Collider Signatures of Higher Curvature Gravity

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We explore the phenomenological implications at colliders for the existence of higher-curvature gravity as extensions to both the Randall-Sundrum(RS) and Arkani-Hamed, Dimopoulos and Dvali(ADD) scenarios. Such terms are expected to arise on rather general grounds from ultraviolet completions of General Relativity, *e.g.*, from string theory. In the Randall-Sundrum model these terms shift the mass spectrum and couplings of the graviton tower. In the case of ADD they can lead to a threshold for the production of long-lived black holes.

1. INTRODUCTION

The Einstein-Hilbert(EH) action

$$S = \int d^{4+n}x \, \sqrt{-g} \left[\frac{M_*^{n+2}}{2} R - \Lambda\right], \tag{1}$$

is the basis for General Relativity(GR) in 4d as well as the ADD[1] and RS[2] models in extra dimensions. As is well known EH is at best an effective action below the scale M_*. As energies approaching M_* are reached additional terms may be generated in the effective action arising from the UV-completion of GR, *e.g.*, string theory. If M_* is not far above the TeV scale then these additional terms may make their presence known at future colliders: the LHC and ILC. From a bottom-up point of view it is not so clear what form such terms might take as the number of possibilities is vast and so we need some guidance. If we require that the new terms do not produce ghosts in the graviton sector, allow unitarity to be maintained and are 'string-motivated', we arrive at a rather unique set of terms called Lovelock invariants[3].

These Lovelock invariants come in fixed order, m, which we denote here as $\mathcal{L}_m$, that describes the number of powers of the curvature tensor, contracted in various ways, out of which they are constructed. Apart from normalization factors we can express the $\mathcal{L}_m$ as

$$\mathcal{L}_m \sim \delta^{A_1B_1...A_mB_m}_{C_1D_1...C_mD_m} \; R_{A_1B_1}{}^{C_1D_1}.....R_{A_mB_m}{}^{C_mD_m} \,, \tag{2}$$

where $\delta^{A_1B_1...A_mB_m}_{C_1D_1...C_mD_m}$ is the totally antisymmetric product of Kronecker deltas and $R_{AB}{}^{CD}$ is the D-dimensional curvature tensor. Fortunately, as can be seen by this definition, the number of such invariants that can exist in any given dimension $D = 4 + n$ is highly constrained. For a space with an even number of dimensions, $D = 2m$, the Lovelock invariant is a topological one and leads to a total derivative, *i.e.*, a surface term, in the action. All of the higher order invariants, $D \leq 2m - 1$, vanish identically. For $D \geq 2m + 1$, the $\mathcal{L}_m$ are dynamical objects that once added the action can alter the field equations normally associated with the EH term. However it can be shown that the addition of any or all of the $\mathcal{L}_m$ to the EH action still results in a theory with only second order equations of motion as is the case for ordinary Einstein gravity. Furthermore, variation of the new action leads to modifications of Einstein's equations by the addition of new terms which are second-rank symmetric tensors with vanishing covariant derivatives and which depend only on the metric and its first and second derivatives, *i.e.*, they have the same general properties as the Einstein tensor itself but are higher order in the curvature. It is these benign properties which provide the Lovelock invariants their unique features.

From the discussion above we see that the most general Lovelock theory in 4-d is just EH! In 5-d, as in the RS case, all of the $\mathcal{L}_{m\geq 3}$ vanish as in 4-d but $\mathcal{L}_2$, which is the Gauss-Bonnet invariant, can now contribute. The generalization is now quite clear: for $D = 5, 6$ only $\mathcal{L}_{0-2}$ can be present. For $D = 7, 8$ only $\mathcal{L}_{0-3}$ can be present while for $D = 9, 10$

only $\mathcal{L}_{0-4}$. Since the ADD model assumes that the compactified space is flat, *i.e.*, a toroidal compactification is assumed, the coefficient of $\mathcal{L}_0$ is taken to be zero in this case. Thus for either the RS or ADD models, there are at most three new pieces to add to the EH action and so the generalized form of the action can be taken to be

$$S = \int d^{4+n}x \, \sqrt{-g} \left(\frac{M_*^{n+2}}{2} \left[R + \frac{\alpha}{M_*^2} \mathcal{L}_2 + \frac{\beta}{M_*^4} \mathcal{L}_3 + \frac{\gamma}{M_*^6} \mathcal{L}_4 \right] - \Lambda \right), \tag{3}$$

where α, β and γ are dimensionless coefficients. Explicit expressions for the $\mathcal{L}_m$ are given elsewhere[3].

