Identification of Large Extra Spatial Dimensions at the LC

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We discuss the possibility to cleanly distinguish, in electron-positron annihilation into fermion pairs at a high energy collider, the indirect manifestations of graviton exchange from those of four-fermion contact interactions. The method is based on cross section asymmetries emphasizing the spin-2 character of graviton exchange and is explicitly applied to the ADD scenario for gravity in extra dimensions. The availability of initial beams longitudinal polarization is also taken into account in the analysis. For typical c.m. energies and time-integrated luminosities foreseen at the International Linear Collider ($\sqrt{s} = 0.5 - 1$ TeV, $\mathcal{L}_{\rm int} = 50 - 1000\,{\rm fb}^{-1}$), a 5σ identification reach of $3.5 - 5.8$ TeV for the mass scale M_H relevant to the ADD model can be obtained, while the size of the reach on the mass scales Λ characterizing four-fermion contact interactions can be of the order of $45 - 65$ TeV.

1. INTRODUCTION

In a variety of proposed new physics (NP) scenarios, non-standard interactions among the familiar Standard Model (SM) particles can be mediated by exchanges of new quantum states with mass scales expected to be much larger than the c.m. energy available at current (and perhaps future) colliders. Accordingly, only indirect manifestations of such high mass scales and their corresponding novel interactions can occur, through deviations of the measured cross sections from the SM predictions. The most convenient theoretical representation of such interactions is provided by the effective interaction framework, where the non-standard Hamiltonian is expanded in a series of specific local operators of increasing dimension, and accordingly the transition amplitudes for processes among the SM particles are power expanded in the (small) ratio between the 'low' c.m. machine energy and the relevant high mass scales. Generally, to limit the number of unknown parameters to be constrained (or determined) experimentally, the lowest-dimensional operator is retained in the expansion, assuming higher powers to be negligible due to the strong suppression by the large mass scale. Clearly, in this situation, NP searches are favoured by the signal enhancement due to the high energies and luminosities available at the planned linear colliders.

A relevant aspect in this regard is that, in principle, different kinds of non-standard interactions may produce in the integrated cross sections similar deviations from the SM and, therefore, it is important to devise suitable observables that, given the expected experimental accuracy, can discriminate among the various, and competing, possible sources of a given deviation. Here, we will focus on the problem of cleanly identifying, in high energy $e^+e^- \to f\bar{f}$ $[f \neq e, t]$ at the International Linear Collider (ILC), signals of the ADD model of gravity in large, compactified, extra spatial dimensions with respect to the effects originating from four-fermion contact interactions. For this purpose, we shall use particular combinations of integrated cross sections, the so-called "center-edge asymmetries", sensitive to the angular dependence of deviations from graviton exchange.

We recall that the differential cross section of the considered processes reads, in terms of helicity cross sections [1] ($z \equiv \cos\theta$; $\alpha, \beta = {\rm L}, {\rm R}$):

$$\frac{d\sigma}{dz} = \frac{1}{4}\sum_{\alpha\beta}\frac{d\sigma_{\alpha\beta}}{dz}; \qquad \frac{d\sigma_{\alpha\beta}}{dz} = N_{\rm colors}\frac{3}{8}\sigma_{\rm pt}|\mathcal{M}_{\alpha\beta}|^2\,(1 \pm z)^2, \tag{1}$$

where '$\pm$' refer to the LL, RR and LR, RL configurations, respectively. The helicity amplitudes can rather generally be expanded into the familiar γ, Z s-channel exchanges plus deviations induced by the novel interaction ($\chi_Z(s)$ is here the Z propagator):

$$\mathcal{M}_{\alpha\beta} \equiv \mathcal{M}^{\rm SM}_{\alpha\beta} + \Delta_{\alpha\beta} = Q_e Q_f + g^e_\alpha\, g^f_\beta\, \chi_Z(s) + \Delta_{\alpha\beta}. \tag{2}$$

