

The Impact of Beamstrahlung on Precision Measurements at CLIC

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We review two commonly used approximate treatments of beamstrahlung at lepton colliders. We discuss the applicability of those approximations and show that they are bound to fail at very high energy e^+e^- colliders such as CLIC. We model the beamstrahlung effects at CLIC by an exact numerical solution to the evolution equation for beamstrahlung and compare to the two approximations. We also discuss the impact of beamstrahlung on precision measurements of new physics parameters. As an illustration we consider Kaluza-Klein lepton production in Universal Extra Dimensions and study the deterioration of the endpoint lepton energy spectrum caused by beamstrahlung.

1. INTRODUCTION

The fixed center of mass energy E_{CM} is a celebrated virtue of lepton colliders. In contrast to hadron colliders, where the parton-level E_{CM} varies from event to event, the additional kinematic constraint allows for a number of new measurements. Some well-known examples are: the presence of sharp endpoints in the distributions of various kinematic observables, the possibility of threshold scans, measurements of the missing mass, the angular distributions at production, etc. The availability of all these new experimental tools allows for precise determination of new physics parameters and is the primary motivation for pursuing the construction of a next generation lepton collider.

One should keep in mind, however, that in reality a fraction of the beam energy is always lost due to the effects of beamstrahlung and bremsstrahlung, so that the true center of mass energy in the event is often displaced from its nominal value. The purpose of the present note is to investigate to what extent these effects can represent a problem for precision measurements of new physics parameters at very high energy e^+e^- colliders such as CLIC. To this end, in Sec. 2 we first review the two commonly used parametrizations of beamstrahlung. Then in Sec. 3 we discuss the applicability of those approximations for lepton colliders with sub-TeV E_{CM} (such as ILC) as well as E_{CM} in the multi-TeV range (such as CLIC). We find that the latter case falls outside the range of validity of the two approximations, and an exact treatment of beamstrahlung effects is required. We solve numerically the evolution equation for beamstrahlung using the exact expression for the Sokolov-Ternov transition probability (see Sec. 2). In Sec. 4 we consider the example of Kaluza-Klein muon production in Universal Extra Dimensions (UED) and use our numerical results from Sec. 3 to investigate the deterioration of the endpoint lepton energy spectrum caused by beamstrahlung. The same analysis can be readily applied to smuon production in supersymmetry as well.

2. ANALYTICAL APPROACHES TO BEAMSTRAHLUNG

For physics purposes we need to know the energy distribution $\psi_e(x, t)$ of the colliding electrons, where $x \equiv E/E_0$ is the electron energy as a fraction of the nominal beam energy E_0 . Neglecting electrons from pair-production, the normalization condition on ψ_e is

$$\int_0^1 dx \psi_e(x, t) = 1. \quad (1)$$

*This talk was given by K. Kong, describing past and ongoing work performed in collaboration with the other authors.

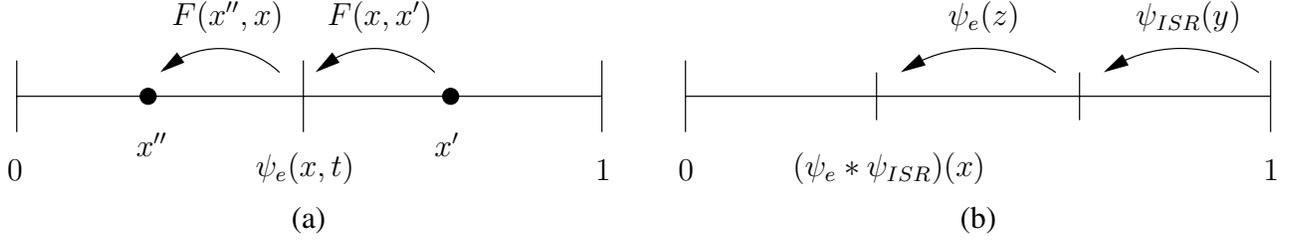


Figure 1: (a) $F(x, x')$ corresponds to the source and $F(x'', x)$ to the sink for the evolution of the spectral function $\psi_e(x, t)$. (b) The final energy of an electron results from a convolution of beamstrahlung and bremsstrahlung (see Sec. 2.3).