2. INFLUENCE ON THE RANDALL-SUNDRUM MODEL

How do these new terms modify the usual RS and ADD model expectations? Let us turn to the RS case first where only the parameter α can be non-zero. In this case a non-zero α will make its presence known by distorting the masses and couplings of the graviton spectrum. Recall that traditional RS model is based on the S^1/Z_2 orbifold with the metric

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \,, \tag{4}$$

with $\sigma = k|y|$ defining the curvature parameter $k \sim M_*$. There are two branes, separated by a distance πr_c, at the orbifold fixed points with the SM living on one of them(the TeV brane) with only gravity in the bulk. The influence of a non-zero α on the phenomenology of this model is correlated with the fact that the space between the two branes has a large constant curvature, *i.e.*, it is AdS_5. The effects of a non-zero α are found to always occur in the combination $\alpha k^2/M_*^2$, explicitly showing the influence of this curvature. Since the ratio k^2/M_*^2 is usually taken to be small(to avoid large curvature!) this damps the effects of the Lovelock terms to some extent.

Defining the useful combination

$$\Omega = \frac{4\alpha k^2/M_*^2}{1 - 4\alpha k^2/M_*^2} \,, \tag{5}$$

it can be shown that $-1/2 \leq \Omega \leq 0$ is required to forbid tachyons in the Kaluza-Klein(KK) graviton spectrum; this forces $\alpha \leq 0$. One finds that $\Omega \neq 0$ causes a shift in the usual RS mass spectrum, *i.e.*, the mass splitting between KK resonances increases, and induces a level dependence in the KK couplings to SM matter on the TeV brane by altering the boundary conditions on these two branes. We find that the interaction of the KK graviton excitations with the SM fields on the TeV brane is now given by

$$\mathcal{L} = \frac{1}{\Lambda_\pi} \sum_n \left[\frac{1 + 2\Omega}{1 + 2\Omega + \Omega^2 x_n^2} \right]^{1/2} h_n^{\mu\nu} T_{\mu\nu} \,, \tag{6}$$

where as usual we define $\Lambda_\pi = \overline{M}_{Pl} e^{-\pi k r_c}$; the KK masses are given as usual by $m_n = x_n k e^{-\pi k r_c}$ with the x_n being the roots of an equation involving Bessel functions. The KK states are generally more massive and more narrow than in the standard RS case; this can be seen in the examples shown in Fig. 1. In the limit $\Omega \to -1/4$ all of the graviton KK states completely decouple as seen in Fig. 2. The value of Ω (and hence α) can be precisely determined at the ILC provided at least the first 2 KK excitations are kinematically accessible; this can be done by measurements of ratios of the masses and widths of these two states[3] which depends only upon Ω. It is likely that such measurements will be sensitive to $\delta\Omega \sim 0.01$ or better. We also see that the KK states now get even more narrow as one moves up the KK tower.

3. INFLUENCE ON THE ADD MODEL

Now let us turn our attention to the ADD case. Since both the bulk and brane are flat in this case higher curvature terms do not quantitatively modify the usual two signatures of ADD[4]: missing energy from graviton

