Updated Constraints on the Minimal Supergravity Model

A. Djouadi
LPTHE, UMR8627–CNRS, F–91405 Orsay Cedex, France
M. Drees
Physikalisches Institut, Bonn, D–53115 Bonn, Germany
J.-L. Kneur*
LPTA, UMR5207–CNRS, F–34095 Montpellier Cedex 5, France

We present an updated analysis of the present constraints on the parameter space of the minimal supergravity model (mSUGRA). New features include, in particular, an improved calculation of the masses of neutral Higgs bosons, constraints from $b \to s\ell^+\ell^-$ decays, and updated $g_\mu - 2$ constraints. We focus on the minimal allowed masses of sparticles and Higgses from various sets of constraints. We find that the direct experimental limits from collider and Dark Matter searches can still be saturated in many cases within this model, even after the quite restrictive WMAP constraint on the Dark Matter relic density. Consequences for sparticle production at the International Linear Collider in this scenario are briefly discussed.

1. INTRODUCTION

The minimal supergravity model (mSUGRA) [1, 2] remains the most widely studied implementation of the supersymmetric version of the Standard Model (MSSM). It can provide a stable gauge hierarchy (for sparticle masses not much above a TeV) [3], a possible Grand Unification of all gauge interactions [4] and, assuming $R-$parity is conserved, a very plausible Dark Matter (DM) candidate [5, 6]. In the general MSSM, the breaking of supersymmetry introduces many unknown parameters. In contrast, the mSUGRA is defined by only four parameters plus a sign:

$$m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ \text{sign}\mu, \tag{1}$$

where m_0 is the soft supersymmetry breaking scalar mass (universal for all flavors, at the scale M_{GUT} of Grand Unification), $m_{1/2}$ the universal supersymmetry breaking gaugino mass, and A_0 the universal supersymmetry breaking trilinear scalar interaction. Finally, $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs doublets (defined at the weak scale), and μ is the supersymmetric higgsino mass parameter. Note that the assumed flavor universality at GUT scale implies that supersymmetric flavor changing neutral current (FCNC) effects occur only radiatively, through renormalization group (RG) evolution. This keeps FCNC at an acceptable level, although flavor changing $b \to s\gamma$ and $b \to s\ell^+\ell^-$ decays do impose important constraints on the parameter space, as we will see. Another very welcome feature of mSUGRA is that it naturally incorporates radiative breaking of the electroweak symmetry [7], i.e. the RG evolution drives the squared mass of one of the Higgs fields to negative values, keeping all squared sfermion masses positive. The (absolute) value of μ is thus determined as function of the other parameters. In spite of these successes, there is a growing perception that the mSUGRA parameter space is getting "squeezed" by ever tightening constraints, mainly from the now quite accurate WMAP determination [8] of the relic density of DM particles. On the other hand, the fact that mSUGRA can accommodate this measurement can be considered as a further success of the model. In any case it seems timely to re–assess the model, taking recent theoretical and experimental developments into account. Besides the WMAP data, these include:

---

*Speaker

- More accurate calculations of leading two–loop corrections to the masses of neutral Higgs bosons [9], which makes it somewhat easier to satisfy the stringent Higgs search limits from LEP;

- The new, somewhat increased central value of the top quark mass [10], which also increases the predicted mass of the lightest neutral Higgs boson;

- Improved limits on radiative b decays and, in particular, first information on $b \to s\ell^+\ell^-$ decays, which excludes scenarios where the sign of the amplitude of $b \to s\gamma$ decays is opposite to the SM prediction [11];

- A growing consensus [12] that the SM prediction for hadronic contributions to the anomalous dipole moment of the muon based on data from e^+e^- colliders is more reliable, which again elevates the discrepancy between the measurement [13] of $g_\mu - 2$ and its SM prediction [14] to the level of ~ 2.5 standard deviations.

