Measurement of Lepton Flavor Violating Yukawa Couplings at ILC

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We discuss lepton flavor violation (LFV) associated with tau leptons in the general framework of the two Higgs doublet model, in which LFV couplings are introduced as a deviation from Model II Yukawa interaction. Parameters of the model are constrained from experimental results and also from requirements of theoretical consistencies such as vacuum stability and perturbative unitarity. Current data for LFV rare tau decays provide substantial upper limits on the LFV Yukawa couplings in the large $\tan\beta$ region, which are comparable with predictions in fundamental theories. Here $\tan\beta$ is the ratio of vacuum expectation values of the two Higgs doublets. A search for the LFV decays $\phi^0 \to \tau^\pm\mu^\mp$ ($\tau^\pm e^\mp$) of neutral Higgs bosons ($\phi^0 = h, H$ and A) at future collider experiments can be useful to further constrain the LFV couplings, especially in the relatively small $\tan\beta$ region ($\tan\beta \lesssim 30$), where rare tau decay data cannot give any strong limit.

1. INTRODUCTION

The structure of the electroweak symmetry breaking sector would directly connect to the property of physics beyond the standard model (SM). Models of such new physics predict extended Higgs sectors with more than one scalar doublets in the low energy effective theory. These extended Higgs models would show distinctive features from the SM phenomenology. The most obvious evidence would be the confirmation of the existence of the extra scalar states such as CP-odd and charged states at future collider experiments. Even if they are too heavy to be directly discovered and only the lightest Higgs boson is found at the experiments, we would be able to explore the extended Higgs sector by searching for deviations from the SM predictions on the couplings with gauge bosons and fermions as well as on the self coupling. Furthermore, search for non-SM interactions can also be useful.

Lepton flavor violation (LFV) is an example for such non-SM phenomena. In particular, LFV in the Yukawa sector can only appear for extended Higgs sectors. Flavor violation between electrons and muons[1] has been tested through rare muon decays such as $\mu \to e\gamma$ and $\mu \to ee^+e^-$, as well as through μ-e conversion. Tau lepton associated LFV has also been studied by rare decays of tau leptons such as $\tau \to \ell_i P^0$[2], $\tau \to \ell_i M^+ M'^-$[3, 4], $\tau \to \ell_i \ell'^+ \ell'^-$[5, 6], and $\tau \to \ell_i \gamma$[7–9], where ℓ_i ($i = 1, 2$) respectively represent an electron and a muon, P^0 does π^0, η and η' mesons, $M^\pm$ ($M'^\pm$) does $\pi^\pm$ and $K^\pm$ mesons, and $\ell'^\pm = e^\pm$ and $\mu^\pm$. The LFV Yukawa couplings can be constrained from the data for these processes especially those with the Higgs boson mediation. For μ-e mixing, the Higgs boson mediated LFV coupling has been discussed in Ref. [10, 11]. Tau lepton associated LFV processes with the Higgs boson mediation have been discussed in models with supersymmetry (SUSY)[12–15] as well as in the two Higgs doublet model (THDM) in some specific scenarios[16–18]. In Ref. [19], tau associated LFV processes have been discussed comprehensively in the framework of 4-Fermi contact interactions. Phenomenological consequences of the LFV Yukawa couplings associated with tau leptons have also been studied for future observables such as B_s decays[13, 20] at (super) B factories[21] and Higgs boson decays[15, 22–24] at CERN LHC[25], an electron-positron linear collider (LC)[26] and a muon collider[27]. Furthermore, deep inelastic scattering processes $\mu N \to \tau X$[28, 29] from intense high energy muons at neutrino factories (or muon colliders) and $eN \to \tau X$[29] by using the electron (positron) beam of a LC would be useful to further explore the tau lepton associated LFV Yukawa couplings.

In this talk, we discuss LFV in Higgs boson decays into a τ-ℓ_i pair in the general framework of the THDM[24]. We

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study experimental upper limits on the tau lepton associated LFV Yukawa couplings to evaluate possible maximal values of the branching fractions. The parameter space of the model is tested by theoretical requirements for vacuum stability[30] and perturbative unitarity[31, 32]. Current data from electroweak precision measurements at LEP[33, 34] and those at the B factories[35] also strongly constrain parameters of the Higgs potential. Under these theoretical bounds and experimental limits on the model, possible maximal values of the LFV couplings of τ-ℓ_i-ϕ^0 are obtained by using the current data for rare tau decays, where ϕ^0 represents two CP-even (h and H) and a CP-odd (A) Higgs bosons. We then evaluate branching ratios of $\phi^0 \to \tau^\pm \ell_i^\mp$ with the maximal allowed values of the LFV couplings of τ-ℓ_i-ϕ^0 in a wide range of the parameter space.

2. LEPTON FLAVOR VIOLATION IN YUKAWA INTERACTION

The Higgs sector of the general THDM is expressed as

$$\begin{aligned} -\mathcal{L}_{\rm Higgs} \;=\; & m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - \left(m_3^2 \Phi_1^\dagger \Phi_2 + \text{H.c.} \right) + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 \left| \Phi_1^\dagger \Phi_2 \right|^2 \\ & + \left\{ \frac{\lambda_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{H.c.} \right\} + \left(\lambda_6 |\Phi_1|^2 \Phi_1^\dagger \Phi_2 + \text{H.c.} \right) + \left(\lambda_7 |\Phi_2|^2 \Phi_1^\dagger \Phi_2 + \text{H.c.} \right), \end{aligned} \quad (1)$$

where Φ_1 and Φ_2 are the scalar iso-doublets with hypercharge 1/2. In Eq. (1), m_3^2, λ_5, λ_6 and λ_7 are complex in general. We here assume that all the parameters m_{1-3}^2 and λ_{1-7} are real. The terms of m_3^2, λ_6 and λ_7 break the discrete symmetry explicitly. As we consider the model in which the discrete symmetry is explicitly broken only in the leptonic Yukawa interaction, we set the hard-breaking coupling constants to be zero in the Higgs potential; i.e., $\lambda_6 = \lambda_7 = 0$[1], and retain only the soft-breaking mass parameter m_3^2.

