Correlation Matrix Method for Pb/Scint Sampling Calorimeter

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We have studied the electromagnetic(EM) and hadronic longitudinal cascade shower fluctuations. The showers were created by electron and pion beams with energies of 1-4 GeV at KEK and observed by a prototype sampling calorimeter of Pb and scintillator sandwich. We investigated the behavior of fluctuations of EM and hadron showers with the use of the correlation matrix.

1. Introduction

The main subject of this talk is the study of EM and hadron shower fluctuations. The average behavior of cascade showers has been investigated by many experiments using incident particles at various energies. By looking at individual cascade showers, we can explain their characteristic features. For this study, we use pulse height deviation at a depth (δ_i), which is defined event-by-event as the deviation of pulse height from the average. Then, we calculate a matrix called "correlation matrix", which represents the strength of the correlation of the pulse height deviations at two depths [1]. We use this matrix as a tool to investigate the shower fluctuation. The details of the correlation matrix are given in section 3. The objectives of this study are the understanding of shower fluctuations using the correlation matrix.

2. Experiment

For this study, we use the data of a prototype calorimeter for future applications in a linear collider [2] from the beam test T411 [3, 4]. The configuration of the calorimeter is shown in Fig. 1. The beamtest was performed at KEK in 1997. The calorimeter has a sandwich structure with 4mm thick Pb plates and 2mm thick scintillator plates. The signal is read out by super layer (S.L.) with each S.L. consisting of 5 sampling layers. There are a total of 42 super layers ($4.9\lambda_I$). A calorimeter with a very detailed sampling as this one used in T411 is well-suited for the study of longitudinal shower fluctuations.

3. Correlation Matrix and Fluctuation

3.1. Correlation Matrix

The correlation matrix is introduced as a tool to investigate the longitudinal fluctuation of cascade shower. At first, we calculate the pulse height deviation of a S.L., which is defined event-by-event as the deviation of pulse height from the average. Event-by-event shower curves and the average shower curve for electrons and pions with an energy of 3 GeV are plotted in Fig. 2. Event-by-event shower fluctuation is much larger for hadron.

The pulse height deviation of the i-th S.L. for the k-th event is defined as follows:

$$\delta_i^{(k)} \equiv p_i^{(k)} - \langle p_i \rangle, \tag{1}$$

where $p_i^{(k)}$ is the pulse height of the i-th S.L. for the k-th event and $\langle p_i \rangle$ means an average over the total number of events. Then we calculate for an ensemble of events the correlation matrix C, which represents the strength of the correlation between pulse height deviations at two S.L.s. The correlation matrix elements are defined as

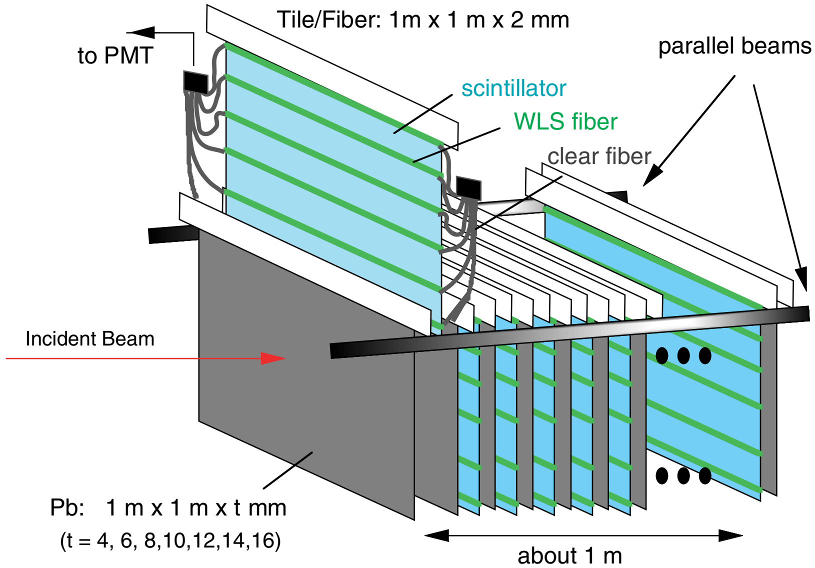

Figure 1: Calorimeter module for the beam test T411.