2. CONSTRAINTS ON mSUGRA PARAMETER SPACE

We use the Fortran code SuSpect [15] to evolve the renormalization group equations (RGE) from the GUT scale where most of the mSUGRA parameters in (1) are defined, down to the electroweak symmetry breaking (EWSB) scale, and to calculate the spectrum of all physical sparticles and Higgs bosons, following the procedure outlined in [16]. This includes the full two–loop RGE for gauge, Yukawa couplings, and all soft supersymmetry breaking terms, and complete one–loop corrections plus leading two–loop corrections [9] to Higgs boson masses. Concerning more precisely the Higgs boson masses, their calculation is performed in the $\overline{DR}$ renormalization scheme, including the full one-loop corrections of ref. [17] plus the leading two-loop corrections controlled by the third generation Yukawa couplings and the strong gauge coupling, derived in ref. [18].
Given a set of input parameters (1), a first theoretical constraint is to require a consistent electroweak symmetry breaking (EWSB) which, apart from determining $|\mu|$ as function of the other parameters, can exclude somewhat large regions of the parameter space in (1), since not all possible input values are compatible with radiative $SU(2) \times U(1)$ symmetry breaking. We also exclude parameter choices such that the scalar potential has deep minima breaking charge and/or color at the electroweak scale [19]. More precisely in SuSpect we impose the "CCB" constraints[20], which exclude very large values of $|A_0|/\sqrt{m_0^2 + m_{1/2}^2}$.
Next, a given input choice has to satisfy several experimental constraints. The ones we consider are:

- the constraints from LEP precision observables on quantum corrections due to superparticles. These include most notably the upper bound on the supersymmetric contribution to the electroweak $\rho-$parameter [21], including 2–loop QCD corrections [22]. However, it turns out that within mSUGRA, this constraint is always superseded by either the LEP Higgs search limit or by the CCB constraint.

- the lower bounds on sparticle and Higgs masses, from direct searches at LEP and Tevatron [23, 24]. Concerning the neutral Higgs mass lower bound, allowing for a theoretical uncertainty [25] in the calculation of m_h of about 3 GeV, we thus require the calculated value of m_h to exceed 111 GeV.

- Recent measurements [13] of the anomalous magnetic moment of the muon lead to the constraint on the supersymmetric contribution [26] to $a_\mu \equiv (g_\mu - 2)/2$

$$-5.7 \cdot 10^{-10} \leq a_{\mu\,\rm SUSY} \leq 4.7 \cdot 10^{-9}. \quad (2)$$

  This range is obtained from combining the 2σ allowed regions using data from e^+e^- annihilation into hadrons and from τ decays, respectively, to estimate the (hadronic) SM contribution to a_μ (see refs.[14] for discussions of this theoretical uncertainty). However, we also study the constraint obtained when requiring a positive MSSM contribution to a_μ,

$$1.06 \cdot 10^{-9} \leq a_{\mu\,\rm SUSY} \leq 4.36 \cdot 10^{-9}\,, \quad (3)$$

corresponding to the 90% CL allowed region when using solely the data from e^+e^- annihilation into hadrons for the evaluation of the SM contribution.

- Next, a very restrictive constraint arises from the determination of the density of non–baryonic Dark Matter from detailed analyses of the anisotropies of the cosmic microwave background (CMB), as obtained in particular by the WMAP experiment [8]. One should nevertheless be aware that the WMAP result is based on several assumptions, which are reasonable but not easy to cross–check independently [27]. In our analysis we thus quite conservatively use the 99% CL region

$$0.087 \le \Omega_{\tilde{\chi}_1^0} h^2 \le 0.138\,, \tag{4}$$

  where $\Omega_{\tilde{\chi}_1^0}$ is the LSP mass density in units of the critical density, and h is Hubble constant in units of 100 km/(s·Mpc). Not surprisingly, this requirement greatly constrains the allowed parameter space. As usual, we assumed that the LSP once was in thermal equilibrium; its relic density is then essentially inversely proportional to its annihilation cross section [6]. Our relic density calculation uses proper thermal averaging of the squared $s-$channel, in particular for Higgses exchange, contribution near the resonance [28], while all other contributions are treated using the standard non–relativistic expansion.

- Allowing for experimental and theoretical errors [23], the branching ratio for radiative b decays should satisfy

$$2.65 \cdot 10^{-4} \le B(b \to s\gamma) \le 4.45 \cdot 10^{-4}. \tag{5}$$

  We evaluate this, including contributions from $tH^\pm$ and $\tilde{t}\tilde{\chi}^\pm$ loops, using the results of ref. [29].

We consider however this last constraint to be not as firm as the other ones discussed above, since it would be affected significantly if allowing for small deviations from universality, or equivalently, small non–diagonal entries in the squark mass matrix [30]. Accordingly some of our results are presented below either with or without this constraint, as explicitly mentioned.

