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The Role of the ILC in the Study of Cosmic Dark Matter

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Though there is strong evidence that dark matter is a major component of the universe, most aspects of dark matter are completely mysterious. We do not know what dark matter is, and we do not know how it is distributed in our galaxy. To resolve these and related questions, we will need information both from particle physics and from astrophysics. In this article, we will describe a path toward the solution of the problems of dark matter, and we will highlight the important role that the ILC has to play in this study.

1. INTRODUCTION

Dark matter is well established as a major component of the universe. We see its gravitational influence at the scales of galaxies and clusters of galaxies and in the dynamics of the plasma that emitted the photons now seen as the cosmic microwave background. These measurements give a consistent estimate that dark matter makes up about 20% of the total energy density of the universe.

Almost every other property of dark matter, however, is a mystery. We do not know what dark matter is made of. The various explanations of dark matter in terms of elementary particles range from particles of mass 10^{-5} eV (axions) through particles of mass 10^{18} GeV ('WIMPzillas'), and even to particles of earth or Jupiter mass (primordial black holes). The paradigm of cosmic structure formation by cold dark matter appears to agree with observations on very large scales, but it is controversial whether this model predicts too large concentrations of mass at the center of galaxies and too many substructures and small companions for the Milky Way.

To address these problems, we need to observe dark matter particles in the galaxy, and to understand those observations, we need to measure the properties of dark matter particles in high-energy physics experiments. Without both halves of the story, we will not be able to reconstruct the full picture.

For many of the possibilities for the identity of the dark matter particle, we may never be able to assemble all of this information. However, there is a general class of candidate dark matter particles for which we can find out experimentally both what they are and where they are. It is possible that the study of these particles could take us over the complete path to the concrete understanding of dark matter. In this talk, we will sketch the particle physics aspects of this study, and the central role that the ILC will play in it.

2. WHY THE WIMP MODEL DESERVES SPECIAL ATTENTION

Among the many particle physics candidates for dark matter, one should receive pride of place. This is the WIMP, which we define to be a massive neutral stable particle that was once in thermal equilibrium in the early universe.

The initial condition of thermal equilibrium allows us to compute the present cosmic density of such a particle, assuming knowledge of the particle's interactions and the extrapolation of standard cosmology back to a temperature comparable to the particle's mass. To perform this computation, one integrates the Boltzmann equation to follow the density of WIMPs as the universe cools to temperatures much lower than the WIMP mass. The WIMPs drop out of thermal equilibrium, and, because of the expansion of the universe, their density becomes so small that further annihilation has a negligible effect. The resulting density is the 'relic density' of the WIMP. To 10% accuracy, this density is given by the relation [1]

$$\Omega_\chi h^2 = \frac{s_0}{\rho_c/h^2}\left(\frac{45}{\pi g_*}\right)^{1/2}\frac{x_f}{m_{\rm Pl}}\frac{1}{\langle\sigma v\rangle} \tag{1}$$

where s_0 is the current entropy density of the universe, ρ_c is the critical density, h is the (scaled) Hubble constant, g_* is the number of relativistic degrees of freedom at the time that the dark matter particle goes out of thermal equilibrium, $m_{\rm Pl}$ is the Planck mass, $x_f \approx 25$, and $\langle\sigma v\rangle$ is the thermal average of the dark matter pair annnihilation cross section times the relative velocity. Most of these quantities are numbers with large exponents. However, combining them and equating the result to $\Omega_\chi \sim 0.2$, we obtain

$$\langle\sigma v\rangle \sim 1 \text{ pb} \tag{2}$$

Interpreting this in terms of a mass, using $\langle\sigma v\rangle = \pi\alpha^2/8m^2$, we find $m = 100$ GeV.

This is a remarkable result, because it places the WIMP at a mass scale where we already expect to find physics beyond the Standard Model. It is highly suggestive that the new physics that is responsible for the breaking of electroweak symmetry also gives rise to a WIMP that is responsible for the dark matter. In fact, it is more than suggestive. In every model of electroweak symmetry breaking, it is possible to add a discrete symmetry that makes the lightest new particle stable. Often, this discrete symmetry is required for other reasons. For example, in supersymmetry, the conserved R parity is needed to eliminate rapid proton decay. In other cases, such as models with TeV-scale extra dimensions, the discrete symmetry is a natural consequence of the underlying geometry. As long as it is generic that the lightest stable particle is neutral, we have a WIMP that is guaranteed to give—to order of magnitude—the correct cosmic density to agree with observations.

If the WIMP model of dark matter is preferred by theory, it is also preferred by experiment, or, at least, by experimenters. Many experiments are now trying to observe dark matter from the galaxy, and more are proposed for the near future. These include 'direct detection' experiments, in which one observes the dark matter as scattering events in sensitive underground detectors, and 'indirect detection', which one observes the products of dark matter annihilation. With a few exceptions, such as the Livermore axion search experiment [2], all of these experiments require that the dark matter particle is a heavy neutral particle with weak-interaction cross sections.

