgs ($H$) and Goldstone ($\chi^0$, $\phi^\pm$) bosons.
  - Check convergence properties of $\tau_W$ expansions.

# $t$ Loops

$$\mathcal{A}_t^{(0)} = \hat{A} N_c Q_t^2 \left\{ \frac{1}{\tau_t} \left[ 1 + \left( 1 - \frac{1}{\tau_t} \right) \arcsin^2 \sqrt{\tau_t} \right] \right\}$$
$$= \hat{A} N_c Q_t^2 \left( \frac{2}{3} + \frac{7}{45}\tau_t + \frac{4}{63}\tau_t^2 + \frac{52}{1575}\tau_t^3 + \frac{1024}{51975}\tau_t^4 + \frac{2432}{189189}\tau_t^5 + \ldots \right),$$

where $\hat{\mathcal{A}} = 2^{1/4} G_F^{1/2} (\alpha/\pi)$ and $\tau_t = (M_H/2M_t)^2$.

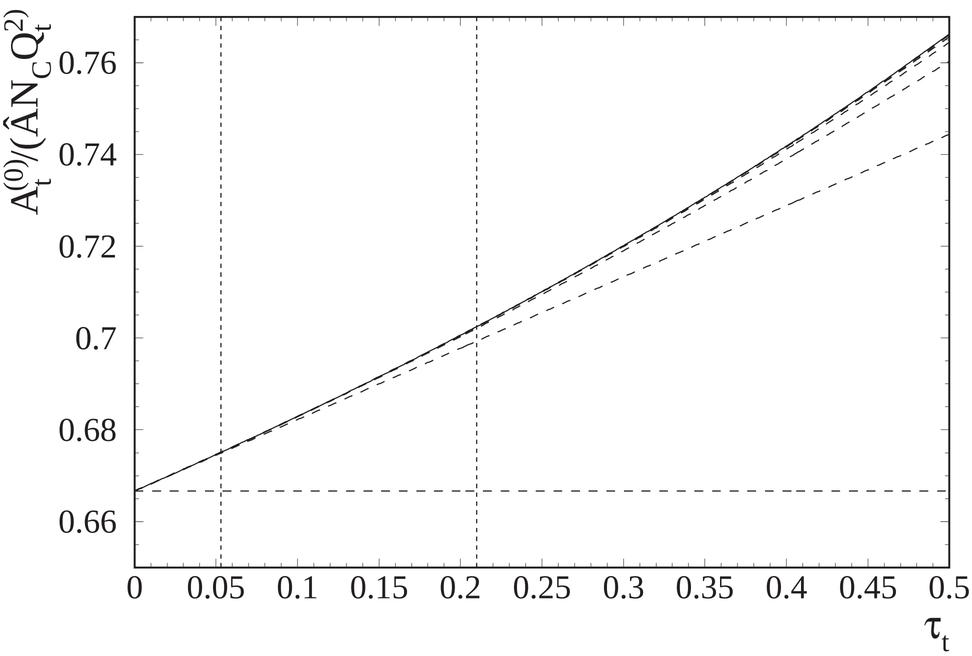


Figure 4: Convergence property of expansion of $\mathcal{A}_t^{(0)}$ in powers of $\tau_t$.

# $W$ Loops

$$\mathcal{A}_W^{(0)} = \hat{\mathcal{A}} \left\{ -\frac{1}{2} \left[ 2 + \frac{3}{\tau_W} + \frac{3}{\tau_W} \left( 2 - \frac{1}{\tau_W} \right) \arcsin^2 \sqrt{\tau_W} \right] \right\}$$
$$= \hat{\mathcal{A}} \left( -\frac{7}{2} - \frac{11}{15}\tau_W - \frac{38}{105}\tau_W^2 - \frac{116}{525}\tau_W^3 - \frac{2624}{17325}\tau_W^4 - \frac{640}{5733}\tau_W^5 + \ldots \right),$$

where $\tau_W = (M_H/2M_W)^2$.