3. RESULTS AND DISCUSSION

In Fig. 1 we show the relevant region of the $(m_{1/2}, m_0)$ plane for $A_0 = 0$, $\mu > 0$, $\tan\beta = 10, 50$ (and $m_t = 178$ GeV). Such a plot is qualitatively consistent with other similar recent analysis[31], but it is useful to cross-check those results with independent MSSM spectrum and relic density calculation codes. The black region for small $m_{1/2}$ is excluded mainly by the lower bounds on chargino masses from direct searches at LEP (there is also a part of this black area excluded by inconsistent EWSB, but it is of relatively small size for these parameter choices, and would become more significant only for larger values of $\tan\beta$ and/or larger values of m_0 typically). The black triangular–shaped exclusion region for (relatively) small m_0 is partly due to the lower bounds on sleptons and squarks from direct searches at LEP and Tevatron, but also from requiring the lightest superparticle (LSP) to be a neutralino (particularly for $\tan\beta = 50$, where major part of this black area corresponds to the lightest tau slepton being the LSP). The violet region for rather small $m_{1/2}, m_0$ is the one excluded by the LEP Higgs mass lower bound, which we take as $m_h \gtrsim$ 111 GeV, allowing for a 3 GeV theoretical uncertainty on the calculation of m_h (the pink strip illustrates this uncertainty, i.e. corresponding to 111 GeV $\lesssim m_h \lesssim$ 114 GeV). The mild evidence for an SM–like Higgs boson with mass ~ 116 GeV [24] favors the red region. The green area is excluded by the $b \to s\gamma$ constraints (which for $\tan\beta = 10$ is superseded by the Higgs exclusion), while the blue area (only visible on the $\tan\beta = 50$ plot) corresponds to the "aggressive" $g_\mu - 2$ range in (3). Finally, in the yellow region the LSP relic density satisfies (4). Notice that, besides the rather familiar "bulk", "stau-coannihilation", and "A-pole" regions (the latter being quite sizeable on the $\tan\beta = 50$ plot), another cosmologically acceptable region of mSUGRA parameter space appears for $m_{1/2} \sim 140 - 150$ GeV and relatively large m_0, which eventually merges with the "focus point" region at much larger m_0, but here in fact corresponds to the LSP annihilation process being enhanced by nearly resonant $s-$channel

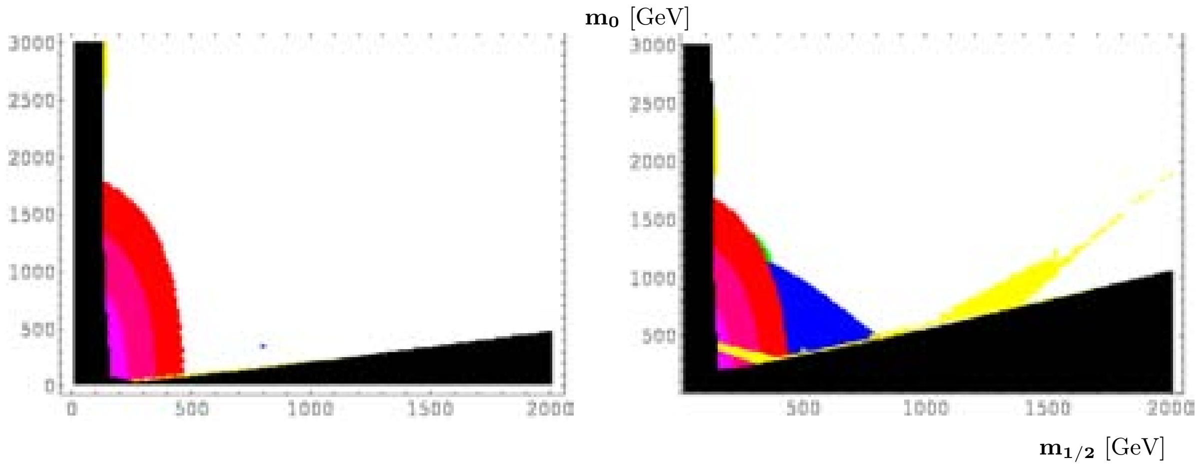

Figure 1: Constraints on the mSUGRA parameter space for $m_t = 178$ GeV, $A_0 = 0$, $\mu > 0$, $\tan\beta = 10$ (left) and $\tan\beta = 50$ (right). The dark area is ruled out by the requirement of consistent EWSB, plus sparticle search limits, as discussed in the text. The violet and green areas are ruled out by, respectively, the LEP Higgs mass lower limit $m_h \gtrsim$ 111 GeV, and the $b \to s\gamma$ constraints. The blue area favors the "aggressive" $g_\mu - 2$ range (3). The red area favors the LEP "evidence" $m_h \sim 116$ GeV, while the pink strip accounts for a 3 GeV theoretical uncertainty on the Higgs mass lower bound. In the yellow strips the neutralino relic density falls in the range (4).

