Sensitivities to the SUSY Scale from Electroweak Precision Observables

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Precision measurements, now and at a future linear electron-positron collider (ILC), can provide indirect information about the possible scale of supersymmetry. Performing a $\chi^2$ analysis, we illustrate the present-day and possible future ILC sensitivities within the constrained minimal supersymmetric extension of the Standard Model (CMSSM), varying the parameters so as to obtain the cold dark matter density allowed by WMAP and other cosmological data. The current data are in very good agreement with the CMSSM prediction for $\tan\beta = 10$, with a clear preference for relatively small values of the universal gaugino mass, $m_{1/2} \sim 300$ GeV. In this case, there would be good prospects for observing supersymmetry directly at both the LHC and the ILC, and some chance already at the Tevatron collider. For $\tan\beta = 50$, the quality of the fit is worse, and somewhat larger $m_{1/2}$ values are favoured. With the prospective ILC accuracies the sensitivity to indirect effects of supersymmetry greatly improves. This may provide indirect access to supersymmetry even at scales beyond the direct reach of the LHC or the ILC.

1. INTRODUCTION

Measurements at low energies may provide interesting indirect information about the masses of particles that are too heavy to be produced directly. A prime example is the use of precision electroweak data from LEP, the SLC, the Tevatron and elsewhere to predict (successfully) the mass of the top quark and to provide an indication of the possible mass of the hypothetical Higgs boson [1]. Predicting the masses of supersymmetric particles is much more difficult than for the top quark or even the Higgs boson, because the renormalizability of the Standard Model and the decoupling theorem imply that many low-energy observables are insensitive to heavy sparticles. Nevertheless, present data on observables such as $M_W$, $\sin^2\theta_{\text{eff}}$, $(g-2)_\mu$ and $\text{BR}(b\rightarrow s\gamma)$ already provide interesting information on the scale of supersymmetry (SUSY), as we discuss in this paper, and have a great potential in view of prospective improvements of experimental and theoretical accuracies [2].

In the future, a linear $e^+e^-$ collider (ILC) will be the best available tool for making many precision measurements [3]. It is important to understand what information ILC measurements may provide about supersymmetry, both for the part of the spectrum directly accessible at the LHC or the ILC and for sparticles that would be too heavy to be produced directly. Comparing the indirect indications with the direct measurements would be an important consistency check on the theoretical framework of supersymmetry.

Improved and more complete calculations of the supersymmetric contributions to a number of low-energy observables such as $M_W$ and $\sin^2\theta_{\text{eff}}$ have recently become available [4]. These, combined with estimates of the experimental accuracies attainable at the ILC and future theoretical uncertainties from unknown higher-order corrections, make now an opportune moment to assess the current sensitivities and the projection to the situation at the ILC [2], see Ref. [5] for previous studies. In order to reduce the large dimensionality of even the minimal supersymmetric extension of the Standard Model, we work here in the framework of the constrained MSSM (CMSSM), in which the soft supersymmetry-breaking scalar and gaugino masses are each assumed to be equal at some GUT input scale. In this case, only four new parameters are needed: the universal gaugino mass $m_{1/2}$, the scalar mass $m_0$, the trilinear soft supersymmetry-breaking parameter $A_0$, and the ratio $\tan\beta$ of Higgs vacuum expectation values. The pseudoscalar Higgs mass $M_A$ and the magnitude of the Higgs mixing parameter $\mu$ can be determined by using the electroweak
vacuum conditions, leaving the sign of $\mu$ as a residual ambiguity.

The non-discovery of supersymmetric particles and the Higgs boson at LEP and other present-day colliders imposes significant lower bounds on $m_{1/2}$ and $m_0$. An important further constraint is provided by the density of dark matter in the Universe, which is tightly constrained by WMAP and other astrophysical and cosmological data [6]. These have the effect within the CMSSM, assuming that the dark matter consists largely of neutralinos [7], of restricting $m_0$ to very narrow allowed strips for any specific choice of $A_0$, tan $\beta$ and the sign of $\mu$ [8]. Thus, the dimensionality of the supersymmetric parameter space is further reduced, and one may explore supersymmetric phenomenology along these ‘WMAP strips’, as has already been done for the direct detection of supersymmetric particles at the LHC and linear colliders of varying energies [9]. A full likelihood analysis of the CMSSM planes incorporating uncertainties in the cosmological relic density was performed in Ref. [10]. We extend this analysis to indirect effects of supersymmetry.