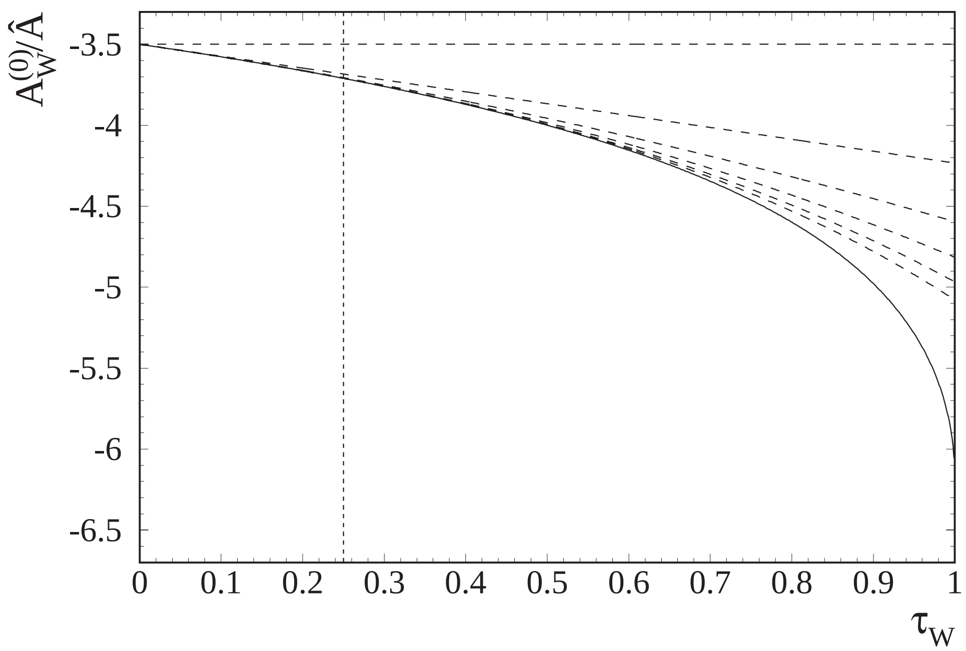


Figure 5: Convergence property of expansion of $\mathcal{A}_W^{(0)}$ in powers of $\tau_W$.

# 3 $\mathcal{O}(G_F M_t^2)$ Correction

Consider all two-loop electroweak diagrams involving a virtual top quark (1690 in $R_\xi$ gauge).
Two classes:

- $t$ loops with virtual $H$ or $\chi$ lines attached to them. $\rightsquigarrow$ Simple Taylor expansion in external momenta.
- $t$ loops with virtual $Z$ lines attached to them. $\rightsquigarrow$ Subleading (below $M_t^2$).
- Diagrams involving $t$, $b$, and $W$ or $\phi$. $\rightsquigarrow$ Nontrivial asymptotic expansion; $M_t^4$ terms occur.