Assuming that the emission of a photon takes place on an infinitesimally short time scale, the interference between successive radiation processes is negligible and the evolution of the electron spectral function ψ_e is described by the following rate equation (see Figure 1a):

$$\frac{\partial \psi_e(x, t)}{\partial t} = - \int_0^x dx'' F(x'', x) \psi_e(x, t) + \int_x^1 dx' F(x, x') \psi_e(x', t). \quad (2)$$

Here $F(x, x')$ is the spectral function of radiation, i.e. $F(x, x')$ is the transition probability (per unit time) for an electron at x' to move into the energy interval $(x, x + dx)$. Obviously, $F(x, x') = 0$ if $x \geq x'$. Pulling out $\psi_e(x, t)$, which is independent of x'' , outside the x'' integral, leaves the integral quantity

$$\nu(x) = \int_0^x dx'' F(x'', x), \quad (3)$$

which represents the average number of photons radiated per unit time by the electron with an instantaneous energy x . The transition probability $F(x, x')$ derived by Sokolov and Ternov [1] is given by

$$F(x, x') = \frac{\nu_{cl} \kappa}{xx'} f(\xi, \eta), \quad (4)$$

$$f(\xi, \eta) = \frac{3}{5\pi} \frac{1}{1 + \xi\eta} \left[\int_\eta^\infty du K_{5/3}(u) + \frac{\xi^2 \eta^2}{1 + \xi\eta} K_{2/3}(\eta) \right], \quad (5)$$

where $\xi \equiv 3x'\Upsilon/2$, $\eta = \kappa(1/x - 1/x')$ and the constant $\kappa \equiv 2/(3\Upsilon)$ is introduced for convenience. K_ν is the modified Bessel function and ν_{cl} is the number of photons per unit time calculated by the classical theory of radiation. By definition, this is also the limit of $\nu(x)$ for $x \rightarrow 0$:

$$\nu_{cl} \equiv \nu(x = 0) = \frac{5}{2\sqrt{3}} \frac{\alpha^2}{r_e \gamma_0} \Upsilon, \quad (6)$$

where r_e is the classical electron radius, α is the fine structure constant, $\gamma_0 = E_0/(m_e c^2)$ and the dimensionless Lorentz invariant parameter

$$\Upsilon = \frac{5}{6} \frac{r_e^2 \gamma_0 N}{\alpha \sigma_z (\sigma_x + \sigma_y)} \quad (7)$$

has a value specific to any given collider, as it depends on the total number of particles N in a gaussian bunch and the rms sizes $\sigma_x, \sigma_y, \sigma_z$ of the gaussian beam.

2.1. Yokoya-Chen (YC) approximation

It is clear that the Sokolov-Ternov (ST) spectral function (4) is too complicated to allow a general analytic solution to Eq. (2). Therefore, one has to resort to some kind of an approximation. In 1989 Yokoya and Chen [2, 3] suggested the following simple approximation to (5):

$$f(\xi, \eta) \approx \frac{1}{\Gamma(1/3)} \eta^{-2/3} e^{-\eta}, \quad (8)$$

with the additional assumption

$$\nu(x) = \int_0^x dx'' F(x, x'') \approx \nu_{cl} = \text{constant}. \quad (9)$$

One can then solve the rate equation (2) by Laplace transformation and obtain an analytical solution:

$$\psi(x, t) = e^{-\nu_{cl}t} \left[\delta(1-x) + \frac{e^{-\eta_x}}{1-x} h(\eta_x^{1/3} \nu_{cl}t) \right], \quad (10)$$

where $\eta_x \equiv \kappa(1/x - 1)$ and

$$h(u) = \frac{1}{2\pi i} \int_{\lambda-i\infty}^{\lambda+i\infty} \exp(up^{-1/3} + p) dp = \sum_{n=1}^{\infty} \frac{u^n}{n! \Gamma(n/3)} \quad (11)$$