We consider the following observables: the $W$ boson mass, $M_W$, the effective weak mixing angle at the $Z$ boson resonance, $\sin^2\theta_{\text{eff}}$, the anomalous magnetic moment of the muon, $(g-2)_\mu$ (we use the SM prediction based on the $e^+e^-$ data for the hadronic vacuum polarization contribution, see the discussion in Ref. [2]) and the rare $b$ decays BR($b \to s\gamma$) and BR($B_s \to \mu^+\mu^-$), as well as the mass of the lightest $CP$-even Higgs boson, $M_h$, and the Higgs branching ratios BR($h \to bb$)/BR($h \to WW^*$). A detailed analysis of the sensitivity of each observable to indirect effects of supersymmetry, taking into account the present and prospective future experimental and theoretical uncertainties, can be found in Ref. [2]. We briefly summarize here the main results on the combined sensitivities at present and for the ILC and update some of the results of Ref. [2], which were obtained for $m_t = 178.0 \pm 4.3$ GeV, using the current experimental central value of $m_t = 172.7 \pm 2.9$ GeV [11].

2. PRESENT SITUATION

We first investigate the combined sensitivity of the four low-energy observables for which experimental measurements exist at present, namely $M_W$, $\sin^2\theta_{\text{eff}}$, $(g-2)_\mu$ and BR($b \to s\gamma$). The branching ratio BR($B_s \to \mu^+\mu^-$), for which at present only an upper bound exists, has been discussed separately in Ref. [2]. We begin with an analysis of the sensitivity to $m_{1/2}$ moving along the WMAP strips with fixed values of $A_0$ and tan $\beta$. The experimental central values, the present experimental errors and theoretical uncertainties are as described in Ref. [2]. The experimental uncertainties, the intrinsic errors from unknown higher-order corrections and the parametric uncertainties have been added quadratically, except for BR($b \to s\gamma$), where they have been added linearly. Assuming that the four observables are uncorrelated, a $\chi^2$ fit has been performed with $\chi^2 \equiv \sum_{n=1}^N (R_n^{\exp} - R_n^{\text{theo}})^2 / (\sigma_n)^2$. Here $R_n^{\exp}$ denotes the experimental central value of the $n$th observable, so that $N = 4$ for the set of observables included in this fit, $R_n^{\text{theo}}$ is the corresponding CMSSM prediction and $\sigma_n$ denotes the combined error, as specified above. We have rejected all points of the CMSSM parameter space with either $M_h < 111$ GeV [12, 13] (taking into account theoretical uncertainties from unknown higher orders) or a chargino mass lighter than 103 GeV [14].

The results for tan $\beta = 10$ are shown in Fig. 1 using $m_t = 178.0 \pm 4.3$ GeV (left) and $m_t = 172.7 \pm 2.9$ GeV [11] (right). They indicate that, already at the present level of experimental accuracies, the electroweak precision observables combined with the WMAP constraint provide a sensitive probe of the CMSSM, yielding interesting information about its parameter space. For tan $\beta = 10$, the CMSSM provides a very good description of the data, resulting in a remarkably small minimum $\chi^2$ value. The fit shows a clear preference for relatively small values of $m_{1/2}$, with a best-fit value of about $m_{1/2} = 300$ GeV. This minimum is even more pronounced and located at slightly lower $m_{1/2}$ values for the lower top-quark mass value, where especially $M_W$ and $\sin^2\theta_{\text{eff}}$ favour lower $m_{1/2}$ values. The best fit is obtained for $A_0 \leq 0$, while positive values of $A_0$ result in a somewhat lower fit quality. The fit yields an upper bound on $m_{1/2}$ of about 600 (450) GeV at the 90% C.L. (corresponding to $\Delta \chi^2 \leq 4.61$) for $m_t = 178.0 \pm 4.3$ GeV ($m_t = 172.7 \pm 2.9$ GeV). Some of the principal contributions to the increase in $\chi^2$ when $m_{1/2}$ increases for tan $\beta = 10$ are as follows (evaluated for $m_t = 178.0 \pm 4.3$ GeV). For $A_0 = -m_{1/2}$, $m_{1/2} = 900$ GeV, we find that $(g-2)_\mu$ contributes about 5 to $\Delta \chi^2$, $M_W$ nearly 1 and $\sin^2\theta_{\text{eff}}$ about 0.2, whereas the contribution of BR($b \to s\gamma$) is negligible. On the other hand, for $A_0 = +2m_{1/2}$, which is disfavoured for tan $\beta = 10$, the minimum in $\chi^2$ is due to a combination of the four observables, but $(g-2)_\mu$ again gives the largest contribution for large $m_{1/2}$. For
$m_t = 172.7 \pm 2.9 \text{ GeV}$ the relative contribution of $M_W$ and $\sin^2 \theta_{\text{eff}}$ to $\Delta \chi^2$ increases, so that also for $A_0 = -m_{1/2}$ the $\Delta \chi^2$ is more evenly distributed between the observables.