It may be useful to also reacall that the SM cross section can be decomposed into z-even and z-odd parts through the total and forward-backward cross sections as

$$\frac{d\sigma^{\rm SM}}{dz} = \frac{3}{8}\sigma^{\rm SM}\left(1+z^2\right) + \sigma^{\rm SM}_{\rm FB}\, z. \tag{3}$$

In the ADD large extra dimension scenario [2], only gravity can propagate in at least two extra spatial dimensions compactified to a radius R of the millimeter size or less, while the SM particles live in the ordinary four-dimensional spacetime and their mutual gravitational interactions are represented by the exchange of a tower of graviton Kakuza-Klein (KK) states $\vec{n}$, very weakly [gravitationally] coupled and with evenly spaced (and almost continuous) mas spectrum $m^2_{\vec{n}} = \vec{n}^2/R^2$ [3]. The summation over the KK spectrum requires the introduction of a ultaviolet cut-off mass scale M_H, expected in the (multi) TeV region, and the interaction can be represented by a dimension-8 effective Lagrangian of the form [4]

$$\mathcal{L}^{\rm ADD} = i\frac{4\lambda}{M_H^4} T^{\mu\nu} T_{\mu\nu}, \tag{4}$$

where $T_{\mu\nu}$ is the energy-momentum tensor and $\lambda = \pm 1$.[1] The corresponding deviations in Eq. (2) are z-dependent:

$$\Delta_{\rm LL}({\rm ADD}) = \Delta_{\rm RR}({\rm ADD}) = f_G(1-2z), \quad \Delta_{\rm LR}({\rm ADD}) = \Delta_{\rm RL}({\rm ADD}) = -f_G(1+2z), \tag{5}$$

where $f_G = \lambda\, s^2/(4\pi\alpha_{\rm e.m.} M_H^4)$ represents the strength of the interaction associated with spin-2 graviton exchange.

The four-fermion contact interaction scenario (CI) can be represented by the following vector-vector dimension-6 effective Lagrangian, and corresponding helicity amplitudes deviations from the SM ($|\eta_{\alpha\beta}| = 1, 0$) [6]:

$$\mathcal{L}_{\rm CI} = 4\pi \sum_{\alpha,\beta} \frac{\eta_{\alpha\beta}}{\Lambda^2_{\alpha\beta}} \left(\bar{e}_\alpha \gamma_\mu e_\alpha\right)\left(\bar{f}_\beta \gamma^\mu f_\beta\right); \qquad \Delta_{\alpha\beta}({\rm CI}) = \pm\frac{s}{\alpha_{\rm e.m.}}\frac{1}{\Lambda^2_{\alpha\beta}}. \tag{6}$$

As one can see, in this case amplitudes deviations are z-independent. Actually, although originally inspired by fermion compositeness remnant binding forces, $\mathcal{L}_{\rm CI}$ should more generally be considered as an effective, "low energy" representation of a variety of non-standard interactions acting at energy scales Λ much larger than the process Mandelstam variables, for example the exchanges of very heavy Z's [7], leptoquarks [8] and even scalar particle exchanges in the t-channel, such as sneutrinos [9] in the contact interaction limit.

Clearly, suitable observables are needed to discriminate signals of the different kinds of NP models.

2. CENTER-EDGE ASYMMETRY

We consider the difference between the "central" and "edge" parts of the cross section:

$$\sigma_{\rm CE}(z^*) \equiv \sigma_{\rm C} - \sigma_{\rm E} = \left[\int_{-z^*}^{z^*} - \left(\int_{-1}^{-z^*} + \int_{z^*}^{1}\right)\right] \frac{d\sigma}{dz}\, dz, \tag{7}$$

with $0 < z^* < 1$, and define the asymmetry $A_{\rm CE}$ by the ratio [10]

$$A_{\rm CE} = \frac{\sigma_{\rm CE}}{\sigma}. \tag{8}$$

[1] In principle, a smooth cutoff procedure based on the "minimal length scale" can be applied in the sum over KK states [5].