Thus, there are very likely to be WIMP candidates for dark matter. Only for these candidates can we perform the crucial experiments that identify the dark matter and tell us the distribution of dark matter in the galaxy. It would be wonderful if a single type of WIMP could account for all of the dark matter in the universe. But whether this is true or not, we ought to settle the issue experimentally. Let us discuss how this can be done.

3. THE IMPORTANCE OF THE LHC IN DARK MATTER STUDIES

There is another common feature of WIMP models based on models of electroweak symmetry breaking. Since these models must somehow generate the quark masses, there are typically new particles with nonzero color. In supersymmetry, for example, we have the squarks. These particles carry the conserved discrete quantum number and so eventually decay to the WIMP. These colored states can be produced copiously in proton-proton collisions. Thus, with the extra assumption that such particles exist and have masses below about 2 TeV, the LHC will produce huge numbers of WIMPs.

To a first approximation, the cross sections for production of pairs of colored particles at the LHC depend only on the mass of the particle. For colored particles of mass below 1 TeV, these cross sections are tens of pb. In models with a conserved discrete quantum number, the pair-production events will contain several jets coming from the decays of the primary particles, plus two WIMPs that will exit a particle physics detector unobserved. These events have the 'jets plus missing energy' signature that is often considered to be characteristic of supersymmetry. In fact, this signature appears in all models of WIMP dark matter that contain new colored particles satisfying the assumptions just described.

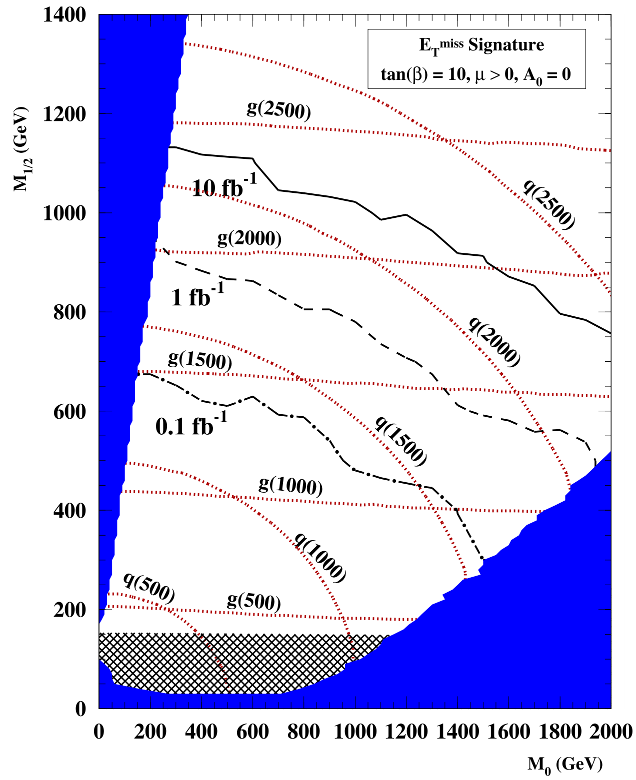

Figure 1: Regions of the mSUGRA parameter space giving the discovery of the missing-energy signature at the LHC with various levels of integrated luminosity, from [3]. To use this result more generally, follow the marked curves of constant squark and gluino masses.

Since the event rates for missing energy events are determined mainly by the primary particle masses, we can estimate these from the results for supersymmetry, which have been worked out in detail. From Fig. 1 [3], we see that, for primary particle masses below 1 TeV, the integrated luminosity required to discover the missing energy signature is amazingly low, about 100 pb^{-1}, or 1% of the LHC first-year design luminosity. It is often noted that it may be a long time before we see signs of the Higgs boson at the LHC. But for WIMP dark matter, the situation is completely different. We will know almost immediately whether the LHC is producing a WIMP dark matter candidate with a mass at the weak interaction scale.

However, this statement comes with an important qualification. Although it will be obvious that the LHC is producing a particle candidate for dark matter, there are scenarios where it might be very difficult to determine, even qualitatively, the identity of that candidate. Consider, for example, the four possible models illustrated on the left-hand side of Fig. 2. This figure shows the decay chain of the colored primary in models of supersymmetry in which the WIMP might be a neutralino or a sneutrino and in models of TeV-scale extra dimensions in which the WIMP might be the partner of the $U(1)$ gauge boson or of a neutrino [4]. In all four cases, the observable leptons and jets in the decay chain are the same. Because at least two unobservable WIMPs are produced in each event, it is not possible to reconstruct the detailed kinematics. Within specific models, characteristic features of the model can be used at the LHC to exclude some of the possibilities [5, 6]. But it is likely that a number of options will remain viable until it is possible to do experiments of a much more incisive type.

Within specific models with a small number of parameters, the LHC data can be used in a powerful way to estimate the dark matter relic density [7]. However, this method goes only so far if we do not know the model. In addition, as well will see in a moment, many models of dark matter—in particular, models in much of the space of supersymmetry preferred by the WMAP result—pose special problems for the LHC in carrying out model-indepedent predictions of the relic density [8].