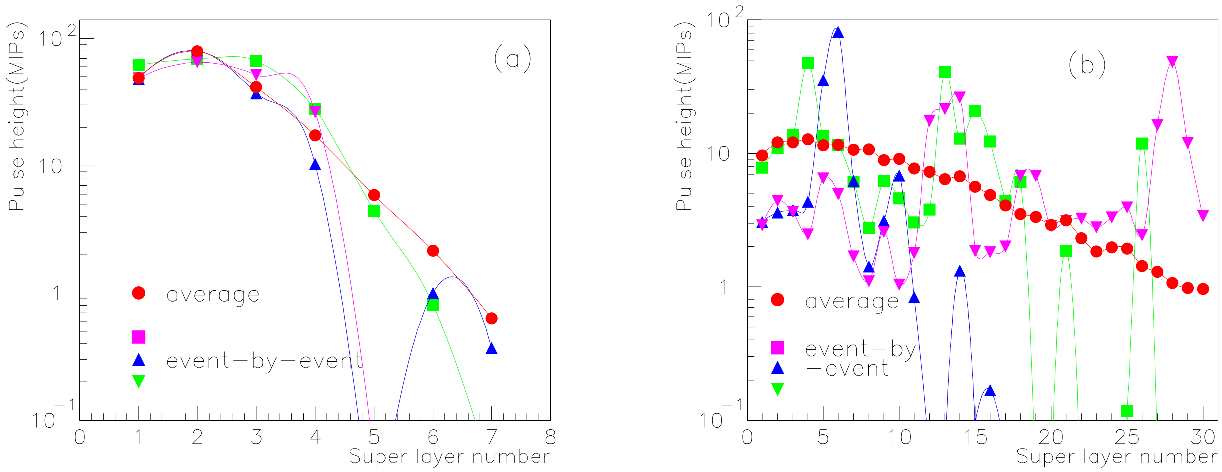

Figure 2: Shower curve of average and event-by-event for (a) EM and (b) hadron shower with the energy of 3 GeV.

$$C_{ij} \equiv \frac{1}{N} \sum_{k=1}^{N} \delta_i^{(k)} \delta_j^{(k)} = \langle \delta_i \ \ \delta_j \rangle, \tag{2}$$

where C_{ij} is the i-th row j-th column element of the correlation matrix. Fig. 3 shows C_{ij} 3-dimentional plot and C_{ij} versus i curves for 3 GeV electrons and pions. From these figures, we can see some features:

(1) A S.L. has positive correlation to the S.L.s nearby and has negative correlation to the S.L.s far off.

(2) For pions, there are two component correlations. One is sharp peak like shape of short distance correlation. The other is gentle wave like shape of long distance correlation.

These features reproduce that of the pulse height deviations at two different S.L.s (Fig.4).