There are eight degrees of freedom in the two Higgs doublet fields. Three of them are absorbed by the weak gauge bosons via the Higgs mechanism, and remaining five are physical states. After the diagonalization of the mass matrices, they correspond to two CP-even (h and H), a CP-odd (A), and a pair of charged ($H^\pm$) Higgs bosons. We define such that h is lighter than H. The eight real parameters m_{1-3}^2 and λ_{1-5} can be described by the same number of physical parameters; i.e., the vacuum expectation value v ($\simeq 246$ GeV), the Higgs boson masses m_h, m_H, m_A and $m_{H^\pm}$, the mixing angle α between the CP-even Higgs bosons, the ratio $\tan\beta$ ($\equiv \langle \Phi_2^0 \rangle / \langle \Phi_1^0 \rangle$) of the vacuum expectation values for two Higgs doublets, and the soft-breaking scale M ($\equiv \sqrt{m_3^2 / \sin\beta \cos\beta}$) for the discrete symmetry. The quartic couplings are expressed in terms of physical parameters in Ref. [36]

Parameters of the Higgs sector are constrained from requirements of theoretical consistencies and also from the current experimental results. We here take into account two kinds of theoretical conditions; i.e., vacuum stability[30] and perturbative unitarity[31, 32] at the tree level. The condition for tree-level unitarity, which we employ in Refs. [32, 36], is described as $\left| \langle \phi_3 \phi_4 | a^0 | \phi_1 \phi_2 \rangle \right| < \xi$, where $\langle \phi_3 \phi_4 | a^0 | \phi_1 \phi_2 \rangle$ is the s-wave amplitude for the process of $\phi_1 \phi_2 \to \phi_3 \phi_4$ with ϕ_a (a=1-4) denoting Higgs bosons and longitudinal components of weak gauge bosons. We take the criterion ξ to be 1 (and also 1/2 for comparison). The experimental constraints are provided by the LEP precision data[33], the $b \to s\gamma$ results[35], and the direct search results for the Higgs bosons[33]. The LEP precision data provide the strong constraint on the new physics structure via the gauge-boson two-point functions. The constraint on ρ parameter indicates that the Higgs sector is approximately custodial $SU(2)$ symmetric. This requirement is satisfied when (i) $m_{H^\pm} \simeq m_A$, (ii) $m_{H^\pm} \simeq m_H$ with $\sin^2(\alpha - \beta) \simeq 1$, and (iii) $m_{H^\pm} \simeq m_h$ with $\cos^2(\alpha - \beta) \simeq 1$. It is known that in Model II, the $b \to s\gamma$ result gives the lower bound on the charged Higgs boson mass. We here take into account this bound by requiring $m_{H^\pm} \gtrsim 350$ GeV.

Next, we consider the Yukawa interaction for charged leptons as

$$-\mathcal{L}_{\rm lepton} = \overline{\ell}_{Ri} \left\{ Y_{\ell_i} \delta_{ij} \Phi_1 + \left(Y_{\ell_i} \epsilon_{ij}^L + \epsilon_{ij}^R Y_{\ell_j} \right) \Phi_2 \right\} \cdot L_j + \text{H.c.}, \quad (2)$$

[1] Even in such a case, λ_6 and λ_7 are effectively induced at the loop level. They are suppressed by the loop factor, so that we here neglect these small effects.

where ℓ_{Ri} (i=1-3) are right-handed charged leptons, and L_i (i=1-3) denote the lepton doublets and Y_{ℓ_i} are the Yukawa couplings for ℓ_i. This interaction is reduced to be of Model II[37] in the limit $\epsilon_{ij}^{L,R} \to 0$ with the discrete symmetry under $e_R^i \to +e_R^i$, $L_i \to +L_i$, $\Phi_1 \to +\Phi_1$, and $\Phi_2 \to -\Phi_2$. Nonzero values of $\epsilon_{ij}^{L,R}$ ($i \neq j$) yield the LFV Yukawa couplings after the diagonalization of the mass matrix. We note that in supersymmetric standard models, the Yukawa interaction for leptons is of Model II at the tree level, and $\epsilon_{ij}^{L,R}$ can be induced at the loop level due to slepton mixing[12, 13, 38]. For the quark sector, Model II Yukawa interactions are assumed to suppress FCNC, imposing the invariance under the transformation of $u_R^i \to -u_R^i$, $d_R^i \to +d_R^i$, $q_L^i \to +q_L^i$, $\Phi_1 \to +\Phi_1$, and $\Phi_2 \to -\Phi_2$.

The tau lepton associated LFV interactions in Eq. (2) can be reduced in the mass eigenbasis of each field to

$$\begin{aligned} -\mathcal{L}_{\tau\mathrm{LFV}} &= \frac{m_\tau}{v\cos^2\beta}\left(\kappa_{3i}^L \overline{\tau} \mathrm{P}_L \ell_i + \kappa_{i3}^R \overline{\ell}_i \mathrm{P}_L \tau\right)\left\{\cos(\alpha-\beta)\,h + \sin(\alpha-\beta)\,H - \mathrm{i}A\right\} \\ &\quad + \frac{\sqrt{2}m_\tau}{v\cos^2\beta}\left(\kappa_{3i}^L \overline{\tau} \mathrm{P}_L \nu_i + \kappa_{i3}^R \overline{\ell_i} \mathrm{P}_L \nu_\tau\right) H^- + \mathrm{H.c.}, \end{aligned} \tag{3}$$

where P_L is the projection operator to the left-handed field, and ℓ_1 and ℓ_2 respectively represent e and μ. In general, the LFV parameters $\kappa_{ij}^{L,R}$ can be expressed in terms of $\epsilon_{ij}^{L,R}$ and $\tan\beta$.[2] We here take these $\kappa_{ij}^{L,R}$ as effective couplings, and investigate their phenomenological consequences. We note that Eq. (3) is exact in the limit of $m_{\ell_i} \to 0$. The terms of κ_{i3}^L and κ_{3i}^R ($i = 1, 2$) are proportional to m_{ℓ_i}, so that they decouple in this approximation.

