Physics Gain of a Precise m_t Measurement

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The top quark mass is currently measured to $\delta m_t^{\rm exp,Tevatron} = 2.9$ GeV and will be measured at the LHC to a precision of $\delta m_t^{\rm exp,LHC} \approx 1$–2 GeV. We show that even this impressive precision will not be sufficient for many future physics applications. These include electroweak precision observables, Higgs physics in extensions of the Standard Model as well as cold dark matter predictions in Supersymmetry. The desired experimental precision can only be reached at the ILC with $\delta m_t^{\rm exp,ILC} \approx 100$ MeV.

1. INTRODUCTION

The mass of the top quark, m_t, is a fundamental parameter of the electroweak theory. It is by far the heaviest of all quark masses and it is also larger than the masses of all other known fundamental particles. The large value of m_t gives rise to a large coupling between the top quark and the Higgs boson and is furthermore important for flavor physics. It could therefore provide a window to new physics. The correct prediction of m_t will be a crucial test for any fundamental theory. The top-quark mass also plays an important role in electroweak precision physics, as a consequence in particular of non-decoupling effects being proportional to powers of m_t. A precise knowledge of m_t is therefore indispensable in order to have sensitivity to possible effects of new physics in electroweak precision tests.

The current world average for the top-quark mass from the measurement at the Tevatron is $m_t = 172.7 \pm 2.9$ GeV [1]. The prospective accuracy at the LHC is $\delta m_t^{\rm exp} = 1$–2 GeV [2], while at the ILC a very precise determination of m_t with an accuracy of $\delta m_t^{\rm exp} \lesssim 100$ MeV will be possible [3, 4]. This error contains both the experimental error of the mass parameter extracted from the $t\bar{t}$ threshold measurements at the ILC and the envisaged theoretical uncertainty from its transition into a suitable short-distance mass (like the $\overline{\rm MS}$ mass).

In the following we show for some examples that in many physics applications the experimental error on m_t achievable at the LHC would be the limiting factor, demonstrating the need for the ILC precision. More examples can be found in Ref. [5].

2. ELECTROWEAK PRECISION OBSERVABLES

Electroweak precision observables (EWPO) can be used to perform internal consistency checks of the model under consideration and to obtain indirect constraints on unknown model parameters. This is done by comparing experimental results of the EWPO with their theory prediction within, for example, the Standard Model (SM) or the Minimal Supersymmetric Standard Model (MSSM).

The two most prominent EWPO are the mass of the W boson, M_W, and the effective leptonic weak mixing angle, $\sin^2\theta_{\rm eff}$. Their current experimental uncertainties and the prospective precisions with further data from the Tevatron, the LHC and the ILC (including the GigaZ option) are summarized in Tab. I, see Refs. [6, 7] for further details.

In addition to the experimental uncertainties there are two sources of theoretical uncertainties: those from unknown higher-order corrections ("intrinsic" theoretical uncertainties), and those from experimental errors of the input parameters ("parametric" theoretical uncertainties). The current and estimated future intrinsic uncertainties within

the SM are [6, 8]

$$\Delta M_W^{\rm intr,today,SM} \approx 4 \text{ MeV}, \qquad \Delta \sin^2\theta_{\rm eff}^{\rm intr,today,SM} \approx 5\times 10^{-5} \ , \tag{1}$$

$$\Delta M_W^{\rm intr,future,SM} \approx 2 \text{ MeV}, \qquad \Delta \sin^2\theta_{\rm eff}^{\rm intr,future,SM} \approx 2\times 10^{-5} \ , \tag{2}$$

while in the MSSM the current intrinsic uncertainties are estimated to [7, 9, 10]

$$\Delta M_W^{\rm intr,today,MSSM} \approx (5-9) \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff}^{\rm intr,today,MSSM} \approx (5-7)\times 10^{-5} \ , \tag{3}$$

depending on the supersymmetric (SUSY) mass scale. In the future one expects that they can be brought down to the level of the SM, see Eq. (2).

The parametric errors of M_W and $\sin^2\theta_{\rm eff}$ induced by the top quark mass, the uncertainty of $\Delta\alpha_{\rm had}$ (we assume a future determination of $\delta(\Delta\alpha_{\rm had}) = 5\times 10^{-5}$ [11]) and the experimental uncertainty of the Z boson mass, $\delta M_Z =$ 2.1 MeV, are collected in Tab. II.