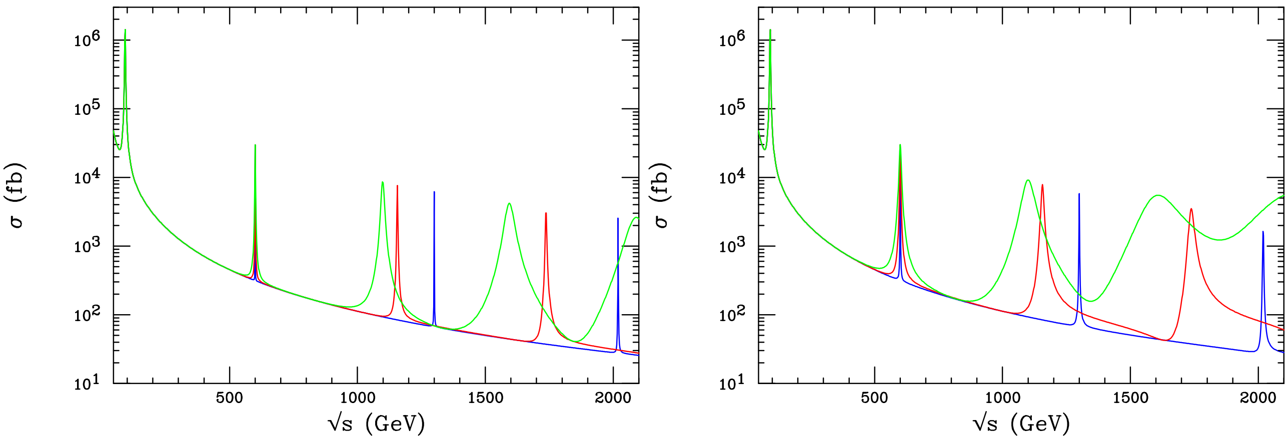

Figure 1: Cross section for $e^+e^- \to \mu^+\mu^-$ assuming $m_1 = 600$ GeV and $k/\overline{M}_{Pl} = 0.05$(left) or 0.1(right). The usual RS model prediction with $\Omega = 0$ yields the lightest spectrum(green); choosing $\Omega = -0.2$(red) and -0.4(blue) shifts the spectrum to ever larger masses.

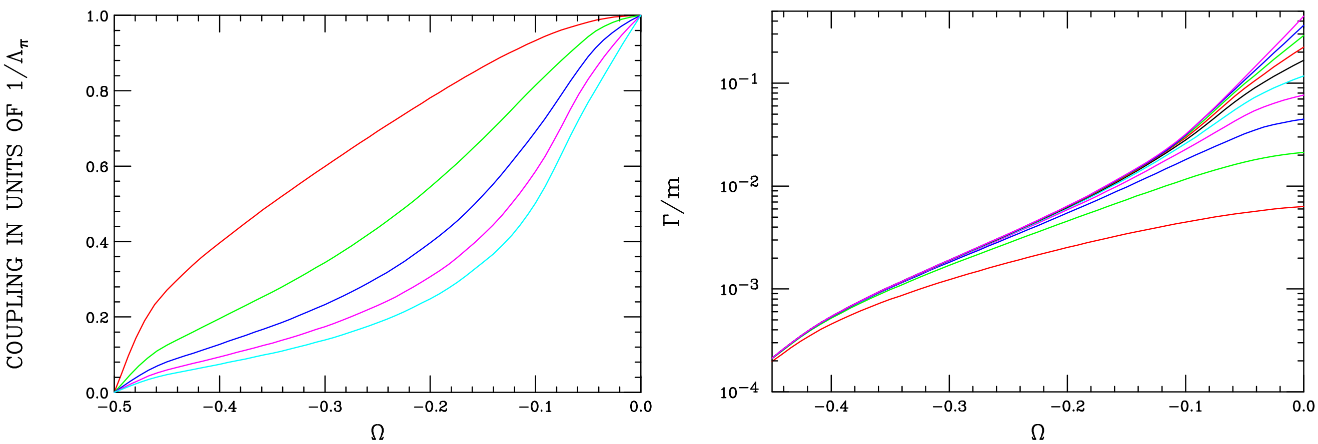

Figure 2: (Left)Coupling strengths of the first KK graviton states, from top to bottom, in units of $1/\Lambda_\pi$ as functions of the parameter Ω. Note that in the RS limit all states have the same coupling. (Right)The ratio Γ/m as a function of Ω for the first ten KK states. The KK number goes up as we go from the bottom to the top of the figure. $k/\overline{M}_{Pl} = 0.05$ has been chosen for purposes of demonstration.