We briefly discuss relationship between $\kappa_{ij}^{L,R}$ and new physics models beyond the cut-off scale of the *effective* THDM. When a new physics model is specified at the high energy scale, $\kappa_{ij}^{L,R}$ can be predicted as a function of the model parameters. For example, in the MSSM, slepton mixing can be a source of LFV. Notice that the induced LFV Higgs interactions do not necessarily decouple in the limit where the SUSY particles are sufficiently heavy, because their couplings only depend on the ratio of the SUSY parameters. Therefore, the Higgs associated LFV processes can become important in a scenario with the soft-SUSY-breaking scale m_{SUSY} to be much higher than the electroweak one. In the MSSM, predicted values of $|\kappa_{3i}^L|^2$ can be as large as of $\mathcal{O}(10^{-6})$ when m_{SUSY} is a few TeV[15, 26]. In the MSSM with right-handed neutrinos, left-handed slepton mixing may be a consequence of running effects of the neutrino Yukawa couplings between the scale of the grand unification and that of the right-handed neutrinos[38]. The parameters κ_{3i}^L are mainly induced by mixing of left-handed sleptons[12–15, 23, 26]. The LFV Yukawa interactions can also appear effectively in the Zee model[39]. The LFV parameters $\kappa_{ij}^{L,R}$ are induced through flavor violating couplings in the charged scalar interactions with leptons.

3. BOUND ON LFV YUKAWA COUPLINGS FROM RARE TAU DECAYS

rare tau decays

In order to constrain the LFV parameters $|\kappa_{3i}^L|$ and $|\kappa_{i3}^R|$, we take into account the data for rare tau decay processes such as $\tau \to \ell_i P^0$, $\tau \to \ell_i M^+ M'^-$, $\tau \to \ell_i \ell'^+ \ell'^-$, and $\tau \to \ell_i \gamma$, where P^0 represents π^0, η and η' mesons, $M^\pm$ ($M'^\pm$) does $\pi^\pm$ and $K^\pm$ mesons, and $\ell'^\pm = e^\pm$ and $\mu^\pm$. The list of the current data from the B factories are summarized in Table I [2–9]. These bounds may be improved at the super B factory around a digit[21]. In our analysis, we take the underlined data in Table I as our numerical inputs. The branching ratios for these rare τ LFV decays are summarized in Ref. [24].

Since branching ratios for $\tau^- \to \ell_i^- P^0$, $\tau^- \to \ell_i^- M^+ M^-$, $\tau^- \to \ell_i^- \ell'^+ \ell'^-$ and $\tau^- \to \ell_i^- \gamma$ depend on different combinations of the Higgs boson masses, independent information can be obtained for the model parameters by

[2] LFV parameters $\kappa_{ij}^{L,R}$ can be expressed as

$$\kappa_{ij}^X = -\frac{\epsilon_{ij}^X}{\left\{1 + (\epsilon_{33}^L + \epsilon_{33}^R)\tan\beta\right\}^2} \qquad (X = L, R), \tag{4}$$

for the case of $\epsilon_{ij}^{L,R}\tan\beta \ll \mathcal{O}(1)$ which is satisfied in the case of the minimal supersymmetric standard model (MSSM)[12, 13].

Mode	Belle (90% CL)	BaBar (90% CL)
$\tau^- \to e^-\pi^0$	$\underline{1.9 \times 10^{-7}}$[2]	
$\tau^- \to e^-\eta$	$\underline{2.4 \times 10^{-7}}$[2]	
$\tau^- \to e^-\eta'$	$\underline{10 \times 10^{-7}}$[2]	
$\tau^- \to \mu^-\pi^0$	$\underline{4.1 \times 10^{-7}}$[2]	
$\tau^- \to \mu^-\eta$	$\underline{1.5 \times 10^{-7}}$[2]	
$\tau^- \to \mu^-\eta'$	$\underline{4.7 \times 10^{-7}}$[2]	
$\tau^- \to e^-\pi^+\pi^-$	8.4×10^{-7}[3]	$\underline{1.2 \times 10^{-7}}$[4]
$\tau^- \to e^-\pi^+K^-$	5.7×10^{-7}[3]	$\underline{3.2 \times 10^{-7}}$[4]
$\tau^- \to e^-K^+\pi^-$	5.6×10^{-7}[3]	$\underline{1.7 \times 10^{-7}}$[4]
$\tau^- \to e^-K^+K^-$	3.0×10^{-7}[3]	$\underline{1.4 \times 10^{-7}}$[4]
$\tau^- \to \mu^-\pi^+\pi^-$	$\underline{2.8 \times 10^{-7}}$[3]	2.9×10^{-7}[4]
$\tau^- \to \mu^-\pi^+K^-$	6.3×10^{-7}[3]	$\underline{2.6 \times 10^{-7}}$[4]
$\tau^- \to \mu^-K^+\pi^-$	15.5×10^{-7}[3]	$\underline{3.2 \times 10^{-7}}$[4]
$\tau^- \to \mu^-K^+K^-$	11.7×10^{-7}[3]	$\underline{2.5 \times 10^{-7}}$[4]
$\tau^- \to e^-e^+e^-$	3.5×10^{-7}[5]	$\underline{2.0 \times 10^{-7}}$[6]
$\tau^- \to e^-\mu^+\mu^-$	$\underline{2.0 \times 10^{-7}}$[5]	3.3×10^{-7}[6]
$\tau^- \to \mu^-e^+e^-$	$\underline{1.9 \times 10^{-7}}$[5]	2.7×10^{-7}[6]
$\tau^- \to \mu^-\mu^+\mu^-$	2.0×10^{-7}[5]	$\underline{1.9 \times 10^{-7}}$[6]
$\tau \to e\gamma$	$\underline{3.9 \times 10^{-7}}$[7]	
$\tau \to \mu\gamma$	3.1×10^{-7}[8]	$\underline{6.8 \times 10^{-8}}$[9]

Table I: Current experimental limits on branching ratios of the LFV rare tau decays.

measuring each of them. When all the masses of Higgs bosons are large, these decay processes decouple by a factor of $1/m_{\text{Higgs}}^4$. Although these branching ratios are complicated functions of the mixing angles, each of them can be simply expressed to be proportional to $\tan^6\beta$ for $\tan\beta \gg 1$ in the SM-like region ($\sin(\alpha-\beta) \sim -1$ [34, 36, 40]). This $\tan^6\beta$ dependence is a common feature of the tau-associated LFV processes with the Higgs-mediated 4-Fermi interactions.