Figure 1: The results of $\chi^2$ fits based on the current experimental results for the precision observables $M_W$, $\sin^2 \theta_{\text{eff}}$, $(g-2)_\mu$ and $\text{BR}(b \to s\gamma)$ are shown as functions of $m_{1/2}$ in the CMSSM parameter space with CDM constraints for $\tan \beta = 10$ and different values of $A_0$. The left plot shows the results for $m_t = 178.0 \pm 4.3 \text{ GeV}$, and the right plot shows the results for $m_t = 172.7 \pm 2.9 \text{ GeV}$.

Figure 2: The $\chi^2$ contours in the CMSSM with $\tan \beta = 10$ for the mass of the lighter chargino (left), $m_{\tilde{\chi}^+_1} \approx m_{\tilde{\chi}^0_2}$, and the lighter stau (right), $m_{\tilde{\tau}_1}$, based on the fits to the parameter space shown in the right plot of Fig. 1 (updated from Ref. [2] for $m_t = 172.7 \pm 2.9 \text{ GeV}$).

In Fig. 2 the fit results of Fig. 1 (right plot, $m_t = 172.7 \pm 2.9 \text{ GeV}$) for $\tan \beta = 10$ are expressed in terms of the masses of the lighter chargino, $m_{\tilde{\chi}^+_1} \approx m_{\tilde{\chi}^0_2}$, and the mass of the lighter stau, $m_{\tilde{\tau}_1}$ (updated from Ref. [2]). The best-fit value for $m_{\tilde{\chi}^+_1}$ is about 200 GeV, while for $m_{\tilde{\tau}_1}$ the best-fit value is even below 150 GeV. The best-fit values for the masses of the neutralinos, charginos and sleptons [2] offer good prospects of direct sparticle detection at both the ILC [3] and the LHC [15], allowing a detailed determination of their properties [16]. There are also some prospects for detecting the associated production of charginos and neutralinos at the Tevatron collider, via their trilepton decay signature, in particular. This is estimated to be sensitive to $m_{1/2} \lesssim 250 \text{ GeV}$ [17], covering much of the region below the best-fit value of $m_{1/2}$ that we find for $\tan \beta = 10$.

For $\tan \beta = 50$ the overall fit quality is worse than for $\tan \beta = 10$, and the sensitivity to $m_{1/2}$ from the precision observables is lower, as is shown in Fig. 3 (left). This is related to the fact that, whereas $M_W$ and $\sin^2 \theta_{\text{eff}}$ prefer small values of $m_{1/2}$ also for $\tan \beta = 50$, the CMSSM predictions for $(g-2)_\mu$ and $\text{BR}(b \to s\gamma)$ for high $\tan \beta$ are in better agreement with the data for larger $m_{1/2}$ values, see the discussion in Ref. [2]. Also in this case the best fit is obtained for negative values of $A_0$, but the preferred values for $m_{1/2}$ are 200–300 GeV higher than for $\tan \beta = 10$. 
Figure 3: The results of a $\chi^2$ fit based on the current experimental results for the precision observables $M_W$, $\sin^2 \theta_{\text{eff}}$, $(g-2)_\mu$, and $\text{BR}(b \to s\gamma)$ are shown as functions of $m_{1/2}$ in the CMSSM parameter space with CDM constraints for different values of $A_0$, $m_t = 178.0 \pm 4.3$ GeV and $\tan \beta = 50$ (left). The corresponding $\chi^2$ contours for the mass of the lighter stau, $m_{\tilde{\tau}_1}$, are shown in the right plot.