emission and new dimension-8 contact interaction operators from KK exchange. All of the usual ADD relationships, such as $\overline{M}^2_{Pl} = V_n M_*^{n+2}$, together with the KK graviton mass spectrum and couplings are left completely unaltered by the higher curvature terms. The presence of Lovelock terms in the action will never be probed by observables associated with such processes. The last ADD signature is TeV-scale black hole(BH)[5] production; here we might expect some modifications as the region of space near BH are highly curved. We remind the reader that BH are expected to form in the collision of two partons once energies above $\sim M_*$ are reached with a cross section given by $\hat{\sigma} \simeq \pi R^2 \theta(\sqrt{\hat{s}} - M_*)$, where here R is the D-dimensional Schwartzschild radius corresponding to the value of $M_{BH} \simeq \sqrt{\hat{s}}$; note the unphysical step function turn-on. This cross section does not correctly model the turn on of BH production but assumes that it starts immediately once M_* is reached. Once formed, the BH, now described by a temperature T, should decay rapidly by Hawking radiation into a number of SM particles. The presence of Lovelock terms in the action alters the usual relationships between the BH mass, radius and temperature, *e.g.*, allowing for Lovelock terms, R is obtained by solving

$$M_{BH}/M_* \;=\; c\Big[x^{n+1} + \alpha n(n+1)x^{n-1} + \beta n(n+1)(n-1)(n-2)x^{n-3}$$

$$+ \;\; \gamma n(n+1)(n-1)(n-2)(n-3)(n-4)x^{n-5}\Big] \,, \tag{7}$$

where $x = M_* R$ and c is a constant given by $c = \frac{(n+2)\pi^{(n+3)/2}}{\Gamma(\frac{n+3}{2})}$. Other BH properties are also modified by the Lovelock parameters as we will discuss below. The influence of a non-zero α on R and T are explicitly shown in Fig. 3. Here we see that O(1) corrections to the usual BH quantities are possible for non-zero α. *If* we think of the Lovelock terms as arising from a perturbative-like expansion of the full action, as in string theory, the coefficients must grow smaller for the higher order terms. When we examine the expressions above we see that for a perturbative expansion to make sense we must have all of $\alpha n^2, \beta n^4$ and $\gamma n^6 < 1$. Since n can be as large as 6 within the usual ADD scenario we might expect that $\alpha \sim 10^{-2}$, $\beta \sim 10^{-3} - 10^{-4}$ and $\gamma \sim 10^{-5}$ with wide margins allowed for errors in these estimates. In the ADD case we will assume that the Lovelock parameters are positive quantities.