The experimental upper limit on $|\kappa_{3i}|^2 (\equiv |\kappa_{3i}^L|^2 + |\kappa_{i3}^R|^2)$ can be obtained by using the experimental results given in Table I and analytic expressions of the decay branching ratios for rare tau LFV decay processes in Ref. [24]. For description, let us consider the bound from the $\tau \to \mu\eta$ results[14];

$$|\kappa_{32}|^2 \leq \left(|\kappa_{32}^{\max}|^2\right)_{\tau\to\mu\eta} \equiv \frac{256\pi \mathrm{Br}(\tau\to\mu\eta)_{\exp} m_A^4}{9G_F^2 m_\tau^3 m_\eta^4 F_\eta^2 \tau_\tau \left(1-\frac{m_\eta^2}{m_\tau^2}\right)^2} \frac{\cos^6\beta}{\sin^2\beta}, \tag{5}$$

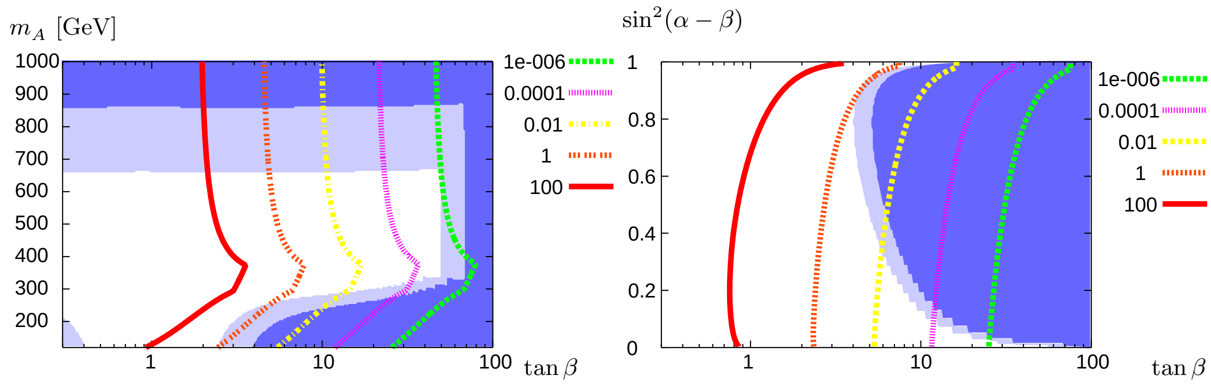
where $\mathrm{Br}(\tau\to\mu\eta)_{\exp}$ is the experimental upper limit on the branching ratio of $\tau \to \mu\eta$ in Table I. In particular, for $\tan\beta \gg 1$, the right-hand-side can be expressed by

$$\left(|\kappa_{32}^{\max}|^2\right)_{\tau\to\mu\eta} \simeq 2.3 \times 10^{-4} \times \left(\frac{m_A}{350[\mathrm{GeV}]}\right)^4 \left(\frac{30}{\tan\beta}\right)^6. \tag{6}$$

It can be easily seen that the bound $(|\kappa_{32}^{\max}|^2)_{\tau\to\mu\eta}$ is rapidly relaxed in the region with small $\tan\beta$ and large m_A. In a similar way to Eq. (5), the maximal allowed value $\left(|\kappa_{3i}^{\max}|^2\right)_{\text{mode}}$ can be calculated for each mode. The combined upper limit $|\kappa_{3i}^{\max}|^2$ is then given by

$$|\kappa_{3i}^{\max}|^2 \equiv \mathrm{MIN}\left\{\left(|\kappa_{3i}^{\max}|^2\right)_{\tau\to\ell\eta}, \left(|\kappa_{3i}^{\max}|^2\right)_{\tau\to\ell\mu^+\mu^-}, \left(|\kappa_{3i}^{\max}|^2\right)_{\tau\to\ell K^+K^-}, \left(|\kappa_{3i}^{\max}|^2\right)_{\tau\to\ell\gamma}, \cdots\right\}. \tag{7}$$

As shown below, $\tau \to \ell_i\eta$ and $\tau \to \ell_i\gamma$ give the strongest upper limits on $|\kappa_{3i}|^2$ in a wide range of the parameter space. In addition, in some parameter regions $\tau \to \ell_i K^+K^-$ and $\tau \to \ell_i\mu^+\mu^-$ can also give similar limits on $|\kappa_{3i}|^2$ to those from the above two processes.



(a) (b)

Figure 1: Contours of $|\kappa_{32}^{\max}|^2$, the possible maximal value of $|\kappa_{32}|^2$ from the rare tau decay results, are shown (a) in the $\tan\beta$-m_A plane and (b) in the $\tan\beta$-$\sin^2(\alpha-\beta)$ plane. The parameters are taken to be (a) $m_h = 120$ GeV, $m_H = m_{H^\pm} = 350$ GeV and $\sin(\alpha-\beta) = -0.9999$, and (b) $m_h = 120$ GeV and $m_H = m_A = m_{H^\pm} = 350$ GeV. The remaining parameter M is scanned from 0 to 1,000 GeV. The dark (light) shaded area indicates the excluded region by the theoretical requirements of vacuum stability and perturbative unitarity with $\xi = 1$ ($\xi = 1/2$).

In Figs. 1-(a) and 1-(b), contour plots for $|\kappa_{32}^{\max}|^2$ are shown under the rare tau decay results in the $\tan\beta$-m_A plane and the $\tan\beta$-$\sin^2(\alpha-\beta)$ plane, respectively. The combined excluded region from the theoretical conditions of vacuum stability and pertrbative unitarity is indicated by the dark shaded area for the criterion $\xi = 1$ and by the light one for $\xi = 1/2$. In Fig. 1-(a), parameters of the Higgs sector are taken to be $m_h = 120$ GeV, $m_H = m_{H^\pm} = 350$ GeV and $\sin(\alpha-\beta) = -0.9999$. In Fig. 1-(b), those are $m_h = 120$ GeV and $m_H = m_A = m_{H^\pm} = 350$ GeV. The value of $|\kappa_{32}^{\max}|^2$ is independent of M, the soft-breaking scale of the discrete symmetry. On the other hand, theoretical bounds from vacuum stability and perturbative unitarity are sensitive to M. Therefore, we evaluate such a theoretical allowed region by scanning M to be from 0 to 1000 GeV. We also take into account the constraint from the ρ parameter measurement and the $b \to s\gamma$ result by taking $\sin(\alpha-\beta) \simeq -1$ and $m_H = m_{H^\pm}$ with $m_{H^\pm} \gtrsim 350$ GeV for Fig 1-(a), and $m_A = m_{H^\pm}$ with $m_{H^\pm} \gtrsim 350$ GeV for Fig 1-(b). From the both figures, it is easily found that the value of $|\kappa_{32}^{\max}|^2$ can extensively be larger for smaller $\tan\beta$ in the allowed region under the theoretical constraints. For $\tan\beta \lesssim 10$ (30), $|\kappa_{32}^{\max}|^2$ can be $\mathcal{O}(0.1)$ ($\mathcal{O}(10^{-4})$). Among the rare tau decay processes, $\tau \to \mu\eta$ and $\tau \to \mu\gamma$ provide the most stringent constraints on $|\kappa_{32}|^2$. While $\tau \to \mu\eta$ is mediated only by A, $\tau \to \mu\gamma$ depends on the masses of h, H, A and $H^\pm$. For $\sin^2(\alpha-\beta) \sim 1$ and $m_A \sim m_H$, the branching ratio of $\tau \to \mu\gamma$ is suppressed because of the cancellation between the one-loop diagrams of A and H. Therefore, $|\kappa_{32}|^2$ is bounded most strongly by the $\tau \to \mu\eta$ result for this case[3]. When m_A differs from m_H or when $\sin^2(\alpha-\beta)$ is to some extent smaller than unity, the one-loop induced $\tau \to \mu\gamma$ process becomes important, and gives the most stringent bound on $|\kappa_{32}|^2$ of all the rare tau decay processes.