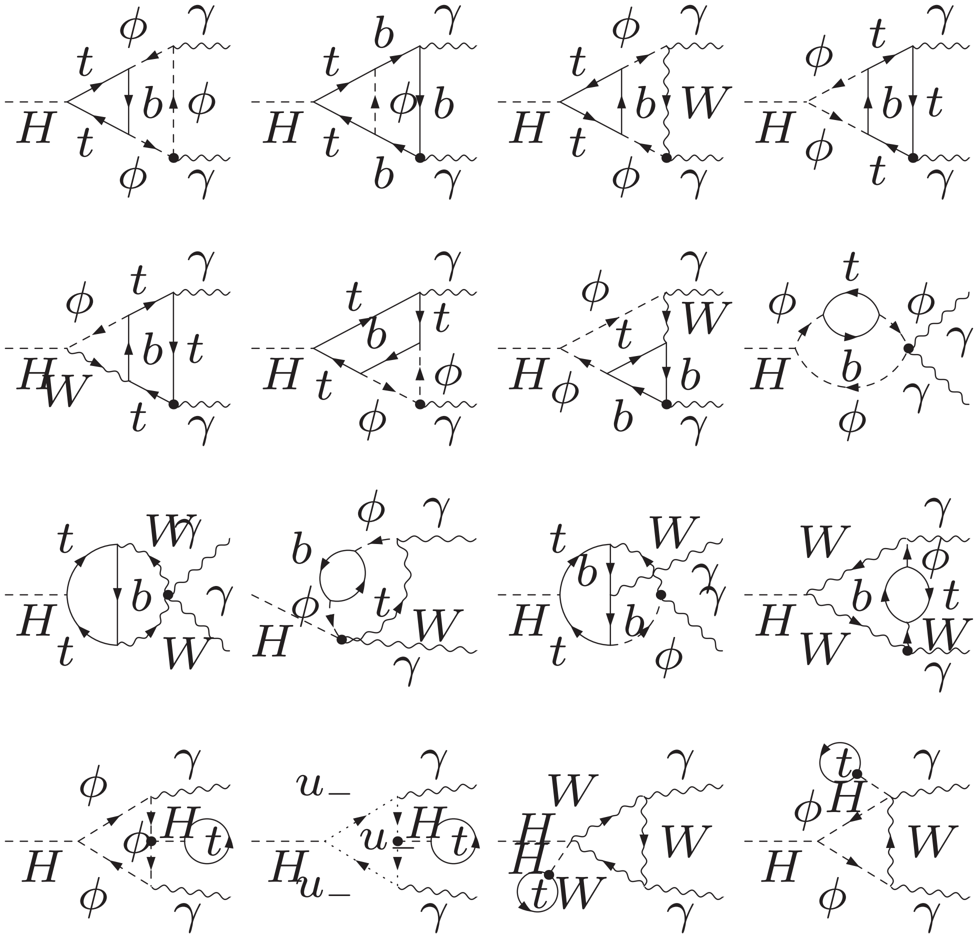


Figure 6: Sample diagrams for $\Gamma(H \to \gamma\gamma)$ at $\mathcal{O}(G_F M_t^2)$.

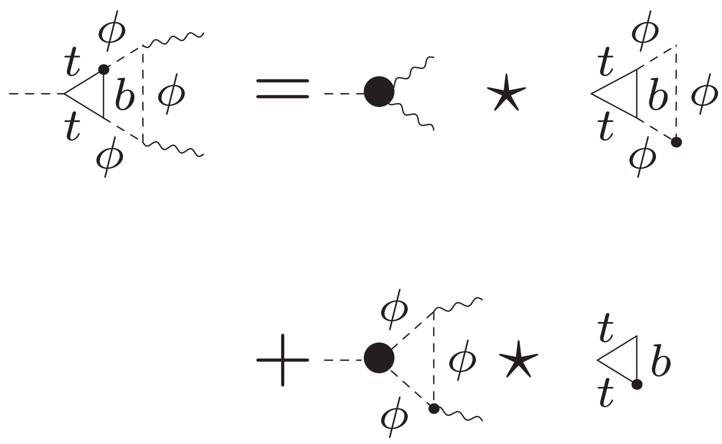


Figure 7: Diagrammatic asymptotic expansion of a diagram that produces $M_t^4$ terms.

$R_\xi$ gauge for $W$ boson:

$$\begin{array}{rl} (i) & M_t^2 \gg M_W^2 = \xi_W M_W^2 \gg M_H^2 \\ (ii) & M_t^2 \gg M_W^2 \gg \xi_W M_W^2 \gg M_H^2 \\ (iii) & M_t^2 \gg \xi_W M_W^2 \gg M_W^2 \gg M_H^2 \\ (iv) & \xi_W M_W^2 \gg M_t^2 \gg M_W^2 \gg M_H^2 \end{array}$$

Final result ($x_t = G_F M_t^2/(8\pi^2\sqrt{2})$):

$$\begin{aligned} \mathcal{A}_{tW}^{(1)} &= \mathcal{A}_u^{(1)} + \mathcal{A}_{H,\chi}^{(1)} + \mathcal{A}_{W,\phi}^{(1)} \\ &= \hat{\mathcal{A}} N_c x_t \left( \frac{367}{108} + \frac{11}{18}\tau_W + \frac{19}{63}\tau_W^2 + \frac{58}{315}\tau_W^3 \right. \\ &\quad \left. + \frac{1312}{10395}\tau_W^4 + \cdots \right) \end{aligned}$$