This asymmetry is sensitive only to the $z = \cos\theta$-even terms of the differential cross section. Indeed, Eq. (6) shows that in the four-fermion interaction case the differential cross section will have exactly the same angular dpendence as the SM one [see Eq. (3)], therefore:

$$A_{\rm CE}^{\rm CI}(z^*) = A_{\rm CE}^{\rm SM} = \frac{1}{2}\, z^*\,(z^{*2}+3) - 1, \tag{9}$$

independent of $\sqrt{s}$, flavour of final fermions and longitudinal polarization. Accordingly, as regards the deviation from the SM prediction, $\Delta A_{\rm CE} \equiv A_{\rm CE} - A_{\rm CE}^{\rm SM}$, for any value of z^*:

$$\Delta A_{\rm CE}^{\rm CI} \equiv A_{\rm CE}^{\rm CI} - A_{\rm CE}^{\rm SM} = 0. \tag{10}$$

Also, from Eq. (9) one notices that for the particular value $z_0^* = (\sqrt{2}+1)^{1/3} - (\sqrt{2}-1)^{1/3} \simeq 0.60$ ($\theta \simeq 53.4^\circ$), one has $A_{\rm CE}^{\rm SM}(z_0^*) = A_{\rm CE}^{\rm CI}(z_0^*) = 0$.

By contrast, Eq. (5) implies that finite deviations of $A_{\rm CE}$ occur for the graviton-exchange ADD scenario for all z^*, and these are dependent on the flavour of the final states f. Indeed, at the leading order in the graviton coupling f_G, i.e., retaining in the differential cross section interference terms with the SM only:

$$\Delta A_{\rm CE}^{\rm ADD}(z^*) \equiv A_{\rm CE}^{\rm ADD} - A_{\rm CE}^{\rm SM} \propto f_G z^* \left(1 - z^{*2}\right). \tag{11}$$

In conclusion, the asymmetry $A_{\rm CE}$ is "blind" to conventional four-fermion contact interactions at all z^* (no deviation from the SM in this case), whereas it is sensitive to ADD graviton exchange effects and, accordingly, can cleanly identify the new physics represented by this scenario. Moreover, the maximal senstitivty could be obtained by measuring $A_{\rm CE}$ around z_0^* where the SM and the CI contributions are both vanishing.[2]

3. CENTER-EDGE FORWARD-BACKWARD ASYMMETRY

With $0 < z^* < 1$, we now consider the analogue of Eq. (7)

$$\sigma_{\rm CE,FB} \equiv (\sigma_{\rm C,FB} - \sigma_{\rm E,FB}) = \left[\left(\int_0^{z^*} - \int_{-z^*}^{0}\right) - \left(\int_{z^*}^{1} - \int_{-1}^{-z^*}\right)\right] \frac{d\sigma}{dz}\, dz. \tag{12}$$

This asymmetry is defined by the ratio, sensitive to $z = \cos\theta$-odd terms only [12]:

$$A_{\rm CE,FB} = \frac{\sigma_{\rm CE,FB}}{\sigma}. \tag{13}$$

For the case of four-fermion contact interactions, Eq. (6), due to the identical angular dependence as in the SM, one immediately finds the relations

$$A_{\rm CE,FB}^{\rm SM}(z^*) = A_{\rm FB}^{\rm SM}\,(-1+2z^{*2}) \qquad \Longrightarrow \qquad A_{\rm CE,FB}^{\rm CI}(z^*) = A_{\rm FB}^{\rm CI}\,(-1+2z^{*2}). \tag{14}$$

Correspondingly, in general contact interactions determine finite deviations of $A_{CE,FB}^{\rm CI}$ from the SM predictions, as indicated by Eq. (14). However, at the value $z_{\rm CI}^* = 1/\sqrt{2}$ ($\theta = 45^\circ$) one has $A_{\rm CE,FB}^{\rm SM}(z_{\rm CI}^*) = A_{\rm CE,FB}^{\rm CI}(z_{\rm CI}^*) = \Delta A_{\rm CE,FB}^{\rm CI}(z_{\rm CI}^*) = 0$, i.e., no deviation there. Consequently, $A_{\rm CE,FB} \neq 0$ at this value of z^* would definitely signal the presence of NP different from four-fermion contact interactions.