The value of $|\kappa_{32}^{\max}|^2$ can be much larger than 100 in a wide range of the parameter region. One might think that such large values of $|\kappa_{3i}|$ cannot be consistent with the unitarity argument for the LFV Yukawa couplings. However, it should be emphasized that the above figures show the contour plots for $|\kappa_{32}^{\max}|^2$ under the rare tau decay results, and not for $|\kappa_{32}|^2$. The region of $|\kappa_{32}^{\max}|^2 \gtrsim 1$ should be taken as the area where $|\kappa_{32}|^2$ can be as large as $\mathcal{O}(10^{-2}$-$10^{-4})$ easily. It is concluded that current results of the tau LFV decays do not give any substantial upper limit on $|\kappa_{32}|^2$ except for high $\tan\beta$ region ($\tan\beta \gtrsim 30$).

Finally, we comment on the bound on $|\kappa_{31}|^2$, the LFV parameters for τ-e mixing. Similar to τ-μ mixing, we can discuss $|\kappa_{31}^{\max}|^2$ comparing the data of $\tau \to e\eta$, $\tau \to e\mu^+\mu^-$, $\tau \to eK^+K^-$ and $\tau \to e\gamma$ listed in Table I with the

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[3]The MSSM result approximately corresponds to this case[14].

formulas given in Ref. [24]. These formulas for τ-e mixing are common with τ-μ mixing except for the factor of $|\kappa_{3i}|^2$, so that difference in contours of $|\kappa_{31}^{\max}|^2$ from those of $|\kappa_{32}^{\max}|^2$ only comes from that in the data. In Table I, the experimental limit for the branching ratio of $\tau \to e\eta$ is about 1.5 times weaker than that of $\tau \to \mu\eta$, while that of $\tau \to e\gamma$ is 5.7 times relaxed as compared to that of $\tau \to \mu\gamma$. Moreover, the upper limit on $\mathrm{Br}(\tau^- \to e^- K^+ K^-)$ is 1.8 times stronger than that on $\mathrm{Br}(\tau^- \to \mu^- K^+ K^-)$. We have numerically confirmed that there are some regions where $\tau^- \to e^- K^+ K^-$ can give the most stringent bound on $|\kappa_{31}|^2$. Therefore, $|\kappa_{31}^{\max}|^2$ is determined from one of $\tau \to e\eta$, $\tau \to e\gamma$ and $\tau^- \to e^- K^+ K^-$ depending on parameter regions.

4. LEPTON FLAVOR VIOLATING HIGGS BOSON DECAYS

As shown in the previous section, the LFV Yukawa couplings can be tested only in the large $\tan\beta$ region by searching for rare tau decays. In order to cover the region unconstrained by rare tau decay results, we here consider LFV via the decay of the neutral Higgs bosons; i.e., $\phi^0 \to \tau^\pm \ell_i^\mp$ ($\phi^0 = h, H$ and A). Branching ratios for these decays are calculated[15, 23–26] to be

$$\mathrm{Br}(h \to \tau^- \ell_i^+) = \frac{1}{16\pi}\frac{m_\tau^2 \cos^2(\alpha-\beta)}{v^2\cos^4\beta}|\kappa_{3i}|^2 \frac{m_h\left(1-\frac{m_\tau^2}{m_h^2}\right)^2}{\Gamma(h\to \mathrm{all})}, \quad (8)$$

$$\mathrm{Br}(H \to \tau^- \ell_i^+) = \frac{1}{16\pi}\frac{m_\tau^2 \sin^2(\alpha-\beta)}{v^2\cos^4\beta}|\kappa_{3i}|^2 \frac{m_H\left(1-\frac{m_\tau^2}{m_H^2}\right)^2}{\Gamma(H\to \mathrm{all})}, \quad (9)$$

$$\mathrm{Br}(A \to \tau^- \ell_i^+) = \frac{1}{16\pi}\frac{m_\tau^2}{v^2\cos^4\beta}|\kappa_{3i}|^2 \frac{m_A\left(1-\frac{m_\tau^2}{m_A^2}\right)^2}{\Gamma(A\to \mathrm{all})}, \quad (10)$$

where $\Gamma(\phi^0 \to \mathrm{all})$ is the total width for corresponding neutral Higgs boson ϕ^0. We here neglect terms of $\mathcal{O}(m_{\ell_i}^2/m_{\phi^0}^2)$. Branching ratios for $\phi^0 \to \tau^+\ell_i^-$ coincide with those for $\phi^0 \to \tau^-\ell_i^+$ given in Eqs. (8), (9) and (10). In the following, we concentrate on the decays into a τ-μ pair. We take the values of the SM parameters as $\alpha_{\mathrm{em}} = 0.007297$, $G_F = 1.166\times 10^{-5}$ GeV^{-2}, $m_Z = 91.19$ GeV, $m_\tau = 1.777$ GeV, $m_\mu = 0.1057$ GeV, $m_b = 4.1$ GeV, $m_t = 174.3$ GeV, $m_c = 1.15$ GeV, $m_s = 0.120$ GeV.