In the case $\tan \beta = 50$ (for $m_t = 178 \pm 4.3$ GeV) the best-fit values for the LSP mass and the lighter stau are still below about 250 GeV [2]. The fit result for the stau mass is shown in the right plot of Fig. 3. The minimum $\chi^2$ for the other masses is shifted upwards compared to the case with $\tan \beta = 10$. The best-fit values are obtained in the region 400–600 GeV. Correspondingly, these sparticles would be harder to detect. At the ILC with $\sqrt{s} \lesssim 1$ TeV, the best prospects would be for the production of $\tilde{\chi}^0_1 \tilde{\chi}^0_1$ or of $\tilde{\tau}_1 \tilde{\tau}_1$. Other particles can only be produced if they turn out to be on the light side of the $\chi^2$ function.

For the case of $\tan \beta = 10$, all the coloured particles should be accessible at the LHC [2]. However, among them, only $t_3$ has a substantial part of its $\chi^2$-favoured spectrum below 500 GeV, which would allow its detection at the ILC. The same applies for the mass of the $A$ boson. The Tevatron collider has a sensitivity to $m_{t_3} \lesssim 450$ GeV [17], which is about our best-fit value for $\tan \beta = 10$. For the $\tan \beta = 50$ case (with $m_t = 178.0 \pm 4.3$ GeV) the particles are mostly inaccessible at the ILC, though the LHC has good prospects. However, at the 90% C.L. the coloured sparticle masses might even exceed $\sim 3$ TeV, which would render their detection difficult. Concerning the heavy Higgs bosons, their masses may well be below $\sim 1$ TeV. In the case of large $\tan \beta$, this might allow their detection via the process $\bar{b}b \to \bar{b}bH/A \to \bar{b}b \tau^+\tau^-$ [18].

While in the fits presented above we kept $A_0/m_{1/2}$ fixed, we now analyse the combined sensitivity of the precision observables $M_W$, $\sin^2 \theta_{\text{eff}}$, $\text{BR}(b \to s\gamma)$ and $(g-2)_\mu$ in a scan over the $(m_{1/2}, A_0)$ parameter plane. In order to perform this scan, we have evaluated the observables for a finite grid in the $(m_{1/2}, A_0, m_0)$ parameter space, fixing $m_0$ using the WMAP constraint. We restrict ourselves here to the $\tan \beta = 10$ case with $m_t = 178 \pm 4.3$ GeV. Fig. 4 (left) shows the WMAP-allowed regions in the $(m_{1/2}, A_0)$ plane. The current best-fit value obtained via $\chi^2$ fit is indicated. The coloured regions around the best-fit value correspond to the 68% and 90% C.L. regions (corresponding to $\Delta \chi^2 \leq 2.30, 4.61$, respectively). The precision data yield sensitive constraints on the available parameter space for $m_{1/2}$ within the WMAP-allowed region. The precision data are less sensitive to $A_0$. The 90% C.L. region contains all the WMAP-allowed $A_0$ values in this region of $m_{1/2}$ values. As expected from the discussion above, the best fit is obtained for negative $A_0$ and relatively small values of $m_{1/2}$. At the 68% C.L., the fit yields an upper bound on $m_{1/2}$ of about 450 GeV. This bound is weakened to about 600 GeV at the 90% C.L. As discussed above, the overall fit quality is worse for $\tan \beta = 50$, and the sensitivity to $m_{1/2}$ is less pronounced (plot not shown here). In this case the best fit is obtained for $m_{1/2} \approx 500$ GeV and negative $A_0$. The upper bound on $m_{1/2}$ increases

\[1\] A similar analysis within the CMSSM as the one presented here has recently been carried out in Ref. [19], where a fit has been performed involving all CMSSM parameters. In this analysis, which did not take into account the electroweak precision observables $M_W$ and $\sin^2 \theta_{\text{eff}}$, a preference for larger values of $\tan \beta$ and somewhat larger $m_{1/2}$ has been reported.
to nearly 1 TeV at the 68% C.L.