# 4 Numerical Analysis

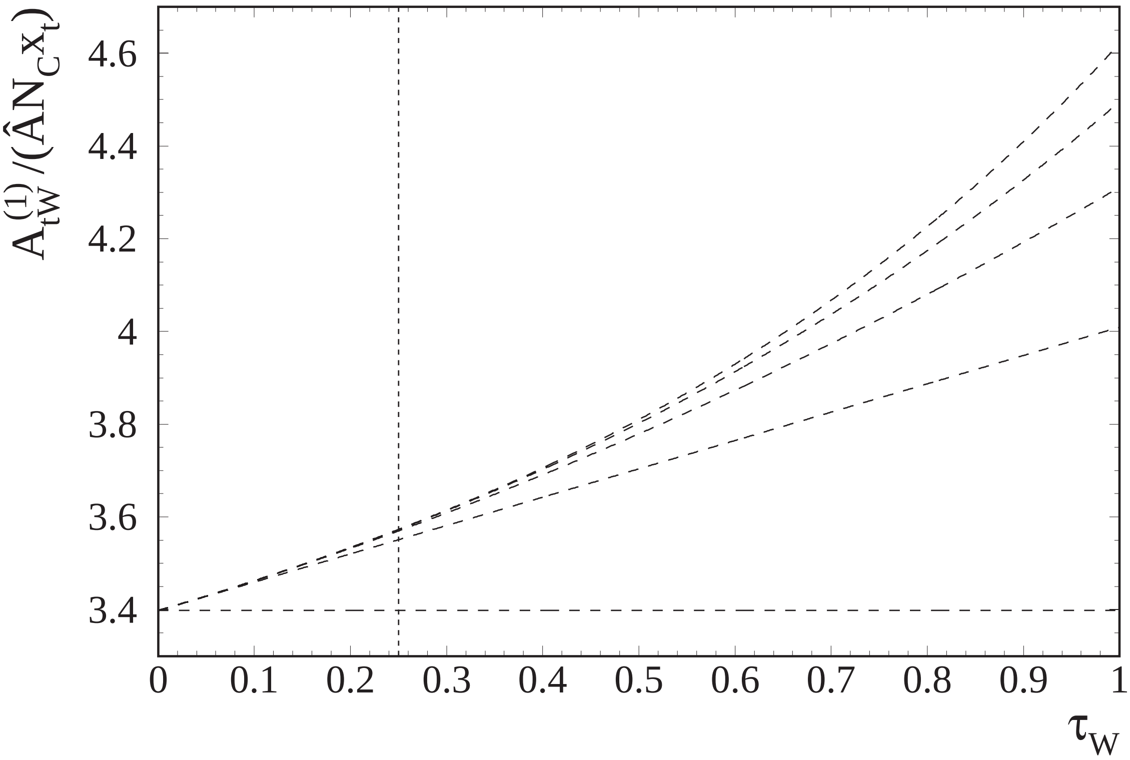


Figure 8: Convergence property of expansion of $\mathcal{A}_{tW}^{(1)}$ in powers of $\tau_W$.

| $M_H$ [GeV] | 120 | 140 | $2M_W$ |
|---|---|---|---|
| $\mathcal{A}_W^{(0)}$ | 0.4% | 1.1% | 3.1% |
| $\mathcal{A}_{tW}^{(1)}$ | 0.3% | 1.0% | 2.8% |

Table 1: Relative deviation of best approximation from second best one.