A search for the LFV decays $h \to \tau^\pm \ell_i^\mp$ can give important information for extended Higgs sectors and thus for the structure of new physics, even when only h is found and any other direct signals for the extended Higgs sector are not obtained by experiments. We here evaluate the possible maximal value of the branching ratio $\mathrm{Br}(h \to \tau^-\mu^+)_{\max}$ under the results of the rare tau decay search, by inserting $|\kappa_{32}^{\max}|^2$ of Eq. (7) into the $|\kappa_{32}|^2$ in Eq. (8).

In Figs. 2-(a) and 2-(b), contours of $\mathrm{Br}(h \to \tau^\pm\mu^\mp)_{\max}$, which is twice of $\mathrm{Br}(h \to \tau^-\mu^+)_{\max}$, are shown in the $\tan\beta$-m_A plane and in the $\tan\beta$-$\sin^2(\alpha-\beta)$ plane, respectively. The parameters are taken to be the same as those for Figs. 1-(a) and 1-(b), respectively; i.e., (a) $m_h = 120$ GeV, $m_H = m_{H^\pm} = 350$ GeV and $\sin(\alpha-\beta) = -0.9999$, and (b) $m_h = 120$ GeV and $m_H = m_A = m_{H^\pm} = 350$ GeV, with M to be scanned from 0 to 1000 GeV. We again show the excluded area from requirements of tree-level unitarity and vacuum stability as in the same way as Figs. 1-(a) and 1-(b). For low and moderate values of $\tan\beta$ ($\tan\beta \lesssim 30$), where rare tau decay results cannot give substantial upper limit on $|\kappa_{32}|^2$, $\mathrm{Br}(h \to \tau^\pm\mu^\mp)_{\max}$ can be sufficiently large. We find that the possible maximal values of the branching ratio can be greater than $\mathcal{O}(10^{-3})$ in a wide rage of the theoretically allowed region.

For relatively lower $\tan\beta$ values, the experimental upper limits on $|\kappa_{32}|^2$ from rare tau decays are weaker, and $\mathrm{Br}(h \to \tau^\pm\mu^\mp)$ can be sufficiently large ($\gtrsim \mathcal{O}(10^{-3})$ for $m_h \sim 120$ GeV). It is expected that a sufficient number of such light h can be produced at future colliders such as CERN LHC, currently planned International Linear Collider (ILC) and CERN CLIC. It has been pointed out that the decay process $h \to \tau^\pm\mu^\mp$ can easily be detected at ILC with the luminosity of 1 ab^{-1}, when $m_h \sim 120$ GeV and $\mathrm{Br}(h \to \tau^\pm\mu^\mp) \gtrsim \mathcal{O}(10^{-3})$ via the Higgsstrahlung process by using the recoil momentum of Z boson[26]. Therefore, the LFV search via the decay $h \to \tau^\pm\mu^\mp$ at ILC is complementary to that via rare tau decays at (super) B factories, and the both cover a wide region of the parameter space of the lepton flavor violating THDM.

Next we discuss branching ratios for the LFV decays of heavier Higgs bosons, $H/A \to \tau^\pm\ell_i^\mp$, using Eqs. (9) and (10) under the current data of LFV rare tau decays. In the THDM, there are many possible decay modes for H

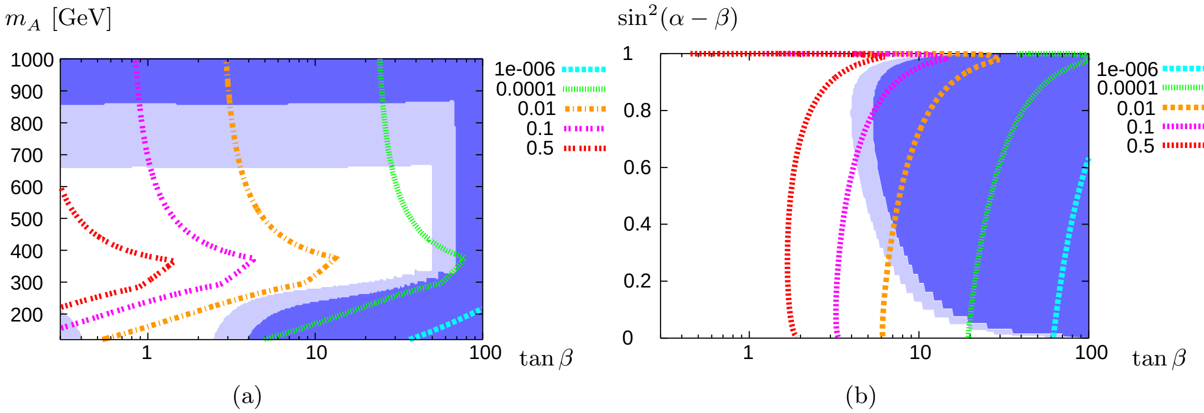

Figure 2: Contour plots of $\mathrm{Br}(h \to \tau^{\pm}\mu^{\mp})_{\max}$, the possible maximal values for the branching ratio under the tau rare decay results, are shown (a) in the $\tan\beta$-m_A plane and (b) in the $\tan\beta$-$\sin(\alpha-\beta)$ plane. The parameters are taken as the same as Figs. 1-(a) and 1-(b), respectively. The dark (light) shaded area indicates the excluded region by the theoretical requirements of vacuum stability and perturbative unitarity with $\xi = 1$ ($\xi = 1/2$).

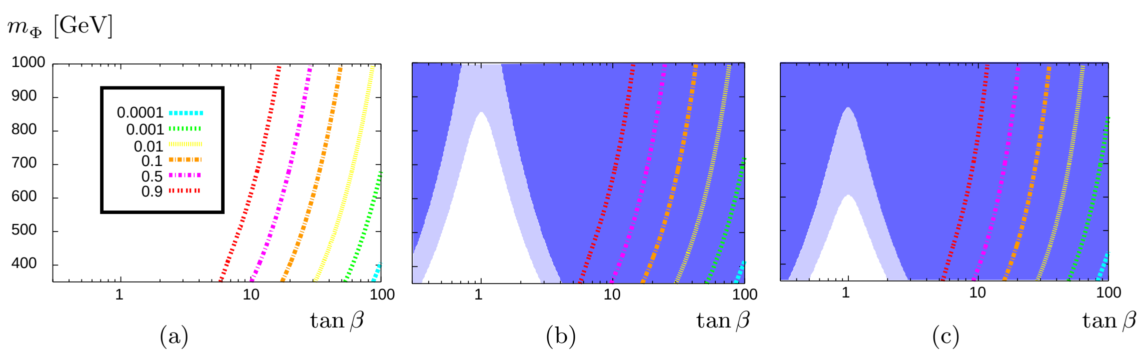

Figure 3: Contour plots of $\mathrm{Br}(H \to \tau^{\pm}\mu^{\mp})_{\max}$ are shown in the $\tan\beta$-m_Φ plane ($m_\Phi \equiv m_H = m_A = m_{H^\pm}$) for $m_h = 120$ GeV and $\sin(\alpha-\beta) = -1$ with (a) $M = m_\Phi$, (b) $M = m_\Phi/\sqrt{2}$, and (c) $M = 0$. The dark (light) shaded area indicates the excluded region by the theoretical requirements of vacuum stability and perturbative unitarity with $\xi = 1$ ($\xi = 1/2$).