In order to keep the theoretical uncertainty induced by m_t at a level comparable to or smaller than the other parametric and intrinsic uncertainties, δm_t has to be of $\mathcal{O}(0.1$ GeV) in the case of M_W, and about 0.5 GeV in the case of $\sin^2\theta_{\rm eff}$. If the experimental error of m_t remains substantially larger, it would constitute the limiting factor of the theoretical uncertainty. Using the EWPO to distinguish different models from each other or to determine indirectly the unknown model parameters the ILC precision on m_t is crucial, in particular in view of the precision measurement of $\sin^2\theta_{\rm eff}$ at GigaZ [5].

Table I: Expected experimental accuracies of M_W and $\sin^2\theta_{\rm eff}$ at the Tevatron, the LHC and the ILC/GigaZ.

	today	Tev./LHC	ILC	GigaZ
$\delta\sin^2\theta_{\rm eff}$ [10^{-5}]	17	17	–	1.3
δM_W [MeV]	34	15	10	7

Table II: Parametric errors on the prediction of M_W and $\sin^2\theta_{\rm eff}$.

	$\delta m_t = 2.9$ GeV	$\delta m_t = 2$ GeV	$\delta m_t = 1$ GeV	$\delta m_t = 0.1$ GeV	$\delta(\Delta\alpha_{\rm had})$	δM_Z
$\delta\sin^2\theta_{\rm eff}$ [10^{-5}]	8.7	6	3	0.3	1.8	1.4
ΔM_W [MeV]	17.4	12	6	1	1	2.5

3. HIGGS PHYSICS IN THE MSSM AND OTHER EXTENSIONS OF THE SM

Because of its large mass, the top quark is expected to have a large Yukawa coupling to Higgs bosons, being proportional to m_t. In each model where the Higgs boson mass is not a free parameter but predicted in terms of the the other model parameters (as e.g. in the MSSM), the diagram in Fig. 1 contributes to the Higgs mass. This diagram gives rise to a leading m_t contribution of the form

$$\Delta m_h^2 \sim G_F\ N_C\ C\ m_t^4 \ , \tag{4}$$

where G_F is the Fermi constant, N_C is the color factor and the coefficient C depends on the specific model. Thus the experimental error of m_t necessarily leads to a parametric error in the Higgs boson mass evaluation.

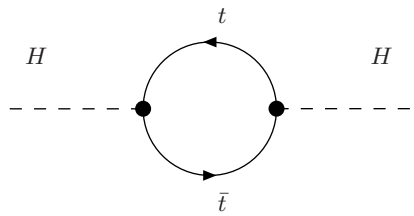

Figure 1: Loop contribution of the top quark to the Higgs boson mass.

Taking the MSSM as a specific example (including also the scalar top contributions and the appropriate renormalization) $N_C\, C$ is given for the light $\mathcal{CP}$-even Higgs boson mass by

$$N_C\, C = \frac{3}{\sqrt{2}\,\pi^2\,\sin^2\beta}\,\log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)\,. \tag{5}$$

Here $m_{\tilde{t}_{1,2}}$ denote the two masses of the scalar tops. The optimistic LHC precision of $\delta m_t = 1$ GeV leads to an uncertainty of $\sim 2.5\%$ in the prediction of m_h, while the ILC will yield a precision of $\sim 0.2\%$. These uncertainties have to be compared with the anticipated precision of the future Higgs boson mass measurements. With a precision of $\delta m_h^{\rm exp,LHC} \approx 0.2$ GeV [12] the relative precision is at the level of $\sim 0.2\%$. It is apparent that only the ILC precision of m_t will yield a parametric error small enough to allow a precise comparison of the Higgs boson mass prediction (where also the intrinsic theoretical uncertainty has to be improved accordingly) and its experimental value.