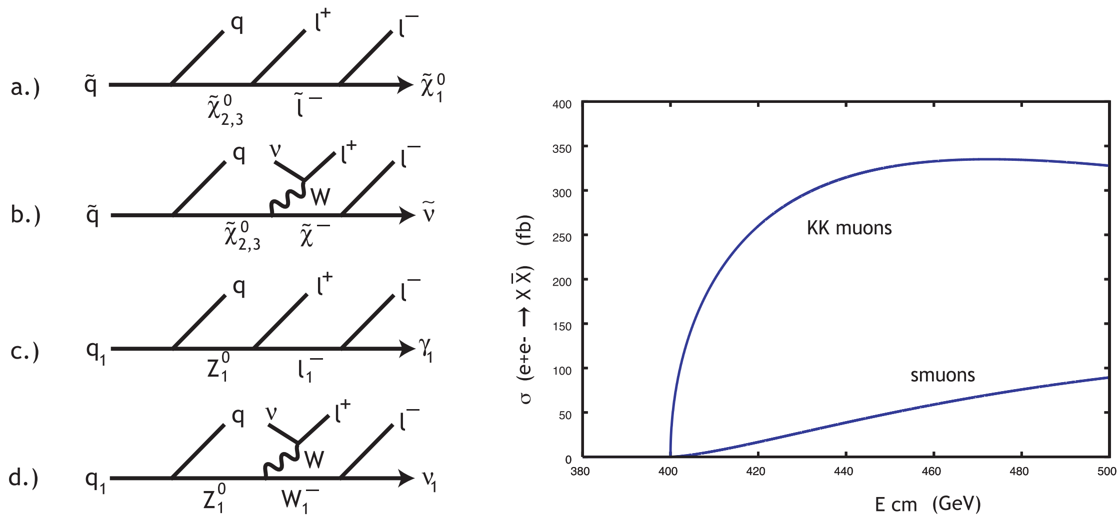

Figure 2: Left: Four possible physics models for missing-energy events at the LHC; Right, discrimination of the models (a) and (c) by a total cross section measurement at the ILC.

4. QUALITATIVE STUDIES OF DARK MATTER AT THE ILC

The problem just described is one that the ILC is well suited to solve. The ILC will provide point-like collisions, tunable centre-of-mass energy, and the availability of powerful analysers such as beam polarisation. These tools will give us the ability to systematically determine the mass spectrum and the interactions of new particles. Even if the ILC may not be able to produce the heaviest new states, the WIMP annihilation cross section depends most strongly on the lightest particles in the new sector. As long as the ILC can reach the first new particle thresholds, it will make the precision measurements of those particles that are most important for predicting the dark matter density.

In models of WIMP dark matter, there is typically a lightest visible particle that decays to the WIMP. In supersymmetry models, this is a slepton or a wino; in extra-dimensional models, it is the Kaluza-Klein recurrence of a lepton or a gauge boson. The various options are distinguished if we can identify the spin and $SU(2) \times U(1)$ quantum numbers of this particle. This plays to a strength of e^+e^- annihilation. The cross section for pair-production in e^+e^- annihilation through a virtual γ and Z has a characteristic energy- and angular-dependence for each value of the spin, and the normalization of the cross section is chacteristic function of the electric charge and weak isospin. This measurement then cleanly separates the various cases. The angular distributions in the decay of the particles provide a check of the spin identification.

To illustrate this, we show on the right-hand side of Fig. 2 the cross sections as a function of energy for the lightest particle that decays to a muon and a WIMP in the models shown in Fig 2(a) and (c). Whereas these models have essentially the same phenomenology at the LHC, we see that they are distinguished in a obvious way at the ILC.

5. QUANTITATIVE ANALYSIS OF WIMP DARK MATTER

Once we have used the ILC results to make the qualitative identification of the model, we are ready to move on to the next level of analysis. A heavy neutral particle observed at accelerators is a candidate for the cosmic dark matter, but this in no way proves that the dark matter is actually composed of this particle. There are three types of observations, though, that would go a long way toward providing that proof. First, we should observe the WIMP in an astrophysical experiment and check that the particle mass seen there is the same at that observed at accelerators. Second, we should determine the parameters of the WIMP model well enough to provide a microscopic prediction of the WIMP relic density. This can be compared to the dark matter density obtained, for example, from the cosmic microwave background. Third, we should check that the microscopic model gives a pattern of WIMP cross sections

Point	m_0	$m_{\frac{1}{2}}$	$\tan\beta$	A_0	sign mu	m_t	reference	$\Omega_\chi h^2$	ILC accuracy
LCC1	100	250	10	-100	+	178	[11]	0.193	$\pm$ 1.0%
LCC2	3280	300	10	0	+	175	[12]	0.110	$\pm$ 3.2%
LCC3	210	360	40	0	+	178	[13]	0.057	$\pm$ 7.5%
LCC4	380	420	53	0	+	178	[14]	0.106	$\pm$ 4.9%

Table I: mSUGRA parameter sets for four illustrative models of neutralino dark matter. Masses are given in GeV.

that is consistent with the result of direct and indirect detection experiments. This last point raises astrophysical questions that we will discuss further in Section 8.