with $\lambda > 0$ and $0 \leq u \leq \infty$. The first term in Eq. (10) represents the electron population unaffected by radiation. The n -th term in the Taylor expansion of the second term in (10) corresponds to the process of n -photon emission. The final electron energy distribution $\psi_e(x)$ due to beamstrahlung is obtained by suitable time averaging of the time-dependent solution (10) to account for the duration of bunch overlap, for example:

$$\psi_e(x) \equiv \frac{2}{l} \int_0^{l/2} dt \psi_e(x, t) = \frac{1}{N_{cl}} \left[(1 - e^{-N_{cl}}) \delta(1-x) + \frac{e^{-\eta_x}}{1-x} \bar{h}(x) \right], \quad (12)$$

where $l = 2\sqrt{3}\sigma_z$ is the effective length of the incoming bunch and $N_{cl} = \nu_{cl}l/2$ is the average number of photons radiated per particle during the entire collision of the e^+e^- beams. The function $\bar{h}(x)$ in the second term is

$$\bar{h}(x) = \sum_{n=0}^{\infty} \frac{\eta_x^{n/3}}{n! \Gamma(n/3)} \gamma(n+1, N_{cl}), \quad (13)$$

where $\gamma(n, x)$ is the incomplete Gamma function

$$\gamma(n, x) = \int_0^x dy e^{-y} y^{n-1}. \quad (14)$$

The resulting solution (12) is in good agreement with ABEL simulation data at least up to $\Upsilon \sim 0.44$ [3].

2.2. Consistent Yokoya-Chen (CYC) approximation

Later on, M. Peskin pointed out [4] that the two assumptions (8) and (9) behind the YC approximation are incompatible with the normalization condition (1) of the energy spectral function. To rectify the problem, Peskin proposed the so-called ‘‘consistent Yokoya-Chen’’ approximation as an alternative to (8):

$$\tilde{f}(\xi, \eta) = \frac{x'}{x} \frac{1}{\Gamma(1/3)} \eta^{-2/3} e^{-\eta}. \quad (15)$$

The rate equation (2) can then be solved in exactly the same way and the resulting analytic solution is

$$\psi_e(x) = e^{-N_\gamma} \left(\delta(x-1) + \frac{e^{-\eta_x}}{x(1-x)} h(N_\gamma \eta_x^{1/3}) \right), \quad (16)$$

where $N_\gamma = \sqrt{3}\sigma_z \nu_{cl} (1 + \Upsilon^{2/3})^{-1/2}$. This solution not only preserves the probability sum rule (1) but also agrees with Guinea Pig simulation for $\Upsilon = 0.104$ (NLC500) and $\Upsilon = 0.299$ (NLC1000) and numerically is not very different from the YC solution (12) for $\Upsilon \ll 1$ [4]. However, for large Υ , the two solutions are significantly different, see Fig. 2 and Sec. 3 below.