In the graviton exchange ADD model, using Eq. (5) one directly finds for the deviation of $A_{\rm CE,FB}(z^*)$ from the SM the following expression to leading order in f_G:

$$\Delta A_{\rm CE,FB}^{\rm ADD}(z^*) = \Delta A_{\rm FB}^{\rm ADD}\left(-1+2z^{*4}\right) \qquad \left[\Delta A_{\rm FB}^{\rm ADD} \propto f_G\right]. \tag{15}$$

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[2] This observable can similarly be applied to identify graviton exchange in lepton-pair production at hadron colliders [11].

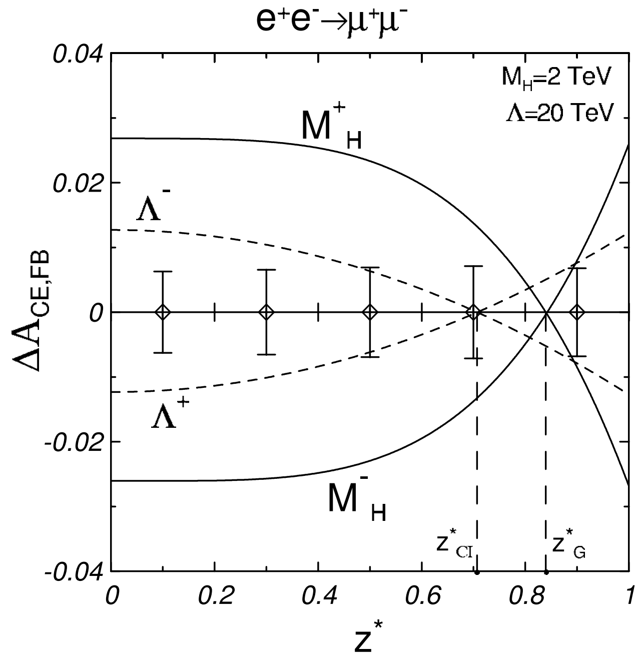

Figure 1: $\Delta A_{\rm CE,FB}(z^*)$ in the CI and the ADD scenarios for the values of Λ and M_H indicated in the text. The $\pm$ superscripts indicate positive and negative interference with the SM, respectively. The vertical bars are the statistical uncertainty at a ILC with $\sqrt{s} = 0.5$ TeV and $\mathcal{L}_{\rm int} = 50\,fb^{-1}$.

Eq. (15) shows that $\Delta A^{\rm ADD}_{\rm CE,FB}(z^*_{\rm G}) = 0$ for $z^*_{\rm G} = 2^{-1/4} \simeq 0.84$ ($\theta \simeq 33^\circ$), i.e., no deviation from graviton exchange at that value of z^* but possible deviations from contact interactions can occur there. The behaviour with z^* of $\Delta A_{\rm CE,FB}(z^*)$ in the two NP scenarios considered here is shown in the case of annihilation into muon pairs in Fig. 1, for $M_H = 2\,\text{TeV}$ and $\Lambda = 20\,\text{TeV}$ as illustrative examples. The '+' and '-' superscripts indicate positive or negative interference with the SM. To show their statistical significance, deviations are compared to the statistical uncertainty expected at the ILC with $\sqrt{s} = 0.5\,\text{TeV}$ and $\mathcal{L}_{\rm int} = 50\,\text{fb}^{-1}$, that are typical planned values [13].