In Fig. 2 the effects of the current top quark mass uncertainty on the m_h prediction [13] are compared to the ILC precision in two benchmark scenarios, the $m_h^{\rm max}$ and the no-mixing scenario [14]. The plot shows m_h as a function of $\tan\beta$, the ratio of the two vacuum expectation values of the two MSSM Higgs doublets. Also indicated is a hypothetical m_h measurement at the LHC, while no intrinsic theoretical uncertainty from unknown higher-order corrections is included. Currently this error is estimated to $\delta m_h^{\rm intr,today} \approx 3$ GeV [7, 15, 16, 17]. In the future one can hope for an improvement down to $\lesssim 0.5$ GeV [7, 17]. If the intrinsic error could be reduced even to ~ 0.1 GeV, its effect in the plot would be roughly as big as the one induced by $\delta m_t = 0.1$ GeV. The inclusion of an intrinsic uncertainty of ~ 1 GeV would lead to a significant widening of the inner band (δm_t from ILC) of predicted m_h values. In this case the intrinsic uncertainty would dominate, implying that a reduction of $\delta m_t = 1$ GeV to $\delta m_t = 0.1$ GeV would lead only to a moderate improvement of the overall uncertainty on m_h.

Confronting the theoretical prediction of m_h with a precise measurement of the Higgs boson mass constitutes a very sensitive test of the MSSM, which allows to obtain constraints on the model parameters, in this case $\tan\beta$. However, the sensitivity of the m_h prediction on $\tan\beta$ shown in Fig. 2 cannot directly be translated into a prospective indirect determination of $\tan\beta$, since fixed values are assumed for all other SUSY parameters. In a realistic situation the anticipated experimental errors of the other SUSY parameters have to be taken into account. For examples including these parametric errors see Refs. [5, 18].

4. COSMOLOGY

In this section we focus on the framework of the constrained MSSM (CMSSM), in which the soft supersymmetry-breaking scalar and gaugino masses are each assumed to be equal at some Grand Unification Theory (GUT) input scale. In this case, the new independent MSSM parameters are just four in number: the universal gaugino mass $m_{1/2}$, the scalar mass m_0, the trilinear soft supersymmetry-breaking parameter A_0, and $\tan\beta$. The pseudoscalar Higgs mass M_A and the magnitude of the Higgs mixing parameter μ can be determined by using the electroweak vacuum conditions, leaving the sign of μ as a residual ambiguity.

The non-discovery of supersymmetric particles and the Higgs boson at LEP and other present-day colliders imposes significant lower bounds on $m_{1/2}$ and m_0. An important further constraint is provided by the density of cold dark matter (CDM) in the Universe, which is tightly constrained by WMAP and other astrophysical and cosmological data [19], leading to $0.094 < \Omega_{\rm CDM}\, h^2 < 0.129$. This has the effect within the CMSSM, assuming that the dark