# Comparison With $\mathcal{O}(\alpha_s)$ Correction

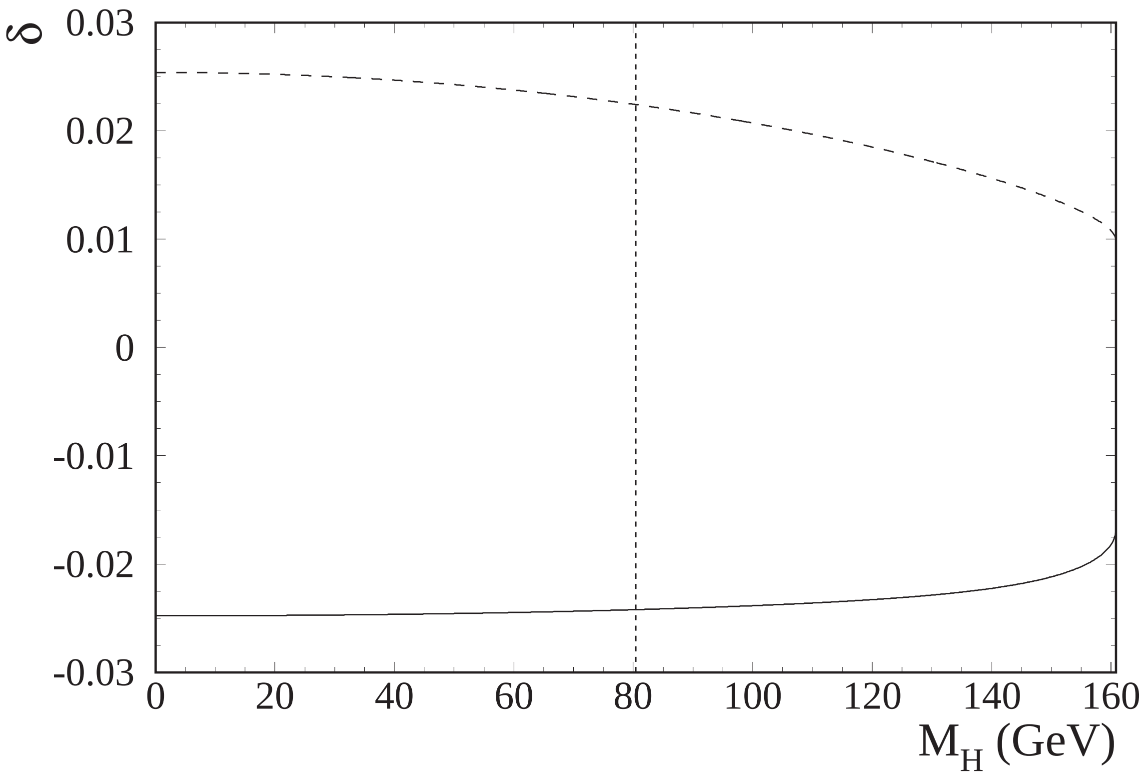


Figure 9: $\mathcal{O}(G_F M_t^2)$ (solid) and $\mathcal{O}(\alpha_s)$ (dashed) corrections to $\Gamma(H \to \gamma\gamma)$.

$\mathcal{O}(\alpha_s)$ gluon correction:


H. Zheng, D. Wu, Phys. Rev. D 42 (1990) 3760;

A. Djouadi, M. Spira, J.J. van der Bij, P.M. Zerwas, Phys. Lett. B 257 (1991) 187;

S. Dawson, R.P. Kauffman, Phys. Rev. D 47 (1993) 1264;

A. Djouadi, M. Spira, P.M. Zerwas, Phys. Lett. B 311 (1993) 255;

K. Melnikov, O.I. Yakovlev, Phys. Lett. B 312 (1993) 179;

M. Inoue, R. Najima, T. Oka, J. Saito, Mod. Phys. Lett. A 9 (1994) 1189;
J. Fleischer, O.V. Tarasov, Z. Phys. C 64 (1994) 413;
J. Fleischer, O.V. Tarasov, V.O. Tarasov, Phys. Lett. B 584 (2004) 294.

$\mathcal{O}(n_f G_F M_W^2)$ light-fermion correction: approx. $-2\%$ – $-1\%$
U. Aglietti, R. Bonciani, G. Degrassi, A. Vicini, Phys. Lett. **B595** (2004) 432.