The above considerations, and Fig. 1, then suggest the following kind of analysis. The measurement of $A_{\rm CE,FB}$ at $z^* \approx z^*_{\rm CI}$ or below has maximal sensitivity to the ADD graviton exchange scenario, with no (or minimal) contamination from CI; this measurement can be combined with the measurement of $A_{\rm CE}$ to further enhance the identification reach on M_H. Instead, the measurement of $A_{\rm CE,FB}$ in an interval around $z^* \approx z^*_{\rm G}$ should have maximal sensitivity to four-fermion CI, hence to the scales Λ, with least contamination from ADD effects. Inclusion of beams longitudinal polarization is easily obtained and results, for our basic observables $A_{\rm CE}$ and $A_{\rm CE,FB}$, into the same z^* dependence as found above times a factor accounting for the initial spin configurations [12].

4. IDENTIFICATION REACHES ON THE MASS SCALES

Basically, in order to evaluate the potential identification reach on the fundamental mass parameters M_H and Λ at the ILC, as achieved by measurements of the asymmetries defined above, one should compare the deviations from the SM predictions to the expected experimental uncertainties on those observables. One can apply a conventional χ^2 analysis, where the χ^2 can be formally defined as

$$\chi^2 = \frac{\left(\Delta\mathcal{O}^f\right)^2}{\left(\delta\mathcal{O}^f\right)^2}, \tag{16}$$

with $\mathcal{O} = A_{\rm CE}, A_{\rm CE,FB}$, $\Delta\mathcal{O}^f$ are the deviations of the asymmetries previously discussed, and $\delta\mathcal{O}^f$ are the experimental uncertainties. In practice, several final states f and $A_{\rm CE}$ with $A_{\rm CE,FB}$ themselves can be appropriately combined

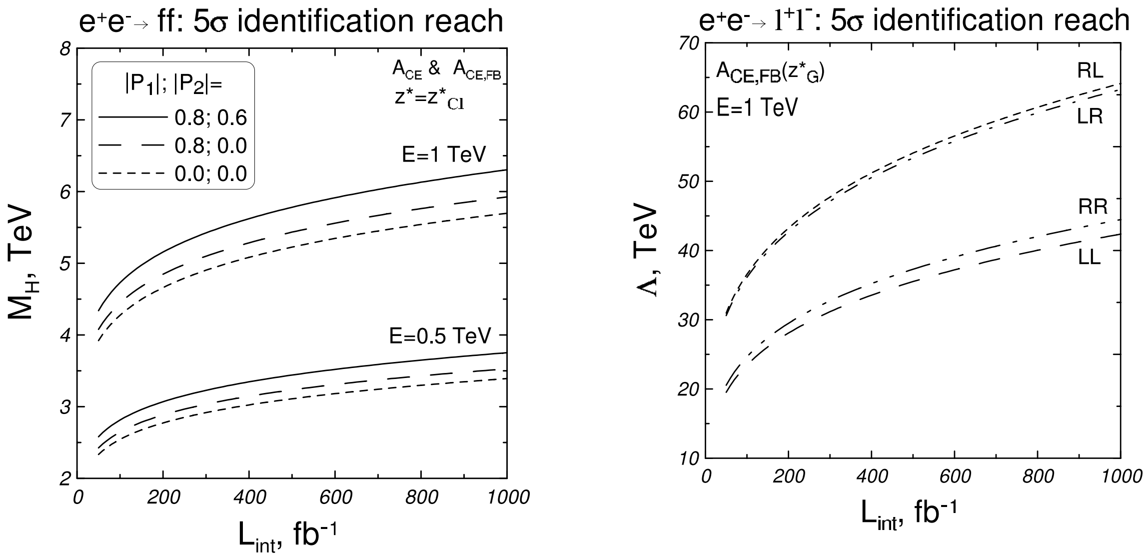

Figure 2: Left panel: 5σ identification reach on the mass scale M_H *vs.* integrated luminosity. Right panel: 5σ reach on the mass scales $\Lambda_{\alpha\beta}$ *vs.* integrated luminosity, labels attached to the curves indicate the helicity configurations $\alpha\beta = \mathrm{LL}, \mathrm{RR}, \mathrm{LR}, \mathrm{RL}$, see Eq. (6).

in Eq. (16). Constraints on M_H and Λs will then follow from the condition $\chi^2 \leq \chi^2_{\mathrm{C.L.}}$, where the actual value of $\chi^2_{\mathrm{C.L.}}$ depends on the desired confidence level.