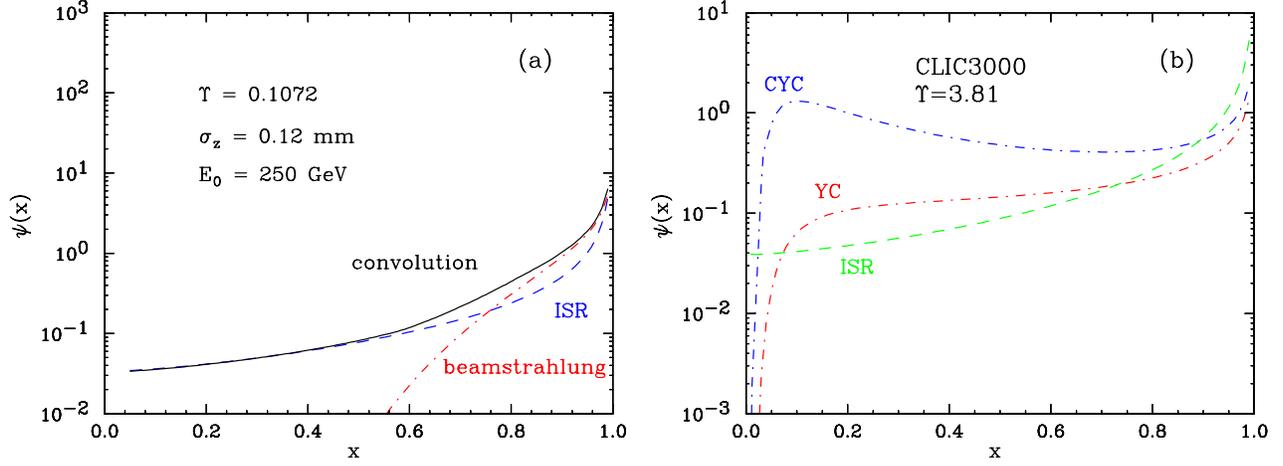


Figure 2: (a) Electron energy distributions due to beamstrahlung (dot-dashed, red), ISR (dashed, blue) and their convolution (solid), for an ILC with the parameters as shown. The YC and CYC approximations (12) and (16) are almost identical in this case. (b) Electron energy distributions due to beamstrahlung (dot-dashed) and ISR (dashed, green), for CLIC with the shown parameters. The YC (CYC) solution is shown in red (blue).

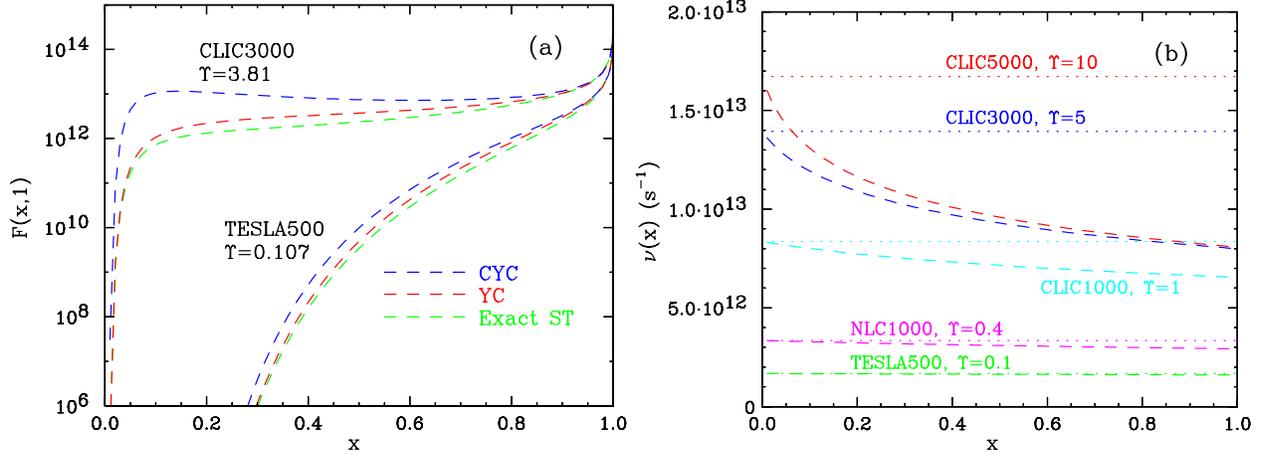


Figure 3: (a) The transition probability $F(x,1)$ as a function of x , using the exact expression (5) or the approximations (8) and (15), for two different values of Υ . (b) The function $\nu(x)$ plotted for various values of Υ .

2.3. Convolution with bremsstrahlung

Another source of electron energy losses is bremsstrahlung, or initial state radiation (ISR), parametrized with its own energy distribution function ψ_{ISR} [5]. The probability to end up with an electron of energy x is a convolution of ISR and beamstrahlung (see Figs. 1b and 2a):

$$\psi_{ISR} * \psi_e = \psi_e * \psi_{ISR} = \int_0^1 dy dz \psi_{ISR}(y) \psi_e(z) \delta(x - yz) = \int_x^1 \frac{dz}{z} \psi_{ISR}\left(\frac{x}{z}\right) \psi_e(z). \quad (17)$$

3. VALIDITY RANGE OF THE APPROXIMATE SOLUTIONS

In order to test the validity of the YC and CYC approximations, in Fig. 3a we plot the transition probability $F(x,1)$ as a function of x , using the exact expression (5) or the approximations (8) and (15), for two different values of Υ . First of all, we notice that as Υ gets large, one enters the quantum regime of radiation, where emitting hard