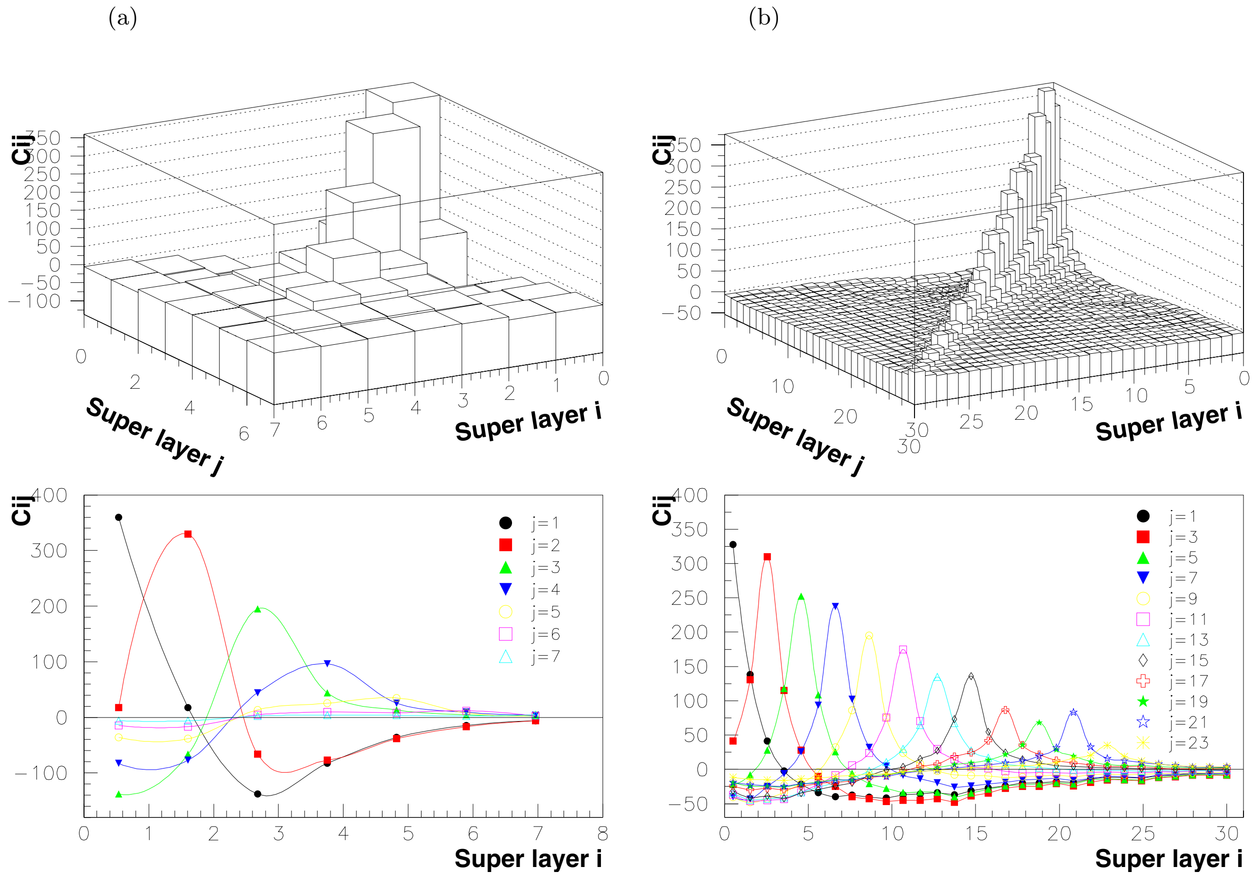

Figure 3: Correlation matrix elements C_{ij} with respect to i for 3 GeV (a) electrons and (b) pions. Upper figures are 3D plots. Lower figures are cross sectional plots.

3.2. Diagonalization of Correlation Matrix

Fig. 4 shows two-dimensional scatter plots of pulse height deviations at two different S.L.s for 3 GeV electrons and pions. These figures show that the pulse height fluctuation of a S.L. is correlated to those of other S.L.s. In these figures, positive and negative correlation are observed. Positive correlation comes from the layers near each other, but negative is from far layers. From the correlated fluctuation δ_i, δ_j of a S.L., we can derive uncorrelated independent fluctuations $\tilde{\delta}_i$, $\tilde{\delta}_j$ by diagonalizing the symmetric matrix C.