There are many reasonable scenarios in which these tests would fail. The WIMP observed at the LHC and the ILC could make up only a fraction of the cosmic dark matter. The WIMP could decay to a 'super-WIMP' [9] with very small astrophysical cross sections, leading to a decreased dark matter density and zero signal in direct and indirect detection experiments. The WIMP could be produced in the early universe through a mechanism that operates out of thermal equilibrium, leading to a larger density, or the density could be diluted by entropy production after the dark matter density is established, that is, in the period between 10^{-10} sec and 10 sec after the Big Bang. In all of these cases, the results of the experiments that we will describe provide a starting point for analyzing the difficulty and charting out the full theory of dark matter. And, in case the two values of the DM density will actually agree, it would be striking evidence that we would have understood the origin of dark matter. This would be a great triumph for both particle physics and cosmology. Before guesssing that this hypothesis is too optimistic, one should remember that the simplest hypotheses for nuclear physics in the early universe beautifully explain the primordial element abundances [10].

Dark matter detection experiments might measure the mass of the dark matter particle to 10-20% accuracy. The LHC should already measure the mass of the WIMP to comparable accuracy, setting up a first confrontation of particle physics and astrophysical results. To go beyond this level, it is necessary to determine the WIMP interaction cross sections. It is not so easy to experimentally determine the cross sections of an unobservable particle. To see that this can be done, we would now like to specialize to the case of supersymmetry and neutralino WIMP dark matter, for which we have carried out explicit model analyses.

6. FIXING THE NEUTRALINO RELIC DENSITY

Typical models in the parameter space of minimal supersymmetry predict too large densities of neutralino dark matter. In such models, the annihilation cross sections are suppressed, either because the particles exchanged are heavy or because the important couplings for annihilation are suppressed by small mixing angles. Models of supersymmetry that produce the observed dark matter relic density do so because some specific mechanism leads enhanced neutralino annihilation. The enhancement of this cross section might be due, for example, to the presence of light sleptons, leading to $\chi\chi \to \ell^+\ell^-$, to sizable gaugino-Higgino mixing, enhancing $\chi\chi \to W^+W^-$ and $\chi\chi \to ZZ$, or to an accidental degeneracy $m_A \approx 2m_\chi$, leading to annihilation through the A^0 Higgs boson as an s-channel resonance.

Each of these mechanisms can be elucidated by specific experiments on the supersymmetric particles. Essentially, we must determine with high accuracy the masses and couplings of the specific particles that enter the key annihilation reactions, and we must exclude the importance of competing annihilation processes. This requires, for the first goal, precision measurements on the lightest particles in the supersymmetry spectrum and, for the second goal, the model-independent exclusion of the possibility that other particles are comparably light. Both goals are very difficult for the LHC. The LHC can often make precise measurements of some particles in the spectrum, but it is difficult for the LHC experiments to assemble the complete set of parameters needed to reconstruct annihilation cross section. And, it is typical that supersymmetry spectra contain light particles that are very difficult to observe in the hadron collider environment. The ILC, in contrast, provides just the right setting to obtain both types of measurements.

Observable	LCC1	LCC2	LCC3	LCC4
$M(\tilde{\chi}_1^0)$	$\pm$ 0.05	$\pm$ 0.7	$\pm$ 0.1	$\pm$ 1.4
$M(\tilde{e}_R)$	$\pm$ 0.05	-	$\pm$ 1.0	$\pm$ 0.6
$M(\tilde{\tau}_1)$	$\pm$ 0.3	-	$\pm$ 0.5	$\pm$ 0.9
$M(\tilde{\tau}_2)$	$\pm$ 1.1	-	-	-
$M(\tilde{\chi}_1^+)$	$\pm$ 0.55	$\pm$ 0.7		$\pm$ 0.6
$M(\tilde{\tau}_1) - M(\tilde{\chi}_1^0)$		-	$\pm$ 1.0	$\pm$ 1.0
$M(\tilde{\tau}_2) - M(\tilde{\chi}_1^0)$			$\pm$ 1.1	
$M(\tilde{\chi}_2^0) - M(\tilde{\chi}_1^0)$	$\pm$ 0.7	$\pm$ 0.4	$\pm$ 2.0	$\pm$ 1.8
$M(\tilde{\chi}_3^0) - M(\tilde{\chi}_1^0)$		$\pm$ 0.3	$\pm$ 0.5	$\pm$ 2.0
$M(\tilde{\chi}_1^+) - M(\tilde{\chi}_1^0)$		$\pm$ 0.3		$\pm$ 2.0
$M(\tilde{\chi}_2^+) - M(\tilde{\chi}_1^+)$			$\pm$ 2.0	$\pm$ 2.0
$M(A^0)$				$\pm$ 0.8
$\Gamma(A^0)$				$\pm$ 1.2

Table II: Summary of the main mass constraints from the ILC for the four benchmark points.

Again, it is not necessary for the ILC to match the energy of the LHC, only that it provides enough energy to see the lightest charged particles of the new sector.