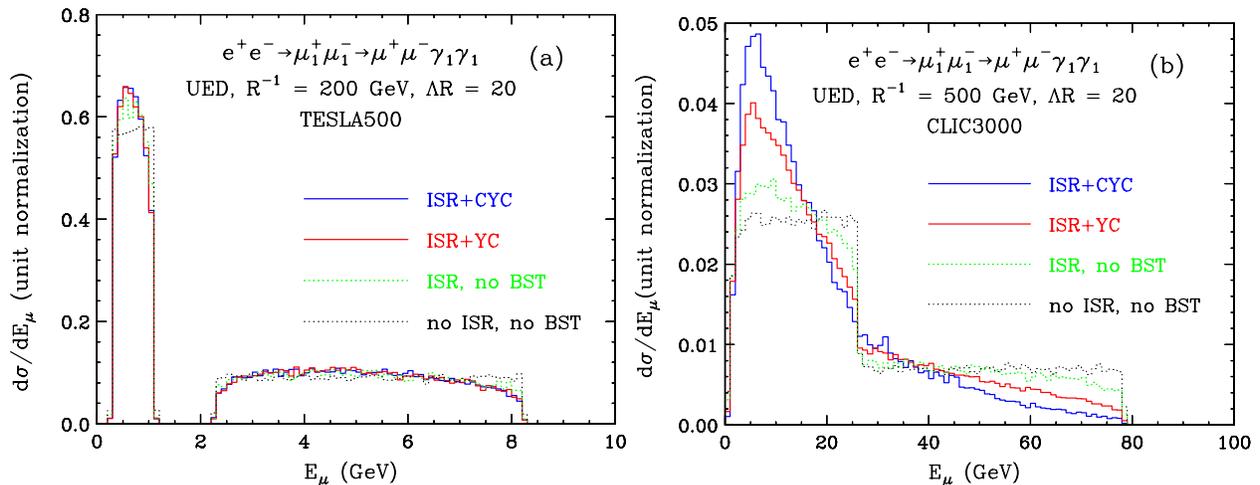


Figure 4: (a) Muon energy spectrum from KK muon production in UED at an e^+e^- collider with $E_{CM} = 500$ GeV and $\Upsilon = 0.107$. The left (right) box-like distribution is due to $SU(2)$ -singlet ($SU(2)$ -doublet) μ_1^\pm production. The dotted lines ignore the beamstrahlung, with the effects from ISR shown in green. The red (blue) lines include beamstrahlung according to the YC (CYC) approximation. (b) The same as (a), but for $R^{-1} = 500$ GeV at CLIC with $E_{CM} = 3000$ GeV and $\Upsilon = 3.81$.

photons becomes more and more likely. The large tail at low x appearing in the case of CLIC could become an annoying spoiler for physics studies — an issue we will look at in more detail in the next Section. We also see that fixing the normalization problem of the YC approximation comes at a price: the improved CYC approximation is numerically further away from the exact ST spectral function (4) and the discrepancy becomes more pronounced at large Υ . Conversely, while the YC approximation does a better job in approximating (4), it not only suffers from the normalization problem, but also relies on the approximation $\nu(x) \approx \nu_{cl} = const$, which fails at large Υ , as evidenced in Fig. 3b. There we plotted $\nu(x)$ and the corresponding ν_{cl} for different values of Υ . We see that for small Υ the $\nu(x)$ function (3) is pretty constant and the YC approximation is valid, but for the larger Υ values which are relevant at CLIC, $\nu(x)$ is far from being a constant, and the YC approximation will fail.

Fig. 3 reveals that in order to get a reliable parametrization of beamstrahlung for large Υ (i.e. at very high energy e^+e^- colliders such as CLIC), we need to improve on the two existing approximations. We tried, but were unable to find a better approximation to (5), which would still allow for an analytic solution in closed form. We therefore resorted to a numerical solution of the original rate equation (2) with the full ST formula (4). This solution was then used for the physics study presented in Sec. 4.