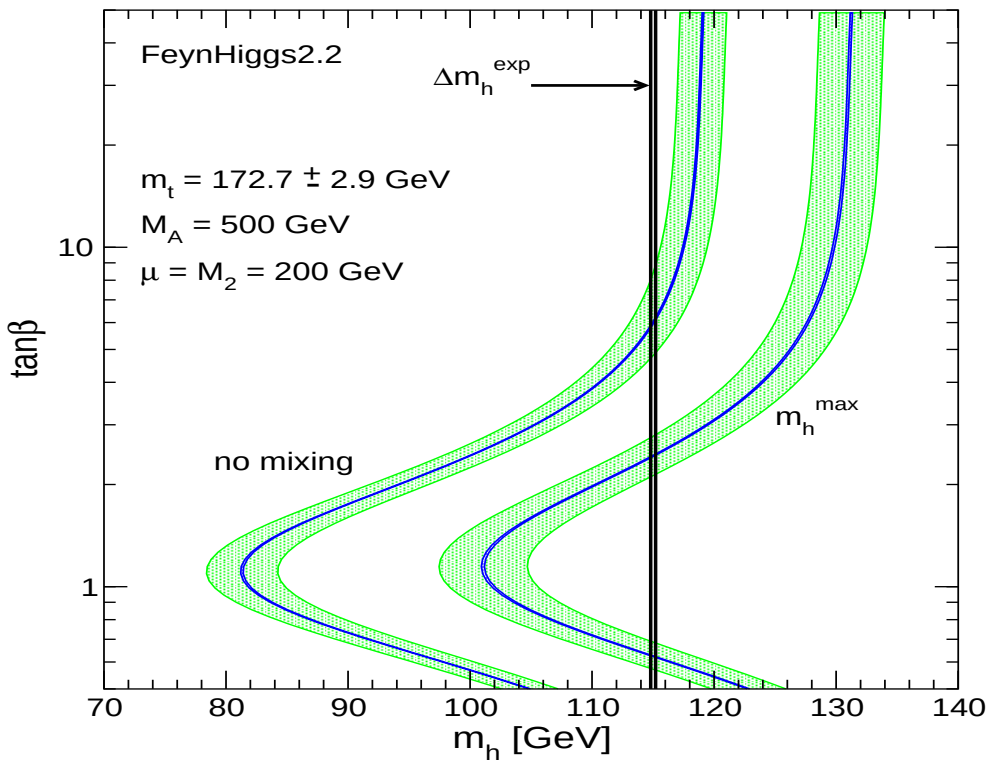

Figure 2: m_h as a function of $\tan\beta$ in the $m_h^{\max}$ and the no-mixing scenario. The light (green) shaded area corresponds to the current top quark mass uncertainty of $\delta m_t^{\exp} = 2.9$ GeV, the dark (blue) shaded one to the anticipated ILC accuracy, $\delta m_t^{\exp} = 0.1$ GeV. Also shown is a possible future m_h measurement at the LHC of $\delta m_h = 0.2$ GeV.

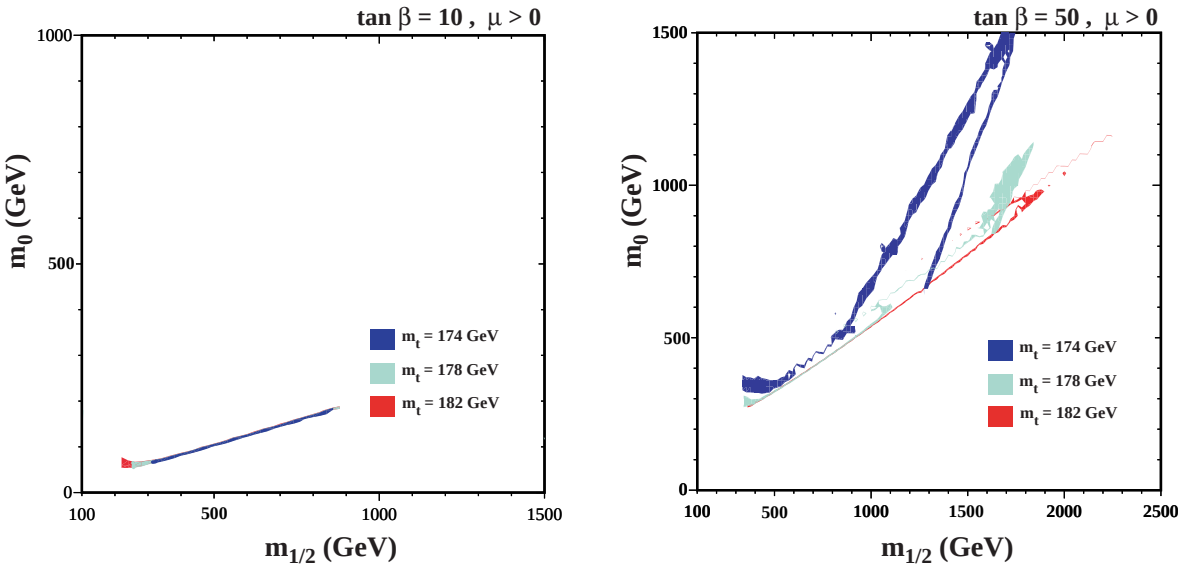

Figure 3: The WMAP strips for $\mu > 0$, $A_0 = 0$ and $\tan\beta = 10$ (left) or $\tan\beta = 50$ (right). The strips are shown for three top quark mass values, $m_t = 174, 178, 182$ GeV [23].

matter consists largely of neutralinos [20], of restricting m_0 to very narrow allowed strips for any specific choice of A_0, $\tan\beta$ and the sign of μ [21, 22]. It is then possible to restrict phenomenological analysis to these 'WMAP strips', see e.g. Ref. [23] and references therein.