(lightest) Higgs h-exchange. This "h-pole" region has been investigated very recently in some detail in [32] (see also refs. [33] for some previous discussions) and appears, for other configurations of parameters, in a rather significant region of the mSUGRA parameter space where one has $2m_{\tilde{\chi}_1^0} \lesssim m_h$. This possibility seemed to be essentially excluded a few years ago by the combination of rising lower bounds on m_h and $m_{\tilde{\chi}_1^0}$ from searches at LEP [23], but it is resurrected essentially by the above mentioned theoretical improvements in the Higgs mass calculation together with the increased top mass central experimental value. A very interesting consequence of this scenario for sparticle searches at future colliders, and at ILC in particular, is that it implies quite stringent *upper* bounds on the masses of the LSP and thus, through the mSUGRA universality relations, on the masses of the lightest chargino and the gluino. Significant upper bounds on most sparticles and Higgses are further obtained in this scenario when combining the WMAP contraint with the "agressive" $g_\mu - 2$ constraint (3) (see ref. [32] for a detailed discussions).

We refrain from illustrating here other possible ways of scanning mSUGRA parameters, like other values of m_t, $\tan\beta$, $A_0 \neq 0$, etc. In fact, rather than the "size of allowed parameter space", as in Fig. 1, it is perhaps more meaningful to search for the *minimal* sparticle masses allowed by the above set of given constraints, when scanning over the full mSUGRA parameter space [34]. This is illustrated in Table 1[1], where the accounted present experimental uncertainty on the top quark mass value has also a strong impact on these lower mass bound results. As one can see from Table 1, even though the WMAP constraint on the DM relic density are quite stringent for the mSUGRA model, they do not affect significantly the direct experimental limits on most sparticle masses, which can still be saturated within the model. But of course, the various lower mass bounds as obtained in Table 1 corresponds sometimes to very different regions of the mSUGRA input parameter space. We refer to ref. [34] for a detailed discussion on these issues.

---

[1] Note that the numbers in Table 1 were obtained by using a former SuSpect version including only the one-loop RGE for squark and slepton mass terms, but these results should be only mildly affected once including[34] the full two-loop scalar soft terms RGE.

Quantity	Set I	Set II	Set III	Set IV	Set V	Set VI
$m_{\tilde{e}_R} \simeq m_{\tilde{\mu}_R}$ [GeV]	103	103	104	103	103	104
$m_{\tilde{e}_L} \simeq m_{\tilde{\mu}_L}$ [GeV]	152	157	157	152	157	157
$m_{\tilde{\tau}_1}$ [GeV]	99	99	99	99	99	99
$m_{\tilde{\tau}_2}$ [GeV]	155	160	160	155	160	160
$m_{\tilde{\nu}_\tau}$ [GeV]	129	136	136	129	136	136
$m_{\tilde{\chi}_1^\pm}$ [GeV]	105	105	105	105	105	105
$m_{\tilde{\chi}_2^\pm}$ [GeV]	218	218	229	218	218	229
$m_{\tilde{\chi}_1^0}$ [GeV]	50	51	53	52	53	53
$m_{\tilde{\chi}_2^0}$ [GeV]	105	105	105	105	105	105
$m_{\tilde{\chi}_3^0}$ [GeV]	136	136	137	159	159	195
$m_{\tilde{\chi}_4^0}$ [GeV]	217	217	227	217	217	227
$m_{\tilde{g}}$ [GeV]	360	374	394	360	394	407
$m_{\tilde{d}_R} \simeq m_{\tilde{s}_R}$ [GeV]	401	444	444	401	444	444
$m_{\tilde{d}_L} \simeq m_{\tilde{s}_L}$ [GeV]	421	466	466	421	466	466
$m_{\tilde{b}_1}$ [GeV]	298	414	414	301	414	414
$m_{\tilde{b}_2}$ [GeV]	393	445	445	397	445	445
$m_{\tilde{t}_1}$ [GeV]	102	102	103	102	220	228
$m_{\tilde{t}_2}$ [GeV]	417	500	500	417	500	500
m_h [GeV]	91	91	91	91	91	91
m_H [GeV]	111	111	111	111	111	111
$m_{H^\pm}$ [GeV]	128	128	128	128	128	128