depending on the mass spectrum. In the numerical analysis, we included contributions from one-loop induced $Z\gamma$, $\gamma\gamma$ and gg modes in addition to all the tree level modes. Those for A are one-loop induced modes of hh, hH, HH, $h\gamma$, $H\gamma$, $W^{\pm}W^{\mp}$, ZZ, $Z\gamma$, $\gamma\gamma$ and gg in addition to all the tree level modes. The branching ratios for $H/A \to \tau^{\pm}\ell_i^{\mp}$ are sensitive to the masses of all the Higgs bosons. Here we consider the case of $\sin(\alpha-\beta) = -1$ and $m_H = m_A = m_{H^\pm}$ ($\equiv m_\Phi$). As discussed in Sec. II, the ρ parameter constraint is satisfied for this choice. From the $b \to s\gamma$ results, m_Φ is taken to be greater than 350 GeV. As also discussed in Sec. II, M determines the decoupling property of heavier Higgs bosons. Although the branching ratios $\mathrm{Br}(H/A \to \tau^{\pm}\ell_i^{\mp})$ are insensitive to M in the present parameter set, its value strongly affects the allowed parameter region under the theoretical conditions of vacuum stability and perturbative unitarity. Notice that couplings of H are similar to those of A for $\sin(\alpha-\beta) = -1$ where there are no HVV couplings. Hence we show the results only for the LFV decays of H below. In a general case, the branching ratio of $H \to \tau^{\pm}\mu^{\mp}$ tends to be smaller than that of $A \to \tau^{\pm}\mu^{\mp}$ due to the contribution from the modes $H \to VV$.

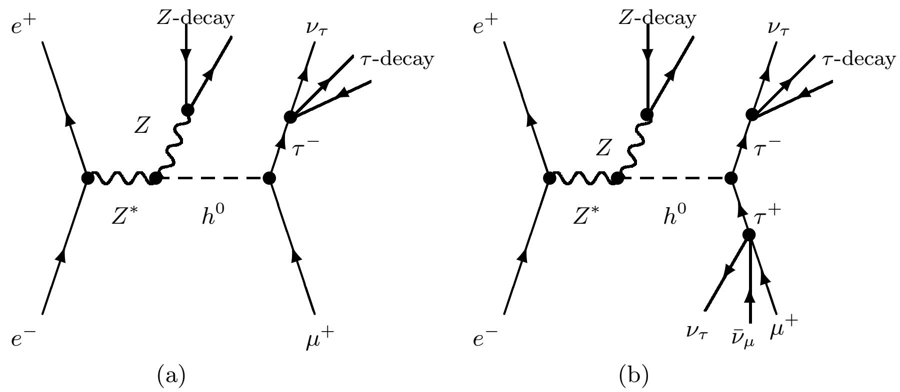

Figure 4: The Feynman diagram of the signal event (a), and that of *the fake event* (b).

In Figs. 3-(a), 3-(b) and 3-(c), contour plots of $\mathrm{Br}(H \to \tau^\pm\mu^\mp)_{\max}$, the upper limit of $\mathrm{Br}(H \to \tau^\pm\mu^\mp)$ under the rare tau decay results, are shown in the $\tan\beta$-m_Φ plane for $M = m_\Phi$, $m_\Phi/\sqrt{2}$ and 0, respectively. As expected, the contours are insensitive to the values of M, and approximately the same in Figs. 3-(a), 3-(b) and 3-(c). It is shown that $\mathrm{Br}(H \to \tau^\pm\mu^\mp)_{\max}$ can be larger than 10^{-3} except for large $\tan\beta$ values with relatively small m_Φ. Therefore, it turns out to be no substantial upper limit on the $\mathrm{Br}(H \to \tau^\pm\mu^\mp)$ in the relatively low $\tan\beta$ region ($\tan\beta \lesssim 20$) from the LFV rare tau decay results. When M is smaller than m_Φ, where the heavier Higgs boson partially receive their masses from the vacuum expectation value, the allowed parameter region is strongly constrained by the requirements of vacuum stability and perturbative unitarity. In particular, for $M = 0$ (Fig. 3-(c)), the allowed region is limited only the area of around $\tan\beta \sim 1$ and $m_\Phi \lesssim 600$ GeV.

The extra Higgs bosons (H, A and $H^\pm$) are expected to be searched at the LHC. The signal of $gg \to H/A \to \tau^\pm\mu^\mp$ may be detectable at LHC with high luminosity (100 fb^{-1}) when $\mathrm{Br}(H/A \to \tau^\pm\mu^\mp)$ is greater than 10^{-2} for $m_{H/A} \sim 350$ GeV and $\tan\beta = 45$[25]. However the rate is rapidly reduced for smaller values of $\tan\beta$ and for larger values of $m_{H/A}$. Further feasibility study is necessary.

5. SEARCH FOR LFV DECAYS OF HIGGS BOSONS AT A LINEAR COLLIDER

Let us consider the LFV Higgs decay $h^0 \to \tau^\pm\mu^\mp$ at a LC in the situation where the heavier Higgs bosons nearly decouple from the gauge bosons; i.e., $\sin(\alpha - \beta) \simeq -1$. The lightest Higgs boson then approximately behaves as the SM one. The main production modes of the lightest Higgs boson at a LC are the Higgsstrahlung $e^+e^- \to Z^* \to Zh^0$ and the W fusion $e^+e^- \to (W^{+*}\bar{\nu}_e)(W^{-*}\nu_e) \to h^0\nu_e\bar{\nu}_e$. For a light h^0 with the mass $m_h \sim 120$ GeV, the former production mechanism is dominant at low collision energies ($\sqrt{s} <$ 400-500 GeV), while the latter dominates at higher energies. For our purpose, the Higgsstrahlung process is useful because of its simple kinematic structure. The signal process is then $e^+e^- \to Z^* \to Zh^0 \to Z\tau^\pm\mu^\mp$. We can detect the outgoing muon with high efficiency, and its momentum can be measured precisely by event-by-event. The momentum of the Z boson can be reconstructed from those of its leptonic $\ell^+\ell^-$ ($\ell^\pm = e^\pm$ and $\mu^\pm$) or hadronic (jj) decay products. Therefore, we can identify the signal event without measuring τ momentum directly, as long as the beam spread rate for $\sqrt{s}$ is sufficiently low.