To illustrate these considerations, a number of specific points in the parameter space of minimal supergravity (mSUGRA) have been chosen for detailed study. Points in mSUGRA are specified by four parameters and a sign. The parameters chosen are listed in Table I. The supersymmetry spectra associated with these parameters were computed using ISAJET 7.69 [15]. Predictions for the dark matter relic density were computed using Micromegas 1.3 [16]; DarkSUSY [17] gives similar results.

The LCC points are chosen to illustrate the various scenarios for the neutralino relic density, in models in which the lightest charged supersymmetric particles can be studied at 500 GeV in the center of mass. LCC1 is chosen as the point SPS1a whose collider observables are studied in great detail in [11]. At this point, t-channel exchange of light sleptons dominates the annihilation cross section. LCC2 is chosen as a point in the 'focus point region' of mSUGRA, with very heavy squarks and sleptons. At this point, the annihilation to W^+W^- and Z^0Z^0 is the dominant mode. The ILC measurement capabilities at this point have been studied in [12]. LCC3 is a point with relatively heavy sleptons, but with the lightest slepton ($\tilde{\tau}_1$) having a mass close to that of the neutralino. In this circumstance, the $\tilde{\tau}_1$ is almost as abundant as the neutralinos at the time that the relic abundance is established, and the dominant processes for supersymmetric particle annihilation are $\tilde{\chi}_1^0\tilde{\tau}_1$ and $\tilde{\tau}_1\tilde{\tau}_1$ annihlation. This situation is call 'coannihilation' [18]. The ILC measurement capabilities at this point have been studied in [13], and the specific issue of measuring the $\tilde{\tau}_1$ mass with precision has been addressed in [19]. LCC4 is a point at which the A^0 Higgs boson is relatively close to the neutralino pair threshold, so that annihilation through the A^0 resonance dominates the annihilation process. The ILC measurement capabilities at this point have been studied in [14].

The ILC analyses just cited are based on parametric simulation that includes realistic detector performances and effects of the ILC beam characteristics. The studies have assumed that the ILC will be able to provide collisions at centre-of-mass energies from 0.3 TeV to 0.5 TeV with an integrated luminosity of 500 fb^{-1} in the first phase of operation. Several of the studies have also assumed a second phase of running at 1 TeV in the center of mass, with an additional data set of 1 ab^{-1}. We show in Table II the estimated accuracies on masses and mass differences derived from these studies.

To assess the abilities of the LHC and ILC collider experiments to predict the dark matter relic density in these models, we have carried out broad scans of the parameter space of supersymmetric models. Previous studies (e.g., [7]) have converted collider measurements to predictions for $\Omega_\chi h^2$ using the assumption that the underlying model belongs to the 4-parameter space of mSUGRA models. We believe that this assumption is much too restrictive to realistically assess the impact of a set of collider measurements. In our analysis, we have described the benchmark points in terms of 24 effective MSSM parameters at the electroweak scale. These parameters sweep out the most

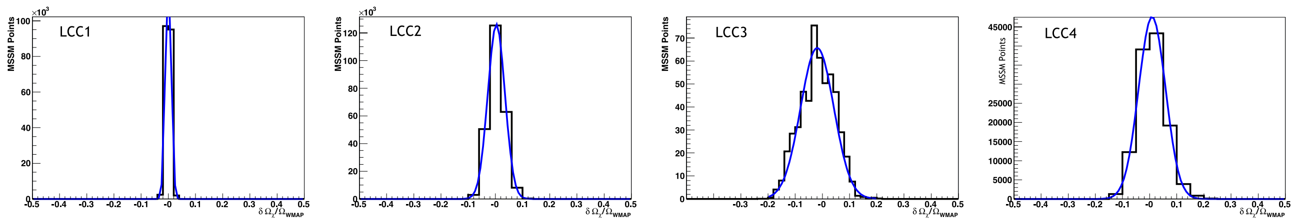

Figure 3: Probabilility distribution of predictions for the neutralino relic density $\Omega_\chi h^2$ based on expected measurements at the LHC and at the ILC, in the four supersymmetry models presented in Table I.

general models within the MSSM in which flavor and CP are conserved. We have then carried out a scan over these 24 parameters to find the full set of MSSM models that would be consistent with measurements made at each benchmark point. We have used two strategies for the scans. In the first, we have made a flat scan in which the MSSM parameters have been independently varied over wide ranges. Each scan point has been weighted by the likelihood that the masses $\{m_j\}$ and other spectral information at the benchmark point can be reproduced by the predictions of the scan point within the errors:

$$\mathcal{L}(P_i) = \prod_j \exp\left\{ -\frac{(m_j(P_i) - m_j(P_0))^2}{2\sigma_j^2} \right\} \ , \tag{3}$$

where P_i is the scan point, P_0 is the benchmark point, and σ_j is the expected experimental accuracy of the measurement. The ILC accuracies for measurements at the various benchmark points, taken from the studies referred to in the previous paragraph, are displayed in Table II. These weights can then be used to build a probability density function for the dark matter relic density, and for cross sections of interest in astrophysics.