Varying the value of m_t has a significant effect on the regions of CMSSM parameter space allowed by CDM, particularly in the 'funnels' where neutralinos annihilate rapidly via the H, A poles. Because of the constraints from the anomalous magnetic moment of the muon [24, 25] we focus here on the case with $\mu > 0$.

Plotted in Fig. 3 [23] is the region in the $m_{1/2}$–m_0 plane for fixed $\tan\beta, A_0$ and $\mu > 0$ for which the relic density is in the WMAP range. We have applied cuts based on the lower limit to the Higgs mass [26, 27], $b \to s\gamma$ [28], and require that the LSP be a neutralino rather than the stau, see Ref. [23] for further details. The thin strips correspond to the relic density being determined by either the coannihilation between nearly degenerate $\tilde{\tau}$'s and χ's or, as seen at high $\tan\beta$, by rapid annihilation when $m_\chi \approx M_A/2$. One can see in the left plot of Fig. 3 that the change in the WMAP strips for $\mu > 0$ and $\tan\beta = 10$ is moderate as m_t is varied, reflecting the fact that the allowed strip is dominated by annihilation of the neutralino LSP χ with the lighter stau. The main effect of varying m_t is that the truncation at low $m_{1/2}$, due to the Higgs mass constraint, becomes more important at low m_t. This effect is not visible in the right plot of Fig. 3 with $\tan\beta = 50$, where the cutoff at low $m_{1/2}$ is due to the $b \to s\gamma$ constraint, and rapid $\chi\chi \to A, H$ annihilation is important at large $m_{1/2}$. The allowed regions at larger $m_{1/2}$ vary significantly with m_t for $\tan\beta = 50$, because the A, H masses and hence the rapid-annihilation regions are very sensitive to m_t through the renormalization group (RG) running.

Thus for large $\tan\beta$ a precise determination of m_t is indispensable to connect the GUT scale parameters with the CDM measurement. An uncertainty of $\delta m_t^{\rm exp} = 1$–2 GeV would sweep out a large part of the $m_{1/2}$–m_0 plane. A precise determination of $\delta m_t^{\rm exp} = 0.1$ GeV, on the other hand, would result in very thin and precisely determined strips that give m_0 as a function of $m_{1/2}$ (depending on the precision of A_0 and $\tan\beta$ and the theory uncertainty in the RG running), see also Ref. [29].

5. CONCLUSIONS

We have investigated the impact of the experimental error of the top quark mass on various physics scenarios. Especially we have compared the parametric error induced by the LHC uncertainty of $\delta m_t^{\rm exp,LHC} \approx 1$–$2$ GeV with the one of the ILC, $\delta m_t^{\rm exp,ILC} \approx 0.1$ GeV.

Concerning electroweak precision observables such as M_W and $\sin^2\theta_{\rm eff}$ the parametric error induced by $\delta m_t^{\rm exp}$ has been investigated. It will match the intrinsic error of M_W and $\sin^2\theta_{\rm eff}$ and their other parametric errors only if the ILC precision of $\delta m_t^{\rm exp} \sim 0.1$ GeV can be reached. Otherwise the parametric error from m_t will dominate the future uncertainties and hamper the otherwise powerful consistency checks of the model under investigation.

The large Yukawa coupling of the top quark can induce large corrections to the prediction of the Higgs boson mass and result in corresponding parametric uncertainties. As has been discussed for the specific example of the MSSM, the prospective experimental error of m_h at the LHC can only be matched if $\delta m_t^{\rm exp} \sim 0.1$ GeV can be achieved.

Furthermore, the prediction of the cold dark matter abundance in the CMSSM has been analyzed. An m_t uncertainty enters the CDM prediction in particular through renormalization group running from the GUT scale to the electroweak scale. For large values of $\tan\beta$ the m_t error can induce a large uncertainty in the CDM prediction. The precision of $\delta m_t^{\rm exp} \sim 0.1$ GeV is desirable for fully exploiting the restrictions of the CDM measurements on the CMSSM parameter space, permitting thorough consistency checks of the model.

Summarizing, the already impressive LHC precision on m_t will not be sufficient to match the required future precisions in various physics models and scenarios. Only the ILC precision will be able to reach the desired accuracy.

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