Depending on the Z decay channel, the signal events are separated into two categories, $jj\tau^\pm\mu^\mp$ and $\ell^+\ell^-\tau^\pm\mu^\mp$. The energy resolution of the Z boson from hadronic jets jj is expected to be $0.3\sqrt{E_Z}$ GeV and that from $\ell^+\ell^-$ is $0.1\sqrt{E_Z}$ GeV. We assume that the detection efficiencies of the Z boson and the muon are 100 %, the rate of the beam energy spread is expected to be 0.1 % level, the muon momentum is measured with high precision and the mass of the lightest Higgs boson will have been determined in the 50 MeV level. We also expect that the effect of the initial state radiation is small for the collider energies that we consider ($\sqrt{s} \sim$ 250-300 GeV). Taking into account all these numbers, we expect that the tau momentum can be determined indirectly within 3 GeV for $jj\tau^\pm\mu^\mp$ and 1 GeV for $\ell^+\ell^-\tau^\pm\mu^\mp$.

Let us evaluate the number of the signal event. We assume that the energy $\sqrt{s}$ is tuned depending on the mass of the lightest Higgs boson: i.e., we take the optimal $\sqrt{s}$ to product the lightest Higgs boson through the Higgsstrahlung process. (It is approximately given by $\sqrt{s} \sim m_Z + \sqrt{2}m_h$.) The production cross section of $e^+e^- \to Zh^0$ is about 220 fb for $m_h = 123$ GeV. Then, we obtain 2.2×10^5 Higgs events if the integrated luminosity is 1 ab^{-1}. When $|\kappa_{32}|^2$ is 8.4×10^{-6}, about 118 events of $jj\tau^\pm\mu^\mp$ and 11 events of $\ell^+\ell^-\tau^\pm\mu^\mp$ can be produced.

Next, we consider the background. For the signal with the Higgs boson mass of ~ 120 GeV, the main background comes from $e^+e^- \to Z\tau^+\tau^-$. The number of the $Z\tau^\pm\mu^\mp$ event from $e^+e^- \to Z\tau^+\tau^-$ is estimated about 3.6×10^4. Although the number of the background events is huge, we can expect that a large part of them is effectively suppressed by using the kinematic cuts[26]. The irreducible background comes from the process shown in Fig.4-(b): the Higgs boson decays into a tau pair, and one of the tau decays into a muon and missings ($e^+e^- \to Zh^0 \to Z\tau^+\tau^- \to Z\tau^\pm\mu^\mp$+missings). We can not distinguish the signal event $h^0 \to \tau^\pm\mu^\mp$ with the event of Fig.4-(b) when the muon emitted from the tau lepton carries the similar momentum to that of the parent, because it leaves the same track on the detector as the signal event. We refer this kind of the background as *the fake signal*. As examined in Ref. [26], the number of the fake signal strongly depends on the precision of the tau momentum determination by the recoil method. We expect that it is attained with the similar precision to that of the Higgs boson mass reconstructed by the recoil momentum. We here take the uncertainty of the tau momentum as 3 GeV for $jj\tau^\pm\mu^\mp$ and as 1 GeV for $\ell^+\ell^-\tau^\pm\mu^\mp$.

Finally, we estimate the statistical significance ($S/\sqrt{B}$) for each channel. The number of the fake events is evaluated in Ref. [26], which is 460 for $jj\tau^\pm\mu^\mp$ and 15 for $\ell^+\ell^-\tau^\pm\mu^\mp$. Therefore, when $|\kappa_{32}|^2$ is 8.4×10^{-6} with $m_h = 123$ GeV, the significance can become 5.5 and 3.0 for $jj\tau^\pm\mu^\mp$ and $\ell^+\ell^-\tau^\pm\mu^\mp$, respectively, taking into account the constraint from the rare tau decay results. The combined significance can reach to 6.3. When $|\kappa_{32}|^2$ is 3.8×10^{-6} with $m_h = 123$ GeV, the number of the signal becomes smaller, and the combined significance amounts to be as large as 2.0.

6. SUMMARY

Lepton flavor violating decays of Higgs bosons have been studied in the general THDM, in which LFV couplings are introduced as a deviation from Model II Yukawa interaction in the lepton sector. The model parameters are constrained by requirements of tree-level unitarity and vacuum stability, and also from the experimental results. The parameters $|\kappa_{3i}|^2$ in LFV Yukawa interactions are bounded from above by using the current data for rare tau LFV decays. In the large $\tan\beta$ region ($\tan\beta \gtrsim 30$), the upper limit on $|\kappa_{3i}|^2$ due to the rare tau decay data turns out to be substantial and comparable with the value predicted by assuming some fundamental theories such as SUSY. For smaller values of $\tan\beta$, the upper limit is rapidly relaxed, and no more substantial constraint is obtained from the rare tau decay results.

We have shown that a search for the LFV decays $\phi^0 \to \tau^\pm\ell_i^\mp$ of neutral Higgs bosons ($\phi^0 = h, H$ and A) can be useful to further constrain the LFV Yukawa couplings at future collider experiments. In particular, the decays of the lightest Higgs boson can be one of the important probes to find the evidence for the extended Higgs sector even when the SM-like situation would be preferred by the data at forthcoming collider experiments. The branching ratio for $h \to \tau^\pm\mu^\mp$ can be larger than $\mathcal{O}(10^{-3})$ except for the high $\tan\beta$ region under the constraints from the current experimental data and also from the theoretical requirements. At ILC (and in case at LHC), these branching fractions can be tested. Therefore, we conclude that the search of LFV in the Higgs boson decay at future colliders can further constrain the LFV Yukawa couplings in the parameter region where rare tau decay data cannot reach.

Note added: Recently similar work was done in Ref. [41] on the experimental upper bound on $|\kappa_{3i}|^2$.

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