Figure 4: The results of $\chi^2$ fits for $\tan \beta = 10$ based on the current experimental results (left) and for the anticipated ILC precision (right) for $M_W$, $\sin^2 \theta_{\text{eff}}$, $(g-2)_\mu$, and BR($b \to s\gamma$) (including also $M_h$ and BR($h \to b\bar{b}$)/BR($h \to WW^*$) in the right plot) are shown in the ($m_{1/2}, A_0$) planes of the CMSSM with the WMAP constraint. The current best-fit point is indicated in both plots. In the right plot two further hypothetical future ‘best-fit’ values are shown for illustration. The coloured regions correspond to the 68% and 90% C.L. regions, respectively.

3. ILC PRECISION

We now turn to the analysis of the future sensitivities of the precision observables, based on the prospective experimental accuracies at the ILC and the estimates of future theoretical uncertainties discussed in Ref. [2]. We perform a $\chi^2$ fit for the combined sensitivity of the observables $M_W$, $\sin^2 \theta_{\text{eff}}$, $(g-2)_\mu$, BR($b \to s\gamma$), $M_h$ and BR($h \to b\bar{b}$)/BR($h \to WW^*$) in the ($m_{1/2}, A_0$) plane of the CMSSM assuming ILC accuracies. The right plot of Fig. 4 shows the corresponding fit results for $\tan \beta = 10$. The WMAP-allowed region and the best-fit point according to the current situation (see the left plot of Fig. 4) are indicated. Two further hypothetical future ‘best-fit’ points have been chosen for illustration. For all the ‘best-fit’ points, the assumed central experimental values of the observables have been chosen such that they precisely coincide with the ‘best-fit’ points.

The comparison of Fig. 4 (right) with the result of the current fit (left) shows that the ILC experimental precision will lead to a drastic improvement in the sensitivity to $m_{1/2}$ and $A_0$ when confronting precision data with the CMSSM predictions. The comparison of these indirect predictions for $m_{1/2}$ and $A_0$ with the information from the direct detection of supersymmetric particles would provide a stringent test of the CMSSM framework at the loop level. A discrepancy could indicate that supersymmetry is realised in a more complicated way than is assumed in the CMSSM. For the best-fit values of the current fit the ILC precision would allow one to narrow down the allowed CMSSM parameter space to very small regions around each of the ‘best-fit’ points according to the ILC accuracies.

The comparison of Fig. 4 (right) with the result of the current fit (left) shows that the ILC experimental precision will lead to a drastic improvement in the sensitivity to $m_{1/2}$ and $A_0$ when confronting precision data with the CMSSM predictions. The comparison of these indirect predictions for $m_{1/2}$ and $A_0$ with the information from the direct detection of supersymmetric particles would provide a stringent test of the CMSSM framework at the loop level. A discrepancy could indicate that supersymmetry is realised in a more complicated way than is assumed in the CMSSM. For the best-fit values of the current fit the ILC precision would allow one to narrow down the allowed CMSSM parameter space to very small regions in the ($m_{1/2}, A_0$) plane. For the example shown here with best-fit values around $m_{1/2} = 300$ GeV it is possible to constrain particle masses within about $\pm 10\%$ at the 95% C.L. from the comparison of the precision data with the theory predictions. Because of the decoupling property of supersymmetric theories, the indirect constraints become weaker for increasing $m_{1/2}$. The additional hypothetical ‘best-fit’ points shown in Fig. 4 (right) illustrate the indirect sensitivity to the CMSSM parameters in scenarios where the precision observables prefer larger values of $m_{1/2}$. We find that for a ‘best-fit’ value of $m_{1/2}$ as large as 1 TeV, which would lie close to the LHC limit and beyond the direct-detection reach of the ILC, the precision data would still allow one to

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2We have checked explicitly that assuming future experimental values of the observables with values distributed statistically around the present ‘best-fit’ points with the estimated future errors does not degrade significantly the qualities of the fits.
establish an upper bound on $m_{1/2}$ within the WMAP-allowed region. Thus, this indirect sensitivity to $m_{1/2}$ could give important hints for supersymmetry searches at higher-energy colliders. For ‘best-fit’ values of $m_{1/2}$ in excess of 1.5 TeV, on the other hand, the indirect effects of heavy sparticles become so small that they are difficult to resolve even with ILC accuracies [2]. Whilst the present analysis has been restricted to the CMSSM, similar conclusions are expected to apply also in models beyond the CMSSM. Such an analysis is currently in preparation.

References


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