4. MUON ENERGY SPECTRUM FROM KK-MUON PRODUCTION

In order to study the effect of beamstrahlung on physical observables, we consider the production of level 1 Kaluza-Klein muons μ_1^\pm in UED [6, 7], which then decay to the lightest KK particle (LKP), in this case the KK partner γ_1 of the photon. The signature is two opposite sign muons and missing energy, which is analogous to smuon production in supersymmetry models with stable neutralino LSP. At hadron colliders, the two scenarios can be confused [7–9], but lepton colliders allow for an easy and straightforward discrimination [9].

Two important observables are the lower, E_{min} , and upper, E_{max} , endpoints of the muon energy spectrum (see Fig. 4). They can be used to extract the masses of the particles involved in the decay as follows:

$$E_{max/min} = \frac{1}{2} M_{\mu_1} \left(1 - \frac{M_{\gamma_1}^2}{M_{\mu_1}^2} \right) \gamma (1 \pm \beta), \quad (18)$$

where M_{μ_1} and M_{γ_1} are the KK muon and LKP masses, respectively, $\gamma = 1/(1 - \beta^2)^{1/2}$, and β is the μ_1 boost. Fig. 4a confirms that at low E_{CM} the effects from beamstrahlung are rather small, and furthermore, the YC and

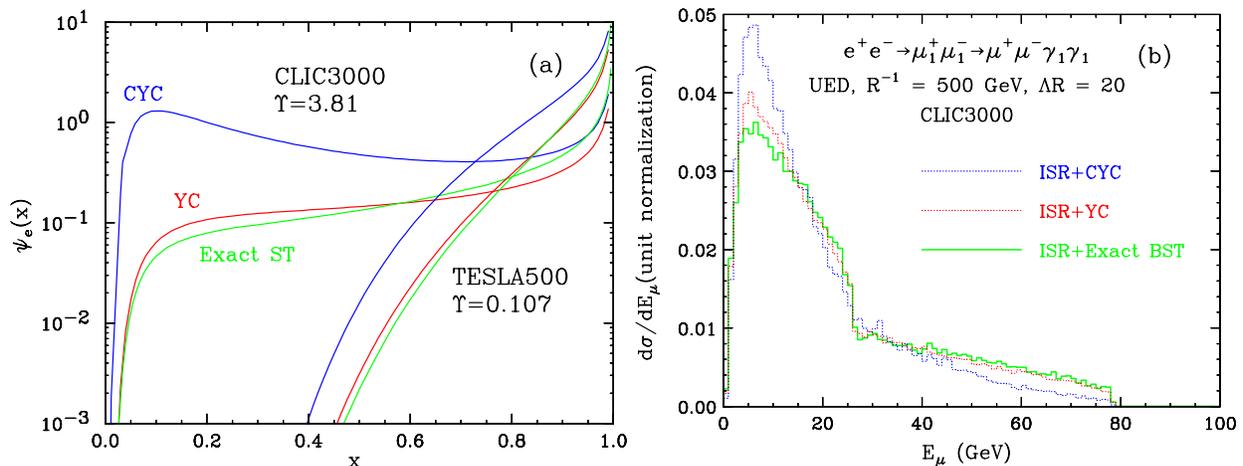


Figure 5: (a) Electron energy distributions resulting from the numerical solution (green) or the YC (red) and CYC (blue) approximations, for $\Upsilon = 3.81$ and $\Upsilon = 0.107$. (b) The same as Fig. 4b, but including the exact solution for beamstrahlung.

CYC approximations are in agreement. However, Fig. 4b reveals that at higher energy colliders, beamstrahlung tends to wash out the endpoint, the degree of deterioration being quite sensitive to the approximation used.

Since we already expect neither approximations to be reliable at large E_{CM} (where Υ is also large), we solved Eq. (2) numerically. The result, together with the YC and CYC approximations, is shown in Fig. 5a, for $\Upsilon = 0.107$ (TESLA500) and $\Upsilon = 3.81$ (CLIC3000) [10]. Notice how the improved CYC approximation considerably overestimates the effect of beamstrahlung at low x . We then implemented the numerical solution in the CompHEP event generator [11]. The resulting true muon energy distribution including both types of beam energy losses is shown in Fig. 5b. We see that beamstrahlung still degrades the kinematic endpoint, although not as much as the CYC and YC approximations would suggest.

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