Table I: Lower bounds on sparticle and Higgs masses in mSUGRA under six different sets of assumptions. In all cases the minimal set of constraints are imposed. In Set II the constraint from $b \to s\gamma$ decays (including the sign of the decay amplitude) is added. The DM constraint (4) is added in set III. Sets IV–VI are like Sets I–III, but with the more aggressive $g_\mu - 2$ constraint (3). All limits have been obtained by scanning the full parameter space for 171 GeV $\leq m_t \leq$ 185 GeV.

4. CONCLUSIONS AND PROSPECTS FOR SPARTICLE PRODUCTION AT ILC

From Table 1 it appears that within mSUGRA the prospects are still promising for producing relatively light sparticles and heavier Higgs bosons at the LHC and the international linear e^+e^- collider (ILC), even when taking into account the stringent WMAP constraints. Although a detailed study is well beyond the scope of the present analysis, we just briefly mention that, for instance, the WMAP–favored stau-coannihilation and a substantial part of the focus point region can be covered at ILC via the production of chargino pairs (see e.g. the first ref. in [33] for a detailed analysis), while the resurrected h-pole region can also be covered easily since it implies quite stringent upper bounds on charginos and neutralinos [32]. In conclusion, even for the first stage ILC energy of $\sqrt{s} \sim 500$ GeV, at least the charginos, neutralinos and charged ($\tilde{\tau}$) sleptons could be copiously produced in regions of the mSUGRA parameter space not excluded by present constraints.

References

[1] A.H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49 (1982) 970; R. Barbieri, S. Ferrara and C.A Savoy, Phys. Lett. B119 (1982) 343; L. Hall, J. Lykken and S. Weinberg, Phys. Rev. D27 (1983) 2359.

[2] H.P. Nilles, Phys. Rep. 110 (1984) 1.

[3] E. Witten, Nucl. Phys. **B188**, 513 (1981); N. Sakai, Z. Phys. **C11**, 153 (1981); S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981); R.K. Kaul and P. Majumdar, Nucl. Phys. **B199**, 36 (1982).

[4] J. Ellis , S. Kelley and D.V. Nanopoulos, Phys. Lett. B260 (1991) 131; U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. B260 (1991) 447; P. Langacker and M. Luo, Phys. Rev.D 44 (1991) 817; C. Giunti, C.W. Kim and U.W. Lee, Mod. Phys. Lett. A6 (1991) 1745.

[5] H. Goldberg, Phys. Rev. Lett. **50**, 1419 (1983); J. Ellis, J. Hagelin, D.V. Nanopoulos, K. Olive and M. Srednicki, Nucl. Phys. **B238**, 453 (1984).

[6] For a review, see G. Jungman, M. Kamionkowski and K. Griest, Phys. Rep. 267 (1996) 195, hep-ph/9506380.

[7] L.E. Ibáñez and G.G. Ross, Phys. Lett. **110B**, 215 (1982); L.E. Ibáñez, Phys. Lett. **118B**, 73 (1982); J. Ellis, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. **121B**, 123 (1983); L. Alvarez-Gaumé, J. Polchinski and M.B. Wise, Nucl. Phys. **B221**, 495 (1983).

[8] WMAP Collab., D.N. Spergel et al., Astrophys. J. Suppl. **148**, 175 (2003), astro–ph/0302209.

[9] B.C. Allanach, A. Djouadi, J.-L. Kneur, W. Porod and P. Slavich, JHEP **0409**, 044 (2004), hep–ph/0406166, and references therein.

[10] D0 Collab. V.M. Abazov et al., Nature **429**, 638-642 (2004), hep–ex/0406031.

[11] P. Gambino, U. Haisch and M. Misiak, hep–ph/0410155.

[12] See e.g. M. Davier, talk at Tau 04, Nara, Japan, Sept. 2004, Nucl. Phys. Proc. Suppl. 144 (2005) 250.