We will here consider an ILC with c.m. energy of either 0.5 Tev or 1 TeV and in both cases electron and positron longitudinal polarizations $|P_1| = 0.80$ and $|P_2| = 0.60$, and will plot the numerical results on M_H and Λ for time-integrated luminosity in the range $50 - 1000$ fb^{-1}. Regarding systematic uncertainties, one expects them to largely cancel in the ratios (8) and (13), and thus the statistical uncertainties to dominate the experimental uncertainty. Indeed, the major sources of systematic uncertainties are found to originate from the errors in the luminosity and in the degree of initial beams longitudinal polarization, for which we assume $\Delta\mathcal{L}_{\mathrm{int}}/\mathcal{L}_{\mathrm{int}} = \Delta P_1/P_1 = \Delta P_2/P_2 = 0.5\%$ (more details can be found in [12]).

In Fig. 2 [left panel], the 5σ identification reach on the mass scale M_H relevant to graviton exchange is shown as a function of the luminosity, and for different longitudinal beam polarization configurations. Here, the final annihilation $f\bar{f}$ channels with $f = \mu, \tau, b, c$ have been summed over, and the asymmetries $A_{\mathrm{CE}}(z^*_{\mathrm{CI}})$ and $A_{\mathrm{CE,FB}}(z^*_{\mathrm{CI}})$ have been combined. As pointed out in previous sections, this provides the maximal sensitivity to the ADD graviton exchange scenario, with no contamination from four-fermion contact interactions. One can see that the identification reach on M_H at the 5σ level is of the order of $3.5 - 5.8$ TeV for energies between 0.5 TeV and 1 TeV and a luminosity of about 500 fb^{-1}, and can potentially increase to $(6.3 - 7.5) \times E_{\mathrm{c.m.}}$ for the highest luminosity. This should be compared with the current limit from LEP and Tevatron, $M_H \geq 1.10 - 1.28$ TeV [14]. Also, one can notice the (slow) scaling of $M_H \sim \left(s^3\mathcal{L}_{\mathrm{int}}\right)^{1/8}$, reflecting the (high) dimension of the effective interaction of Eq. (4).

In the right panel of Fig. 2, we show the 5σ reach on the four-fermion interaction mass scales Λ as a function of luminosity, at the ILC c.m. energy of 1 TeV. Here, the observable $A_{\mathrm{CE,FB}}(z^*_{\mathrm{G}})$ is used, and only the final l^+l^- pairs with $l = \mu, \tau$ are combined in the χ^2. Also, the longitudinal polarizations are chosen as $P_1 = 0.80$ and $P_2 = -0.60$, to disentangle the various helicity combinations of Eq.(6). It can be seen that the limits on Λs scale as $\sim (s\mathcal{L}_{\mathrm{int}})^{1/4}$, faster than for M_H, due to the (lower) dimension-6 of the effective interaction (6). According to the previous discussion, maximal sensitivity to four-fermion CI, with least (or no) contamination from ADD graviton exchange is expected. The potential 5σ reach on Λs of the linear collider ranges up to 45 TeV and 65 TeV for c.m. energies of 0.5 TeV and 1 Tev, respectively, depending on the particular helicity configurations. Current bounds, of the order of 10 TeV and depending on the CI model considered, are reviewed in [15]. Also, the limits on Λ obtained here may potentially improve the constraints on a very heavy sneutrino parameters [12].

Acknowledgments

The work of NP was supported by funds of the University of Trieste and of the MIUR (Italian Ministry for University and Research).

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