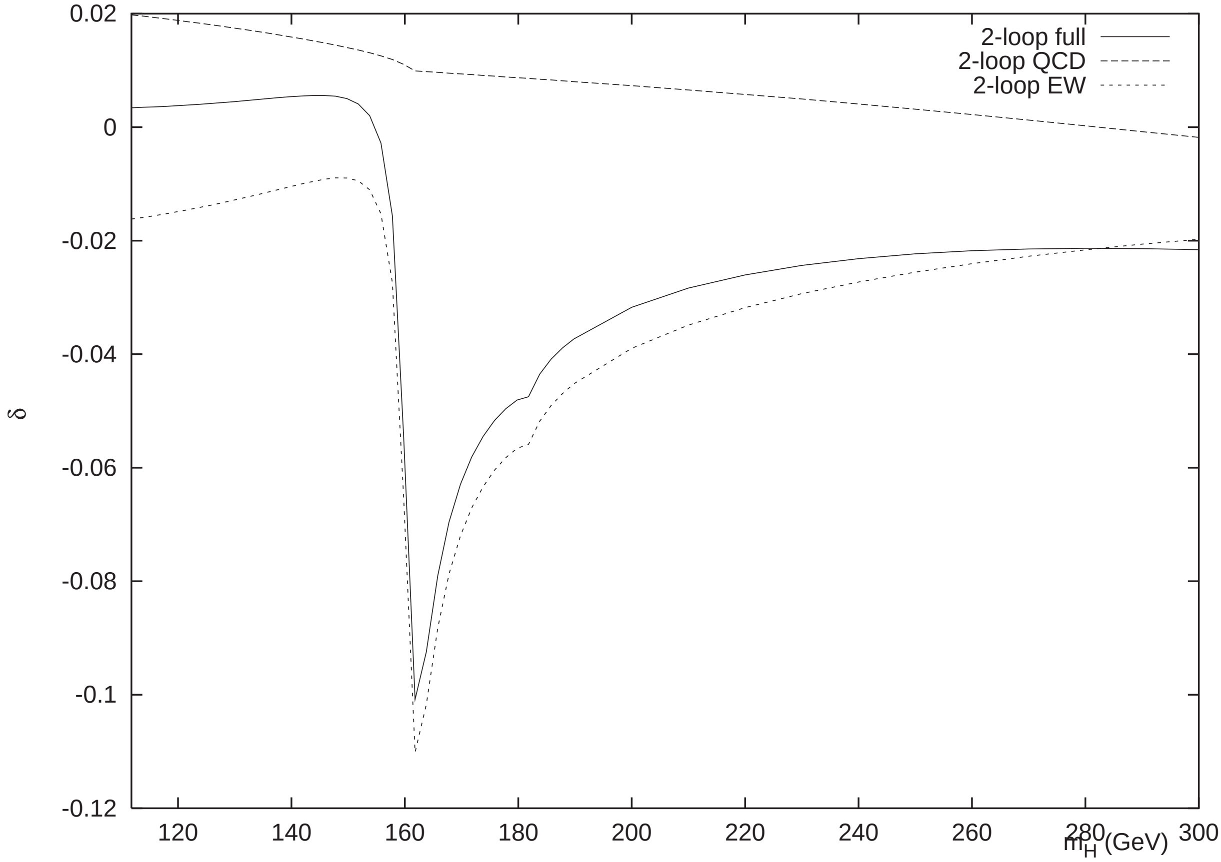


Figure 10: $\mathcal{O}(n_f G_F M_W^2)$ (dotted) and $\mathcal{O}(\alpha_s)$ (dashed) corrections to $\Gamma(H \to \gamma\gamma)$.

# 5 Conclusions

- Dominant two-loop electroweak $\mathcal{O}(G_F M_t^2)$ correction to $\Gamma(H \to \gamma\gamma)$ for $M_W \lesssim M_H \lesssim 2M_W$ available as expansion in $\tau_W = (M_H/2M_W)^2$ through $\mathcal{O}(\tau_W^4)$.
- Reduction by approx. $-2.5\%$ – $-2\%$.
- Positive QCD correction slightly overcompensated.
- Net effect of known corrections $-2\%$ – $-1\%$.