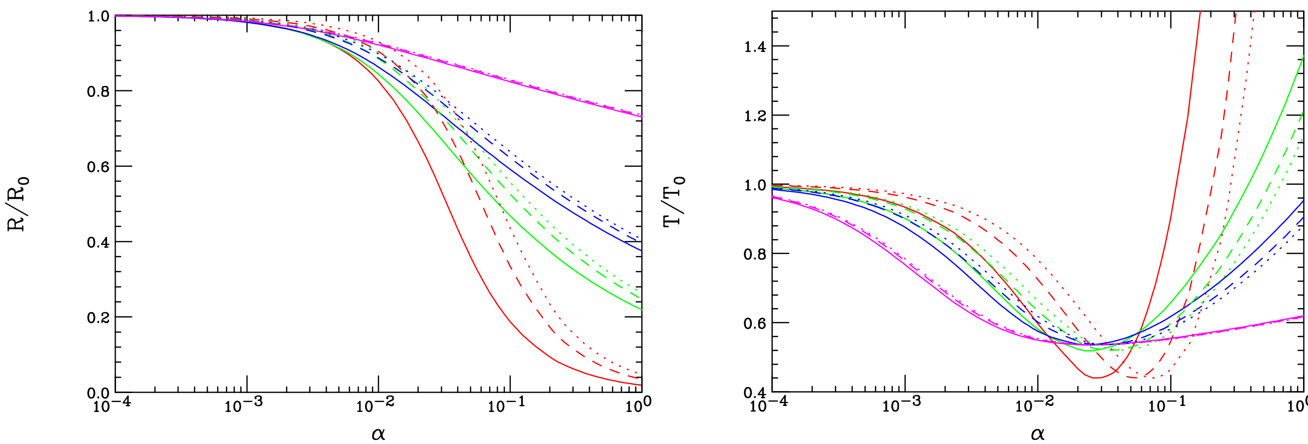

Figure 3: Influence of $\alpha \neq 0$ on the BH mass-Schwarzschild radius(left) and temperature(right) relationship for $n = 2, 4, 6$ and 20, corresponding to the red, green, blue and magenta sets of curves, respectively. The solid, dashed and dotted curves in each case correspond to $m = M_{BH}/M_* = 2, 5, 8$, respectively. Here $\beta = \gamma = 0$ and quantities with an index "0" label the predictions from the EH action.

When $\beta(\gamma)$ becomes non-zero, new *qualitative* changes in BH properties become possible for the case of $n = 3(5)$ as is shown in Fig. 4. Here we see that with $n = 3$ for a critical value of β, both R and T are driven to zero. In fact one can show that a threshold behavior occurs, *i.e.*, for $n = 3$, unless $M_{BH} > 60\pi^3\beta M_*$ no BH horizon forms; for $n = 5$ the threshold occurs at $840\pi^4\gamma M_*$. Furthermore, for values of β and γ in the 'natural' ranges discussed above this tells us that BH will not form below a critical center of mass energy at a collider. The value of this mass threshold as well as the shape of the BH production cross section immediately above threshold are determined solely by the Lovelock parameters.

These threshold shapes can be measured at the LHC as shown in Fig. 5 but precision measurements will require the ILC. It is important to notice that in both cases the cross sections are quite large $\sim 10 - 100$ pb which may yield up to $10^5 - 10^6$ events so that there is no shortage of available statistics. We note that asymptotically, far above threshold, the cross section we obtain becomes those of the simple step function model. Examining Fig. 5 we see that the ILC should be able to precisely determine the various Lovelock parameters with high precision. We note that these thresholds for BH production do not form when n is even.

The BH with $n = 3, 5$ that we have been discussing have another interesting property. BH that result from the EH action have a negative heat capacity. As the EH BH evaporate by Hawking radiation and lose mass they become hotter and evaporate more quickly until they are completely gone. This process occurs quite rapidly for TeV-scale BH, $\sim 10^{-25}$ sec or less. In the Lovelock case for $n = 3, 5$, the heat capacity is positive so that BH will cool as they lose mass. In this case they will Hawking radiate until they reach the threshold mass where they become (semi-classically!) stable, *i.e.*, if we start with a BH with a mass of, say, 1.5 times the threshold value and ask how long it will take for it to Hawking radiate down to the threshold mass we obtain infinity. This can be seen in Fig. 6.