The flat scan method offers a even sampling of the parameter phase space, but it is quite inefficient, because the high accuracies of measurements at the ILC select out models in narrow regions of the parameter space. A more efficient way to select points is the Markov Chain Monte Carlo algorithm [20, 21]. In collaboration with Baltz and Wizansky, we have adopted this strategy to compute probability densities for the prediction of dark matter properties. In this method, one steps from one point P_i in the MSSM parameter space to the next point P_{i+1} if the likelihood (3) increases; if the likelihood decreases, one makes the step with the probability $\mathcal{L}(P_{i+1})/\mathcal{L}(P_i)$. This rule produces an ensemble of points that are generated with probabilty proportional to $\mathcal{L}(P_i)$. The Markov Chain method offers a more effective sampling of the parameter phase space compared to the flat scan. A detailed discussion of our analysis and its results will be given in [22].

This analysis leads to estimates of the precision of the prediction of the neutralino relic density from the ILC measurements for the four benchmark points. These estimates are given in the right-hand column of Table I. The accuracies range from 1% in the most straightforward case to 7.5% in the model with coannihilation. The scan data for the four points, and fits to Gaussian distributions, are shown in Fig. 3.

7. COMPARISON OF ILC AND LHC

Using the methods described in the previous section, it is possible to compare predictions for the neutralino relic density from measurements at the ILC to the determinations from LHC measurements. The two analyses can be carried out in parallel, by writing the suite of supersymmetry spectroscopy measurements expected at each benchmark point, constructing the likelihood function, and then following one of the scan strategies described above.

It is important to note that, to apply this analysis to the LHC, we must begin from the assumption that the underlying physics model is supersymmetry. As we have emphasized in Section 3, this assumption would need to be justified by data from the ILC. To keep this in mind, we have labeled the curves from the LHC analysis 'LHC (after Q)', that is, after qualitative identification of the model.

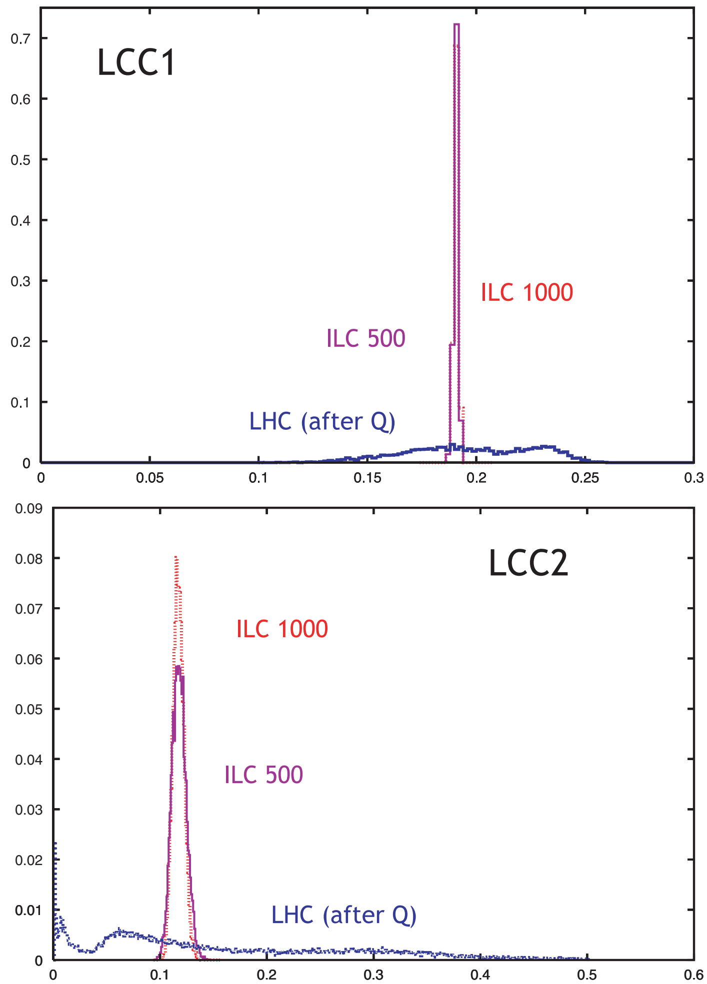

Figure 4: Probability distribution of predictions for $\Omega_\chi h^2$ from collider measurements, using expected measurements from future colliders. In each plot, the three distributions represent the predictions using data from the LHC, from the 500 GeV ILC (peaked solid histograms), and the 1 TeV ILC (dotted histograms). The terminology 'LHC (after Q)' is explained in the text.

In Fig. 4, we show the comparison of the determination of $\Omega_\chi h^2$ from collider data for the reference points LCC1 and LCC2. The figures are constructed by choosing an appropriate supersymmetry parameter point P_0, writing, for LHC and for ILC, a suite of measurements, with errors, that would be expected in that model, and then scanning the 24-dimensional parameter space of the flavor- and CP-conserving MSSM to identify models consistent with this set of measurements. Each model appears with a weight proportional to its likelihood, assuming Gaussian errors in the measurements.