$$\tilde{\boldsymbol{\delta}} \equiv T\boldsymbol{\delta} \tag{3}$$

$$TCT^T \equiv \Lambda = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix} \tag{4}$$
$$(\lambda_1 > \cdots > \lambda_n)$$

$$T = \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_n \end{pmatrix} \tag{5}$$

$$\boldsymbol{x}_i = \begin{pmatrix} x_{1i} & \cdots & x_{ni} \end{pmatrix}$$

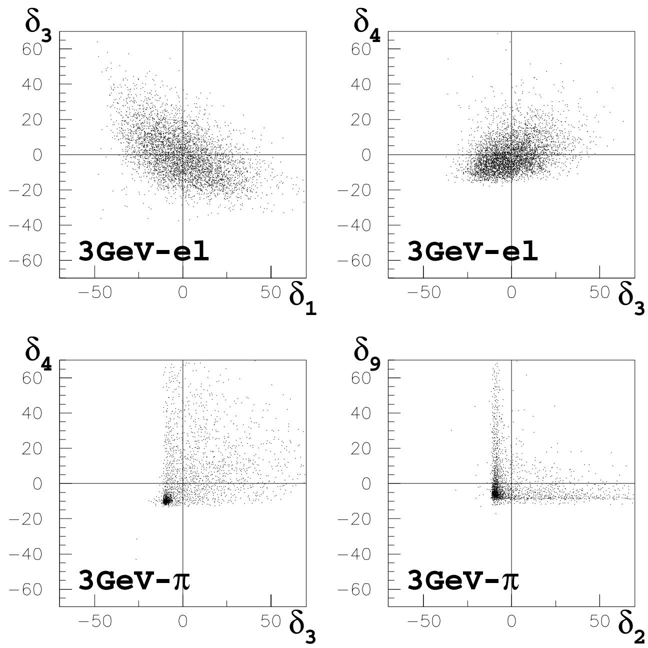

Figure 4: Two-dimensional scatter plots of pulse height deviations at two different depths for 3 GeV electrons and pions.

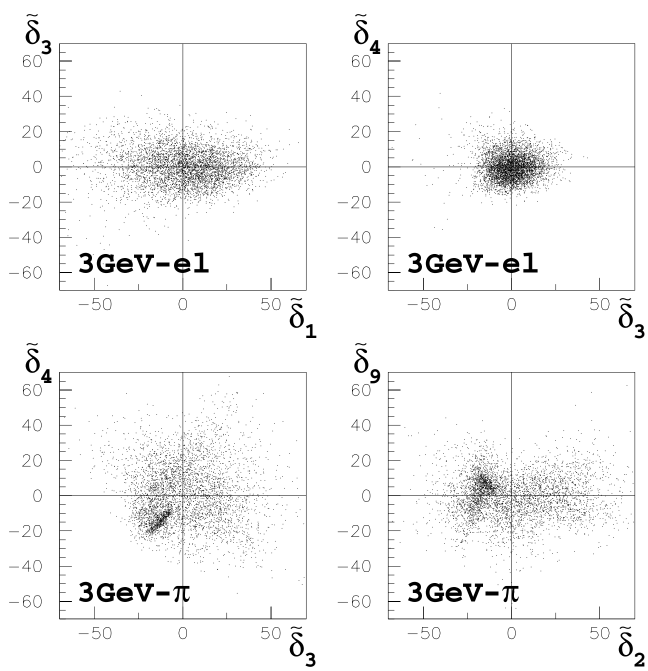

Figure 5: Two-dimensional scatter plots of independent pulse height deviations at two different depths for 3 GeV electrons and pions.