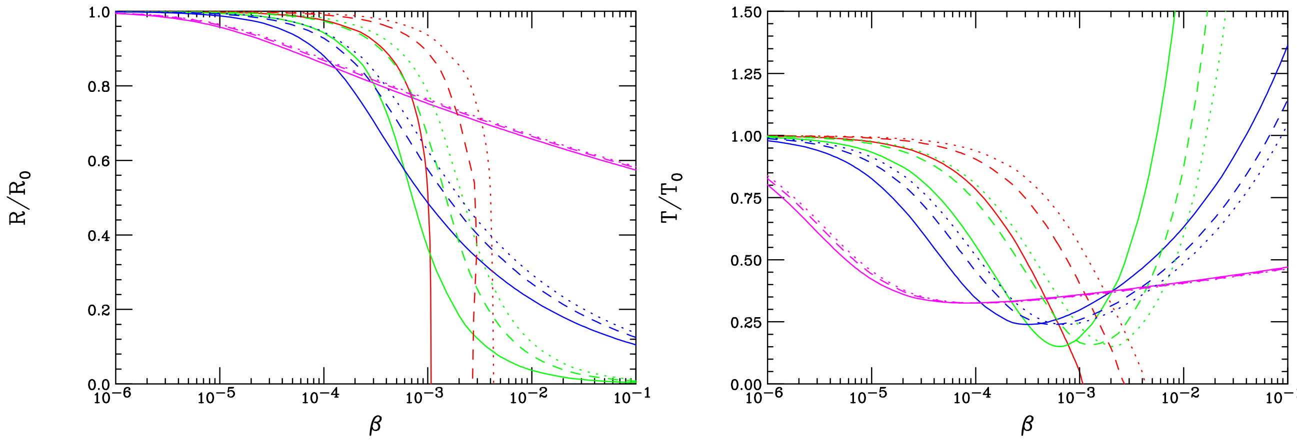

Figure 4: Same as the previous figure but now as a function of β with $\alpha = \gamma = 0$ and $n = 2 \rightarrow n = 3$ since $\mathcal{L}_3$ vanishes when $n = 2$.

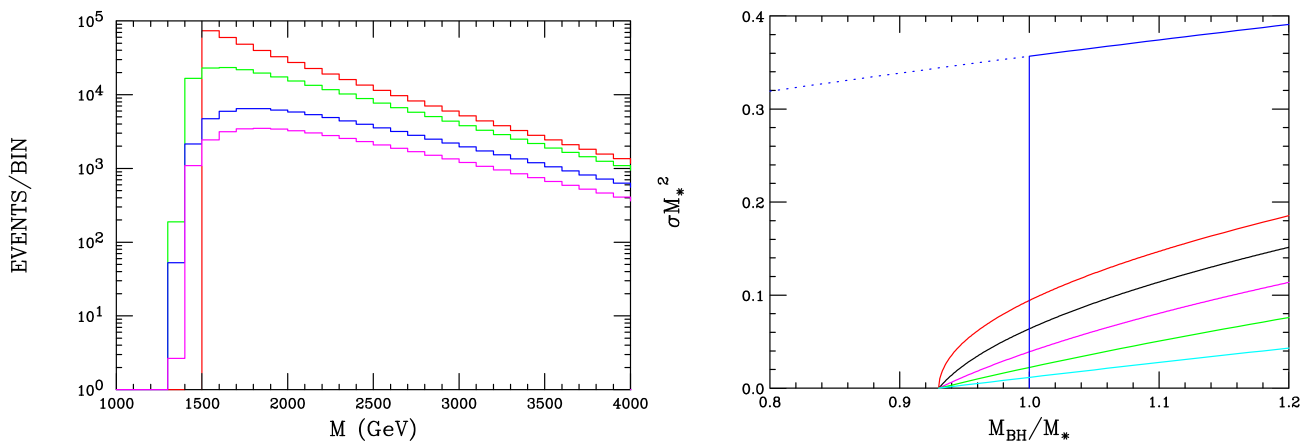

Figure 5: Threshold behavior of the BH cross section at the LHC(left) and ILC(right) for $n = 3$ and $\beta = 0.0005$. In the LHC case the top curve assumes the absence of Lovelock terms, while the subsequent ones correspond to the above β value with $\alpha = 0(0.01, 0.02)$; a luminosity(bin width) of 100 fb^{-1} (100 GeV) has been assumed. In the ILC case a scaled cross section is presented with $\sqrt{s} = M_{BH}$. Here, the top blue line in the naive θ-function that is usually assumed in the absence of Lovelock terms. The subsequent curves correspond to β as given above with $\alpha = 0, 0.002, 0.005, 0.01$ and 0.02 from top to bottom, respectively

It would be interesting to study the properties of these long-lived BH if they were embedded in media of various densities.

4. SUMMARY AND CONCLUSION

In this note we have briefly summarized the influence of higher-order Lovelock curvature terms on the phenomenology of the familiar RS and ADD models. In the case of RS model the dominant effect is a modification to the Kaluza-Klein graviton mass spectrum and their associated couplings to matter on the TeV brane. The mass spacing between the KK states increases, their couplings become KK level dependent and weaker in overall strength. Since KK masses and widths can be measured with very high precision at the ILC given sufficient center of mass energy, the value of the single possibly non-zero Lovelock parameter in this case should be well determined.