The point LCC1 is identical to the point SPS1a that was studied in detail for collider experiments in [11]. The point is unusual in that squarks produced at the LHC decay through a cascade of two-body decays that include on-shell sleptons. Identification of the endpoints gives enough kinematic constraints to determine all of the light slepton and neutralino masses. In addition, an A^0 boson light enough to provide significant resonant anninhilation would be directly observed through its decay to $\tau^+\tau^-$; thus, the presence of such a light A^0 can be excluded from the LHC data. Nevertheless, the accuracy of these measurements estimates the dark matter density only up to about 20% accuracy. At the ILC, the masses of light sleptons and neutralinos are determined to parts per mil. This gives much stronger constraints, and a determination of the relic density to the level of 1%.

The point LCC2 is one in which the dominant neutralino annihilation is to W^+W^-, Z^0Z^0, and Z^0h^0. In the limit that the neutralino is purely the supersymmetric partner of the $U(1)$ gauge boson, these annihilation reactions are

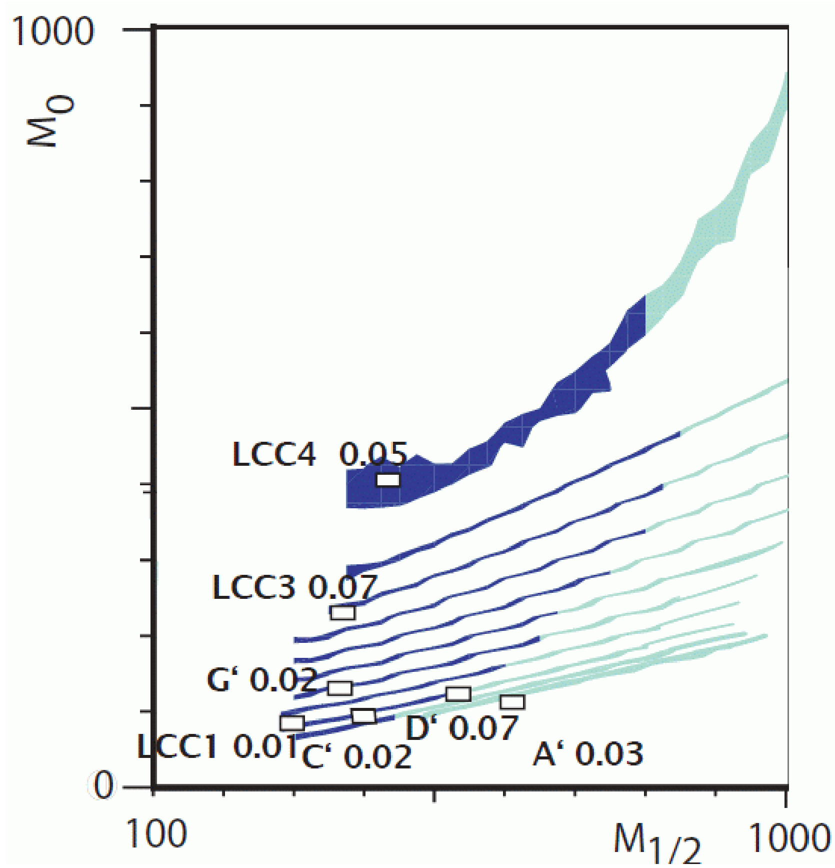

Figure 5: Estimated accuracies $\delta\Omega_\chi/\Omega_\chi$ for the neutralino dark matter relic density as a function of the mSUGRA parameters.

forbidden. So the value of the annihilation cross sections is controlled by the size of the gaugino-Higgsino mixing angles. These must be infered from the details of the chargino and neutralino spectrum. In e^+e^- annihilation, we obtain additional constraints from the polarized e^+e^- pair-production cross sections.

The point LCC2 corresponds to the more generic situation in which the LHC can make a limited set of precision supersymmetry spectroscopy measurements. The squarks and sleptons are very heavy at this point. However, the gluino has a mass of about 850 GeV, so gluino pairs are copiously produced at the LHC. A gluino decays to $q\bar{q}$ plus a neutralino or chargino. The mass difference between $\widetilde{\chi}^0_2$ and $\widetilde{\chi}^0_1$, and also that between $\widetilde{\chi}^0_3$ and $\widetilde{\chi}^0_1$, is less than m_Z, and so these decays contain dilepton cascades that will allow measurement of neutralino mass differences to the 1% level. Still, it turns out to be difficult at the LHC to exclude scenarios with large mixing angles or relatively light staus that lead to rapid annihilation and low values of $\Omega_\chi h^2$. As a result, the predictions for $\Omega_\chi h^2$ from the LHC data cover a very broad range. At the ILC, measurements of the production of the lightest charginos and neutralinos, with mass measurements to part per mil, lead to a prediction of the relic density with few-percent accuracy.

The situation is similar at other points of the MSSM parameter space. The determination of the WIMP annihilation cross section requires control over the couplings and masses of the light particles that decay to the WIMP. The energy reach of the LHC is ultimately less important that the ILC's ability to study these particles in a precise way.

The variation of the error on $\Omega_\chi h^2$ from ILC data over the parameter space of mSUGRA models is shown in Fig. 5. The estimates plotted for the LCC points are from Table I; those for the points A', C', D', G' are from [19].