where λ_i's are eigenvalues of the matrix C. T is an orthogonal transformation matrix. $\boldsymbol{x}_l$ means the l-th eigenvector corresponding to the eigenvalue λ_l. Two-dimensional scatter plots of independent pulse height deviations ($\tilde{\boldsymbol{\delta}}_i$ vs. $\tilde{\boldsymbol{\delta}}_j$) for 3 GeV electrons and pions are shown in Fig. 5. The figures for electrons show no correlation between $\tilde{\delta}_i$ and $\tilde{\delta}_j$. For pions, a few parts remain correlated, the overall correlation is negligible. The eigenvalues and eigenvectors of the correlation matrix C were obtained from the experimental data. Eigenvalue λ_l vs. l for 3 GeV pions are shown in Fig. 6. The elements of eigenvector $\boldsymbol{x}_l(i)$ are plotted against the element number i in Fig. 7. First eigenvector corresponds to largest eigenvalue. The vibration of the curve increases as the value of l becomes bigger for pions (as if a base vibration, a 2nd vibration, a 3rd vibration, and so on). We cannot see such a feature for electrons because the number of elements is small. Each eigenvector represents a mode of shower fluctuations.

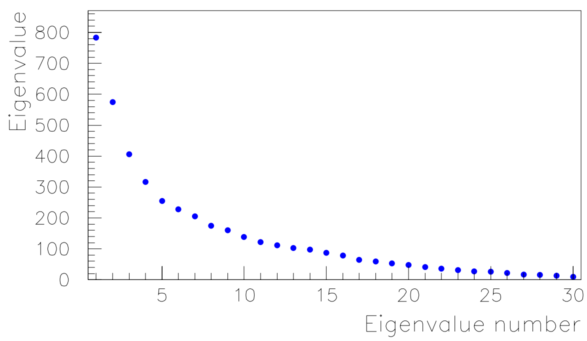

Figure 6: Eigenvalue λ_i of the correlation matrix for 3 GeV pions.

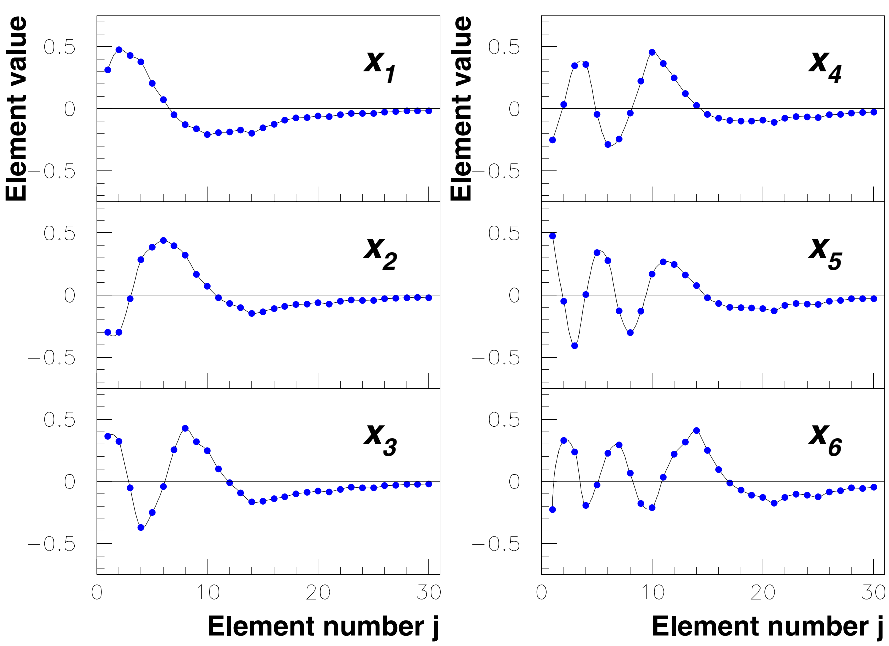

Figure 7: Elements of eigenvectors $\boldsymbol{x}_l$ ($l = 1 \sim 6$) of the correlation matrix for 3 GeV pions.

4. Summary

We studied the longitudinal cascade shower fluctuations. The general properties of the fluctuations were observed for both EM and hadron showers. Independent fluctuations and basic vibrations of the shower development were obtained by diagonalizing the correlation matrix.

References

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[2] K. Abe *et al.*, Particle Physics Experiments at JLC, KEK Report 2001-11 (2001), **hep-ph/0109166**.
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