In the case of the ADD model the modifications are quite different. First, the usual ADD signatures, *i.e.*, missing

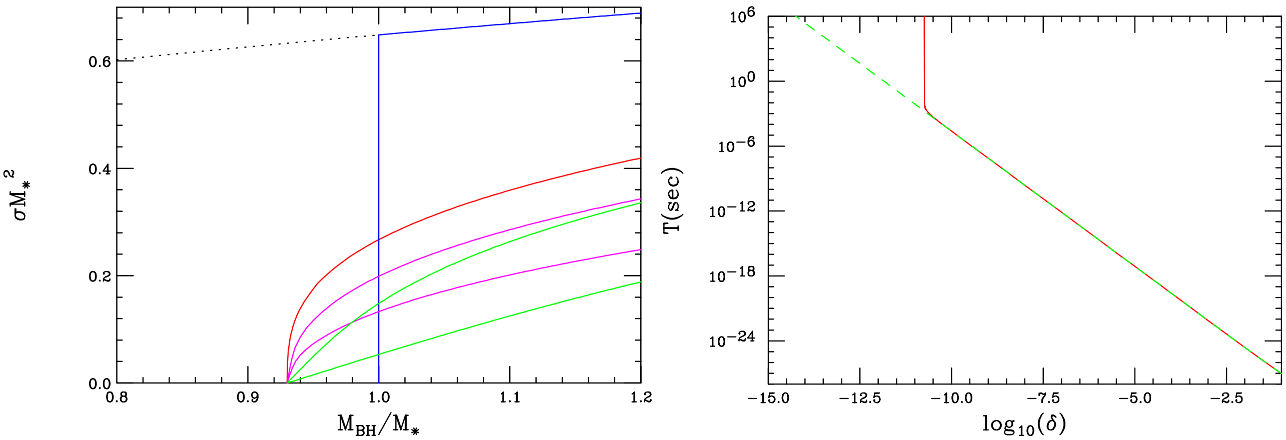

Figure 6: (Left)Close up of the BH production threshold at the ILC for $n = 5$; all curves have $\gamma \simeq 1.14 \cdot 10^{-5}$ and, from top to bottom, have $\alpha, \beta = 0, 0(0.003, 0; 0, 0.00003; 0.01, 0; 0, 0.0001)$, respectively. (Right) Decay of an $n = 3$ BH with $\beta = 0.0005$ and an initial mass of 1.5 times the threshold value(green dash). Here $M_{BH}/M_* = M_{thresh}/M_* + \delta$.

energy and dimension-8 contact interactions, remain unaltered at the quantitative level. The presence of Lovelock invariants in the action does modify the production as well as the properties of TeV scale black holes that are produced with large cross sections in high energy collisions in this scenario. Similarly to the shifts in the KK properties in the RS model there are comparable O(1) modifications in the nature of BH due to Lovelock terms. There are, however, some qualitative changes for the case of n odd: long-lived BH are possible and a mass threshold now exists below which BH horizons will not form. Both of these possibilities can be probed in detail at future colliders and it may be possible to determine the values of the Lovelock parameters with reasonable precision by measuring the shape of the BH production cross section near threshold.

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References

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Rev. D **59**, 086004 (1999) [arXiv:hep-ph/9807344] and Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398].

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999)

[3] For more details of this analysis and original references, see T. G. Rizzo, JHEP **0501**, 028 (2005) [arXiv:hep-ph/0412087]; arXiv:hep-ph/0503163.

[4] J. Hewett and M. Spiropulu, Ann. Rev. Nucl. Part. Sci. **52**, 397 (2002) [arXiv:hep-ph/0205106].

[5] T. Banks and W. Fischler, hep-th/9906038; S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. **87**, 161602 (2001) [arXiv:hep-ph/0106295]; S. B. Giddings and S. Thomas, Phys. Rev. D **65**, 056010 (2002) [arXiv:hep-ph/0106219].