8. FIXING THE NEUTRALINO DETECTION CROSS SECTIONS

If we can understand the underlying physics associated with the WIMP, we can also determine the cross sections for direct and indirect detection of WIMPs in astrophysical experiments. For these quantities, the comparison of microscopic and astrophysical results brings in new issues. Astrophysical detection rates depend on the basic cross sections, but they also depend on how dark matter is distributed in the galaxy.

From dynamical studies and from gravitational lensing, we now understand that the distribution of dark matter in clusters of galaxies and on super-galactic scales is rather smooth, and is in accord with simulations of the formation of cosmic structure. However, a lingering puzzle of the cold dark matter model of structure formation is that it seems to predict a great deal of structure in the dark matter on scales smaller than that of the galaxy. The cold dark matter model has been claimed to conflict with observations in predicting a greater density of dark matter at the center of the galaxy than is observed, and a larger number of dwarf galaxy companions of the Milky Way. This situation is reviewed in [23, 24]. It is unclear whether the predictions are wrong because the simulations of dark matter in the galaxy are not sufficiently complete, or whether the predictions are correct but the resulting structure is not visible to current experiments.

To obtain a first idea of the issues involved, consider the problem of observing dark matter in the galaxy indirectly through the flux of gamma rays from dark matter annihilation. This flux is given by the formula

$$E_\gamma \frac{d\Phi}{dE_\gamma} = E_\gamma \frac{d(\sigma v)}{dE_\gamma} \cdot \frac{1}{4\pi m_\chi^2} \cdot \int dz \ \rho^2(z) \tag{4}$$

The first two factors here are essentially microscopic quantities, and we might hope to determine them at colliders. The last quantity, proportional to the square of the mass density of dark matter, is determined by astrophysics. Many papers pretend that this quantity can be taken as known. But, in fact, for the dark matter density at the galactic center, this integral varies by five orders of magnitude, for example, among the default density profiles of DarkSUSY [17]. It is actually an ill-posed problem to try to determine both the microscopic properties of dark matter and the distribution of dark matter from the same data set.

The data needed to determine the detection cross sections at colliders is similar to that needed to fix the annihilation cross section that enters the calculation of the relic abundance. The cross section in (4) is relatively straightforward to analyze: The gamma ray spectrum has almost the same form if WIMP pairs annihilate to W^+W^-, Z^0Z^0, or $q\bar{q}$, since in all three cases the gammas come from π^0 decay in jets. The main difficulty is in determining the magnitude of the annihilation cross section. It is tempting to put the value equal to that from (1). This is a good approximation if the WIMPs annihilate in the s-wave and if co-annihilation with other states is not important in setting the relic density. Thus, the calculation of the cross section entering (4) is, to a great extent, a matter of qualitatively distinguishing physics scenarios. Some of these distinctions could be made already at the LHC, and the distinctions would be straightforward to recognize from the ILC data.

For the direct detection cross section, there is less astrophysical uncertainty but more uncertainty from the microscopic physics. Because the cross section depends on only one power of the density, and because we live in a non-exceptional part of the galaxy, the uncertainty from astrophysics might be only a factor of two. However, the detection cross section can have important contributions from s-channel squarks, whose properties will not be well constrained at the ILC, so it might not be possible to fix this cross section microscopically to the high accuracy with which we can predict the relic density. Still, if squarks are relatively light or, on the other hand, too heavy to give large contributions, we will be able to make firm predictions for this cross section.

It is generally the case in astrophysics that observable quantities are convolutions of microscopic cross sections with densities that are determined by cosmic processes. The study of dark matter is no different. In fifteen years, with the ILC data and with data from the coming generations of underground and high-energy astrophysics experiments, we will have a large set of varied and complementary measurements. These may well solve the current questions about the distribution of dark matter in the galaxy. At the very least, they will take us a long way from our current state of ignorance.

9. CONCLUSIONS

In this article, we have discussed WIMP models of dark matter and the possibility that we can elucidate these models experimentally. To confront these models with experiment, to find out whether WIMPs exist and whether they provide all or any of the dark matter, many steps are required. We must:

1. Discover missing-energy events at a collider and estimate the mass of the WIMP.
2. Observe dark matter particles in the galaxy, and determine whether their mass is the same as that observed in collider experiments
3. Determine the qualitative physics model that leads to the missing-energy signature.
4. Determine the parameters of this model that predict the WIMP relic density.
5. Determine the parameters of this model that predict the direct and indirect detection cross sections
6. Measure products of cross sections and densities from astrophysical observations to build the picture of dark matter in the galaxy.

If dark matter is composed of a single type of WIMP, this program of measurements should lead us to a complete understanding of what this particle is and how it is distributed in the galaxy. If the composition of dark matter is more complex, we will only learn this by carrying out this program and finding that it does not sum to a complete picture. Hopefully, further evidence from the microscopic theory will suggest other necessary ingredients.

Both high-energy physics and astrophysics measurements are required for this program. From the high-energy physics side, the first step should be achieved at the LHC. To make further progress, however, we will need the capabilities of the ILC. When the program is complete, astrophysicists will see the ILC as a crucial tool for our understanding of the universe.

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