[13] Muon $g-2$ Collab., G.W. Bennett et al., Phys. Rev. Lett. **89**, 101804 (2002), Erratum–ibid. **89**, 129903 (2002), hep–ex/0208001, and Phys. Rev. Lett. **92**, 161802 (2004), hep–ex/0401008.

[14] M. Davier, S. Eidelman, A. Höcker and Z. Zhang, Eur. Phys. J. **C31**, 503 (2003), hep–ph/0308213; K. Hagiwara, A.D. Martin, D. Nomura and T. Teubner, Phys. Rev. **D69**, 093003 (2004), hep–ph/0312250; J.F. de Troconiz and F.J. Yndurain, hep–ph/0402285; M. Passera, hep–ph/0411168.

[15] A. Djouadi, J.-L. Kneur and G. Moultaka, hep–ph/0211331.

[16] A. Djouadi, M. Drees and J.-L. Kneur, JHEP **0108**, 055 (2001), hep–ph/0107316.

[17] D. M. Pierce, J. A. Bagger, K. T. Matchev and R. J. Zhang, Nucl. Phys. B491 (1997) 3.

[18] G. Degrassi, P. Slavich and F. Zwirner, Nucl. Phys. B611 (2001) 403; A. Brignole, G. Degrassi, P. Slavich and F. Zwirner, Nucl. Phys. B631 (2002) 195; A. Dedes and P. Slavich, Nucl. Phys. B657 (2003) 333; A. Dedes, G. Degrassi and P. Slavich, Nucl. Phys. B672 (2003) 144.

[19] J.M. Frère, D.R.T. Jones and S. Raby, Nucl. Phys. **B222**, 11 (1983); M. Claudson, L. Hall and I. Hinchliffe, Nucl. Phys. **B228**, 501 (1983).

[20] J.A. Casas, A. Lleyda and C. Muñoz, Nucl. Phys. B471 (1996) 3, hep–ph/9507294.

[21] R. Barbieri and L. Maiani, Nucl. Phys. B224, 32 (1983); C.S. Lim, T. Inami and N. Sakai, Phys. Rev. D29, 1488 (1984); E. Eliasson, Phys. Lett. 147B, 65 (1984); M. Drees and K. Hagiwara, Phys. Rev. D42 (1990) 1709.

[22] A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger and G. Weiglein, Phys. Rev. Lett. 78 (1997) 3626, hep–ph/9612363, and Phys. Rev. D57 (1998) 4179, hep–ph/9710438.

[23] Particle Data Group, S. Eidelman et al, Phys. Lett. **B592**, 1 (2004). For an up–to–date summary of sparticle search limits from the LEP experiments, see `http://lepsusy.web.cern.ch/lepsusy/` .

[24] The ALEPH, DELPHI, L3 and OPAL Collab.s, Phys. Lett. **B565**, 61 (2003), hep–ex/0306033.

[25] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, Eur. Phys. J. C28, 133 (2003).

[26] S.P. Martin and J.D. Wells, Phys. Rev. D64 (2001) 035003, hep-ph/0103067; G. Degrassi and G.F. Giudice, Phys. Rev. D58, 053007 (1998), hep–ph/9803384.

[27] See e.g. M. Drees, talk at *SUSY2004*, Tsukuba, Japan, June 2004, hep–ph/0410113.

[28] K. Griest and D. Seckel, Phys. Rev. D43, 3191 (1991).

[29] G. Degrassi, P. Gambino and G.F. Giudice, JHEP 0012 (2000) 009, hep-ph/0009337.

[30] K. Okumura and L. Roszkowski, Phys. Rev. Lett. 92, 161801 (2004), hep–ph/0208101.

[31] See e.g. J.R. Ellis, S. Heinemeyer, K.A. Olive and G. Weiglein, hep–ph/0411216, and refs. therein.

[32] A. Djouadi, M. Drees and J.-L. Kneur, hep-ph/0504090.

[33] H. Baer, A. Belyaev, T. Krupovnickas and X. Tata, JHEP **0402**, 007 (2004), hep–ph/0311351; H. Baer, T. Krupovnickas and X. Tata, JHEP **0406**, 061 (2004), hep–ph/0405058.

[34] A. Djouadi, M. Drees, J.-L. Kneur and P. Slavich, in preparation.